Major Project Report

"INTERACTION EFFECT OF ARBITRARILY ORIENTED TWO CRACKS IN ISOTROPIC PLATE"

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By

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Certificate

This is to certify that the major project report (Part-I) entitled "**Interaction effect of arbitrarily oriented two cracks in isotropic plate"** submitted by **Umesh N. Pujari** (04mme012) towards the partial fulfillment of the requirements for Semester III of Master of Technology (Mechanical) in the field of CAD/CAM of Nirma University of Science and Technology is the record of work carried out by him under our supervision and guidance. The work submitted has in our opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other University or Institution for award of any degree or diploma.

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Abstract

This report is aimed to obtain the general solution for interaction effect of arbitrarily oriented two cracks in isotropic material subjected to various loading. The cracks are present due to inclusions or irregularities in the material. The stress analysis is based on the theory of elasticity by using the Muskhelishvili's complex variable method. The complex stress function is determined for the plane stress condition to determine the stress intensity factor.

It starts with a general idea about the complex functions and its use in stress analysis for the less complex shape. The proposed method helps to study interaction effect of two arbitrarily oriented cracks.

Also the FEM solutions for stress intensity factor for the same case using ANSYS software are to be determined and result were compared for different loading condition as well as different crack orientation.

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Nomenclature

a	Crack length
ν	Poisson's ratio
E	Modulus of elasticity
P	Uniform pressure or tension load
z	Complex number ($z = x + iy$)
$\bar{z}$	Complex conjugate of z
i	Imaginary unit ($i = \sqrt{-1}$)
Re	Real part
Im	Imaginary part
ζ	Complex number in a mapped plane ($\zeta = \rho e^{i\theta}$)
Z_0	Center distance between two cracks in Z plane
C_0, C	Center distance between two cracks in ζ plane
t	Boundary value of complex number ζ on unit circle
$f(t)$	Boundary value of stresses
K_I	Mode I stress intensity factor
K_{II}	Mode II stress intensity factor
K_{III}	Mode III stress intensity factor
λ	Biaxial load factor
α	Angular orientation of first crack w.r.t. direction of loading λP or X-axis

β	Angular orientation of second crack w.r.t. direction of loading λP or X-axis
σ_x, σ_y, τ_{xy}	Stress components in Cartesian co-ordinates
σ_r, σ_θ, $\tau_{r\theta}$	Stress components in Polar co-ordinates
$\phi(z), \psi(z)$	Complex stress functions in Z-plane
$\phi'(z), \psi'(z)$	First derivative of stress functions w.r.t. Z
$\phi''(z), \psi''(z)$	Second derivation of stress function w.r.t. Z
$\phi(\zeta)$	Complex stress functions in ζ-plane
$\phi'(\zeta), \psi'(\zeta)$	First derivative of stress functions w.r.t. ζ
$\phi''(\zeta), \psi''(\zeta)$	Second derivative of stress functions w.r.t. ζ
$\omega(z)$	Mapping function
0	Function related to a plate without any crack
1	Function with respect to origin of first crack for single crack solution.
2	Function with respect to origin of second crack for single crack solution.
11	Function with respect to origin of first crack O_1
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CHAPTER 1
INTRODUCTION

1.1 INTRODUCTION

The failure of materials is a challenging interdisciplinary problem of both technological and fundamental interests. From the technological point of view, the understanding of the failure mechanisms of materials under various external conditions may improve dramatically the integrity of structures in a wide range of applications. From the theoretical point of view, the understanding of the way materials fail entails the development of new mathematical methodologies and necessitates the introduction of new concepts in non-linear and solid state physics.

During most of its historical development, the science of Mechanics of Materials relied principally on closed-form (not computational) mathematical theorists. Much of their work represents mathematical intuition and skill of a very high order, challenging even for advanced researchers of today.

Fracture mechanics is based on the implicit assumption that there exists a crack in structural component. The crack may be man made such as a hole, notch, a slot etc. The crack may appear due to manufacturing defects like slag, inclusions in a weldment or heat affected zones due to uneven cooling. A dangerous crack may be nucleated and grown during the service of the component. The presence of theses cracks may weaken the structure and lead to a reduction in its operational life. In order to ensure safety and reliability, it is necessary to be able to predict the behavior of cracks under service conditions. That is, to predict how fast the cracks grow and how strong the cracked structure is; both depend upon the stress intensity factor which governs the stress field at the crack tip.

About 40-50 years ago when analysis for the growth of a crack was not available, a reasonably high factor of safety was chosen to account for unforeseen factors. A large part of the ambiguity has been cleared with the development of stress analysis by theory of elasticity. This enable designer to use much lower factor of safety, thus reducing cost of structural components. At the same time weight of component is reduced and reliability is enhanced.

The designer is frequently required to predict the fatigue behavior of cracks emanating from these discontinuities so as to assess the residual life and to avoid catastrophic failure. However, the problem is complicated by the fact that these critical load bearing structures contain residual stresses and interacting stress concentration features such as holes, in homogeneities and inclusions. To accurately describe the fracture behavior of structures, it is necessary to account for the complex geometry of the crack in the calculation of the stress intensity factors (SIFs). The engineering design demands for stress intensity factor solutions for various kinds of geometries are increasing rapidly, especially in the aeronautical industry, the chemical industry and nuclear power industry.

1.2 Aim of the Project:

The presence of cracks introduces stresses in the components. The crack tip stress intensity factor has a major influence on the structural integrity of every machine. The method which can help to analyze the interaction effect between two cracks is worth developing. The present study derives motivation from such an issue. The aim of the present work is to derive the close form solution that can determine the stress intensity factors at the tips of two cracks in an infinite isotropic plate and to study the interaction effect between them for various geometrical parameters and loading conditions. The analysis is based on the two-dimensional theory of elasticity by using the Muskhelishvili's complex variable approach. The complex stress functions are used to evaluate the stress intensity factors (SIFs) at the crack tips.

The investigations have been carried out with the following objectives.

1. The two cracks are arbitrarily oriented in the infinite isotropic plate
2. To present an important method for analyzing the interaction effect of two cracks for an infinite plate subjected to various loading conditions.
3. Derivation of solutions for complex stress functions that can determine the interaction effect on stress intensity factors for two cracks for an infinite plate subjected to various loading conditions.

4. Analysis of stress intensity factors for the above cases for various geometrical parameters.

5. Solution of particular problems using FEM analysis.

The stress intensity factors at the crack tips and their variations with respect to the geometrical parameters are presented for the specific cases.

The following cases have been studied

The infinite plate with two cracks subjected to

1. Uniformly distributed arbitrary biaxial load at infinity,

The interaction effects of following parameters will be studied.

1. Effect of crack length variations,
2. Effect of inclination between cracks,
3. Effect of variation of centre distance between cracks.

1.3 Methodology

The present study is concerned with the derivation of solutions that can determine the stress intensity factors at the tips of two cracks in an infinite isotropic plate and the interaction effect between two cracks for various loading conditions. The two cracks are arbitrarily oriented. The analysis is based on the two dimensional theory of elasticity by using the Muskhelishvili's complex variable approach. The complex stress functions are used to evaluate the stress intensity factors at the crack tips. In the present solution the problem of the infinite plate with two cracks subjected to various loading conditions will be solved using Muskhelishvili [1] complex stress functions. In the present method the solution in terms of two complex functions is to be obtained without a hole or discontinuity in an infinite plate for the given loading condition. The results of the analysis will be compared with the results using FEM

Chapter 2
Literature Review

2.1 Introduction

Analytical methods of stress analysis greatly facilitate the parametric study and provide the more accurate solutions. Various design methods have been proposed by a number of researchers for analyzing stresses and deflections in plates. During the last decade many authors have proposed analytical, experimental or numerical techniques to analyze the stresses and deflections. Some of the methods to determine the stress intensity factor are,

2.2 Stress intensity factor evaluation methods,

1) **Close form solutions:**

Complex function theory (conformal mapping, boundary collocation method, Laurent series expansion, integral transforms (Fourier, Mellin, Hanckel transforms) eigen function expansion) limited to very simple cases

2) **Computational solutions** (FEM, BEM, FDM …)

3) **Experimental solutions** (photo elasticity, moiré interferometry….)

2.3 Solutions for In-plane loading

Ukadgaonkar and Awasare alone have given the mathematical equations more explicitly for each stage of solution. Uniaxial, biaxial and shear stresses are considered on isotropic plates containing circular [2], elliptical, triangular [3] and rectangular holes. An elliptical hole is considered for the anisotropic case. Solutions for biaxial and shear stresses are obtained by superposition of the solutions of uniaxial loading.

2.4 Mapping function:

The research on stress concentration problems is ongoing. Application of complex stress functions by Kolosov and Muskhelishvili has enabled the solutions of various boundary value problems in a much simple way. V.G. Ukadgaonkar and D.K.N. Rao give the stress distribution around triangular holes in anisotropic plates

Xiangqiao Yan fined the solution for a numerical analysis of cracks emanating from a square hole in a rectangular plate under biaxial loads by the boundary element method.. Based on the displacement field around the crack tip, the following formulas exist for the stress intensity factors (SIFs) K_I and K_{II} at the crack tips. [4]

$$K_I = \frac{G\sqrt{2\pi}}{4(1-\nu)} \lim_{r\to 0} \{Dy\ (r)/r^{0.5}\}$$

$$K_{II} = \frac{G\sqrt{2\pi}}{4(1-\nu)} \lim_{r\to 0} \{Dx\,(r)/r^{0.5}\}$$

Where Dy(r) and Dx(r) are the normal and shear components of displacement discontinuity at a distance r from the crack tip(s).

A closed form solution for the infinite plate containing Hole with cusps and applied by concentrated forces by complex variable function. He has given that here are several approaches to solve this problem.

First of them was given by Muskhelishvilli. The kernel of the first approach is that, making an operation like, $(1/2\pi i)\int_{\gamma}[...](d\sigma/\sigma-\zeta)$ on the boundary condition equation and using the behaviors of the complex variable function, the complex potentials $\phi(\zeta)$ and $\psi(\zeta)$ can be always obtained.

Second approach is given by England, Bowie and Wu. The kernel of second approach is that, using the properties of the rational function $\omega(\zeta)$ and the traction free condition on the hole contour, one can always make a continuation of the function from the exterior of unit circle into the interior of unit circle, then the complex potentials can be obtained.

The third approach given by Mulikusi is most effective and interesting. The kernel of third approach is one first defines the complex potentials $\phi(\zeta)$ and $\omega(\zeta)=-\overline{\omega}(1/\zeta)\phi'(\zeta)/\omega'(\zeta)-\psi(\zeta)$ on the whole complex plane. Then by using the conditions concerning applied forces, the principal part of the functions $\omega(\zeta)$ and $\phi(\zeta)$ on the region $\sum_{-}(|\zeta|>1)$ can be easily found.

He has done the elastic analysis for the

1) An infinite plate containing hypocycloid hole [4]

The mapping function for the hypocycloid crack is,

$$z=\omega(\zeta)=\zeta+\zeta^{-n}/n, \quad \text{(n, integer)}$$

Maps the unit circle and its exterior (on the ζ-plane) into the hypocycloid contour and its exterior (on the z-plane)

An infinite plate containing symmetric aerofoil crack

The mapping function for the aerofoil crack is

$$z=\omega(\zeta)=\zeta+\frac{(1-m)^2}{\zeta-m}, \qquad (0<m<1)$$

Maps the unit circle and its exterior (on the ζ-plane) into the symmetric airfoil crack and its exterior (on the z-plane)

3) An infinite plate containing symmetric lip crack

The mapping function $z=\omega(\zeta)=\zeta-\frac{m^2}{\zeta}+\frac{\left(1-m^2\right)^2}{2}\left(\frac{1}{\zeta-m}+\frac{1}{\zeta+m}\right)$' (0<m<1)

Maps the unit circle and its exterior (on the ζ -plane) into the symmetric lip crack and its exterior (on the z-plane)

4) Mapping function for the triangular hole.[3]

The outside region of the triangular hole is mapped external to the unit circle in the ζ-plane. The mapping function for the triangular hole due to Schwarz-Cristobel Transformation is expressed in the following form

$$z=\omega(\zeta)$$
$$=R\left[\zeta+\varepsilon\left(\frac{m_1}{\zeta^2}+\frac{m_2}{\zeta^5}+\frac{m_3}{\zeta^8}+\frac{m_4}{\zeta^{11}}+\frac{m_5}{\zeta^{14}}+\frac{m_6}{\zeta^{17}}\right)\right],$$

Where m_1=1, m_2=1/5, m_3 =5/4, m_4=1/891, m_5=1/243, m_6=1/37179, where ε varies from -1/3 to1/3

5) Mapping function for the region outside the hole. [2]

The outside region of any shape of hole in z-plane can be conformally mapped external to the unit circle in ζ plane by means of a mapping function given in a general form by

$$z = \omega(\zeta) = R\left(\zeta + \sum_{k=1}^{N} \frac{m_k}{\zeta^k}\right)$$

Where m_k are the constants of the mapping function corresponding to the power of ζ

6) In the elliptic notch case, the following mapping function is introduced [12]

$$z = \omega(\zeta) = R\left(\zeta + \frac{m}{\zeta}\right) \qquad (0 \le m \le 1)$$

Which maps the ellipse contour and its exterior (on the z-plane) into the unit circle unit and its exterior (on the ζ-plane)

Chapter 3
Complex variable formulation

3.1 Introduction:

Earlier methods in the Theory of Elasticity assume stress functions and the method provide the proof of existence of solutions, where as the Complex variable approach deduces the explicit solutions. This method is useful in solving many two dimensional problems of stress concentrations, contact problems, and the problems of Fracture mechanics.

The analysis is based on the two-dimensional theory of elasticity by using the Muskhelishvili's complex variable approach [1]. The complex stress functions are used to evaluate the stress intensity factors (SIFs) at the crack tips. The aim of the present work is to derive the close form solution that can determine the stress intensity factors at the tips of two cracks in an infinite isotropic plate. Further these solutions are used to obtain the interaction effect between two cracks using Schwarz's alternating method [5] various geometrical parameters and loading conditions.

In many problems of practical interest, it is convenient to use stress functions as complex functions of two variables. We will see that these have the ability to satisfy the governing equations automatically, leaving only adjustments needed to match the boundary conditions. For this reason, complex-variable methods play an important role in theoretical stress analysis, and even in this introductory treatment we wish to illustrate the power of the method.

3.1.2 Analytic Function:

A complex function is said to be analytic on a region S if it is complex differentiable at every point in S. If a complex function is analytic on a region S, it is infinitely differentiable in S. A complex function may fail to be analytic at one or more points through the presence of singularities, or along lines or line segments through the presence of branch cuts. [6]

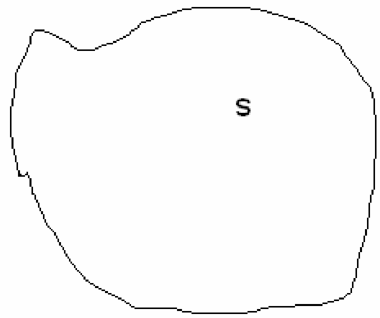

Fig3.2: Region S closed bounded

Points of no analyticity are called singular points. They are important for certain applications in physics and engineering.

3.1.3 Harmonic Function:

A function z of two real variables x and y is said to be harmonic in a given domain of the xy plane if throughout that domain, it has continuous partial derivatives of the first and second order and satisfies the partial differential equation. Harmonic functions are called potential functions in physics and engineering

Let $f(z) = f(x + iy) = u(x, y) + iv(x, y)$ be an **analytic** function in the domain S. If all second-order partial derivatives of u and v are continuous, then both u(x,y) and v(x,y) are **harmonic** function in S.

Most applications of conformal mapping involve harmonic functions, which are solutions to Laplace's equation, [6]

$$\nabla^2 \phi = 0$$

From Cauchy Riemann condition it is easy to show that the real and imaginary parts of an analytic function are harmonic, but the converse is also true: Every harmonic function is the real part of an analytic function, $\phi = \mathrm{Re}\,\phi$ the complex potential.

3.2 Airy's stress function (ϕ):

A state of plane stress or strain in a two-dimensional solid, free from body forces, may be specified by three mutually orthogonal stress components. These may be stated as first-order differential equations. If the three stress components are equal to the appropriate second order partial derivatives of an arbitrary function, ϕ , then they satisfy the equilibrium conditions.

In spite of the fact that Airy spoke in italics he was sufficiently well-understood for the function, ϕ to be called an **Airy stress function**. [7]

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}$$

$$\sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}$$

$$\tau_{xy} = \frac{\partial^2 \phi}{\partial x \partial y} \tag{3.1}$$

3.3 Biharmonic Equation:

The equation of equilibrium condition is as follows,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \tag{3.2}$$

By plugging in the equations associated with Airy's Stress Function into the equilibrium condition, we can illustrate that the functions do indeed satisfy equilibrium.

$$\frac{\partial^3 \phi}{\partial x \partial y^2} - \frac{\partial^3 \phi}{\partial x \partial y^2} = 0$$

The compatibility condition is given by

$$\frac{\partial^2 \varepsilon_{xx}}{\partial y^2} + \frac{\partial^2 \varepsilon_{yy}}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = 0 \tag{3.3}$$

By substitute the components of Airy's Stress Function into our compatibility condition and simplify.

$$\frac{\partial^4 \phi}{\partial x^4} + 2\frac{\partial^4 \phi}{\partial x^2 y^2} + \frac{\partial^4 \phi}{\partial y^4} = 0 \tag{3.4}$$

Equation (3.4) is also known in mathematics as the *Biharmonic Equation* and can be expressed compactly as: $\nabla^2\nabla^2\phi=0$ (3.5)

Where ∇^2 is the Laplacian Operator

3.3.1 Representation of biharmonic equation:

If P is a harmonic function then according to Cauchy Riemann condition P + iQ is an analytic function. [5]

Let us put, $\nabla^2\phi=P$

$$\nabla^4\phi=\nabla^2\nabla^2\phi=0$$

So it's integral also analytic.

$$\phi(z)=R+iI=\frac{1}{4}\int_0^z(P+iQ)\,dx$$

from this $\nabla^2 R=0$ and $\nabla^2 I=0$

.

If u_1 is another harmonic function then $\phi = xR + yI + u_1$ which satisfies the biharmonic equation. Then we can write $\phi=\mathrm{Re}\left[\bar{z}\,\phi(z)+\psi(z)\right]$

where $\mathrm{Re}\,[\psi(z)]=u_1$ so that

$$2\phi = z\overline{\phi(z)}+\bar{z}\,\phi(z)+\psi(z)+\overline{\psi(z)} \tag{3.6}$$

3.4 Complex Variable Formulation:

From equation (3.1) we have,

$$\sigma_y - i\tau_{xy} = \frac{\partial^2\phi}{\partial X^2} + i\frac{\partial^2\phi}{\partial X\partial y}$$

(3.7)

$$\sigma_y + i\tau_{xy} = \frac{\partial^2\phi}{\partial y^2} - i\frac{\partial^2\phi}{\partial X\partial y}$$

Let,

$$f = \frac{\partial \phi}{\partial X} + i\frac{\partial \phi}{\partial y} = \phi(z) + z\overline{\phi'(z)} - \overline{\psi'(z)} \tag{3.8}$$

By solving the equations (3.6) and (3.7) we get

$$\sigma_X + \sigma_y = 2[\phi'(z) + \overline{\phi'(z)}] = 4\,\text{Re}[\phi'(z)]$$

And

$$\sigma_y - \sigma_X + 2i\tau_{xy} = 2[\bar{z}\phi''(z) + \psi''(z)] \tag{3.9}$$

From above two equations we get the stresses for the plane stress condition. [5]

3.5 Governing Equations for Mapping Procedure:

For the problems on the stress distribution around triangular holes, rectangular holes, non-circular holes, the conformal mapping procedure is followed. In the solution of such problem the crack in the Z plane is mapped in to the circle in the ζ plane, which has its center at the origin $\zeta = 0$ and radius equal to unity. The mapping function to map an infinite plate containing a crack in the Z plane, to a region outside a unit circle in ζ plane is given by Sih [5]

$$z = \omega(\zeta) = \frac{a}{2}\left(\zeta + \frac{1}{\zeta}\right) \tag{3.10}$$

Where a is the crack length.

Hence $\phi(z)$ and $\psi(z)$ can be written as,

$$\phi(\omega(\zeta)) = \phi(\zeta), \qquad \psi(\omega(\zeta)) = \psi(\zeta)$$

The stresses and displacement can be written as,

$$\sigma_y + \sigma_x = 4\,\text{Re}\left[\frac{\phi'(\zeta)}{\omega'(\zeta)}\right] = 4\text{Re}[\phi'(z)]$$

$$\sigma_y - \sigma_x + 2i\tau_{xy} = \frac{2}{\omega'(\zeta)}\left[\overline{\omega(\zeta)}\left[\frac{\phi'(\zeta)}{\omega(\zeta)}\right]' + \psi'(\zeta)\right] = 2[\bar{z}\phi''(z) + \psi'(z)] \tag{3.11}$$

The Stress function in the ζ plane is given by [5],

$$\phi(\zeta) = \frac{-(F_\zeta + i'F_\eta)}{2\pi(1+\kappa)}\log\zeta + (B'+iC')\frac{a}{2}\zeta + \phi_1(\zeta)$$

$$\psi(\zeta) = \kappa\frac{(F_\zeta + i'F_\eta)}{2\pi(1+\kappa)}\log\zeta + (B'+iC')\frac{a}{2}\zeta + \psi_1(\zeta)$$

(3.12)

Where,

i) F_ξ and F_η are the resultant forces in the ξ and η directions respectively on the boundary of the unit circle in the mapped plane.

ii) The constants B, C, B', C' indicate the stresses and rigid body rotation at infinity and are given by

$$B = \frac{\sigma_x + \sigma_y}{4} = \frac{\sigma_1 + \sigma_2}{4} \quad (3.13)$$

$$B'+iC' = \frac{1}{2}(\sigma_y - \sigma_x + 2i\tau_{xy}) = -\frac{1}{2}(\sigma_1 - \sigma_2)e^{-2i\alpha} \quad (3.14)$$

$$C = \frac{2G}{1+\kappa}\omega^\infty$$

Where, ω^∞ denote the rigid body rotation at infinity, for the analysis of stresses the constant C can be set to zero,

G = modulus of rigidity,

$\kappa = 3 - 4\nu$ for plane strain,

$\kappa = \frac{3-\nu}{1+\nu}$ For plane stress,

ν = Poisson's ratio,

σ_x, σ_y, τ_{xy} are the stresses at infinity.

σ_1, σ_2 are the principal stresses at infinity and α the angle mode by σ_1 with x axis. In the absence of boundary stresses at the edge of the crack, and if the rigid body rotation C = 0 then equations (3.12) becomes,

$$\phi(\zeta)=\phi_0(\zeta)+\phi_1(\zeta) \tag{3.15}$$

$$\psi(\zeta)=\psi_0(\zeta)+\psi_1(\zeta)$$

Where,

$$\phi_0(\zeta)=(B+iC)(\frac{a}{2})(\zeta)=(B)(\frac{a}{2})(\zeta)$$

$$\psi_0(\zeta)=(B'+iC')(\frac{a}{2})(\zeta) \tag{3.16}$$

We Have,

$$\omega(\zeta)=\frac{a}{2}\left(\zeta+\frac{1}{\zeta}\right)$$

$$\overline{\omega(\zeta)}=\frac{a}{2}\left(\frac{1}{\zeta}+\zeta\right)$$

$$\omega'(\zeta)=\frac{a}{2}\left(1-\frac{1}{\zeta^2}\right)$$

$$\overline{\omega'(\zeta)}=\frac{a}{2}\left(1-\zeta^2\right) \tag{3.17}$$

$\phi_1(\zeta)$ And $\psi_1(\zeta)$ are the analytic stress functions and given by [5],

$$\phi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{f_0(t)}{(t-\zeta)}dt$$

$$\psi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{\overline{f_0(t)}}{(t-\zeta)}dt-\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\phi_1'(\zeta) \tag{3.18}$$

Therefore $\psi_1(\zeta)$ can be written as,

$$\psi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{\overline{f_0(t)}}{(t-\zeta)}dt-\frac{\zeta(1+\zeta^2)}{(\zeta^2-1)}\phi_1'(\zeta)$$

Where t is the boundary value of and ζ, fo(t) is the analytical boundary condition and given by,

$$f_0(t)=\phi_0(t)+\frac{\omega(t)}{\overline{\omega'(t)}}\overline{\phi'_0(t)}+\overline{\psi_0(t)} \tag{3.19}$$

In general stress functions $\phi(t)$ and $\psi(t)$ give the boundary condition as,

$$f(t)=\phi(t)+\frac{\omega(t)}{\overline{\omega'(t)}}\overline{\phi'(t)}+\overline{\psi(t)} \tag{3.20}$$

By substituting the values of eqa (3.17) for $\zeta = t$,

$$f(t)=\phi(t)+\frac{t^2+1}{t(1-t^2)}\overline{\phi'(t)}+\overline{\psi(t)} \tag{3.21}$$

To evaluate equations $\phi_1(\zeta)$ and $\psi_1(\zeta)$ the Cauchy integral theorem and the residue theorem are used.

The stresses in the Cartesian co-ordinates in terms of the stress functions in the mapped plane are given by (ρ and θ are polar co-ordinates).

$$\sigma_y+\sigma_x=\sigma_\theta+\sigma_\rho=4\operatorname{Re}\left[\frac{\phi'(\zeta)}{\omega'(\zeta)}\right]$$

$$\sigma_y-\sigma_x+2i\tau_{xy}=\frac{2}{\omega'(\zeta)}\left[\overline{\omega(\zeta)}\left[\frac{\phi'(\zeta)}{\omega(\zeta)}\right]'+\psi'(\zeta)\right]$$

In polar coordinates stresses can be given as,

$$\sigma_\theta-\sigma_\rho+2i\tau_{\rho\theta}=(\sigma_y-\sigma_x+2i\tau_{xy})e^{2i\lambda_0}$$

$$e^{2i\lambda_0}=\frac{\zeta^2\omega'(\zeta)}{\rho^2\overline{\omega'(\zeta)}} \tag{3.22}$$

$$\sigma_\theta-\sigma_\rho+2i\tau_{\rho\theta}=(\sigma_y-\sigma_x+2i\tau_{xy})\frac{\zeta^2\omega'(\zeta)}{\rho^2\overline{\omega'(\zeta)}}$$

Where, λ_0 = angle between ρ and X-axes.

3.5.1 Stress Intensity Factors

The mode-I and mode-II stress intensity factors (SIF) for the crack tips are obtained by using the resulting stress function in equation given by Erdogan and Sih [5].

$$K_I - iK_{II} = 2\sqrt{\frac{\pi}{a}}\phi'(\zeta) \pm 1 \tag{3.23}$$

a = crack length,

+ sign for $\zeta = 1$ and $\eta = 0$ and

- sign for $\zeta = -1$ and $\eta = 0$ and

$\phi'(\zeta)$ Stress function given by corresponding approximation solution.

The derivative of $\phi(\zeta)$ in the above equation is obtained from eqn. (3.15).

3.6 Closing Remarks

In this chapter complex variable formulation and governing equations for mapping are given. The analytical solutions of stress intensity factors for a crack in an infinite plate are formulated by using complex variable method.

Chapter 4
General solution for inplane loading with arbitrary biaxial loading condition

4.1 Problem configuration

The arbitrary biaxial loading condition is introduced in boundary conditions to consider several cases of in-plane loads. This condition considers any arbitrary orientation of uniaxial and biaxial stresses or moments as well as shear stress or twisting moments applied at infinity. This condition has been adopted from Ukadgaonkar's [5] solution for the elliptical hole in an isotropic plate. By means of this condition, solutions for biaxial loading or shear or twist can be obtained without the need for superposition of the solutions of the uniaxial loading. This is achieved by merely introducing the biaxial loading factor λ and the orientation angle α in the boundary conditions at infinity. A remotely applied loading is considered about the arbitrary co-ordinate axes $x'oy'$ that makes an angle α with xoy axes in the principle directions of the body.

4.2 Boundary Condition at Infinity

The Boundary condition about the arbitrary coordinate axes $x'oy'$ for in plane loading given in following [2];

$$\sigma_{x'} = \lambda P, \qquad \sigma_{y'} = P, \qquad \tau_{x'} = 0, \qquad at|z| \to \infty \tag{4.1}$$

Fig.4.1 shows the infinite plate with two cracks subjected to arbitrary biaxial loading condition.

By applying the relation of transformation of axes, the boundary conditions in eqns. (4.1) about xoy axes are given by:

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

$$\sigma_{y'} - \sigma_{x'} + i\tau_{xy} = (\sigma_y - \sigma_x + i\tau_{xy})e^{2i\alpha} \tag{4.2}$$

By applying above relations we can write stresses as,

$$\sigma_x = \frac{p}{2}\left[(\lambda+1)+(\lambda-1)\cos 2\alpha\right]$$

$$\sigma_y = \frac{p}{2}\left[(\lambda+1)-(\lambda-1)\cos 2\alpha\right] \tag{4.3}$$

$$\tau_{xy} = \frac{p}{2}\left[(\lambda-1)\sin 2\alpha\right]$$

From equation (3.13), (3.14) and (4.3) the boundary condition about xoy axes can be written explicitly as:

$$B = \mathrm{Re}[\phi'(z)] = \frac{p}{4}(\lambda+1) \tag{4.4}$$

$$B' + iC' = \mathrm{Re}[\psi'(z)] = -\frac{p}{2}(\lambda-1)e^{2i\alpha} \tag{4.5}$$

The boundary conditions in equation (4.4) and (4.5) are useful to determine the stress functions for the crack free plate.

4.2.1 Applications of Arbitrary Biaxial Loading Condition

With reference to Fig. 4.1, the following values of λ and α will be taken in to eqn. (4.1) to obtain different conditions of loading.

1. Inclined uniaxial tension or cylindrical bending : $\lambda = 0$ and $\alpha \neq 0$

(a) Loading along X-axis : $\lambda = 0$ and $\alpha = \pi/2$

(b) Loading along Y-axis : $\lambda = 0$ and $\alpha = 0$

2. Hydrostatic tension or all around moment at infinity : $\lambda = 1$ and $\alpha \neq 0$

(a) Equal biaxial tension or moment : $\lambda = 1$ and $\alpha = 0$ (4.6)

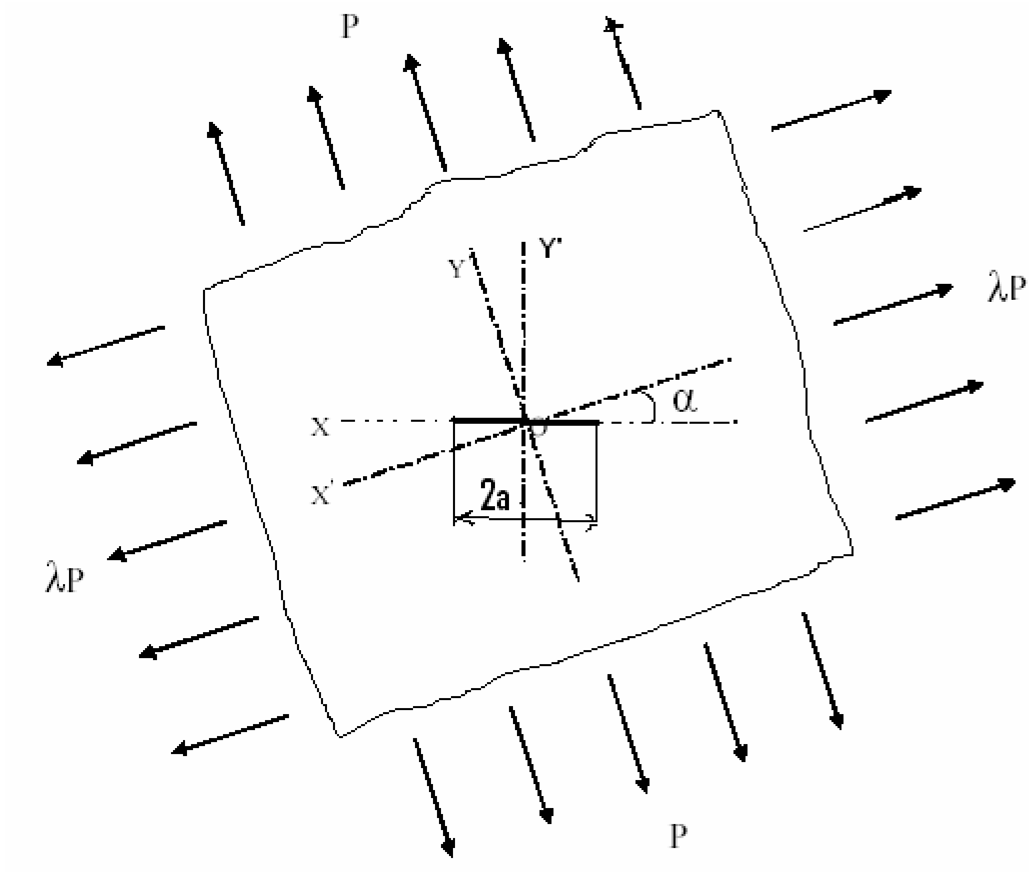

Fig: 4.1 Infinite plate with crack subjected to arbitrary biaxial load.

4.3 Solution for an infinite plate with single crack subjected to arbitrary biaxial load

Conformal mapping technique is applied to map the crack in the z plane to a region outside the unit circle in the ζ plane using following mapping equation.

$$z = \omega(\zeta) = \frac{a}{2}\left(\zeta + \frac{1}{\zeta}\right)$$

Where a is the crack length.

The stress functions are given by equations (3.14).

$$\phi(\zeta) = \phi_0(\zeta) + \phi_1(\zeta) \tag{4.7}$$

$$\psi(\zeta)=\psi_0(\zeta)+\psi_1(\zeta) \tag{4.8}$$

Where $\phi_0(\zeta)$ and $\psi_0(\zeta)$ given by,

$$\phi_0(\zeta)=B\left[\frac{a}{2}\left(\zeta+\frac{1}{\zeta}\right)\right] \tag{4.9}$$

$$\psi_0(\zeta)=(B'+iC')\left[\frac{a}{2}\left(\zeta+\frac{1}{\zeta}\right)\right] \tag{4.10}$$

From above equation (4.9) we get,

$$\overline{\phi_0(\zeta)}=B\left[\frac{a}{2}\left(\frac{1}{\zeta}+\zeta\right)\right] \tag{4.11}$$

$$\overline{\psi_0(\zeta)}=(B'-iC')\left[\frac{a}{2}\left(\frac{1}{\zeta}+\zeta\right)\right] \tag{4.12}$$

$$\phi_0'(\zeta)=B\left[\frac{a}{2}\left(1+\frac{1}{\zeta^2}\right)\right] \tag{4.13}$$

$$\overline{\phi_0'(\zeta)}=B\left[\frac{a}{2}\left(1-\zeta^2\right)\right] \tag{4.14}$$

Substituting stress functions from equation (4.10) and (4.11) in to the boundary condition equation (3.19) we get,

$$f_0(t)=\frac{a}{2}\left[2B\left(\frac{t^2+1}{t}\right)+(B'+iC')\left(t+\frac{1}{t}\right)\right] \tag{4.15}$$

$$\overline{f_0(t)}=\frac{a}{2}\left[2B\left(\frac{t^2+1}{t}\right)+(B'-iC')\left(t+\frac{1}{t}\right)\right] \tag{4.16}$$

Substituting the equations (4.11) and (4.12) in the equations (3.18) so that $\phi_1(\zeta)$ and $\psi_1(\zeta)$ can be obtained.

$$\phi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{f_0(t)}{(t-\zeta)}dt$$

And

$$\psi_1(\zeta)=-\frac{1}{2\pi i}\oint\frac{\overline{f_0(t)}}{(t-\zeta)}dt-\frac{\overline{\omega(\zeta)}}{\omega'(\zeta)}\phi_1'(\zeta)$$

Evaluating the Cauchy's integral,

$$\phi_1(\zeta) = -\frac{a}{2}\frac{\left(2B + \left(B' + iC'\right)\right)}{\zeta} \tag{4.17}$$

And

$$\psi_1(\zeta) = \frac{a}{2}\left[-\frac{\zeta^2 + 1}{\zeta\left(\zeta^2 - 1\right)}\left(2B + \left(B' + iC'\right)\right) + \left(B' + iC'\right)\left(\frac{\zeta}{1+\zeta}\right)\right] \tag{4.18}$$

The Stress functions are obtained as,

$$\phi(\zeta) = \frac{a}{2}\left[B\left(\zeta + \frac{1}{\zeta}\right) - \frac{\left(2B + \left(B' + iC'\right)\right)}{\zeta}\right] \tag{4.19}$$

$$\psi(\zeta) = \frac{a}{2}\left[-\frac{\left(2B + \left(B' - iC'\right)\right)}{\zeta} - \frac{\zeta^2 + 1}{\zeta\left(\zeta^2 - 1\right)}\left(2B + \left(B' + iC'\right)\right) + \left(B' + iC'\right)\left(\frac{\zeta}{1+\zeta}\right)\right] \tag{4.20}$$

These complex stress functions satisfy the stress free boundary condition exactly as $f(t) = 0$.

4.4 Stress Intensity Factors

The mode-I and mode-II stress intensity factors (SIF) for the crack tips are obtained by using the resulting stress function in equation given by Erdogan and Sih [6].

$$K_I - iK_{II} = 2\sqrt{\frac{\pi}{a}}\phi_i'(\zeta_i) \pm 1 \tag{4.21}$$

a = crack length,

+ sign for $\zeta = 1$ and $\eta = 0$ and

- sign for $\zeta = -1$ and $\eta = 0$ and

$\phi_i'(\zeta_i)$ Stress function given by corresponding approximation solution.

The derivative of $\phi(\zeta)$ in the above equation is obtained from equation (4.9)

Chapter 5
The Interaction of Two Cracks Subjected to Arbitrary Biaxial Load

5.1 Introduction

The infinite plate containing two cracks subjected to an arbitrary biaxial load is considered to study their interaction effect. The interaction effect of two cracks for the infinite plate subjected to various loading is studied in the form of complex stress functions, which are obtained by Schwarz's alternating method [7]. The stress intensity factors at the crack tips are evaluated using equation given by Sih and Edrogen and their variations with respect to geometrical parameters are presented for the following cases.

1. Effect of center distance between two cracks,
2. Effect of inclination of cracks of two cracks,
3. Effect of biaxial load factor λ .

5.2 Interaction Effect of Two cracks

The interaction effect of two cracks for the infinite plate subjected to the arbitrary biaxial loading is studied in the form of complex stress functions, which are obtained by Schwarz's alternating method [5] Fig. 5.1 shows two cracks subjected to the arbitrary biaxial load and a, b are the crack lengths, α and β are the angles made by the cracks with the direction of load.

5.2.1 Schwarz's alternating method for multiply connected region:

By using Schwarz's alternating method the problem of multiply connected region can be reduced to the series of simply connected region problem. The Schwarz's alternating method is applied to the doubly connected region as follows.

First the problem of an infinite plate with the first crack is solved for the given boundary condition by using the procedure followed for a single crack solution. The complex stress functions are determined. These stress functions are transformed to the center of the second crack by rotation and translation. The boundary condition at the second crack boundary is determined, by using the transformed stress functions. This boundary condition does not satisfy the required boundary condition at the second crack. Hence a new problem of second crack subjected to a combination of the negative of this boundary condition and the required boundary condition on the crack boundary is solved, which gives the corrected stress functions. The superposition of the transformed and corrected stress functions gives the required second approximation solution for the multiply connected region which is valid near second crack. These stress functions satisfy the boundary condition exactly at the second crack. Similarly starting from the second crack, the second approximation solution around the first crack can be evaluated.

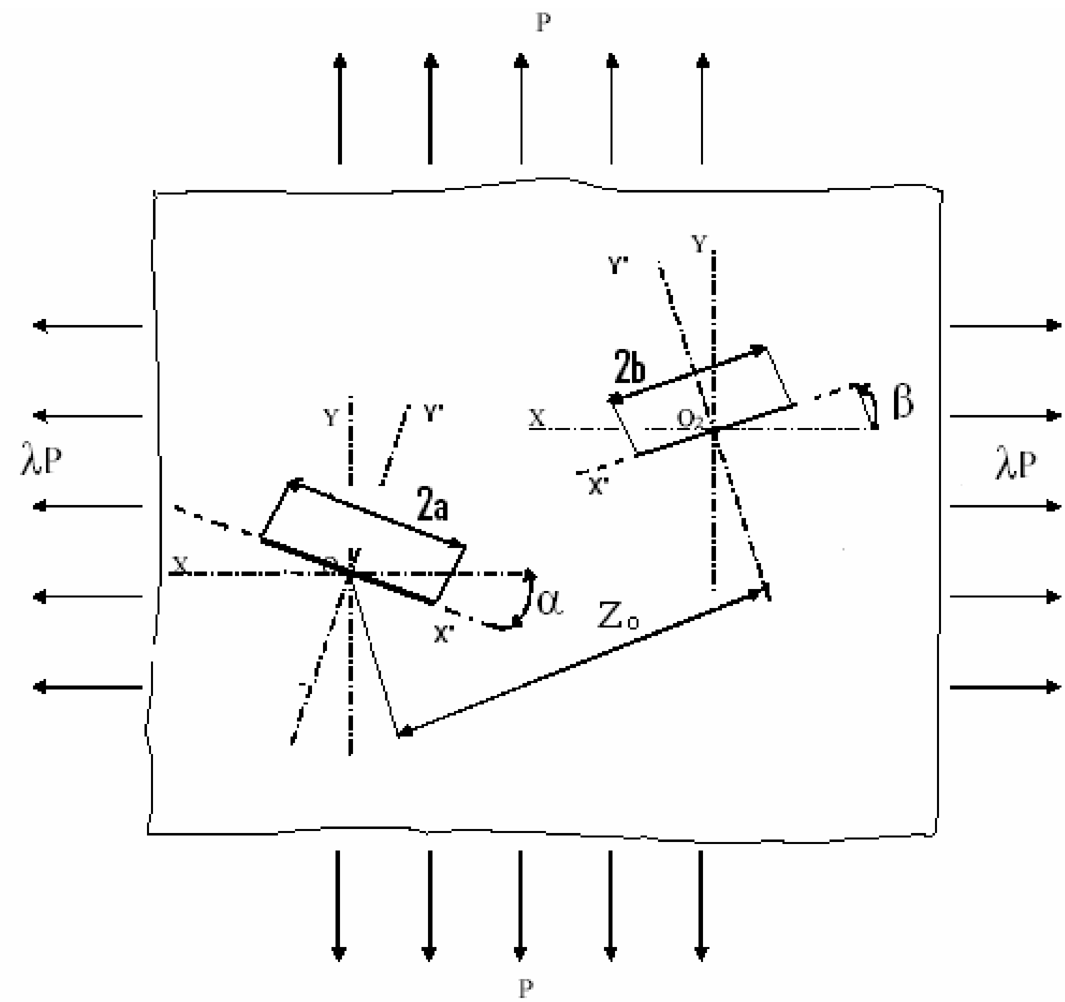

Fig: 5.1 Infinite plate with two cracks subjected to arbitrary biaxial load.

5.3 Determination of stress functions

5.3.1. Scheme of solution

The isotropic plate containing the cracks is subjected to remotely applied tensions $\sigma^{\infty}{}_{x'} = \lambda P \quad \sigma^{\infty}{}_{y}' = \lambda$ at the outer edges as shown in Fig. 5.1. The edges of the crack are free from loading. To determine the stresses around the crack, the solution is split into two stages.

First stage solution

The stress functions $\phi(\zeta)$ and $\psi(\zeta)$ are obtained for a crack free plate due to applied stresses $\sigma^{\infty}{}_{x'}$ and $\sigma^{\infty}{}_{y'}$. The boundary conditions $f_1(t_1), f_2(t_2)$ on a fictitious crack are determined from these stress functions.

Second stage solution

For the second stage solution, the plate with crack is applied by a negative of the boundary conditions $f_1(t_1)$, $f_2(t_2)$ on its crack boundary in the absence of remote loading The stress functions of the second stage solution are determined from these boundary conditions.

5.3.2 First stage solution.

The stress function for the first crack of crack length a, is given by,

For first crack

$$\phi_1(\zeta_1) = \frac{a}{2}\left[B\left(\zeta_1 + \frac{1}{\zeta_1}\right) - \frac{(2B + (B' + iC'))}{\zeta_1}\right]$$

$$\psi_1(\zeta_1) = \frac{a}{2}\left[-\frac{(2B + (B' - iC'))}{\zeta_1} - \frac{\zeta_1{}^2 + 1}{\zeta_1(\zeta_1{}^2 - 1)}(2B + (B' + iC')) + (B' + iC')\left(\zeta_1 + \frac{1}{\zeta_1}\right)\right] \qquad (5.1)$$

The stress functions for the second crack of crack length b is given by,

For second crack

$$\phi_2(\zeta_2) = \frac{b}{2}\left[B\left(\zeta_2 + \frac{1}{\zeta_2}\right) - \frac{(2B + (B' + iC'))}{\zeta_2}\right]$$

$$\psi_2(\zeta_2) = \frac{b}{2}\left[-\frac{(2B+(B'-iC'))}{\zeta_2} - \frac{\zeta_2^{\,2}+1}{\zeta_2(\zeta_2^{\,2}-1)}(2B+(B'+iC'))+(B'+iC')\left(\zeta_2+\frac{1}{\zeta_2}\right)\right] \quad (5.2)$$

The first approximation solution is defined as vectorial addition of the stress components at any point i.e. stress at any point in plane is a vector sum of the individual stress components due to the loading of the first crack boundary and second crack boundary. However, this does not take into account the interaction effect of the other crack in the vicinity [10].

5.3.3 Second stage solution.

For the first crack gives stress functions $\phi_1(\zeta_1)$ and $\psi_1(\zeta_1)$ Starting from the first crack to account interaction effect on the second crack the stress functions given by equations (4.19) and (4.20)are transformed to the second crack center O_2 by rotation through an angle $(\alpha+\beta)$ and translation through distance C_0 in the mapped plane ξ-η plane where C_0 is given by,

$$Z_0 = \frac{a}{2}\left(C_0 + \frac{1}{C_0}\right) \quad (5.3)$$

Where, Zo is the center distance between two cracks in Z plane. This equation has root.

$$C_0 = \frac{Z_0}{a} + \sqrt{\frac{Z_0^{\,2}}{a^2} - 1} \quad (5.4)$$

Transforming the stress function to the center of second crack [7] Ref. Fig.4.3.

$$\phi_{12}(\zeta_2) = \frac{a}{2}\left[\phi_1(\zeta_2 + C_0)e^{i(\alpha+\beta)}\right]e^{-i(\alpha+\beta)}$$

$$\psi_{12}(\zeta_2) = \frac{a}{2}\left[\psi_1(\zeta_2 + C_0)e^{i(\alpha+\beta)}\right]e^{i(\alpha+\beta)} + \overline{C_0}\phi'_{12}(\zeta_2) \quad (5.5)$$

The transformed stress functions are,

$$\phi_{12}(\zeta_2) = \frac{a}{2}\left[B\left((\zeta_2 + C_0) + \frac{e^{-2i(\alpha+\beta)}}{(\zeta_2 + C_0)}\right) - \frac{(2B+(B'+iC'))}{(\zeta_2 + C_0)e^{2i(\alpha+\beta)}}\right] \quad (5.6)$$

$$\psi_{12}(\zeta_2)=\frac{a}{2}\left[\begin{array}{l} -\dfrac{\left(2B+\left(B'-iC'\right)\right)}{\left(\zeta_2+C_0\right)}- \\ \dfrac{\left(\zeta_2+C_0\right)^2 e^{2i(\alpha+\beta)}+1}{\left(\zeta_2+C_0\right)\left(\left(\zeta_2+C_0\right)^2 e^{2i(\alpha+\beta)}-1\right)}\left(2B+\left(B'+iC'\right)\right) \\ +\left(B'+iC'\right)\left(\left(\zeta_2+C_0\right)e^{2i(\alpha+\beta)}+\dfrac{1}{\left(\zeta_2+C_0\right)}\right) \\ +\overline{C_0}\left[B\left(1-\dfrac{1}{\left(\zeta_2+C_0\right)^2 e^{2i(\alpha+\beta)}}\right)+\dfrac{\left(2B+\left(B'+iC'\right)\right)}{\left(\zeta_2+C_0\right)^2 e^{2i(\alpha+\beta)}}\right] \end{array}\right] \tag{5.7}$$

These transformed stress functions give a boundary condition $f_{12}(t_2)$ on the second crack as

$$f_{12}(t_2)=\phi_{12}(t_2)+\frac{\left(t_2{}^2+1\right)}{t_2(1-t_2{}^2)}\overline{\phi_{12}'(t_2)}+\overline{\psi_{12}(t_2)}$$

Boundary condition $f_{12}(t_2)$ on the second crack given as ,

$$f_{12}(t_2)=\frac{a}{2}\left[\begin{array}{l} B\left(\left(t_2+C_0\right)+\dfrac{e^{-2i(\alpha+\beta)}}{\left(t_2+C_0\right)}\right)-\dfrac{\left(2B+\left(B'+iC'\right)\right)}{\left(t_2+C_0\right)e^{2i(\alpha+\beta)}}+ \\ \left(\dfrac{\left(t^2+1\right)}{t_2(1-t_2{}^2)}\right)\left[B\left(1-\dfrac{e^{2i(\alpha+\beta)}}{\left(\overline{t_2}+\overline{C_0}\right)^2}\right)-\dfrac{\left(2B+\left(B'-iC'\right)\right)}{\left(\overline{t_2}+\overline{C_0}\right)^2 e^{-2i(\alpha+\beta)}}\right]+ \\ \left(B'-iC'\right)\left(\left(\overline{t_2}+\overline{C_0}\right)e^{-2i(\alpha+\beta)}+\dfrac{1}{\left(\overline{t_2}+\overline{C_0}\right)}\right)-\dfrac{\left(2B+\left(B'+iC'\right)\right)}{\left(\overline{t_2}+\overline{C_0}\right)} \\ -\left[\dfrac{\left(\left(\overline{t_2}+\overline{C_0}\right)^2 e^{-2i(\alpha+\beta)}+1\right)}{\left(\overline{t_2}+\overline{C_0}\right)\left(\left(\overline{t_2}+\overline{C_0}\right)^2 e^{-2i(\alpha+\beta)}-1\right)}\right]\left(2B+\left(B'-iC'\right)\right) \\ +C_0\left[B\left(1-\dfrac{e^{-2i(\alpha+\beta)}}{\left(\overline{t_2}+\overline{C_0}\right)^2}\right)+\dfrac{\left(2B+\left(B'-iC'\right)\right)}{\left(\overline{t_2}+\overline{C_0}\right)^2 e^{-2i(\alpha+\beta)}}\right] \end{array}\right] \tag{5.8}$$

This will be causing additional loading at the second crack boundary. But the true loading condition at the second crack is a stress free boundary condition. In order to achieve the stress free boundary condition at the second crack, a new problem of an infinite plate with second crack is solved with the boundary condition $f_2(t_2)$ given by

$$f_2(t_2)= -f_{12}(t_2)$$

The corrected stress functions valid near the second crack can be obtained

$$\phi_{22}(\zeta_2)=\frac{a}{2}\left[\begin{array}{l}\dfrac{1}{\zeta_2}+\dfrac{\left(1-2B-\left(B'-iC'\right)\right)e^{i(\alpha+\beta)}}{\overline{C_0}}\\ \left\{\dfrac{-\left(\overline{C_0}^2-1\right)\left(1+\zeta_2\overline{C_0}\right)\left(3+\overline{C_0}^2\right)+\left(1+\overline{C_0}^2\right)\left(\overline{C_0}^2-3+2\zeta_2\overline{C_0}\right)}{\left(\overline{C_0}^2-1\right)^2\left(1+\zeta_2\overline{C_0}\right)^2}\right\}\\ +\dfrac{\left(2B+\left(B'-iC'\right)\right)}{\overline{C_0}\left(1+\zeta_2\overline{C_0}\right)}-\left[\begin{array}{l}\dfrac{1}{\left(1+\zeta_2\overline{C_0}\right)}+\dfrac{\overline{C_0}\left(\overline{C_0}+e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1+\zeta_2\left(\overline{C_0}+e^{i(\alpha+\beta)}\right)\right)}+\\ \dfrac{\overline{C_0}\left(\overline{C_0}-e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1+\zeta_2\left(\overline{C_0}-e^{i(\alpha+\beta)}\right)\right)}\end{array}\right]\\ -\left(\dfrac{B'-iC'}{\overline{C_0}}\right)\left(\dfrac{1}{1+\zeta_2\overline{C_0}}\right)+\\ \dfrac{C_0e^{-2i(\alpha+\beta)}}{\overline{C_0}^2}\left(\dfrac{2\left(1+\zeta_2\overline{C_0}\right)-1}{\left(1+\zeta_2\overline{C_0}\right)^2}\right)\left[1-\left(2B+\left(B'-iC'\right)\right)\right]\end{array}\right] \tag{5.9}$$

Similarly $\psi_{22}(\zeta_2)$ can be obtained

$$\psi_{22}(\zeta_2)=\frac{a}{2}\left[\begin{array}{l}\dfrac{\left(B+\left(B'-iC'\right)\right)e^{2i(\alpha+\beta)}}{\overline{C_0}\left(1+\zeta_2\overline{C_0}\right)}-\dfrac{2\zeta_2}{1-\zeta_2^2}+\\ \dfrac{2e^{-2i(\alpha+\beta)}\left(1+\left(2B+\left(B'-iC'\right)\right)\right)\left(\zeta_2C_0^2+2C_0+\zeta_2\right)}{\left(C_0-1\right)^2\left(C_0+1\right)^2\left(1-\zeta_2^2\right)}\\ -\dfrac{\zeta_2\left(\zeta_2^2+1\right)}{\zeta_2^2+1}\phi'_{22}(\zeta_2)\end{array}\right] \tag{5.10}$$

Where $\phi'_{22}(\zeta_2)$ is derivative of $\phi_{22}(\zeta_2)$

5.3.4 Stress Function for the Second Crack

The stress function $\phi_2(\zeta_2)$ and for the second crack can be obtained by superposing the corresponding transformed stress function $\phi_{12}(\zeta_2)$ equation(5.6) and corrected stress function $\phi_{22}(\zeta_2)$

$$\phi_2(\zeta_2) = \phi_{12}(\zeta_2) + \phi_{22}(\zeta_2)$$

$$\phi_2(\zeta_2) = \frac{a}{2}\left[\begin{array}{l} B\left((\zeta_2 + C_0) + \frac{e^{-2i(\alpha+\beta)}}{(\zeta_2 + C_0)}\right) - \frac{(2B + (B' + iC'))}{(\zeta_2 + C_0)e^{2i(\alpha+\beta)}} + \frac{1}{\zeta_2} + \frac{(1 - 2B - (B' - iC'))e^{i(\alpha+\beta)}}{\overline{C_0}} \\ \left\{\frac{-\left(\overline{C_0}^2 - 1\right)\left(1 + \zeta_2\overline{C_0}\right)\left(3 + \overline{C_0}^2\right) + \left(1 + \overline{C_0}^2\right)\left(\overline{C_0}^2 - 3 + 2\zeta_2\overline{C_0}\right)}{\left(\overline{C_0}^2 - 1\right)^2\left(1 + \zeta_2\overline{C_0}\right)^2}\right\} \\ + \frac{(2B + (B' - iC'))}{\overline{C_0}\left(1 + \zeta_2\overline{C_0}\right)} - \left[\begin{array}{l} \frac{1}{\left(1 + \zeta_2\overline{C_0}\right)} + \frac{\overline{C_0}\left(\overline{C_0} + e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1 + \zeta_2\left(\overline{C_0} + e^{i(\alpha+\beta)}\right)\right)} + \\ \frac{\overline{C_0}\left(\overline{C_0} - e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1 + \zeta_2\left(\overline{C_0} - e^{i(\alpha+\beta)}\right)\right)} \end{array}\right] \\ -\left(\frac{B' - iC'}{\overline{C_0}}\right)\left(\frac{1}{1 + \zeta_2\overline{C_0}}\right) + \frac{C_0 e^{-2i(\alpha+\beta)}}{\overline{C_0}^2}\left(\frac{2\left(1 + \zeta_2\overline{C_0}\right) - 1}{\left(1 + \zeta_2\overline{C_0}\right)^2}\right)\left[1 - (2B + (B' - iC'))\right] \end{array}\right]$$

(5.11)

And the stress function $\psi_2(\zeta_2)$ valid near the second crack can be obtained by superposing the corresponding transformed stress function $\psi_{12}(\zeta_2)$ equation (5.7) and corrected stress function $\psi_{22}(\zeta_2)$ equation (5.13)

$$\psi_2(\zeta_2) = \psi_{12}(\zeta_2) + \psi_{22}(\zeta_2)$$

$$\psi_2(\zeta_2) = \frac{a}{2}\left[\begin{array}{l} -\frac{(2B + (B' - iC'))}{(\zeta_2 + C_0)} - \\ \frac{(\zeta_2 + C_0)^2 e^{2i(\alpha+\beta)} + 1}{(\zeta_2 + C_0)\left((\zeta_2 + C_0)^2 e^{2i(\alpha+\beta)} - 1\right)}(2B + (B' + iC')) \\ + (B' + iC')\left((\zeta_2 + C_0)e^{2i(\alpha+\beta)} + \frac{1}{(\zeta_2 + C_0)}\right) \\ + \overline{C_0}\left[B\left(1 - \frac{1}{(\zeta_2 + C_0)^2 e^{2i(\alpha+\beta)}}\right) + \frac{(2B + (B' + iC'))}{(\zeta_2 + C_0)^2 e^{2i(\alpha+\beta)}}\right] \\ \frac{(B + (B' - iC'))e^{2i(\alpha+\beta)}}{\overline{C_0}\left(1 + \zeta_2\overline{C_0}\right)} - \frac{2\zeta_2}{1 - \zeta_2^2} + \frac{2e^{-2i(\alpha+\beta)}\left(1 + (2B + (B' - iC'))\right)\left(\zeta_2 C_0^2 + 2C_0 + \zeta_2\right)}{(C_0 - 1)^2(C_0 + 1)^2\left(1 - \zeta_2^2\right)} \\ -\frac{\zeta_2\left(\zeta_2^2 + 1\right)}{\zeta_2^2 + 1}\phi'_{22}(\zeta_2) \end{array}\right]$$

(5.12)

Equations (5.11) and (5.12) are the second approximation stress functions valid near the second crack. These stress functions satisfy the stress free boundary condition exactly on the edge of the second crack.

5.3.5 Stress Function for the First Crack

Starting from the second crack stress functions from equations, the stress function valid near the first crack is obtained as

$$\phi_1(\zeta_1) = \phi_{21}(\zeta_1) + \phi_{11}(\zeta_1)$$

$$\psi_1(\zeta_1) = \psi_{21}(\zeta_1) + \psi_{112}(\zeta_1)$$

The stress function for the second crack $\phi_2(\zeta_2)$ and $\psi_2(\zeta_2)$ from the first approximation are given by the equations (4.19), starting from the second crack to account interaction effect on the first crack, the stress functions given by the equations.(4.19) are transformed to the center of the first crack O_1 by rotation through an angle $(\alpha+\beta)$ and translation through distance C_1 in the mapped plane ξ-η plane where C_1 is given by

$$Z_0 = \frac{b}{2}\left(C_1 + \frac{1}{C_1}\right) \tag{5.13}$$

$$C_1 = \frac{Z_0}{b} + \sqrt{\frac{Z_0^{\ 2}}{b^2} - 1} \tag{5.14}$$

Transforming the stress function to the center of first crack

$$\phi_{21}(\zeta_1) = \frac{b}{2}\left[\phi_2\left(\zeta_1 - C_1\right)e^{i(\alpha+\beta)}\right]e^{-i(\alpha+\beta)}$$

$$\psi_{21}(\zeta_1) = \frac{b}{2}\left[\psi_2\left(\zeta_1 - C_1\right)e^{i(\alpha+\beta)}\right]e^{i(\alpha+\beta)} - \overline{C_1}\phi'_{21}(\zeta_1) \tag{5.15}$$

The transformed stress functions are

$$\phi_{21}(\zeta_1) = \frac{b}{2}\left[B\left(\left(\zeta_1 - C_1\right) + \frac{e^{-2i(\alpha+\beta)}}{\left(\zeta_1 - C_1\right)}\right) - \frac{\left(2B + \left(B' + iC'\right)\right)e^{-i(\alpha+\beta)}}{\left(\zeta_1 - C_1\right)}\right] \tag{5.16}$$

$$\psi_{21}(\zeta_1)=\frac{b}{2}\left[\begin{array}{l} -\dfrac{\left(2B+\left(B'-iC'\right)\right)}{\left(\zeta_1-C_1\right)}-\dfrac{\left(\zeta_1-C_1\right)^2 e^{-2i(\alpha+\beta)}+1}{\left(\zeta_1-C_1\right)\left(\left(\zeta_1-C_1\right)^2 e^{-2i(\alpha+\beta)}-1\right)}\left(2B+\left(B'+iC'\right)\right) \\ +\left(B'+iC'\right)\left(\left(\zeta_1-C_1\right)e^{-2i(\alpha+\beta)}+\dfrac{1}{\left(\zeta_1-C_1\right)}\right) \\ -\overline{C_1}\left[B\left(1-\dfrac{e^{-i(\alpha+\beta)}}{\left(\zeta_1-C_1\right)^2}\right)+\dfrac{\left(2B+\left(B'+iC'\right)\right)e^{-i(\alpha+\beta)}}{\left(\zeta_1-C_1\right)^2}\right] \end{array}\right] \tag{5.17}$$

These transformed stress functions give a boundary condition $f_{21}(t_1)$ on the second crack as

$$f_{21}(t_1)=\phi_{21}(t_1)+\frac{\left(t_1{}^2+1\right)}{t_1(1-t_1{}^2)}\overline{\phi_{21}'(t_1)}+\overline{\psi_{21}(t_1)} \tag{5.18}$$

Boundary condition $f_{21}(t_1)$ on the first crack given as ,

$$f_{21}(t_1)=\frac{b}{2}\left[\begin{array}{l} B\left(\left(t_1-C_1\right)+\dfrac{e^{-2i(\alpha+\beta)}}{\left(t_1-C_1\right)}\right)-\dfrac{\left(2B+\left(B'+iC'\right)\right)e^{-i(\alpha+\beta)}}{\left(t_1-C_1\right)}+ \\ \dfrac{\left(t_1{}^2+1\right)}{t_1(1-t_1{}^2)}\left[B\left(1-\dfrac{e^{i(\alpha+\beta)}}{\left(t_1-\overline{C_1}\right)^2}\right)+\dfrac{\left(2B+\left(B'-iC'\right)\right)e^{i(\alpha+\beta)}}{\left(t_1-\overline{C_1}\right)^2}\right] \\ -\dfrac{\left(2B+\left(B'+iC'\right)\right)}{\left(t_1-\overline{C_1}\right)}-\dfrac{\left(t_1-\overline{C_1}\right)^2 e^{2i(\alpha+\beta)}+1}{\left(t_1-\overline{C_1}\right)\left(\left(t_1-\overline{C_1}\right)^2 e^{2i(\alpha+\beta)}-1\right)}\left(2B+\left(B'-iC'\right)\right) \\ +\left(B'-iC'\right)\left(\left(t_1-\overline{C_1}\right)e^{2i(\alpha+\beta)}+\dfrac{1}{\left(t_1-\overline{C_1}\right)}\right)- \\ C_1\left[B\left(1-\dfrac{e^{i(\alpha+\beta)}}{\left(t_1-\overline{C_1}\right)^2}\right)+\dfrac{\left(2B+\left(B'-iC'\right)\right)e^{i(\alpha+\beta)}}{\left(t_1-\overline{C_1}\right)^2}\right] \end{array}\right] \tag{5.19}$$

This will be causing additional loading at the first crack boundary. But the true loading condition at the first crack is a stress free boundary condition. In order to achieve the stress free boundary condition at the first crack, a new problem of an infinite plate with first crack is solved with the boundary condition $f_1(t_1)$ given by,

$f_1(t_1)= -f_{21}(t_1)$

The corrected stress functions valid near the first crack can be obtained as,

$$\phi_{11}(\zeta_1)=\frac{b}{2}\left[\begin{array}{l}\frac{1}{\zeta_1}-\frac{\left(1-2B-\left(B'-iC'\right)\right)e^{-i(\alpha+\beta)}}{\overline{C_1}}\\ \left\{\frac{-\left(\overline{C_1}^2-1\right)\left(1-\zeta_1\overline{C_1}\right)\left(3+\overline{C_1}^2\right)+\left(1+\overline{C_1}^2\right)\left(\overline{C_1}^2-3-2\zeta_1\overline{C_1}\right)}{\left(\overline{C_1}^2-1\right)^2\left(1-\zeta_1\overline{C_1}\right)^2}\right\}\\ -\frac{\left(2B+\left(B'-iC'\right)\right)}{\overline{C_1}\left(1-\zeta_1\overline{C_1}\right)}-\left[\begin{array}{l}\frac{1}{\left(1-\zeta_1\overline{C_1}\right)}-\frac{\overline{C_1}\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1+\zeta_1\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)\right)}-\\ \frac{\overline{C_1}\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)}{e^{-i(\alpha+\beta)}\left(1+\zeta_1\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)\right)}\end{array}\right]\\ +\left(\frac{B'-iC'}{\overline{C_1}}\right)\left(\frac{1}{1-\zeta_1\overline{C_1}}\right)-\\ \frac{C_1e^{-2i(\alpha+\beta)}}{\overline{C_1}^2}\left(\frac{2\left(1-\zeta_1\overline{C_1}\right)-1}{\left(1-\zeta_1\overline{C_1}\right)^2}\right)\left[1-\left(2B+\left(B'-iC'\right)\right)\right]\end{array}\right] \tag{5.20}$$

Similarly $\psi_{11}(\zeta_1)$ can be obtained

$$\psi_{11}(\zeta_1)=\frac{b}{2}\left[\begin{array}{l}\frac{\left(B+\left(B'-iC'\right)\right)e^{-2i(\alpha+\beta)}}{\overline{C_1}\left(1-\zeta_1\overline{C_1}\right)}-\frac{2\zeta_2}{1-\zeta_1^2}+\\ \frac{2e^{2i(\alpha+\beta)}\left(1+\left(2B+\left(B'-iC'\right)\right)\right)\left(\zeta_1C_1^2+2C_1+\zeta_1\right)}{\left(C_1+1\right)^2\left(C_1-1\right)^2\left(1-\zeta_1^2\right)}\\ -\frac{\zeta_1\left(\zeta_1^2+1\right)}{\zeta_1^2+1}\phi'_{11}(\zeta_1)\end{array}\right] \tag{5.21}$$

Where $\phi'_{11}(\zeta_1)$ is derivative of $\phi_{11}(\zeta_1)$

The stress function $\phi_1(\zeta_1)$ for the second crack can be obtained by superposing the corresponding transformed stress function $\phi_{21}(\zeta_1)$ and corrected stress function $\phi_{11}(\zeta_1)$

$\phi_1(\zeta_1)=\phi_{21}(\zeta_1)+\phi_{11}(\zeta_1)$

$$\phi_1(\zeta_1)=\frac{b}{2}\left[\begin{array}{l} B\left((\zeta_1-C_1)+\frac{e^{-2i(\alpha+\beta)}}{(\zeta_1-C_1)}\right)-\frac{(2B+(B'+iC'))e^{-i(\alpha+\beta)}}{(\zeta_1-C_1)}+\frac{1}{\zeta_1}-\frac{(1-2B-(B'-iC'))e^{-i(\alpha+\beta)}}{\overline{C_1}} \\ \left\{\frac{-\left(\overline{C_1}^2-1\right)\left(1-\zeta_1\overline{C_1}\right)\left(3+\overline{C_1}^2\right)+\left(1+\overline{C_1}^2\right)\left(\overline{C_1}^2-3-2\zeta_1\overline{C_1}\right)}{\left(\overline{C_1}^2-1\right)^2\left(1-\zeta_1\overline{C_1}\right)^2}\right\} \\ -\frac{(2B+(B'-iC'))}{\overline{C_1}\left(1-\zeta_1\overline{C_1}\right)}-\left[\begin{array}{l}\frac{1}{\left(1-\zeta_1\overline{C_1}\right)}-\frac{\overline{C_1}\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1+\zeta_1\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)\right)}- \\ \frac{\overline{C_1}\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)}{e^{-i(\alpha+\beta)}\left(1+\zeta_1\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)\right)}\end{array}\right] \\ +\left(\frac{B'-iC'}{\overline{C_1}}\right)\left(\frac{1}{1-\zeta_1\overline{C_1}}\right)-\frac{C_1e^{-2i(\alpha+\beta)}}{\overline{C_1}^2}\left(\frac{2\left(1-\zeta_1\overline{C_1}\right)-1}{\left(1-\zeta_1\overline{C_1}\right)^2}\right)[1-(2B+(B'-iC'))] \end{array}\right] \tag{5.22}$$

Similarly,

$$\psi_1(\zeta_1)=\psi_{21}(\zeta_1)+\psi_{11}(\zeta_1)$$

$$\psi_1(\zeta_1)=\frac{b}{2}\left[\begin{array}{l} -\frac{(2B+(B'-iC'))}{(\zeta_1-C_1)}-\frac{(\zeta_1-C_1)^2e^{-2i(\alpha+\beta)}+1}{(\zeta_1-C_1)\left((\zeta_1-C_1)^2e^{-2i(\alpha+\beta)}-1\right)}(2B+(B'+iC')) \\ +(B'+iC')\left((\zeta_1-C_1)e^{-2i(\alpha+\beta)}+\frac{1}{(\zeta_1-C_1)}\right) \\ -\overline{C_1}\left[B\left(1-\frac{e^{i(\alpha+\beta)}}{(\zeta_1-C_1)^2}\right)+\frac{(2B+(B'+iC'))e^{i(\alpha+\beta)}}{(\zeta_1-C_1)^2}\right] \\ \frac{(B+(B'-iC'))e^{-2i(\alpha+\beta)}}{\overline{C_1}\left(1-\zeta_1\overline{C_1}\right)}-\frac{2\zeta_2}{1-\zeta_1^2}+\frac{2e^{2i(\alpha+\beta)}(1+(2B+(B'-iC')))\left(\zeta_1C_1^2+2C_1+\zeta_1\right)}{(C_1+1)^2(C_1-1)^2\left(1-\zeta_1^2\right)} \\ -\frac{\zeta_1\left(\zeta_1^2+1\right)}{\zeta_1^2+1}\phi'_{11}(\zeta_1) \end{array}\right] \tag{5.23}$$

The equations (5.22) and (5.23) are the second approximation stress functions valid near the first crack and are found to satisfy the stress free boundary condition exactly at the first crack.

While evaluating the Cauchy's integrals following conditions are to be satisfied.

$\overline{C_0}$, $\overline{C_1}$ $\rangle 1$ and $\left(\overline{C_0}+e^{i(\alpha+\beta)}\right)$ and $\left(\overline{C_0}-e^{i(\alpha+\beta)}\right)\rangle 1$

And $\left(\overline{C_1} + e^{-i(\alpha+\beta)}\right)\left(\overline{C_1} - e^{-i(\alpha+\beta)}\right) \rangle 1$ (5.24)

These conditions give a minimum center distance in a mapped plane for the validity of solution.

5.4 Stress Intensity Factors

The mode-I and mode-II stress intensity factors (SIF) for the crack tips are obtained by using the resulting stress function in equation given by Erdogan and Sih [6].

$$K_I - iK_{II} = 2\sqrt{\frac{\pi}{a}}\phi_i'(\zeta_i) \pm 1 \quad (5.25)$$

a = crack length,

+ sign for $\zeta = 1$ and η=0 and

- sign for $\zeta = -1$ and η=0 and

$\phi_i'(\zeta_i)$ Stress function given by corresponding approximation solution.

5.4.1 SIFs for the First Crack

$\phi_1(\zeta_1)$ is the stress function obtained by the second approximation, SIF for the first crack is given by,

$$
K_I - iK_{II} = 2\sqrt{\frac{\pi}{a}\frac{b}{2}}\left[\begin{array}{l}
B\left(1-\dfrac{e^{-2i(\alpha+\beta)}}{(\pm 1 - C_1)^2}\right) + \dfrac{\left(2B+(B'+iC')\right)e^{-i(\alpha+\beta)}}{(\pm 1 - C_1)^2} - 1 - \dfrac{\left(1-2B-(B'-iC')\right)e^{-i(\alpha+\beta)}}{\overline{C_1}} \\
\left[\begin{array}{l}
\dfrac{\left[\left(\overline{C_1}^2-1\right)\left(1-\overline{C_1}\right)\left(3+\overline{C_1}^2\right)+\left(1+\overline{C_1}^2\right)\left(-2\overline{C_1}\right)\right]}{\left(\overline{C_1}^4-1\right)^2\left(1\pm\overline{C_1}\right)^2} - \\
\dfrac{2\left[\left(\overline{C_1}^2-1\right)\left(1\pm\overline{C_1}\right)\left(3+\overline{C_1}^2\right)+\left(1+\overline{C_1}^2\right)\left(\overline{C_1}^2-3\pm 2\overline{C_1}\right)\right]}{\left(\overline{C_1}^4-1\right)^2\left(1\pm\overline{C_1}\right)^3}
\end{array}\right] \\
-\dfrac{\left(2B+(B'-iC')\right)}{\overline{C_1}\left(1\pm\overline{C_1}\right)} - \left[\begin{array}{l}
\dfrac{1}{\left(1\pm\overline{C_1}\right)^2} + \dfrac{\overline{C_1}\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1\pm\left(-\overline{C_1}+e^{-i(\alpha+\beta)}\right)\right)^2} + \\
\dfrac{\overline{C_1}\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)}{e^{-i(\alpha+\beta)}\left(1\pm\left(-\overline{C_1}-e^{-i(\alpha+\beta)}\right)\right)^2}
\end{array}\right] \\
+\left(\dfrac{B'-iC'}{\overline{C_1}}\right)\left(\dfrac{1}{\left(1\pm\overline{C_1}\right)^2}\right) - \dfrac{C_1 e^{-2i(\alpha+\beta)}}{\overline{C_1}^2}\left(\dfrac{2}{\left(1\pm\overline{C_1}\right)}+\dfrac{1}{\left(1\pm\overline{C_1}\right)^3}\right)\left[1-\left(2B+(B'-iC')\right)\right]
\end{array}\right]
\tag{5.26}
$$

The upper line signs are for the tip c and the lower line signs are for tip d as shown in Fig. 5.1

5.4.2 SIFs for the Second Crack

$\phi_2(\zeta_2)$ is the stress function obtained by the second approximation , SIF for the second crack is given by,

$$K_I - iK_{II} = 2\sqrt{\frac{\pi}{b}}\,\frac{a}{2}\left[\begin{array}{l}
B\left(1-\dfrac{e^{-2i(\alpha+\beta)}}{(\pm 1+C_0)^2}\right)-\dfrac{\left(2B+(B'+iC')\right)}{(\pm 1+C_0)^2 e^{2i(\alpha+\beta)}}-1+\dfrac{\left(1-2B-(B'-iC')\right)e^{i(\alpha+\beta)}}{\overline{C_0}} \\
\left[\begin{array}{l}
\dfrac{\left[-\left(\overline{C_0}^2-1\right)\left(\overline{C_0}\right)\left(3+\overline{C_0}^2\right)+\left(1+\overline{C_0}^2\right)\left(2\overline{C_0}\right)\right]}{\left(\overline{C_0}^2-1\right)^2\left(1\pm\overline{C_0}\right)^2}- \\
\dfrac{2\left[-\left(\overline{C_0}^2-1\right)\left(1+\zeta_2\overline{C_0}\right)\left(3+\overline{C_0}^2\right)+\left(1+\overline{C_0}^2\right)\left(\overline{C_0}^2-3\pm 2\overline{C_0}\right)\right]}{\left(\overline{C_0}^2-1\right)^2\left(1\pm\overline{C_0}\right)^3}
\end{array}\right] \\
-\dfrac{\left(2B+(B'-iC')\right)}{\overline{C_0}\left(1\pm\overline{C_0}\right)^2}-\left[\begin{array}{l}
\dfrac{1}{\left(1\pm\overline{C_0}\right)}+\dfrac{\overline{C_0}\left(\overline{C_0}+e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1\pm\left(\overline{C_0}+e^{i(\alpha+\beta)}\right)\right)^2}- \\
\dfrac{\overline{C_0}\left(\overline{C_0}-e^{i(\alpha+\beta)}\right)}{e^{i(\alpha+\beta)}\left(1\pm\left(\overline{C_0}-e^{i(\alpha+\beta)}\right)\right)^2}
\end{array}\right] \\
-\left(\dfrac{B'-iC'}{\overline{C_0}}\right)\left(\dfrac{1}{\left(1\pm\overline{C_0}\right)^2}\right)+ \\
\dfrac{C_0 e^{-2i(\alpha+\beta)}}{\overline{C_0}^2}\left(\dfrac{-2}{\left(1\pm\overline{C_0}\right)^2}+\dfrac{1}{\left(1\pm\overline{C_0}\right)^3}\right)\left[1-\left(2B+(B'-iC')\right)\right]
\end{array}\right] \tag{5.27}$$

The upper line signs are for the tip c and the lower line signs are for tip d as shown in Fig. 5.1

5.5 Results and Discussions

With these generalized functions different problems can be solved by setting the values of crack lengths a, b, angles α, β, loading factor λ, load P and the center distance between the cracks Z_0. The results are obtained for stress intensity factors by using equations (5.35) and (5.36) for some of the cases.

The method of solution has been tested for following cases.

1. Effect of center distance between two cracks,
2. Effect of inclination of cracks,
3. Effect of biaxial load factor λ

For calculating the results an infinite isotropic plate is considered with Young's modulus 2*105 N/mm2 and Poisson's ratio 0.3 the crack length a = b = 5mm, P = 1 N/mm2.

5.6 Single Crack Solution

5.6.1 Effect of crack length on stress intensity factor

The effect of crack length a on K_I is studied for the single crack solution, The values of load factor $\lambda = 0$ and $\alpha = 0$ are taken to determine the results. The results are presented in Table 5.1 and compared with Bowie analysis in Fig. 5.3. It is seen that the results are exact and the results can be compared with the results of the interaction of two cracks in an infinite isotropic plate.

Table 5.1 SIF for uniaxial tension along Y-axis for infinite plate with single crack

Crack Length (a)	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
K1/K0	2.1284	1.7979	1.6515	1.5642	1.5046	1.4607	1.4265	1.3989	1.3761	1.3568
Stress Intensity Factor (k1)	1.886	2.253	2.535	2.773	2.982	3.171	3.345	3.507	3.659	3.803
Crack Length (a)	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75	5
K1/K0	1.3402	1.3257	1.313	1.3016	1.2913	1.2821	1.2737	1.266	1.2589	1.2523
Stress Intensity Factor (k1)	3.939	4.07	4.195	4.316	4.432	4.545	4.654	4.76	4.863	4.963

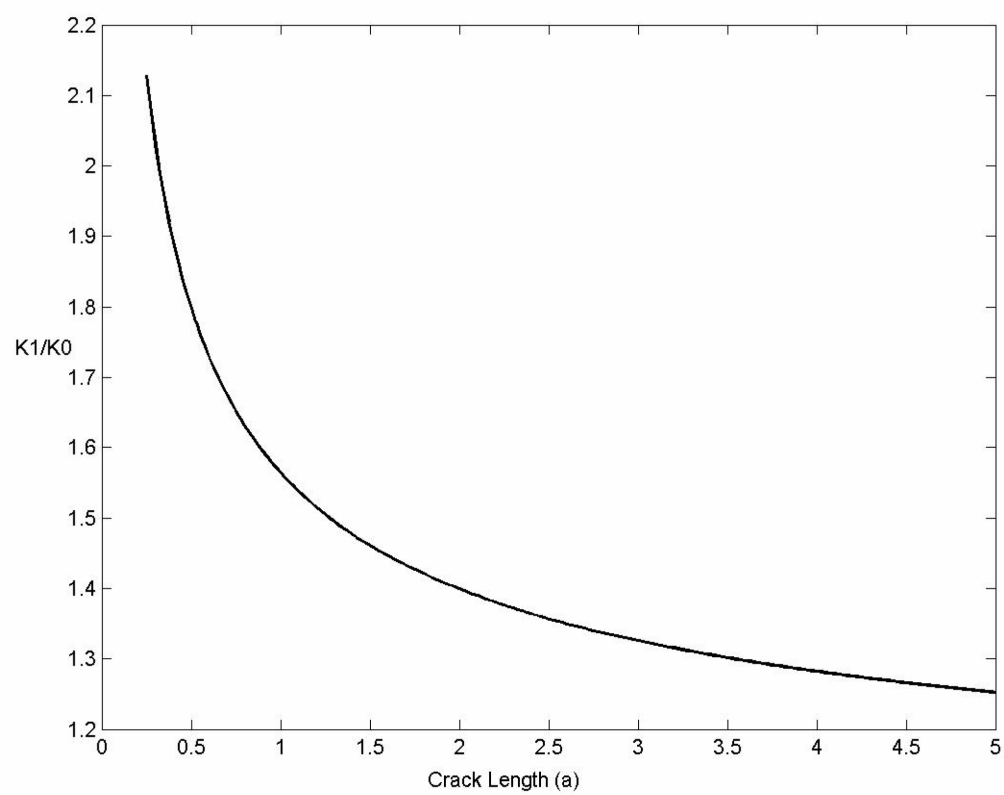

Fig: 5.2 Effect of crack length on KI for single crack solution.

5.6.2 Effect of Biaxial Load Factor

The analysis is studied for Rho and λ . Crack length a=5 and α =0 are taken, results are given in Table 5.2, Fig.5.3 shows the results. From Table 5.2 and Fig.5.3 it is observed that increasing the biaxial load factor λ increases the K_I

Table 5.2 SIF for different loading factor at different values of rho for infinite plate with single crack

Rho	load factor λ=0	K1	λ=0.5	K1	λ=1	K1	λ=2	K1
1.1		3.0835		3.1604		4.7021		3.3911
2.1		1.4891		1.8317		2.5763		2.8596
3.1		1.1629		1.5599		2.1413		2.7509
4.1		1.0444		1.4611		1.9833		2.7114
5.1		0.9884		1.4145		1.9087		2.6928
6.1		0.9577		1.3889		1.8677		2.6825
7.1		0.939		1.3733		1.8428		2.6763

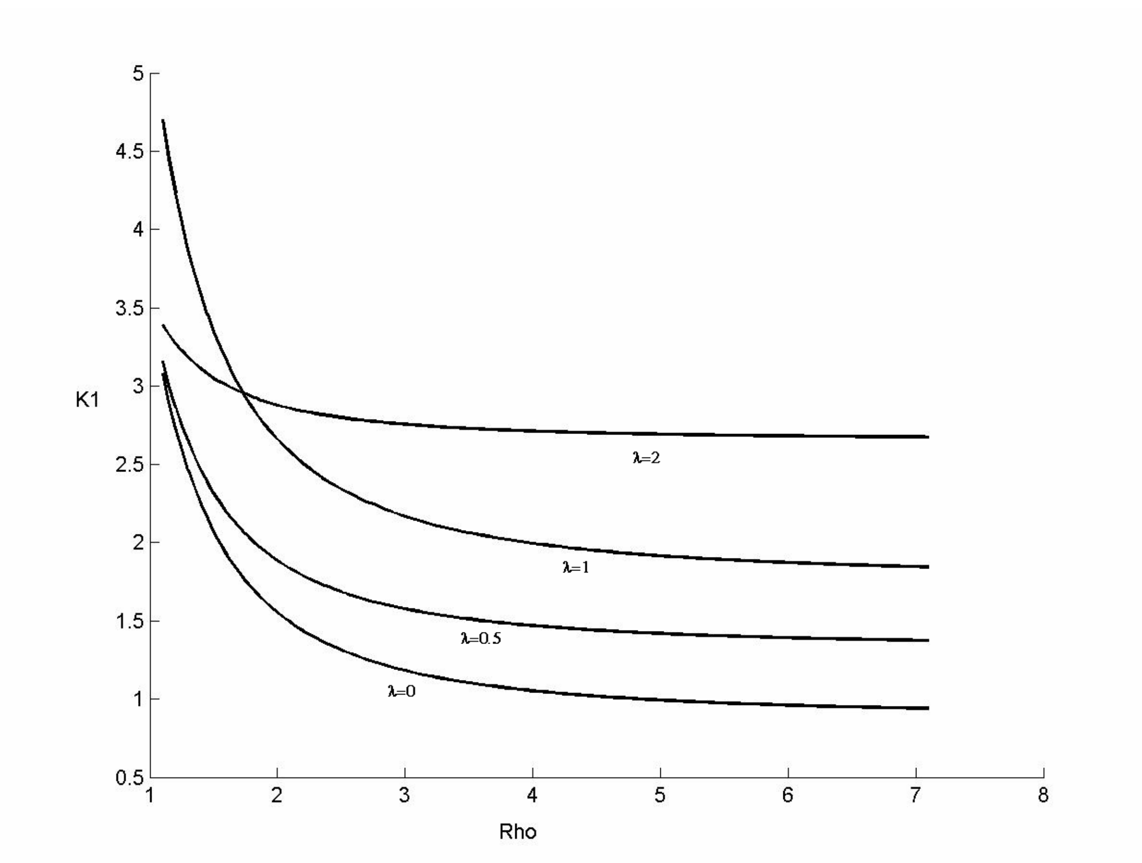

Fig: 5.3 Effect of biaxial load factor on K_I

5.6.3 Effect of Orientation of α

The analysis is studied for Rho and α crack length a=5. results are given in Table 5.3, Fig.5.4 shows the results.

Table 5.3 SIF for different orientation of α and different values of rho for infinite plate with single crack

Rho	α =0 K1	α =30 K1	α =60 K1	α =90 K1
1.1	3.0835	2.3511	0.8862	0.1538
2.1	1.4891	1.2881	0.8862	0.6853
3.1	1.1629	1.0707	0.8862	0.794
4.1	1.0444	0.9917	0.8862	0.8335
5.1	0.9884	0.9544	0.8862	0.8522
6.1	0.9577	0.9339	0.8862	0.8624
7.1	0.939	0.9214	0.8862	0.8686

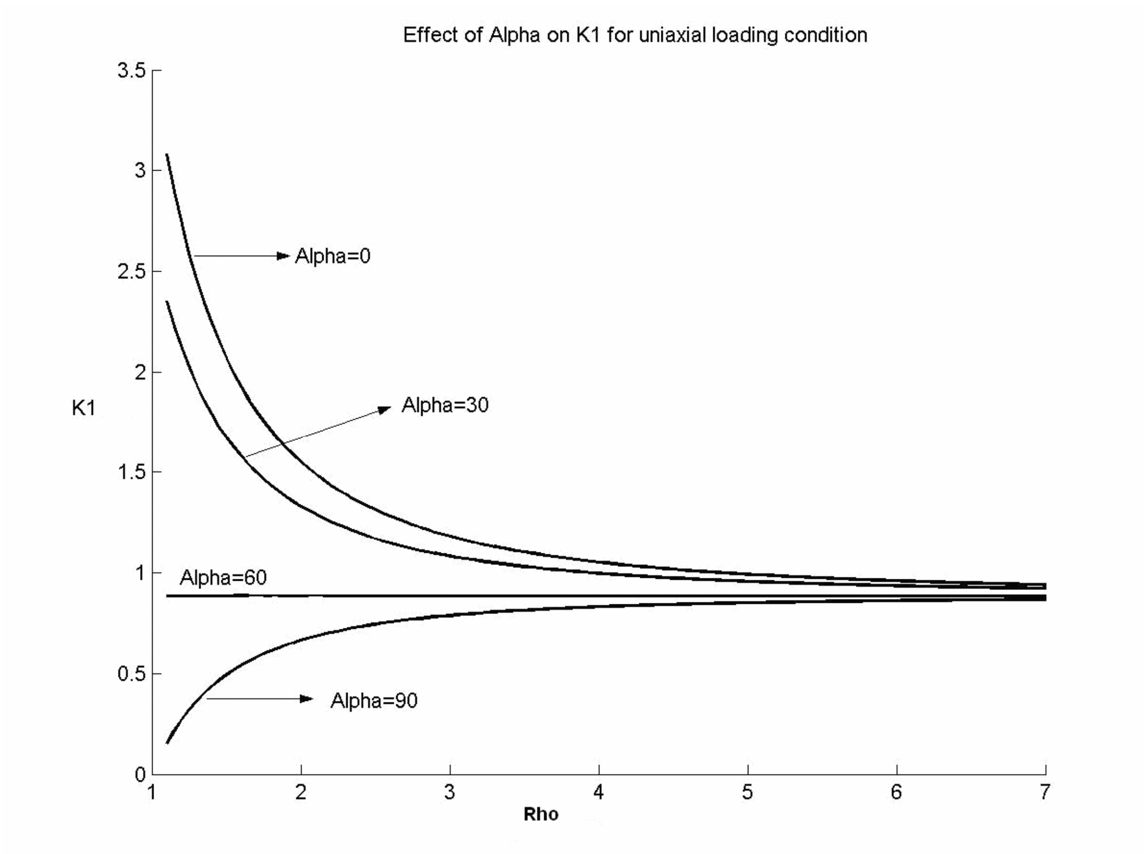

Fig: 5.4 Effect of orientation of α on K_I

5.7 Interaction effect of two cracks

5.7.1 Interaction of Two Equal Collinear Cracks

The analysis is studied for, a = b and α = ß. for the loading along Y-axis as shown. To analyze the effect of center distance between two cracks the results are obtained for 2*(a)/d ratio varying from 0.91 to 0.1. It is observed that the results are matching exactly with the results obtained by Ukadgaonkar

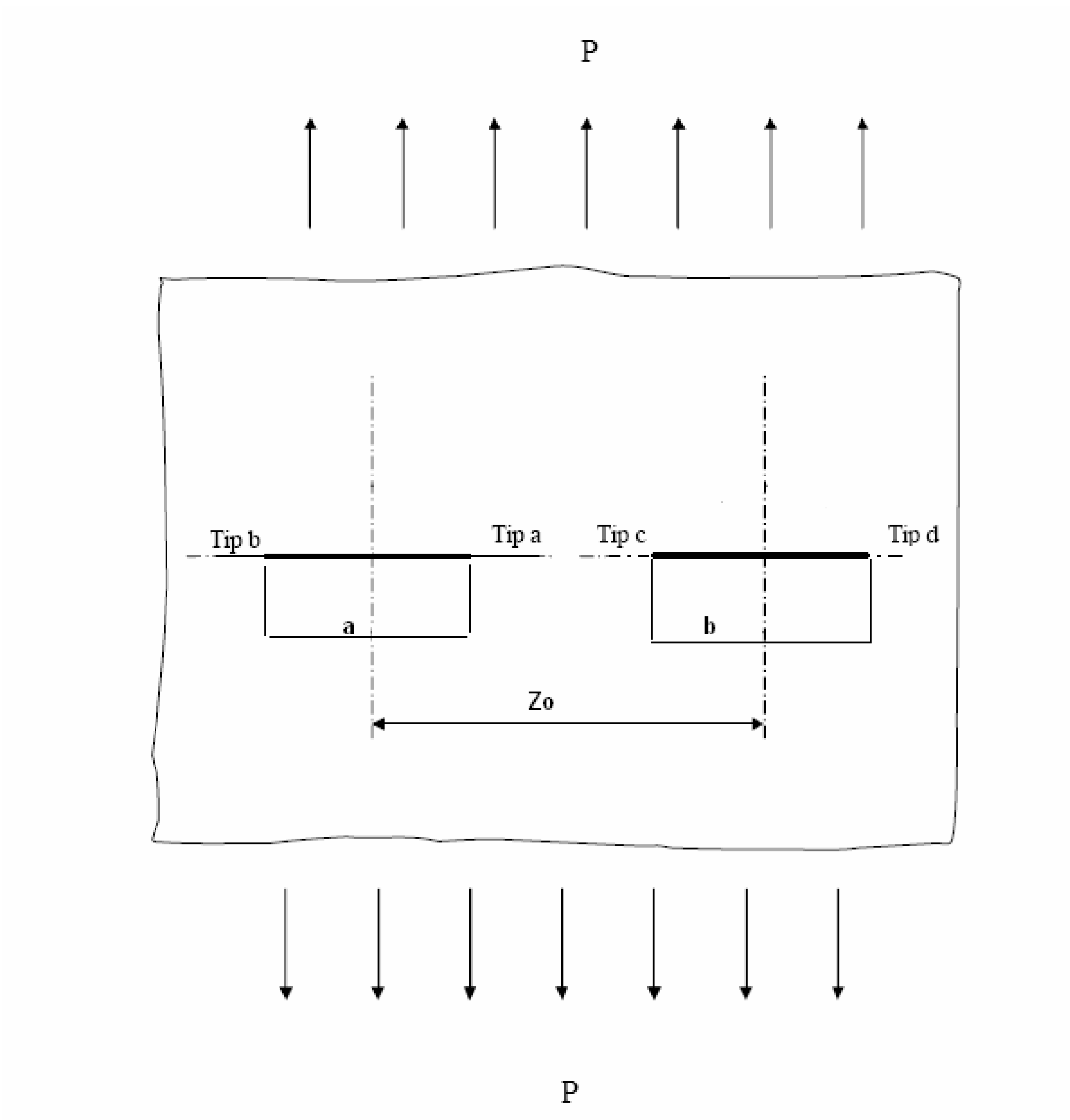

Fig: 5.5 Interaction of two cracks with equal collinear cracks subjected to uniaxial tension along Y-axis

From Table 5.2 and Fig. 5.6 it is observed that as the center distance between two cracks is increased the SIF decreases. When the distance between the crack tips is equal to the crack length i.e. 2*(a)/d = 0.5, the interaction effect is less than 5%. When two cracks are far apart from each other the solution of second approximation reduces to the single crack. For the ratio 2*(a)/d less than 0.5, the SIF for inner and outer crack tips is equal and the interaction effect is negligible. For all the above loading cases, K_{II} is obtained as zero for all the crack tips. K_I for tip a is same as that for tip c and K_I for tip b is same as that of tip d.

2a/zo	Present Method (K1/K0)	Ukadgaonkar (K1/K0)
0.9	1.2457	1.257
0.8	1.1742	1.154
0.7	1.131	1.097
0.6	1.0683	1.06
0.5	1.049	1.036
0.4	1.029	1.02
0.3	1.0192	1.01
0.2	1.0061	1.0043
0.1	1.0013	1.001

Table 5.4 Effect of center distance for two equal collinear cracks subjected to uniform tensile load along Y-axis.

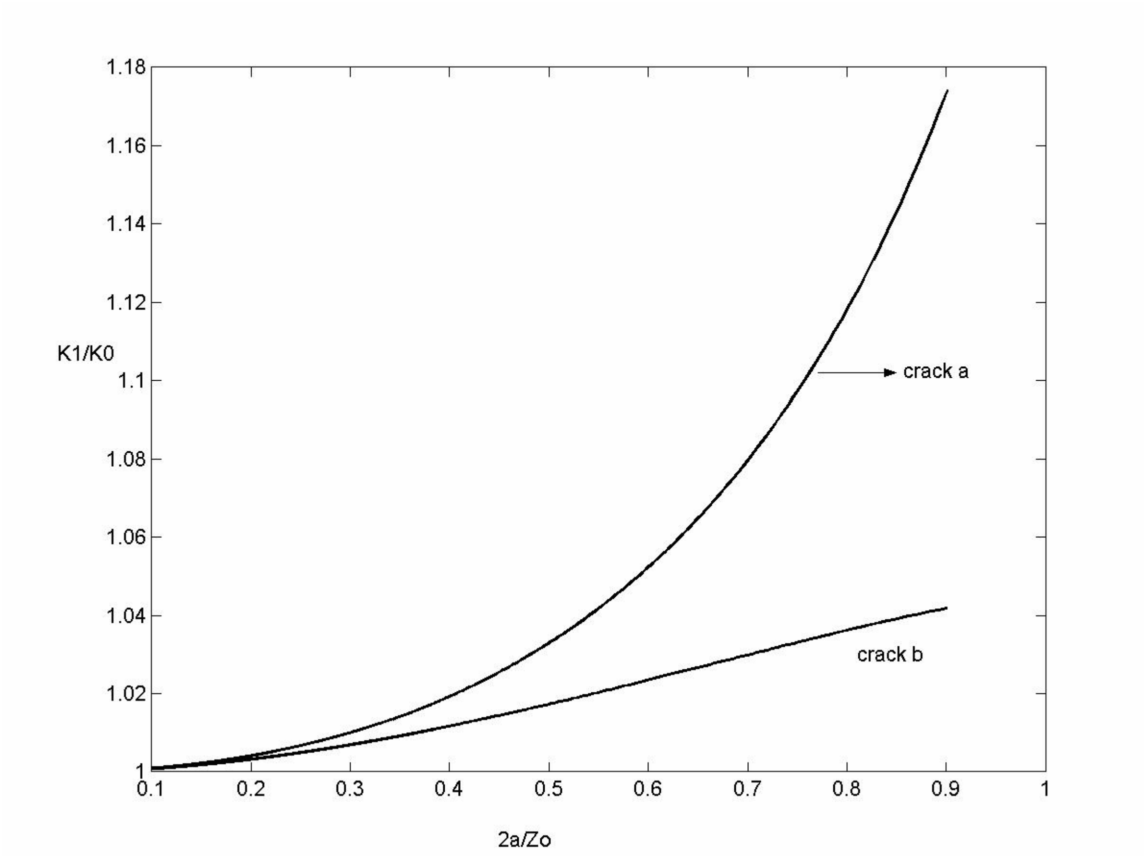

Fig 5.6 Effect of center distance for two equal collinear cracks subjected to uniform tensile load along Y-axis.

5.7.2 Interaction of Two Equally Inclined Cracks Subjected to Uniaxial Tension along Y-Direction

The analysis is studied for a = b, 2*(a)/d = 0.1 and α = ß. Fig. 4.7 shows the loading diagram. Table 4.3 and Fig.4.8 Shows the results as seen from Table 5.3 the results obtained by the present method are in good agreement with those obtained by Isida [89].

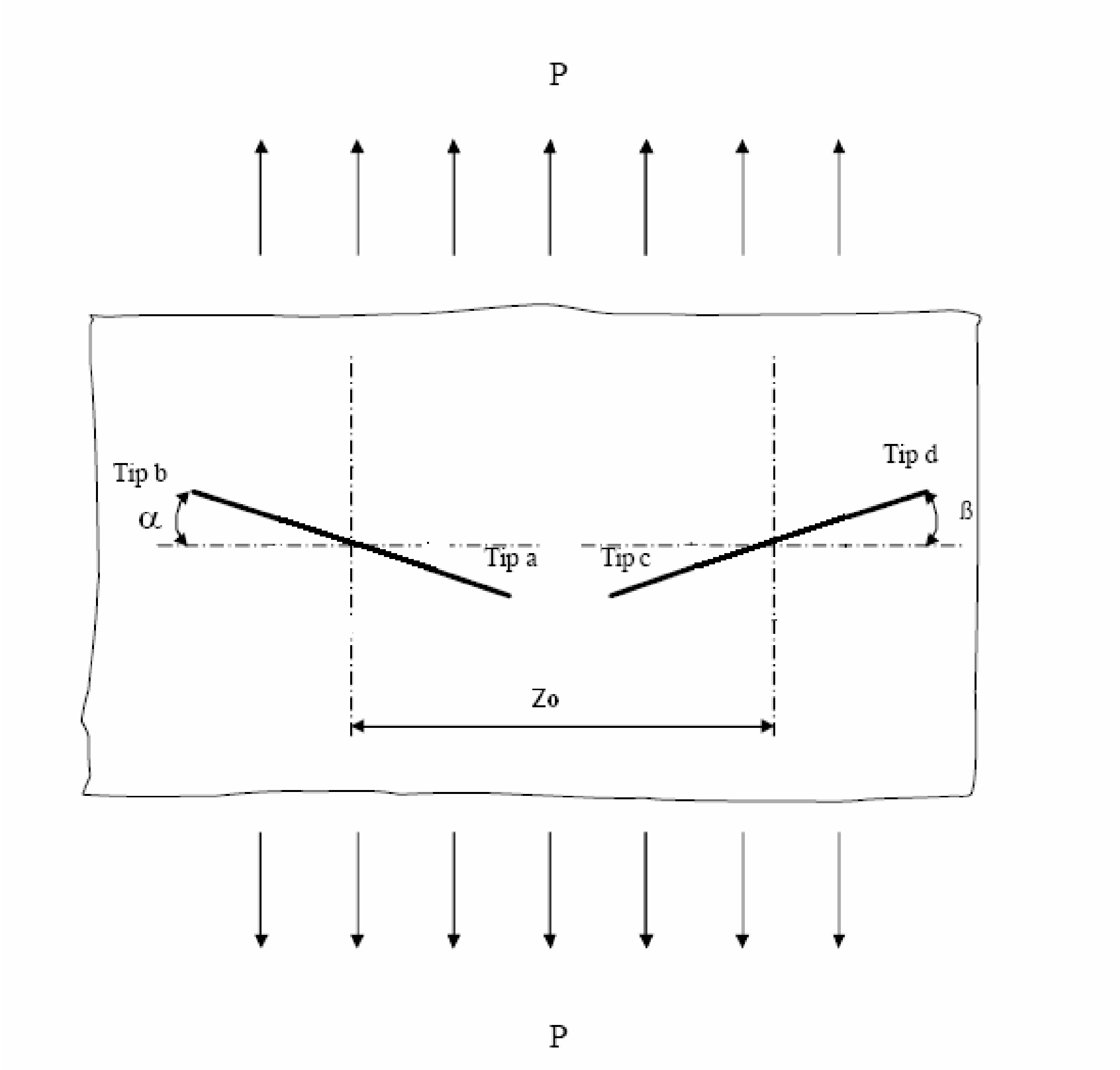

Fig: 5.7 Interaction of two equally inclined cracks subjected to uniaxial tension along Y-axis

It is seen from Fig.4.8 that, for the same center distance as angle ß increases KI decreases and at ß equal to 900 KI is almost zero. For the same angle ß, if the centre distance decreases as the two cracks are placed close to each other, the interaction effect increases and as the centre distance increases the interaction effect decreases and the solution tends to the single crack with two cracks solution.

Crack Length	Alpha(degree)	K1/K0
a=3mm	0	1.3257
	30	1.0757
	60	0.5757
	90	0.3257
a=6mm	0	1.2303
	30	0.9803
	60	0.4803
	90	0.2303
a=9mm	0	1.1881
	30	0.9381
	60	0.4381
	90	0.1881

Table 5.5 Effect of Crack Length and orientation of alpha on two equal cracks subjected to uniform tensile load along Y-axis.

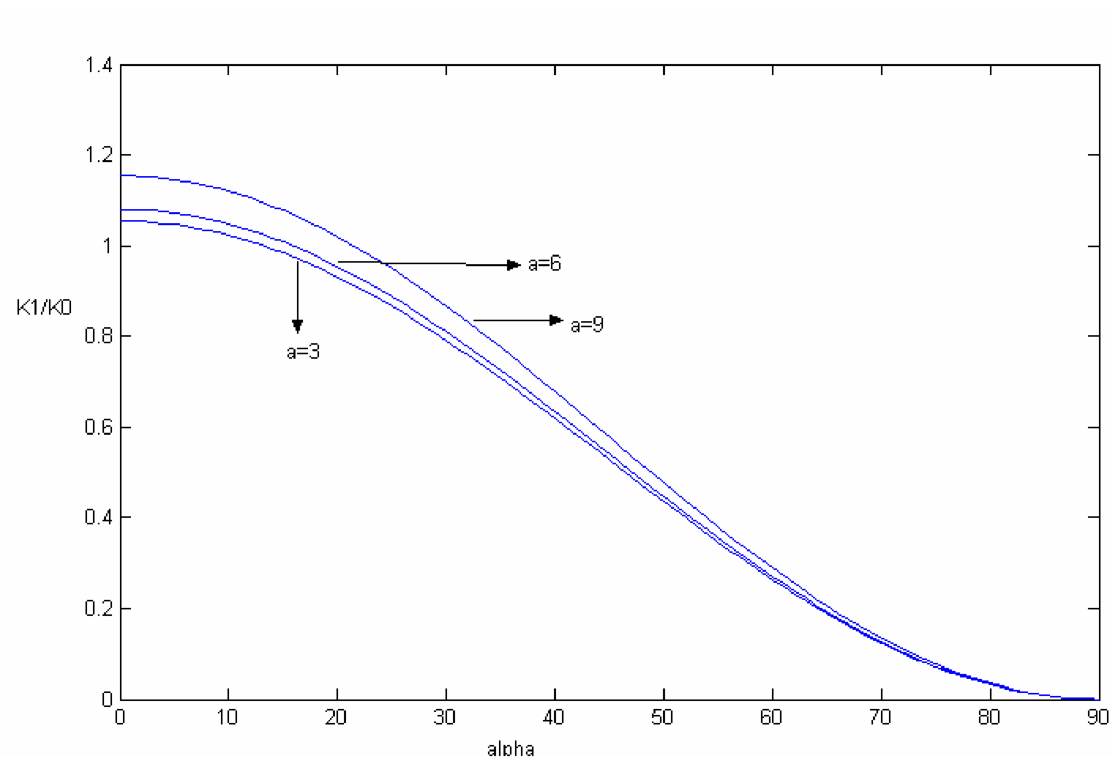

Fig: 5.8 Effect of crack length and orientation of alpha on two equal cracks subjected to uniform tensile load along Y-axis.

5.7.3 Effect of Center Distance

The analysis is studied for a = b and α = ß. Fig. 4.10 shows the loading and results are given in Table 5.4. From Table5.4 it is observed that as the center distance between the two cracks increases the SIF at the inner tips reduces. It indicates that as the two cracks move away from each other the effect of interaction on SIF reduces and the SIF value approaches to the single crack solution.

Table 5.6 Effect of center distance and orientation of alpha on two equal cracks subjected to uniform tensile load along Y-axis.

Alpha	Center Distance=10.25		Center Distance=11.50		Center Distance=25	
	K1	**K1/K0**	**K1**	**K1/K0**	**K1**	**K1/K0**
0	5.1515	1.2998	4.9146	1.24	4.5142	1.139
30	3.9886	1.0064	3.7453	0.945	3.2863	0.8292
60	1.7407	0.4392	1.4322	0.3614	0.8308	0.2096
90	0.6361	0.1605	0.2834	0.0715	-0.397	-0.1002
120	1.7407	0.4392	1.4322	0.3614	0.8308	0.2096
150	3.9886	1.0064	3.7453	0.945	3.2863	0.8292
180	5.1515	1.2998	4.9146	1.24	4.5142	1.139
210	3.9886	1.0064	3.7453	0.945	3.2863	0.8292
240	1.7407	0.4392	1.4322	0.3614	0.8308	0.2096
270	0.6361	0.1605	0.2834	0.0715	-0.397	-0.1002
300	1.7407	0.4392	1.4322	0.3614	0.8308	0.2096
330	3.9886	1.0064	3.7453	0.945	3.2863	0.8292
360	5.1515	1.2998	4.9146	1.24	4.5142	1.139

Closing remarks:

The stress intensity factors at the crack tips are evaluated and their variations with respect to the geometrical parameters are presented for the specific cases in this chapter.. The interaction effects of following parameters have been studied.

1. Effect of center distance between two cracks,
2. Effect of crack length variations of two cracks,
3. Effect of inclination between cracks of two cracks.

Chapter 6
FEM Solutions using ANSYS

6.1 Introduction

The results for uniaxial loading condition for single crack are obtained. The ANSYS 10 software is used to obtain the results by FEM. The modeling, meshing and postprocesssing are done by the ANSYS the results obtained will be compared with the results of analytical solutions.

6.2ANSYS Package

The ANSYS is a comprehensive general-purpose finite element computer program. The ANSYS program has many capabilities ranging from a simple, linear, static analysis to a complex, nonlinear, transient dynamic analysis. A typical analysis in ANSYS involves three distinct steps. [10]

1. **Preprocessing**: Using PREP7 processor, providing data such as the geometry, materials, and element type to the program.
2. **Solution**: Using Solution processor, defining the type of analysis, set boundary conditions, applies loads, and initiate finite element solutions.
3. **Postprocessing**: Using POST1 (for static or steady state problems) or POST26 (for transient problems), reviewing the results of analysis through graphical displays and tabular listings.

6.2.1 Solving Fracture Mechanics Problems Using ANSYS

Fracture mechanics deals with the study of how a crack or flaw in a structure propagates under applied loads. It involves correlating analytical predictions of crack propagation and failure with experimental results. The analytical predictions are made by calculating fracture parameters such as stress intensity factors in the crack region, which you can use to estimate crack growth rate. Typically, the crack length increases with each application of some cyclic load, such as cabin pressurization-depressurization in an airplane. Further, environmental conditions such as temperature or extensive exposure to irradiation can affect the fracture propensity of a given material. Solving fracture mechanics problems using ANSYS involves performing a linear static analysis and then using specialized postprocessing commands to calculate desired fracture parameters. For all the cases an isotropic thin plate with plane stress condition is considered. The Young's modulus for the plate material is 2*105 N/mm2 and the plate thickness is 1mm The Poisson's ratio is 0.3 taken. The external dimensions of the plate are taken large to satisfy infinite plate such that the ratio of size of a plate to the size of the crack is 10.The model is generated for all the cases using keypoints [10]

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6.2.2 Modeling the Crack Region

The most important region in a fracture model is the region around the edge of the crack referred as crack tip in case of 2-D model and crack front in a 3-D model.

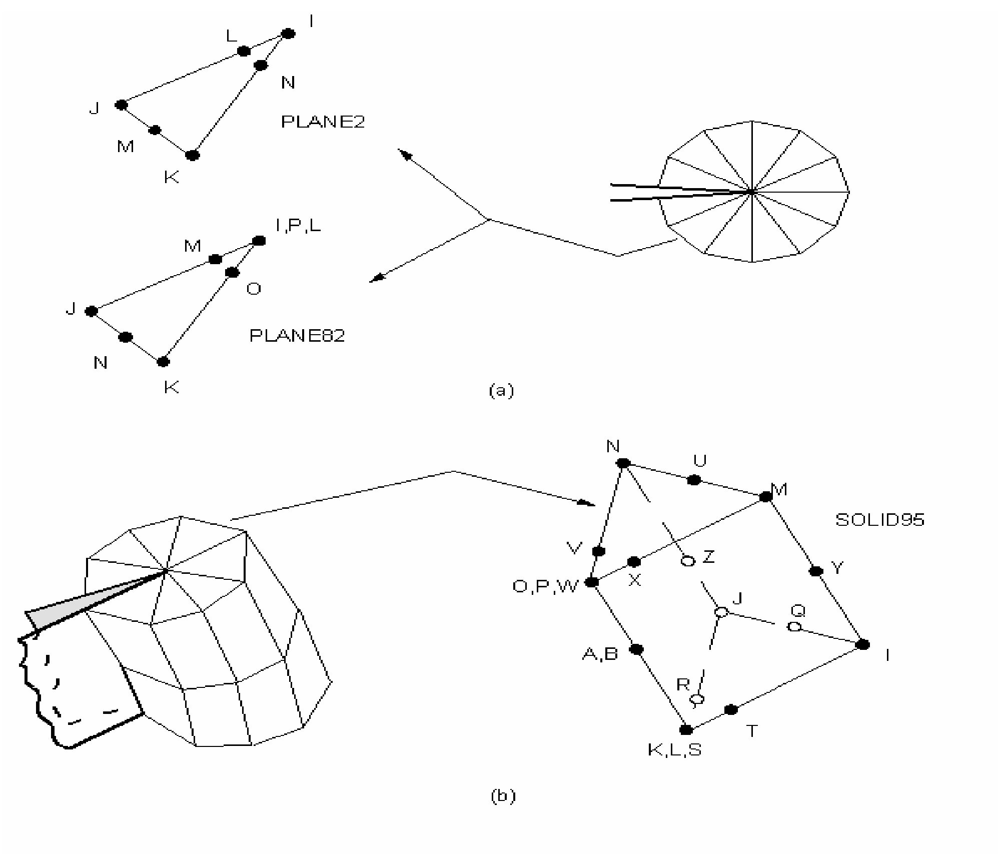

Fig 6.1: Singular element (a) Plane 2 and plane 82for 2Dd model (b)Solid 95 for 3D model

The stresses and the strains are singular at the crack tip, to pick up the singularity in the stresses, the crack faces should be coincident, and the elements around the crack tip (crack front) should be with singular elements. [12]

The recommended element type for two-dimensional fracture model is PLANE2, the six-node triangular solid. The first row of elements around the crack tip should be singular. The PREP7 KSCON command, which assigns element division sizes and generates singular elements around the crack tip. The recommended element for the three-dimensional model is SOLID95, the 20-node element; the first row of the elements around the crack front should be singular elements.

6.2.3 Calculating Stress Intensity Factors

Once the static analysis is complete, using POST1, the general post processor, stress intensity factors (SIFs) can be calculated. The POST1 KCALC command calculates the Stress intensity factors KI, KII and KIII. To use KCALC command properly

Following steps are to be followed.

1. Define a local crack tip or crack front coordinate system, with X parallel to the crack face (perpendicular to the crack front in 3-D models) and Y perpendicular to the crack face.
2. Define a path along the crack face. The first node on the path should be the crack-tip node.
3. Calculate Stress Intensity Factors KI, KII and KIII using KCALC command.

6.3 Single Crack Solution

The infinite plate with single crack

6.3.1 Infinite Plate with single Crack Subjected to Uniaxial Uniform Tensile Load at Infinity along Y-axis i.e. $\alpha = 0$, $\lambda = 0$.

Uniform pressure of 1 N per unit length is applied along Y-axis. The SIFs obtained by ANSYS and the present solution are given in Table 6.1. The corresponding deformation and stress contour plots are shown.

Table 6.1 Infinite plate with crack subjected to uniaxial uniform tensile load.

KI	
ANSYS	Present solution
4.5606	3.803

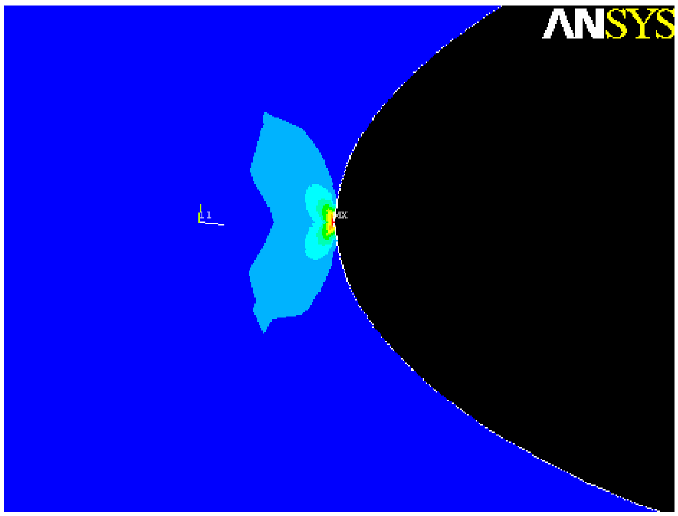

Fig. 6.2: 1st principal stress result for loading along Y-axis

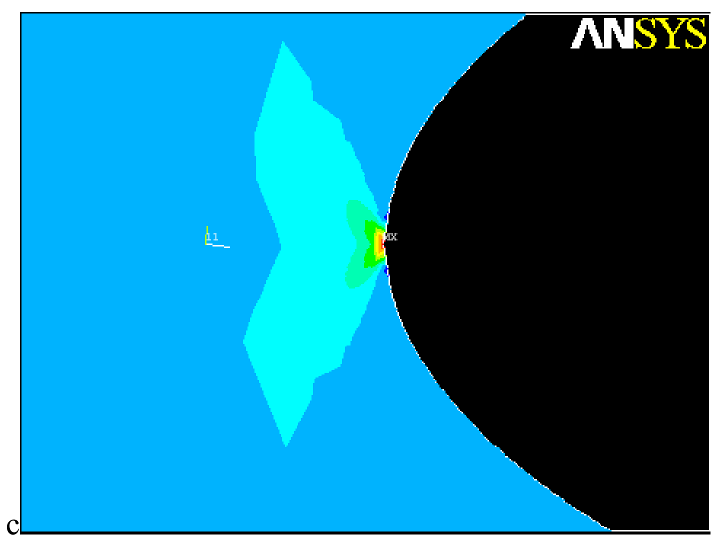

Fig. 6.3 Stress distribution result for biaxial loading

6.4 Interaction of Two Cracks

The infinite plate with two cracks following crack geometries are considered

1. Plate with equal collinear cracks.

The crack length is a = b = 5 mm. The center distance between two cracks is Zo = 11.11 mm. i.e.2a/ Zo = 0.91. The plate size is 100 mm.x100 mm.

2. Plate with unequal collinear cracks.

The crack length for the first crack is a =3mm. and the crack length for the second crack is b = 6 mm. i.e. b/a = 2. The center distance between two cracks is Zo = 11.11 mm. i.e. 2a/ Zo = 0.91. The plate size is 100 mm x100 mm.

3. Plate with inclined cracks.

The crack length is a = b = 5 mm. The center distance between two cracks is Zo = 11.11 mm. i.e. 2a/Zo = 0.91. The angle α = 30 and ß = 0 full model of a plate is considered. The plate size is 100 mm x 100 mm

6.4.1 Infinite Plate with Equal Collinear Cracks Subjected to Uniaxial Uniform Tensile Load at Infinity along Y-axis i.e. α = ß = 0, λ = 0.

Uniform pressure of 1 N per unit length is applied along Y-axis. The SIFs obtained by ANSYS and the present solution are given in Table 6.2. The values are close to each other. The corresponding deformation and stress contour plots are shown.

Table 6.2 Infinite plate with two equal collinear cracks subjected to uniaxial uniform tensile load.

KI			
ANSYS		Present solution	
a tip 4.018	b tip 5.32	a tip 3.963	b tip 4.9724

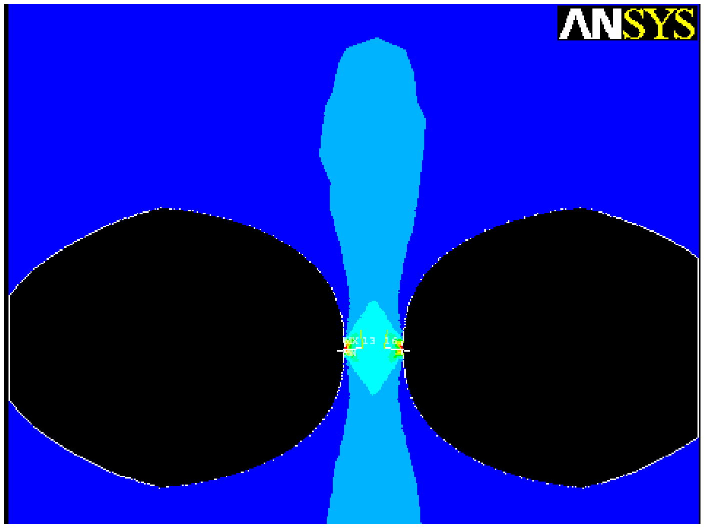

Fig. 6.4: Stress distribution due to Y-axial loading

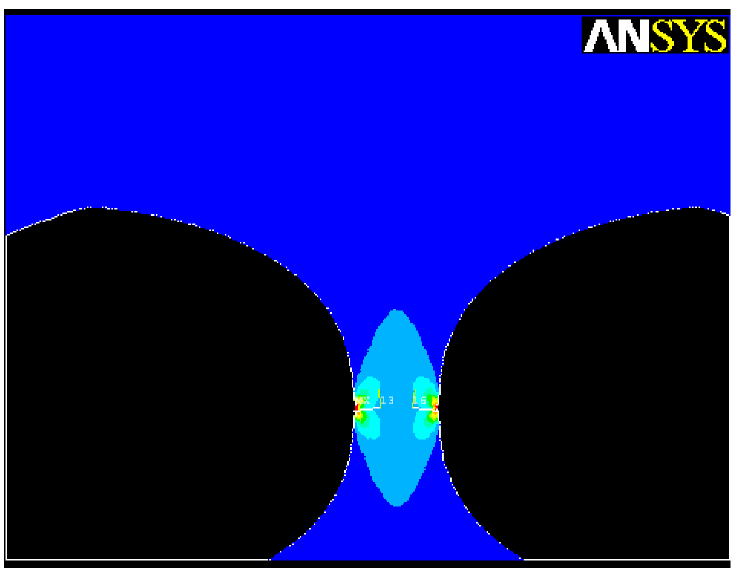

Fig: 6.5 1st principal stress result for loading along Y-axis

6.4.2 Infinite Plate with Equal Collinear Cracks Subjected to Biaxial Uniform Tensile Load at Infinity along X-axis and Y-axis i.e. $\alpha = ß = 0, \lambda = 1$.

Uniform pressure of 1 N per unit length is applied along Y-axis. The SIFs obtained by ANSYS and the present solution are given in Table 6.3.. The corresponding deformation and stress contour plots are shown.

Table 6.3 Infinite plate with two equal collinear cracks subjected to biaxial uniform tensile load.

KI			
ANSYS		Present solution	
a tip	b tip	a tip	b tip
3.9159	5.287	4.6517	3.316

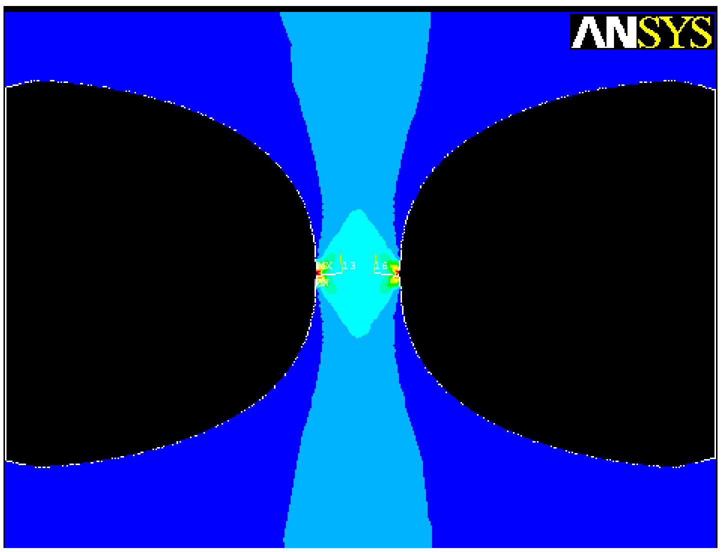

Fig: 6.6 Stress Distribution due to biaxial loading

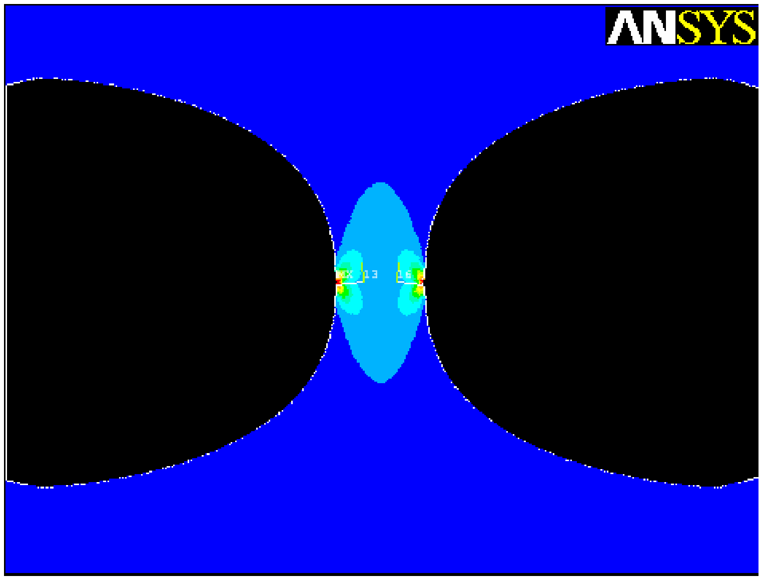

Fig: 6.7 1st principal stress result for biaxial loading

6.4.3 Infinite Plate with Unequal Collinear Cracks Subjected to Biaxial Uniform Tensile Load at Infinity along Y-axis i.e. a=3mm and b=6mm α = ß = 0, λ =1.

Uniform pressure of 1 N per unit length is applied along Y-axis. The SIFs obtained by ANSYS and the present solution are given in Table 6.4. The values are close to each other. The corresponding deformation and stress contour plots are shown.

Table 6.4 Infinite plate with two unequal collinear cracks subjected to biaxial uniform tensile load.

KI			
ANSYS		Present solution	
c tip 6.8617	d tip 4.91	c tip 8.308	d tip 6.4256

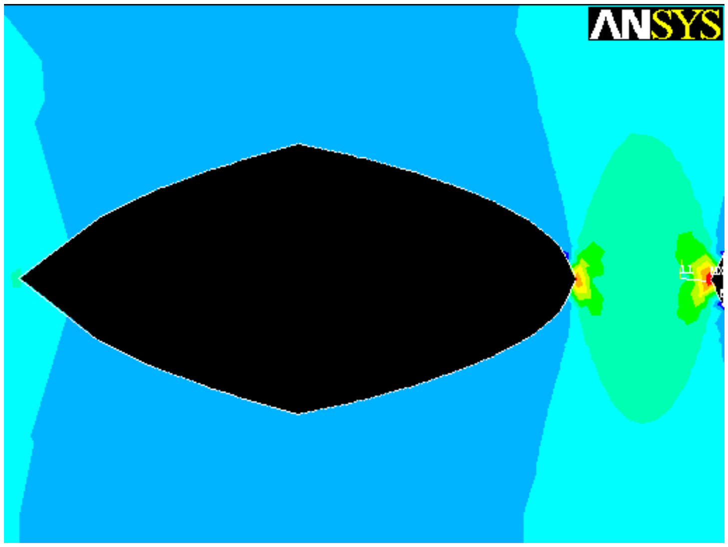

Fig: 6.8 Stress distribution due to Y-axial loading

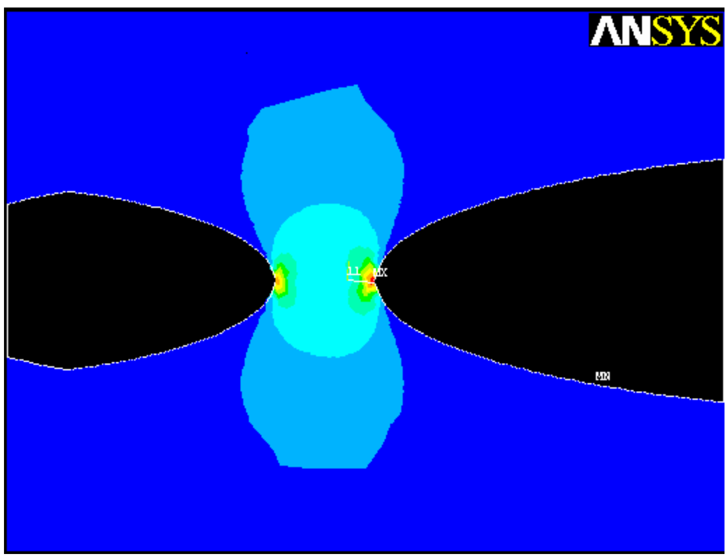

Fig: 6.9 1st principal stress result for biaxial loading

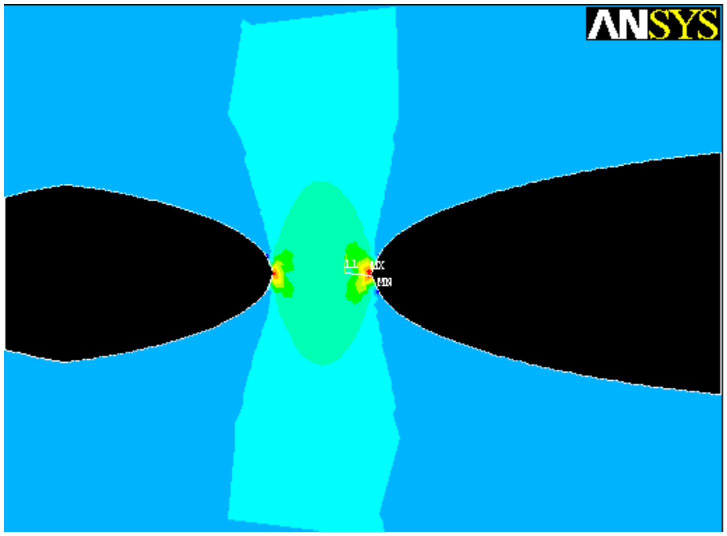

Fig: 6.10 Stress distribution result due to biaxial loading

6.4.4 Infinite Plate with Equal Inclined Cracks Subjected to Uniaxial Uniform Tensile Load at Infinity along Y-axis i.e. α =30 ß = 0, λ = 0.

Uniform pressure of 1 N per unit length is applied along Y-axis. The SIFs obtained by ANSYS and the present solution are given in Table 6.5. The corresponding deformation and stress contour plots are shown.

Table 6.5 Infinite plate with two equal collinear cracks subjected to uniaxial uniform tensile load.

KI			
ANSYS		Present solution	
b tip	c tip	b tip	c tip
3.6159	4.32	3.186	4.471

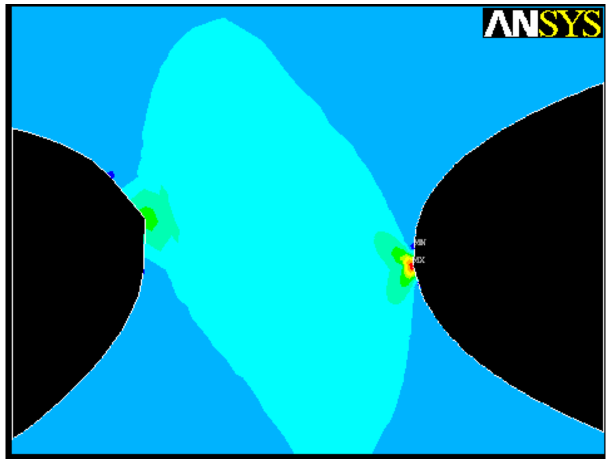

Fig: 6.11 Stress distribution result due to Y-axial loading

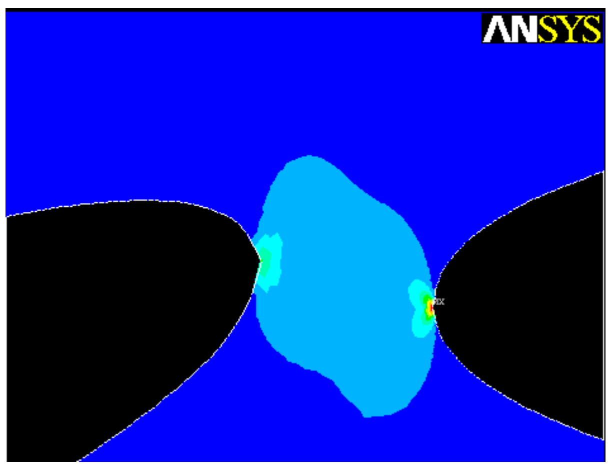

Fig: 6.12 1st principal stress result due to uniaxial loading

6.5 Remarks

The results obtained by FEM and the present solution are compared and they are found to be close to each other for most of the cases. Both analytical and EFM analysis shows that the maximum stress intensity factor is at the inner crack tips. The FEM is highly versatile and handles problems of highly complicated geometry and loading conditions. In fact, the tedium involved in generating the model, discretizing it, applying the loads and boundary conditions and getting solution for the stresses and the stress intensity factors is very high as compared to the analytical solution. The present analytical solutions cannot compete with the solution by FEM; however, they yield the results for the interaction of two cracks for different loading conditions in a simple way.

Chapter 7

Conclusion and Scope for future work

7.1 Introduction

In this chapter results and general discussions have been presented. Finally the main conclusions and contributions of the present work are given. Also the scope for future work has been suggested.

7.2 Results and Discussions

The present work has been carried out with the objectives of analyzing the interaction effect of two cracks in an infinite isotropic plate subjected to different boundary conditions, using Schwarz alternating method. The complex stress functions are derived which satisfies the boundary condition. The stress intensity factors at the crack tips are evaluated and their variations with respect to the geometrical parameters are presented for the specific cases.

The interaction effects of following parameters have been studied.

1. Effect of center distance between two cracks,
2. Effect of crack length variations of two cracks,
3. Effect of inclination between cracks of two cracks.

The results for all the cases have been given and discussed in details at the respective chapters. Here only main results and discussions are given.

When two cracks are present in the infinite plate subjected to a load, the stress intensity factors, both at the inner tips and the outer tips are greater than the stress intensity factors for a single crack in the infinite plate. If the two cracks are very close to each other, coalescence between the cracks occurs as a result of the interaction. In fact as the crack tips approach each other the stress intensity factors at the inner tips increases rapidly. When the cracks are far apart the interaction effect can be neglected and the stress intensity factors for the crack tips will therefore behave in a way, which is similar to that of a single crack in the infinite plate.

For studying the effect of center distance between two cracks, the crack lengths have been assumed to be equal. It is found that as the two cracks come close to each other their interaction on the magnitude of stress intensity factors increases rapidly. The stress intensity factors for the inner crack tips are more than the stress intensity factors for the outer crack tips. As the center distance between two cracks becomes large. The stress intensity factors at the inner and the outer crack tips approach asymptotically to the value of stress intensity factors for the crack tips of the single crack solution.

For studying the effect of crack lengths the length of first crack is assumed as constant and the crack length of the second crack is varied. It is observed that as the length of second crack increases the stress intensity factors of the second crack tips increases rapidly, and due to the interaction effect the stress intensity factors for the first crack tips is also increases but the rate of increase is less. It is also seen that the stress intensity factors for the inner crack tips is more than the stress intensity factors for the outer crack tips. For studying the effect of inclination of the cracks first crack is assured to be parallel to the X-axis and the inclination of the second crack has been varied. It is found that the interaction of the inclination is more if the two cracks are close to each other.

For the case of equal biaxial loading, it is seen that the effect of inclination of cracks is negligible.

For supporting the analytical solution, particular problems have been solved using FEM and the results are in good agreement with that of the analytical solution. The amount of time and labor involved in the present method of analysis is less in comparison with other methods.

This may well encourage the use of proposed method of solution. The present method gives more insight in to the analysis. The wide variety of problems has been solved and the results are found to be very encouraging.

The results presented here are intended both to demonstrate the potential of Schwarz alternating method and provide the SIFs data useful for the design of structures.

7.3 Conclusions

The main conclusions of the thesis are as follows

1. The proposed method of solution helps to study interaction effect of two cracks for wide variety of crack geometries and loading conditions.
2. The method of solution is good enough to obtain sufficiently accurate results.
3. Each general solution consolidates a number of solutions available for a particular type of loading. The scope of the basic formulation adapted for each of these solutions has been enhanced considerably to make the general solution capable of considering large number of crack geometries and the loading conditions.
4. The arbitrary biaxial loading condition facilitates consideration of any orientation and ratio of biaxial loading.
5. The analytical results of present solutions are supported with the results obtained by FEM and they are in good agreement. Both the analytical and FEM analysis show that the stress intensity factors at the inner crack tips is higher than the stress intensity factors at the outer crack tips. The results of the present general solutions are satisfactory and these solutions are extremely useful to study the effect of crack geometries.

7.4 Scope for Future Work

1. Analytical solutions for the complex stress functions can be obtained for more than two cracks.

2. Analytical solutions for the complex stress functions can be obtained for biaxial shear loading.

2. Experimental verification of the present work can be carried out by 2-D photo elasticity.

3. Anisotropic or orthotropic cases can be considered while analyzing the problems of the same geometry and loading conditions.

4. General solutions can be obtained for stresses around cracks in finite width plates considering mechanical loads.

5. Results for all the cases considered in the present work can be obtained by finite element method.

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