

Development, Analysis and Application of a Voltage Stability Index for Online Monitoring

Major Project Report
(Part-II)

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By

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CERTIFICATE

This is to certify that the Major Project Report entitled “**Development, Analysis and Application of a Voltage Stability Index for Online Monitoring**” submitted by **Ms. Shruheti K. Vaghasiya (19MEEE13)** towards the fulfillment of the requirement for semester IV of Master of Technology (Electrical Engineering) in the field of Electrical Power Systems in Nirma University is the record of work carried out by her under our supervision and guidance. The work submitted has in my opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other University or Institute.

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ABSTRACT

Growing demand for electricity and the modernization of power systems in challenging markets has led to power systems running near to their stable limits. Consistent monitoring and control of power systems is therefore urgently required. Voltage stability indices help to determine the current operation of a power system, forecast potential changes to the system's existence, and determine a long-term pattern of growth under predefined circumstances. A new normalized voltage stability measure called the P-index, robust and based on strong theoretical foundations, is proposed in this project. Then after few modifications a different voltage stability index called Q-index is proposed. Also, the generator reactive power limit is a key factor in voltage instability. When the field or armature current limit becomes active, the generator reactive power limit becomes voltage dependent. Further, careful examination of generator reactive power characteristics, reveals the effect of generator reactive power capability on system voltage stability. The analysis of application of the P-index, Q-index and implication of generator reactive limits on IEEE 14 bus system is carried out.

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Chapter 1

Introduction

1.1 Overview

Power system stability is a complex object that has challenged power system engineers for many years. As power system evolved and interconnection economically attractive, the complexity of the stability problems increased. Present trends in the planning and operation of power system have resulted in new kinds of stability problems. Financial and regulatory conditions have caused electric utilities to build power system with less redundancy and operate them closer to transient stability limits interconnections are continuing to grow with more use of new technologies such as multi-terminal HVDC transmission. Modes of instability are becoming increasingly more complex and require a comprehensive concentration of various aspects of system stability. In particular voltage stability and low frequency inter-area oscillations have become greater sources of concern than in the past. Where is this problems used to occur in isolated situation they have now become more common place. The need for analysing the long-term dynamic response following major upset and ensuring proper coordination of protection and control system is also being recognised.

Significant research and development work has been undertaken in last few years to gain a better insight into physical aspects of this new stability problem in to develop analytical tools for their analysis in better system design. Developments in control system theory

and numerical methods have had a significant influence on this work.

It is necessary to estimate the value of maximum allowable loading of buses, which might a variable value due to many factors, reactive power load, reactive power sources location and network topology. This estimation can be achieved using voltage stability indices. Voltage stability indices, which use admittance matrix elements and some system variables such as power flow through lines and buses voltages, need little computational efforts and can give fast diagnosis of voltage stability condition [11]. There are two groups of these indices: line stability indices and bus voltage indices. Line stability indices are calculated depending on the maximum loadability of a two-bus system. Bus Voltage Indices can estimate stability margin, i.e. the distance of the actual state of the system to the stability limits. In addition, they can describe the stability of a certain region or the entire power system [8].

1.2 Problem Statement

Several methods for assessing voltage stability have been proposed in the literature. They're just trying to figure out how close things are to collapsing. However, these strategies have been found to have a variety of drawbacks. Because of the discontinuities induced by system controls, some show nonlinear behaviour. Others are computationally inefficient, making them unsuitable for online use. Some have also been found to have faulty theoretical foundations. It is clear that a simple but reliable voltage stability analysis technique is still needed

1.3 Objective

1. The very first objective is to measure how far the present operating point is to voltage collapse point is and identifies the weak lines and buses in the power system.
2. The second objective used on-line or off-line to help operators in real time operation of power system or in designing and planning operations.
3. Lastly it can also be used for load shedding on the particular load bus can be done

which can reassure that system will operate under stability limits.

1.4 Literature Survey

For the first time, a basic voltage stability indicator was proposed that was both simple to calculate numerically and articulate in its output. The L-index is a number that ranges from 0 (no load on the system) to 1 (maximum load on the system) (voltage collapse) [1]. The indicator L is a quantitative measure of the difference between the current state of the network and the stability threshold. In literature [2] the continuation power flow works on the basis of a predictor-corrector scheme to find a solution direction for a set of system of equations that have already been reformulated to provide a load parameter. It begins with a proven solution and then uses a tangent predictor to approximate a new solution for a different load parameter value. A traditional power flow uses the same Newton-Raphson methodology to correct this calculation. The process' intermediate effects are used to build a voltage stability index and classify the system's most vulnerable areas to voltage failure. The tangent vector $dV/d\lambda$ is useful in the continuation phase since it defines the orientation of the solution path at a corrected solution point. The tangent vector provides the differential change in voltage at each bus for a given differential change in device load.

It was discovered in [3] that as the load on a load bus increases, the value of the diagonal elements dQ_i/dV_i and dP_i/dV_i and dP_i/d_i from their no-load value to their value at any given loading condition can be used as a voltage stability index for the load bus i. Three voltage stability indices are proposed based on these criteria: The suggested index I_i is a far more accurate measure of a highly loaded power system's proximity to the voltage stability boundary. Since it is built on the jacobian matrix components, which are readily accessible from load flow, it is not only more efficient but also much easier to compute. In [4] paper presents on utilizing an established index called FVSI (Fast Voltage Stability Index) to act as a numerical verification of the shedding locations. The research work done shows that the FVSI index can be used and load shedding at these points does improve the stability of the system. Information extracted from system network behavioral studies proved to be useful in determining and

assessing the best solution strategy in mitigating voltage instability during post fault condition. Utilizing FVSI as an index would reduce time in assessing the voltage stability state of the system.

It is revealed in [5] that the basic impedance match technique has problems to predict voltage stability margin when applied to multi-load systems. Power system loads are non-linear and dynamic. They cannot be easily equivalent to thevenin impedance. In order to preserve the elegance and simplicity of the impedance matching idea, this paper proposes the concept of coupled single-port circuit. In this circuit, all the loads are brought outside of the equivalent system. Therefore, the coupling effects among the loads can be dealt with explicitly. The proposed coupled single-port circuit can be obtained by collecting the phasor measurements at the generator buses and the load buses of concern from PMUs. Literature [6] proposes a voltage stability index (VSI) to predict voltage collapse in power system. The index is based on maximum loading capability of the bus combined with a thevenin equivalent method for the aggregated representation. The technique is applied to transmission test system. Using thevenin equivalent calculated by fast load flow or PMU data, the index can be used for online voltage analysis. To apply the index for transmission system a fast method to reduce the system into two node system as in is applied.

The research in paper [7] states that several voltage stability indicators have been developed in an attempt to quantify proximity to voltage collapse. This study proposes a new normalized voltage stability indicator called the P-index that is robust and based on solid theoretical foundations. It was also shown how the P-index can be used to estimate distance to collapse and the amount of load to be shed. The results show that the P-index gives a better indication of proximity to voltage collapse compared to the L-indices and tangent vectors and is more conservative than the coupled single-port circuit method. The performance of the proposed P-index and load shedding scheme were tested using dynamic simulation on the well known Kundur 10-bus system. Voltage collapse of the system was simulated and the results show that the P-index correctly assesses system stability conditions and estimates the amount of load that needs to be shed.

Chapter 2

Power System Stability

2.1 Basic Concept of Power System

Electricity demand is currently rising rapidly, especially in developing countries like India. As a result of the constant demand, the power grid is operating at its maximum capacity. Electric power-sensitive industries such as information technology, communication, and electronics are driving up demand for efficient, affordable, and high-quality power. In this case, satisfying electric power demand is not the only criterion; power system engineers must also ensure that customers receive reliable and high-quality power. These problems emphasise the importance of comprehending the power system's equilibrium.

“Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most of the system variables bounded so that practically the entire system remains intact”. The disturbances mentioned in the definition could be faults, load changes, generator outages, line outages, voltage collapse or some combination of these [8]. Power system stability can be broadly classified into rotor angle, voltage and frequency stability. Each of these three stabilities can be further classified into large disturbance or small disturbance, short term or long term. The classification is depicted in Figure 2.1

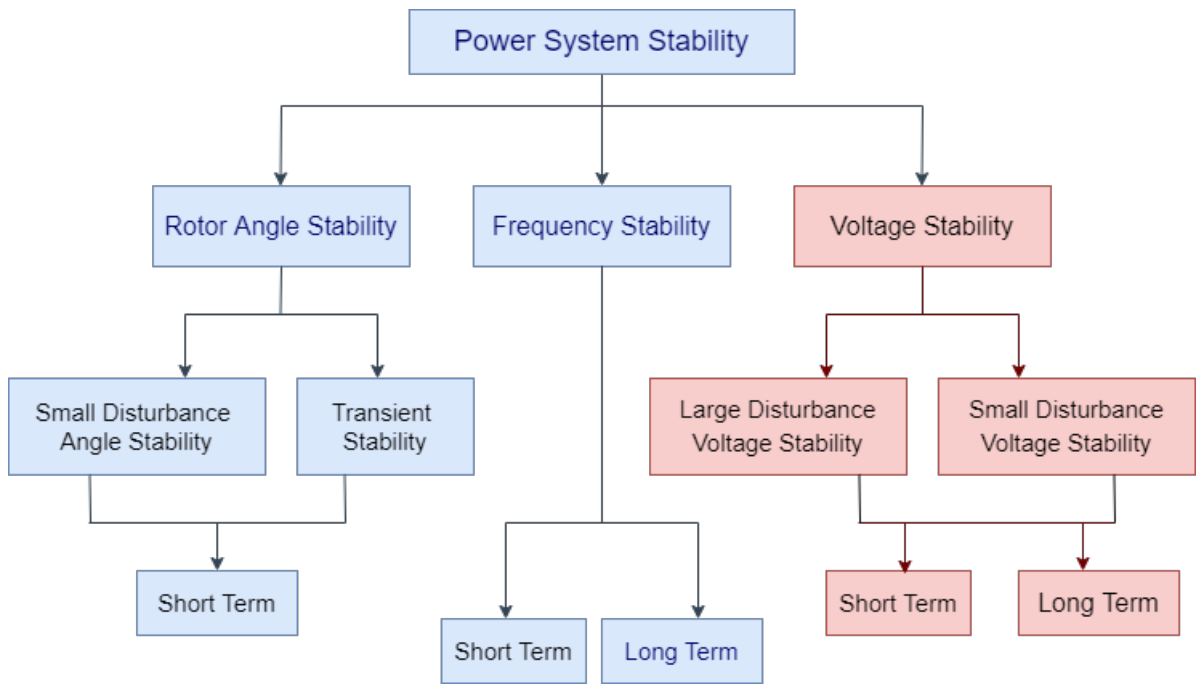


Figure 2.1: Classification of Power System Stability

2.2 Rotor Angle Stability

“It is the ability of the system to remain in synchronism when subjected to a disturbance”.

The rotor angle of a generator depends on the balance between the electromagnetic torque due to the generator electrical power output and mechanical torque due to the input mechanical power through a prime mover. Remaining in synchronism means that all the generators electromagnetic torque is exactly equal to the mechanical torque in the opposite direction [17]. If in a generator the balance between electromagnetic and mechanical torque is disturbed, due to disturbances in the system, then this will lead to oscillations in the rotor angle. Rotor angle stability is further classified into small disturbance angle stability and large disturbance angle stability.

2.3 Frequency Stability

“It refers to the ability of a power system to maintain steady frequency following a severe disturbance between generation and load”. It depends on the ability to restore equilibrium between system generation and load, with minimum loss of load. Frequency instability may lead to sustained frequency swings leading to tripping of generating units or loads. During fre-

quency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds like under frequency control to several minutes, corresponding to the response of devices such as prime mover and hence frequency stability may be a short-term phenomenon or a long-term phenomenon.

2.4 Voltage Stability

"It is the ability of the system to maintain steady state voltages at all the system buses when subjected to a disturbance. If the disturbance is large then it is called as large-disturbance voltage stability and if the disturbance is small it is called as small-disturbance voltage stability". In case voltage fluctuations occur due to fast acting devices like induction motors, power electronic drive, HVDC etc then the time frame for understanding the stability is in the range of 10-20 s and hence can be treated as short term phenomenon. On the other hand if voltage variations are due to slow change in load, over loading of lines, generators hitting reactive power limits, tap changing transformers etc then time frame for voltage stability can stretch from 1 minute to several minutes[9]. The main difference between voltage stability and angle stability is that voltage stability depends on the balance of reactive power demand and generation in the system where as the angle stability mainly depends on the balance between real power generation and demand.

Though, stability is classified into rotor angle, voltage and frequency stability they need not be independent isolated events. A voltage collapse at a bus can lead to large excursions in rotor angle and frequency. Similarly, large frequency deviations can lead to large changes in voltage magnitude [17].

The rotor angle, voltage and frequency stability of each part of the power system, such as the prime mover generator rotor, generator stator, transformers transmission lines, load, regulating equipment and safety systems, should be mathematically interpreted using suitable measurement methods. The entire power structure can be represented by a series of Differential Algebraic Equations (DAE) that can be used to evaluate system stability.

Chapter 3

Voltage Stability

3.1 Introduction

“Voltage stability can be defined as the ability of the system to retain system voltages within acceptable limits when subjected to disturbance”. If the disturbance is large then it is called as large-disturbance voltage stability and if the disturbance is small it is called as small-signal voltage stability. Voltage stability can be a local phenomenon where only a particular bus or buses in a particular region have voltage stability issue and this may not affect the entire system. Voltage stability can be a global phenomenon where many of the system buses experience voltage stability problems which can also trigger angle stability problems and hence can affect the entire system. Some of the voltage stability problems can start as a local problem and escalate to global stability problem. Voltage stability may be classified into two categories. These are:

1. Large-disturbance Voltage Stability
2. Small-disturbance Voltage Stability

Large-disturbance Voltage Stability – It is concerned with a system stability to control voltages following a large disturbance such as system faults, loss of load, or loss of generation. For determination of this form of stability requires the examination of the dynamic performance of the system over a period sufficient to capture of such devices as under load tap

changing transformers, generator field, and current limiters. Large disturbance voltage studies can be studied by using non-linear time domain simulations which include proper modeling.

Small-Disturbances Voltage Stability – The operating state of a power system is said to have small disturbances voltage stability if the system has small disturbances, a voltage near loads does not change or remain close to the pre-disturbance values. The concept of small disturbance stability is related to steady state and be analyzed using a small-signal model of the system.

Voltage stability of a system can be analyzed either by static analysis or dynamic analysis. In static analysis the system is assumed to be in steady state and hence instead of taking the DAE of the system only algebraic equations are considered. This type of analysis is suitable for small disturbances in the system [18]. For large disturbances the DAE are solved and the system response over a certain period of time is observed. It is important that for voltage stability the loads should be properly modeled as each type of load will affect the system voltage stability in a different way. Similarly the tap changing transformers, shunt or series reactive power compensators, generator reactive power limits, line charging capacitance should be included in the system representation to get an accurate picture of voltage stability.

3.2 Definitions of Voltage Stability

In literature several definitions of voltage stability are found which are based on time frames, system states, size of disturbance etc. During voltage instability, a broad spectrum of phenomena will occur.

3.2.1 Definitions according to CIGRE

CIGRE defines voltage stability in a general way similar to other dynamic stability problems. According to CIGRE,

- A power system at a given operating state is small-disturbance voltage stable if, following any small disturbance; voltages near loads are identical or close to the pre-

disturbance values.

- A power system at a given operating state and subject to a given disturbance is voltage stable if voltages near loads approach post-disturbance equilibrium values. The disturbed state is within the region of attraction of the stable post-disturbance equilibrium.
- A power system undergoes voltage collapse if the post-disturbance equilibrium voltages are below acceptable limits

3.2.2 Definitions according to Hills and Hiskens

Hill and Hiskens propose a definition which is divided into a static and dynamic part. For the system to be stable, the static part of the following must be true.

- The voltages must be viable i.e. they must lie within an acceptable band.
- The power system must be in a voltage regular operating point. A regular operating point implies that if reactive power is injected into the system or a voltage source increases its voltage, a voltage increase is expected in the network. For the dynamic behaviour of the phenomena the following are the concepts:
 - *Small disturbance voltage stability*: A power system at a given operating state is small disturbance stable if following any small disturbance, its voltages are identical to or close to their pre-disturbance equilibrium values.
 - *Large disturbance voltage stability*: A power system at a given operating state and subject to a given large disturbance is large disturbance voltage stable if the voltages approach post-disturbance equilibrium values.
 - *Voltage collapse*: A power system at a given operating state and subject to a given large disturbance undergoes voltage collapse if it is voltage unstable or the post disturbance equilibrium values are nonviable
- A power system undergoes voltage collapse if the post-disturbance equilibrium voltages are below acceptable limits

3.2.3 Definitions according to IEEE

According to IEEE, the following formal definitions of terms related to voltage stability are given:

- Voltage Stability is the ability of a system to maintain voltage so that when load admittance is increased, load power will increase, and so that both power and voltage are controllable.
- Voltage Collapse is the process by which voltage instability leads to loss of voltage in a significant part of the system.
- Voltage Security is the ability of a system, not only to operate stably, but also to remain stable (as far as the maintenance of system voltage is concerned) following any reasonably credible contingency or adverse system change.

3.3 Voltage Stability Analysis

Voltage stability analysis can be divided into dynamic and static voltage stability analyses. Both methods involve the examination of how far a system is to voltage instability and the mechanism of voltage instability, i.e., how and why voltage instability arises, the key factors contributing to instability, identification of voltage weak areas the network under study, and identifying the most effective measures of counteracting instability and by extension improving voltage stability [13].

3.3.1 Static Analysis

In static analysis of voltage stability the snapshots of the entire system at different instants is considered and at each instant the system is assumed to be in steady state that the rates of changes of the dynamic variables are zero. Hence, instead of considering all the differential algebraic equations only algebraic power balance equations are considering assuming that the system is in steady state [21]. At each instant whether the system is voltage stable or not and also how far the system is from instability can be assessed. There are two methods for assessing whether system is voltage stable or not. They are sensitivity analysis and modal

analysis.

It is however noted that although voltage stability is a dynamic problem, much of the problem can be solved through static analysis since it yields most of the required information with regards to the voltage stability status of a system. Static analysis also takes less computational time and effort when compared to dynamic analysis and hence easier to carry out. Static voltage stability analysis involves computation of power flow equations. It is basically taking “snapshots” of probable operating conditions which can be used to validly point out the mechanism of voltage collapse for different operating conditions [19].

3.3.2 Dynamic Analysis

Dynamic voltage stability analysis is based differential equations and deals with how bus voltages vary with changes in system operating parameters. It is useful in studying voltage collapse scenarios and understanding the chronology of events that lead to voltage collapse. Dynamic voltage stability methods include small signal stability analysis, time domain simulations, bifurcation analysis, and energy function method.

The voltage stability of a system can also be assessed by the transient simulation over a period of time. The differential algebraic equations of the system should be solved through numerical methods and the transient simulation should be carried out for few minutes to completely observe the interaction of generators, static loads, dynamic loads tap changing transformer etc. The transient simulation should be done for different fault scenarios and the system behaviour should be observed. If the system voltages are restored to acceptable values after fault clearing then the system is voltage stable if not the system is voltage unstable. The simulations needs to be done for few minutes because the time constant involved can be very small like generator exciter or very large like induction motors or tap changing transformers [17]. For voltage stability assessment load modelling is important hence both static and dynamic loads should be modelled. The reactive power compensating devices like series or shunt capacitor and static VAR compensator should also be included in the system model.

3.4 Generator Reactive Power Capability

Generator reactive power capability plays a crucial role in maintaining system voltage stability. Capability Curve of Generator defines the boundaries within which it can deliver reactive power continuously without overheating. Generator rating is specified in terms of MVA and power factor at a particular terminal voltage [23]. Active power delivered by generator is only limited by the power delivering capability of turbine. But the reactive power which a generator can deliver continuously without over heating is governed by three limits: Armature Current Limit, Field Current Limit and End Part Heating Limit.

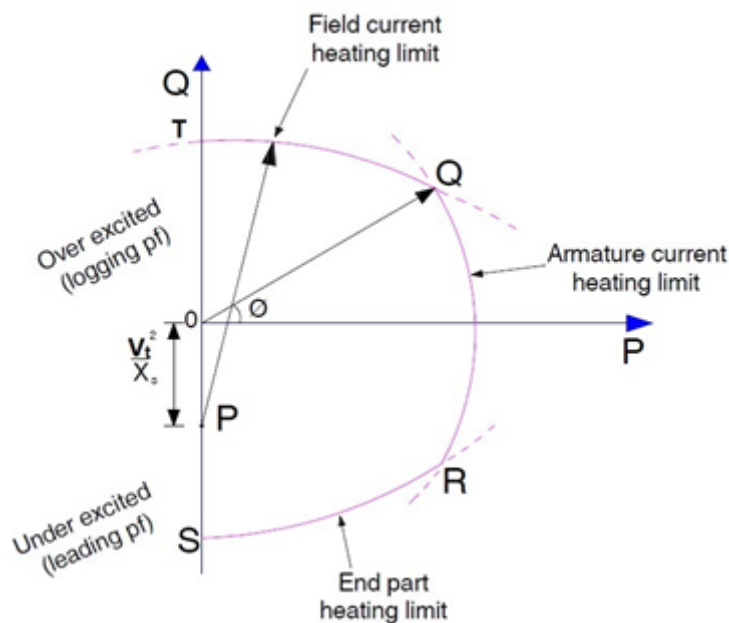


Figure 3.1: Generator Capability Curves

Following points can be observed and noted from the capability curve:

1. Armature heating and field heating limits are shown by curve QR and QT respectively. Both these curve cut at point Q which means the generator shall be operated at this point under lagging load condition.
2. End part heating is shown by curve RS for leading load condition. It shall be noted that armature heating limit and end part heating limit curves cut each other at point R. This point is the point of operation of generator under leading load condition.
3. Generator operation shall always be confined within the capability curve at all the time

else it may damage due to overheating.

4. Capability Curve is supplied by the manufacturer and works as a guideline for operator of generator.
5. Capability Curve is all about heating of different parts. This means that if the cooling of machine is increased then the limits of generator operation will also increase. This means the individual limits will increase and the operation boundary will broaden.

It is well known that generators have maximum and minimum real power capabilities. In addition, they also have maximum and minimum reactive power capabilities. The maximum reactive power capability corresponds to the maximum reactive power that the generator may produce when operating with a lagging power factor[**13**]. The minimum reactive power capability corresponds to the maximum reactive power the generator may absorb when operating with a leading power factor. These limitations are a function of the real power output of the generator, that is as the real power increases, the reactive power limitations move closer to zero.

3.5 Causes of Voltage Instability

Voltage instability phenomena are the ones in which the receiving end voltage decreases well below its normal value and does not come back even after setting restoring mechanisms such as VAR compensators, or continues to oscillate for lack of damping against the disturbances. Voltage collapse is the process by which the voltage falls to a low, unacceptable value as a result of an avalanche of events accompanying voltage instability [**12**]. Once associated with weak systems and long lines, voltage problems are now also a source of concern in highly developed networks as a result of heavier loading. There are three main causes of voltage instability:

1. Load dynamics: Loads are the driving force of voltage instability. Load dynamics are due to the following devices:
 - Load tap changing (LTC) transformer role is to keep the load side voltage in a defined band near the rated voltage by changing the ratio of transformer. As most of

the loads are voltage dependent, a disturbance causing a voltage decrease at a load bus will cause a decrease in the power consumption [14]. This tends to favor stability. However, the LTC will then begin to restore the voltage by changing the ratio step by step with a predefined timing. The increase in voltage will be accompanied by an increase in the power demand which will further weaken the power system stability.

- Thermostat will control the electrical heating. The thermostat acts by regularly switching the heating resistance on and off[18]. In the case of a voltage decrease, the power consumption, hence the heating power, will be reduced. Therefore, the thermostat will tend to supply the load during a longer time interval. The aggregated response of a huge group of this kind of loads is seen as a restoration of the power, comparable to the one of the LTC.
- Induction motors have dynamic characteristics with short time constants. Restoration process occurs following voltage reduction because the motor must continue to supply a mechanical load with a torque more or less constant [14].

2. Transmission system: Each transmission element, line or transformer, has a limited transfer capability. It is dependent on several factors:

- The impedance of the transmission element.
- The power factor of the load.
- The presence of voltage controlled sources (generators or Static Var Compensator-SVC) at one or both extremities of the element and the voltage set point of these sources [15].
- The presence of reactive compensation devices (mechanically switched capacitors or reactors).

3. Generation system: When the power system flows increase, the transmission system consumes more reactive power. The generators must increase their reactive power output. Operating point of generator can be found from it's capability curve. But due to

over-excitation limiter (OEL) and stator current limiter (SCL), voltage can't be controlled after these limiters are activated [15].

3.6 Tools for Voltage Stability Analysis

Different methods exist in the literature for carrying out a steady state voltage stability analysis. The conventional methods [20] can be broadly classified into the following types:

1. P-V curve method
2. Q-V curve method
3. Modal analysis
4. Continuation power flow (CPF) method

3.6.1 P-V Curve Method

In voltage stability analysis, relation between power transfer to the load and voltage of the load bus is not weak. Variation in power transfer from one bus to another bus effects the bus voltages [19]. This can be studied using P-V curve. For a network, load buses (PQ buses) are identified to plot the P-V curves. The load model is taken as constant real power which is represented by Equation $P = P_0(1 + \lambda K_L)$ Where P_0 is the base case load real power, λ is loading factor and K_L is the load increment factor. The power-flow solution of the system is taken as a base case. Steps in P-V curve analysis:

1. Select a load bus, vary the load real power using loading factor λ and load increment factor K_L . Keep the power factor as constant.
2. Compute the power flow solution for the present load condition and record the voltage of the load bus.
3. Increase the loading factor by small amount and repeat step 2 until power flow does not have convergence.
4. P-V curve is plotted using the calculated load bus voltages for increased load values.

- Real power margin is computed by subtracting the base load value from maximum load value at which voltage collapse occurs.

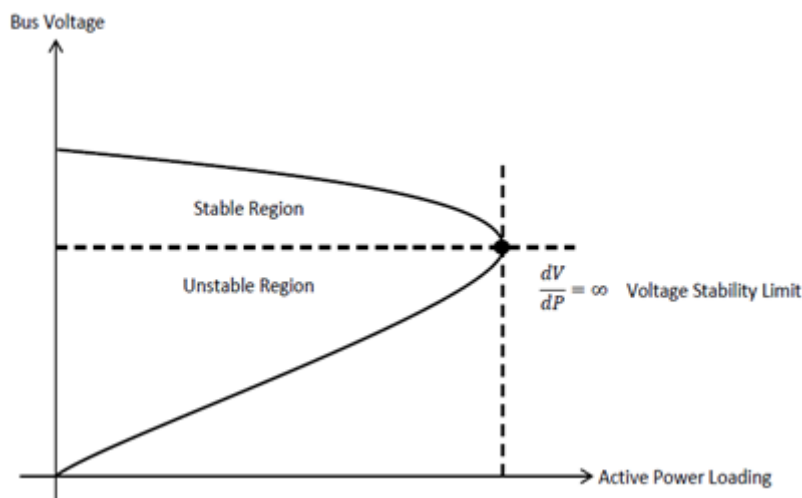


Figure 3.2: PV Curve

In P-V curve shown in Figure 3.2, there are three regions related to real power load P . In the first region up to loadability limit, power flow equation has two solutions for each P of which one is stable voltage and other is unstable voltage. If load is increased, two solutions will coalesce and P is maximum. If load is further increased, power flow equation doesn't have a solution. Voltage corresponding to "maximum loading point" is called as critical voltage.

3.6.2 Q-V Curve Method

The V-Q curves, gives reactive power margin. It shows the reactive power injection or absorption for various scheduled voltages. If reactive power load is scheduled instead of voltages Q-V curves are produced. Q-V curves are a more general method of assessing voltage stability. Many utilities uses Q-V curves to determine the proximity to voltage collapse and to establish system design criteria based on Q and V margins [21]. Q-V curves can be used to check whether the voltage stability of the system can be maintained or not and to take suitable control actions. A typical V-Q curve is shown in figure 3.3.

Near the collapse point of Q-V curve, sensitivities get very large and then reverse sign. Also, it can be seen that the curve shows two possible values of voltage for the same value of

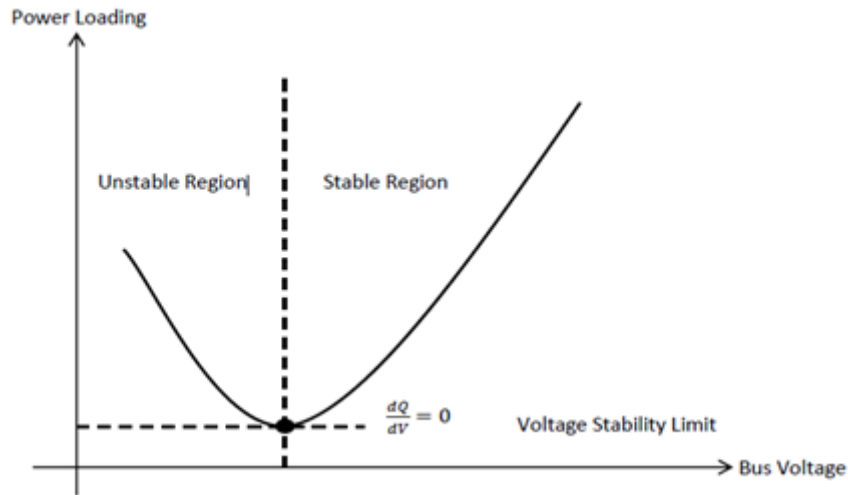


Figure 3.3: QV Curve

power. The power system operated at lower voltage value would require very high current to produce the power. That is why the bottom portion of the curve is classified as an unstable region and system can't be operated in this region. Constant reactive power load model is selected and represented by the following Equation: $Q = Q_0(1 + \lambda K_L)$

Where Q_0 is the base case load reactive power, λ is loading factor and K_L is the load increment factor. The power flow solution of the system is taken as a base case. Steps in Q-V curve analysis:

1. Select a load bus, vary the load reactive power using load demand factor λ and load increment factor K_L . Keep the real power of load as constant.
2. The reactive power output of each generator should be allowed to adjust.
3. Compute the power flow solution for the present load condition and record the voltage of the load bus.
4. Increase the load demand factor λ by small amount and repeat step 3 until power flow does not have convergence.
5. Q-V curve is plotted using the calculated load bus voltages for increased load values.
6. Reactive power margin is computed by subtracting the base load value from maximum load value at which voltage collapse occurs.

3.6.3 Disadvantages of P-V curves and Q-V curves

Though both methods are widely used as index to find the proximity to voltage collapse, but they have few disadvantages.

- In both methods, at a time only one bus is considered for load variation. As there is no information about critical buses, power flow studies are to be done for many buses which takes so much time.
- As the loading on the system approaches critical point, convergence problem occurs in solving the power flow equation.
- These methods doesn't give useful information about the causes of voltage instability.

3.6.4 Modal Analysis

Using Modal analysis proposed by Gao, Morrison and Kundur in 1992 [17], the reactive power margin and voltage instability contributing factors are calculated. Modal analysis depends on power flow Jacobian matrix [21]. Real power is kept constant and reduced Jacobian matrix J_R of the system is calculated. The matrix J_R represents the linearized relationship between the incremental changes in bus voltage (ΔV) and the bus reactive power injection (ΔQ). If the minimum eigenvalue of J_R is greater than zero, the system is voltage stable. Using the left and right eigenvectors corresponding to critical mode, bus participation factors can be calculated. Branch participation factors are calculated from linearized reactive power loss. Buses and Branches with large participation factors are identified as critical buses [21].

3.6.5 Continuation Power Flow (CPF) Method

Continuation power flow (CPF) method proposed by Venkataramana Ajjarapu [10] is used for finding the continuous power flow solutions starting from base load condition to steady state voltage stability limit. The main difference between CPF and conventional power flow method can be observed as the operating point approaches critical point. In conventional power flow as the operating point comes close to critical point, power flow will not con-

verge. In CPF method, divergence problem doesn't arise and it uses predictor-corrector process to find the next operating point. As the critical point is approached, loading factor λ reaches maximum and starts decreasing. The tangent component corresponding to λ is zero at critical point and becomes negative after that. From the tangent vector, information about weak buses can be obtained.

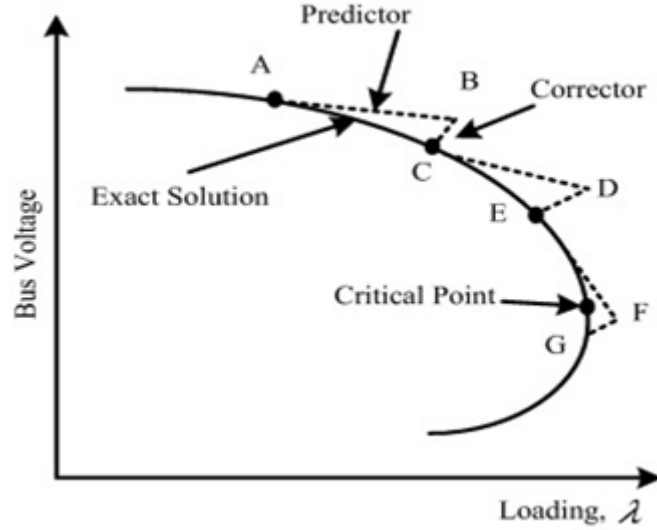


Figure 3.4: CPF Curve

The CPF algorithm overcomes the Jacobian singularity problem by reformulating the power flow problem with locally parametrized continuation techniques [10]. The power flow problem can be represented by a set of nonlinear equation: $(x) = 0$ where $x = [\delta, V]T$. The continuation power flow introduces a load parameter, λ to track the solution of the nonlinear equations. The base case has $\lambda = 0$. The parameterized power flow nonlinear equations can be written as: $(x, \lambda) = 0$. The reformulated power balance equations at a bus k are:

$$\Delta P_k = P_{GK}(\lambda) - P_{LK}(\lambda) - P_K \quad (3.1)$$

$$\Delta Q_k = Q_{GK}(\lambda) - Q_{LK}(\lambda) - Q_K \quad (3.2)$$

where,

$$P_k = |V_k| \sum_{j=1}^n Y_{KJ} |V_j| \cos(\delta_K - \delta_J - \theta_{KJ})$$

$$Q_k = |V_k| \sum_{j=1}^n Y_{KJ} |V_j| \sin(\delta_K - \delta_J - \theta_{KJ})$$

The modified load and generation at bus k are:

$$P_L(\lambda) = P_{LK0}[1 + \lambda K_{LK}] \quad (3.3)$$

$$Q_L(\lambda) = Q_{LK0}[1 + \lambda K_{LK}] \quad (3.4)$$

$$P_G(\lambda) = P_{GK0}[1 + \lambda K_{GK}] \quad (3.5)$$

The tangent vector is calculated from $x = [F\delta, FV, F\lambda]^T = 0$. $F\delta, FV, F\lambda$ are partial derivatives of (δ, V, λ) with respect to δ, V and λ . The $[F\delta, FV]$ is nothing but the original Jacobian matrix [10]. The tangent vector has to be normalized in order to guarantee the non-singularity of the augmented Jacobian. Therefore, the augmented Jacobian should satisfy:

$$\begin{bmatrix} F\delta & FV & F\lambda \\ & e_k & \end{bmatrix} \begin{bmatrix} \\ \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ \\ +1 \end{bmatrix} \quad (3.6)$$

where e_k is a row vector in which the k_{th} element is the only non-zero element. And the augmented Jacobian is defined as: $J_{aug} = \begin{bmatrix} F\delta & FV & F\lambda \\ & e_k & \end{bmatrix}$ After the tangent vector is obtained, the predicted solution for iteration i is given by:

$$\begin{bmatrix} \delta_{predict}^k \\ V_{predict}^k \\ \lambda_{predict}^k \end{bmatrix} = \begin{bmatrix} \delta^{k-1} \\ V^{k-1} \\ \lambda^{k-1} \end{bmatrix} + \sigma \begin{bmatrix} d\delta^{k-1} \\ dV^{k-1} \\ d\lambda^{k-1} \end{bmatrix} \quad (3.7)$$

where σ is the designated step size. The index k and the step size should be chosen appropriately. The corrector then is calculated by:

$$\begin{bmatrix} \Delta\delta^k \\ \Delta V^k \\ \Delta\lambda^k \end{bmatrix} = -J_{aug}^{-1} \begin{bmatrix} \Delta f^k \\ 0 \end{bmatrix} \quad (3.8)$$

The solution after the corrector process is given by:

$$\begin{bmatrix} \delta^k \\ V^k \\ \lambda^k \end{bmatrix} = \begin{bmatrix} \delta_{predict}^k \\ V_{predict}^k \\ \lambda_{predict}^k \end{bmatrix} + \begin{bmatrix} \Delta \delta^k \\ \Delta V^k \\ \Delta \lambda^k \end{bmatrix} \quad (3.9)$$

After we get the tangent vector, we need to verify whether the system has reached the critical point. The sign of the product $dV/d\lambda$ provides the information related to the critical point ($dX = 0$ corresponds to the critical point. If the sign of the product $dV/d\lambda$ is positive then the critical point has been passed).

3.6.6 Comparison of Different Methods

Table 3.1: Comparison of Different Methods of Voltage Stability Analysis

Method	Features	Advantages	Limitations
PV and QV curves	Plots denoting load bus voltage magnitudes for power increased in a particular PQ bus. The stability criterion is the 'distance' between the current operating point and the extremes of curves	Gives a quantitative measurement of the proximity to voltage collapse. QV curves give the reactive power injection/absorption for scheduled voltages, which is useful in sizing of shunt capacitors.	A large system would require a lot of computational effort. Convergence problems occur as loading on the system approaches the voltage collapse point. No information about the causes of voltage instability
CPF	It is a technique used for tracing the whole of a PV curve by finding the next stable operating point for a given load or load change scenario. It utilizes the predictor-corrector method	Overcomes convergence problems that arise with the use of PV and QV curves; hence, one can determine critical points of where voltage collapse occurs accurately	Does not give information about the causes of voltage instability. Bus specific, which makes it computationally intensive and time consuming.
Sensitivity Analysis	Based on the sensitivity matrix derived from load flow. The sensitivity parameters are determined by the relationship between state control variables	Provides a good judge on the voltage stability status of a system. Identifies voltage weak buses in a system	The linear characteristics of the sensitivity index are not good especially for complex power systems; hence, it cannot accurately reflect the critical state of a system.
Modal Analysis	It involves computing the smallest eigenvalues and associated eigenvectors of the reduced Jacobian matrix obtained from the load flow. The eigenvalues and eigenvectors are used to calculate participation factors which identify the cause of instability	Gives information regarding the voltage stability status from both perspective, proximity to voltage collapse and mechanism of instability. Identifies load regions most susceptible to voltage instability, weak buses and critical links in the network.	Eigenvalues do not provide an absolute measure of the proximity to voltage collapse.

Chapter 4

Voltage Stability Indices

4.1 What is Voltage Stability Index ??

Voltage stability index is a scalar magnitude that predicts proximity to voltage instability in assessment of voltage stability of power system. Voltage stability indices are used to assess the current activity of a power grid, forecast potential improvements in the system's nature, and assess a long-term growth pattern under certain conditions. They're useful for calculating how close a given operating point is to the voltage failure point. These indices are plain, easy to use, and cost little to compute. On-line and off-line experiments will also benefit from voltage stability indices [21]. Voltage stability indices are scalar variables that can be used to calculate the difference between the current operating point and the voltage instability point as a function of components, for example, in the nodal admittance matrix, power flow is represented by voltage magnitudes or angles. Voltage stability indices, on the other hand, allow for the detection of critical lines and nodes, as well as the definition of critical areas of voltage stability in order to take corrective steps. Because of the growing usage of measuring instruments in PES, such as PMUs, it is now possible to use actual measurements obtained from the PES to apply voltage stability indices using online measurements for real-time monitoring system.

Thevenin parameters, on the other hand, must be estimated from measurements collected over a time window large enough for operating conditions to adjust but small enough to fulfil the condition of no device disruption. Unfortunately, this situation can never be mitigated [7]. Since the thevenin equivalence specifications are difficult to track, some researchers suggested alternative on-line voltage stability assessment indices and methods that do not require the detection of thevenin equivalence parameters. Any of these indices are dependent on local transmission line phasor measurements.

4.2 L-Index

One of the most popular indices which do not depend on a thevenin equivalent and is well suited for online applications is the L-index [1]. It is simple and can easily be calculated from normal load flow data. Hence, for online monitoring and simplified calculation L-index is derived as voltage stability indicator, which ranges from 0 to 1 as discussed earlier. The main problem with the voltage instability occurred due to the either lack of the reactive power injection in the system or reactive power absorption by the load which can not be satisfied by the generator of particular system [25]. So, we can say that dynamic load connected to the particular load bus is the main problem with the instability of the power system.

4.2.1 Derivation

L-index can be derived via simple two bus system which consists of one generator bus and one load bus with suitable line impedance [1]. The starting point for the subsequent analysis is the line model which can be conceived as the simplest power system and which can also be treated analytically.

It is given by figure whereby node 1 is assumed to supply the load whose voltage behavior is of interest and where node 2 is a generator node. The properties of node 1 can be described in terms of the admittance matrix of the system as below:

$$Y_{11}V_1 + Y_{12}V_2 = I_1 = S_1/V_1 \quad (4.1)$$

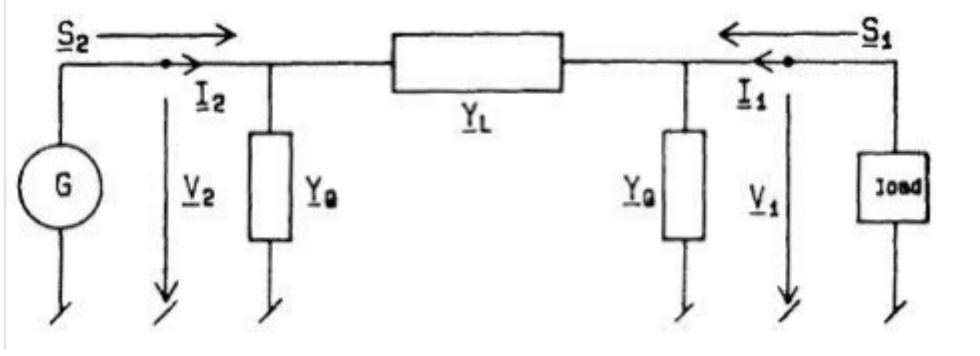


Figure 4.1: Two Bus Line Model

$$Y_{11}V_1V_1' + Y_{12}V_2V_1' = S_1 \quad (4.2)$$

$$V_1^2 + V_0V_1' = S_1/Y_{11} = a_1 + jb_1 \quad (4.3)$$

From the above equations we can derive equation which can represent circle diagram with its axis as reactive power and active power where circle diameter changes with the changes of its per unit value [24]. Power circle formula can be modify to find the voltage stability index needed for the power system in terms of the apparent power or in terms of voltage and impedances of the line in between the buses, which are as follows:

$$|S_1 - Y_{11}V_1^2| = V_0V_1Y_{11} \quad (4.4)$$

$$L = |S_1/(Y_{11} + V_1^2)| = |1 + (V_0/V_1)| \quad (4.5)$$

Equation (4.5) gave L-index whose range is from 0 to 1, but this equation is valid only for two bus linear model, to make this equation valid for n-bus system, it is modified to include all the parameters needed like total number of generator bus in the system and total number of load bus in the n-bus system to make sure voltage stability index that was calculated gives better result than previous voltage stability index which was calculated with the help of load flow analysis of the power system. L-index for n-bus system given as follows:

$$L_j = |S_j/(Y_{jj} + V_j^2)| = |1 + (V_{0j}/V_j)| \quad (4.6)$$

$$L = \text{MAX}_{j \in \alpha_L} \left| 1 - \frac{\sum_{i \in \alpha_G} F_{ji} V_i}{V_j} \right| \quad (4.7)$$

In equation (4.7), F_{ji} can be derived from H matrix of the output of the load flow analysis of the power system, which vary with the load on the bus as well the bus itself. Whereby α_L is the set of consumer nodes and α_G is the set of generator nodes. Thus the important outcome of the presented theory is $L \leq 1$ for system stability.

4.3 P-Index

4.3.1 Introduction

With the advancement in phasor measurement technology, simplifying the entire network to a thevenin equivalent became very common. For stability analysis, thevenin equivalent is considered to be very simple and straightforward, making it ideal for use in real-time power system monitoring. While the thevenin counterpart has received a lot of publicity, it is not without its challenges. It is difficult to monitor the Thevenin equivalence parameters using real-time measurements [22]. At least two measurement sets (snapshots) of local voltage and current phasors are needed to compute the thevenin equivalence parameters. Thevenin parameters, on the other hand, must be estimated from measurements collected over a time window large enough for operating conditions to adjust but small enough to fulfil the condition of no device disruption. Regrettably, this situation can never be remedied.

Since the thevenin equivalence parameters are difficult to track, some researchers suggested alternative on-line voltage stability assessment indices and methods that do not require the detection of thevenin equivalence parameters. Any of these indices are dependent on local transmission line phasor measurements. One of the most popular indices which do not depend on a thevenin equivalent and is well suited for online applications is the P-index

4.3.2 Derivation

A simple radial system [7] is used at first to explain the concept behind the proposed indicator. Consider the two bus system shown in figure 4.2 where the load at bus 2 is $P_L + jQ_L$ and the voltage magnitude is V .

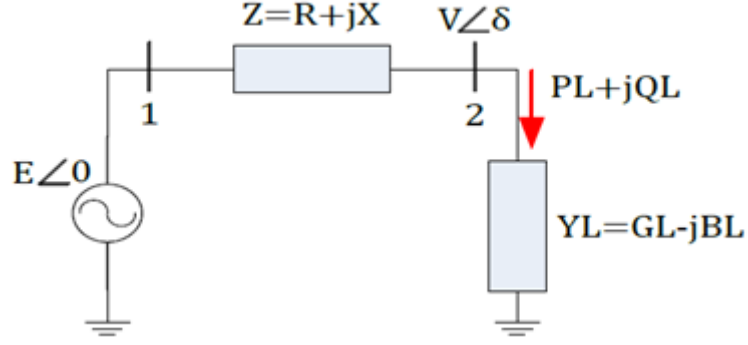


Figure 4.2: Two Bus System

Load admittance is $G_L - jB_L$, where,

$$G_L = P_L/V^2 \quad B_L = Q_L/V^2 \quad (4.8)$$

The active power increment and subsequently admittance and voltage can be written as:

$$\begin{aligned} \Delta P_L &= (V + \Delta V)^2 (G_L + \Delta G_L) V^2 G_L \\ &= (V + \Delta V)^2 \Delta G_L + (2V + \Delta V) G_L \end{aligned} \quad (4.9)$$

The voltage stability index this paper proposes is based on the ratio of the two terms in 4.9, i.e. the ratio of power lost to power gained,

$$P - index = -\frac{(2V + \Delta V)G_L}{(V + \Delta V)^2} * \frac{\Delta V}{\Delta G_L} \quad (4.10)$$

In the limiting case as $\Delta G_L, \Delta V \rightarrow 0$, then,

$$P - index = -\frac{2G_L}{V} * \frac{dV}{dG_L} \quad (4.11)$$

$$\frac{dV}{dG_L} = \frac{dV}{dP_L} * \frac{dP_L}{dG_L} \quad (4.12)$$

Then, by using $P_L = V^2 G_L$ one may write,

$$dP_L = V^2 dG_L + 2VG_L \quad (4.13)$$

or differentiating w.r.t. G_L , we get:

$$\frac{dP_L}{dG_L} = V^2 + 2VG_L \frac{dV}{dG_L} \quad (4.14)$$

Substituting this value in equation 4.12

$$\frac{dV}{dG_L} = \frac{dV}{dP_L} (V^2 + 2VG_L \frac{dV}{dG_L}) \quad (4.15)$$

The equation 4.15 after manipulations can be expressed as:

$$\frac{dV}{dG_L} = \frac{\frac{V^2 dV}{dP_L}}{1 - 2VG_L \frac{dV}{dP_L}} \quad (4.16)$$

Substituting this value in P-index defined in 4.11,

$$P - index = \frac{-2VG_L \frac{dV}{dP_L}}{1 - 2VG_L \frac{dV}{dP_L}} \quad (4.17)$$

In terms of active power,

$$P - index = \frac{-2\frac{P_L}{V} \frac{dV}{dP_L}}{1 - 2\frac{P_L}{V} \frac{dV}{dP_L}} \quad (4.18)$$

To calculate P-index for any load bus j of n-bus system, P-index can be given as:

$$P - index = \frac{-2\frac{P_{Lj}}{V_j} \frac{dV_j}{dP_{Lj}}}{1 - 2\frac{P_{Lj}}{V_j} \frac{dV_j}{dP_{Lj}}} \quad (4.19)$$

It is necessary to find the value of dV_j/dP_{Lj} . This can be calculated from the system Jacobian matrix. If the inverse Jacobian matrix equations are defined as follows (where $\Delta P_L, \Delta Q_L$ are net bus increments):

$$\begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} H, N \\ J, L \end{bmatrix} \begin{bmatrix} \Delta P_L \\ \Delta Q_L \end{bmatrix} \quad (4.20)$$

We may write, for load bus j :

$$\Delta V_j = \sum_{i \in L, G} j_{ji} \Delta P_{Li} + \sum_{i \in L} l_{ji} \Delta Q_{Li} \quad (4.21)$$

or

$$\frac{\Delta V_j}{\Delta P_{Vj}} \rightarrow \frac{dP_{Lj}}{dV_j} = \sum_{i \in L, G} j_{ji} \alpha_{ji} + \sum_{i \in L} l_{ji} \alpha_{ji} \beta_i \quad (4.22)$$

Where,

$$\alpha_{ji} = \frac{\Delta P_{Li}}{\Delta P_{Lj}} = \frac{P_{Li}}{P_{Lj}}$$

$$\beta_{ji} = \frac{\Delta Q_{Li}}{\Delta P_{Li}} = \frac{Q_{Li}}{P_{Li}} = \tan \phi_i, \text{ where } \phi_i \text{ is the power factor angle of the load at bus } i$$

4.4 Proposed Voltage Stability: Q-Index

4.4.1 Voltage and Reactive Power Relation

Voltage control and reactive-power management are two aspects of a single activity that both supports reliability and facilitates commercial transactions across transmission networks. Transmission line impedances also make it necessary to provide reactive power to maintain voltage levels necessary for active power to flow through [24]. Therefore reactive power is essential to move active power through transmission and distribution systems to the customer.

However if reactive power in a system is too high, there is increased heat loss in transmission lines and loads as the current flowing through the system is much higher, creating a potentially hazardous breakdown situation[25]. The power factor of a load tells us what fraction of the apparent power is in the form of real power and performs actual work. A high power factor is desirable since it minimizes the amount of reactive power needed by the load, reducing heat losses and maximizing efficiency. On an alternating-current (AC) power system, voltage

is controlled by managing production and absorption of reactive power. There are three reasons [26] why it is necessary to manage reactive power and control voltage.

- First, both customer and power-system equipment are designed to operate within a range of voltages, usually within $\pm 5\%$ of the nominal voltage. At low voltages, many types of equipment perform poorly; light bulbs provide less illumination, induction motors can overheat and be damaged, and some electronic equipment will not operate at. High voltages can damage equipment and shorten their lifetimes.
- Second, reactive power consumes transmission and generation resources. To maximize the amount of real power that can be transferred across a congested transmission interface, reactive-power flows must be minimized. Similarly, reactive-power production can limit a generator's real-power capability.
- Third, moving reactive power on the transmission system incurs real-power losses. Both capacity and energy must be supplied to compensate these losses.

4.4.2 Derivation

The derivation of Q-index is more of modification of P-index discussed in section 4.3.2. The only difference is that here reactive power is taken into consideration instead of active power. The final equation obtained after this evaluation is called Q-index.

A simple radial system is used at first to explain the concept behind the proposed indicator. Consider the two bus system shown in figure 4.2 where the load at bus 2 is $P_L + jQ_L$ and the voltage magnitude is V .

Load admittance is $G_L - jB_L$, where,

$$G_L = P_L/V^2 \quad B_L = Q_L/V^2 \quad (4.23)$$

The reactive power increment and subsequently admittance and voltage can be written as:

$$\begin{aligned} \Delta Q_L &= (V + \Delta V)^2 (B_L + \Delta B_L) V^2 B_L \\ &= (V + \Delta V)^2 \Delta B_L + (2V + \Delta V) B_L \end{aligned} \quad (4.24)$$

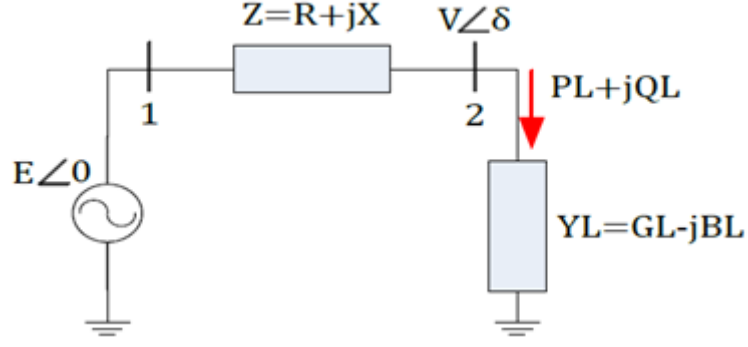


Figure 4.3: Two Bus System

The voltage stability index this paper proposes is based on the ratio of the two terms in 4.24, i.e. the ratio of power lost to power gained,

$$Q - index = -\frac{(2V + \Delta V)B_L}{(V + \Delta V)^2} * \frac{\Delta V}{\Delta B_L} \quad (4.25)$$

In the limiting case as $\Delta B_L, \Delta V \rightarrow 0$, then,

$$Q - index = -\frac{2B_L}{V} * \frac{dV}{dB_L} \quad (4.26)$$

$$\frac{dV}{dB_L} = \frac{dV}{dG_L} * \frac{dP_L}{dB_L} \quad (4.27)$$

Then, by using $Q_L = V^2 B_L$ one may write,

$$dQ_L = V^2 dB_L + 2V B_L \quad (4.28)$$

or differentiating w.r.t. B_L , we get:

$$\frac{dQ_L}{dB_L} = V^2 + 2V B_L \frac{dV}{dB_L} \quad (4.29)$$

Substituting this value in equation 4.12

$$\frac{dV}{dB_L} = \frac{dV}{dQ_L} (V^2 + 2V B_L \frac{dV}{dB_L}) \quad (4.30)$$

The equation 4.30 after manipulations can be expressed as:

$$\frac{dV}{dB_L} = \frac{\frac{V^2 dV}{dQ_L}}{1 - 2VB_L \frac{dV}{dQ_L}} \quad (4.31)$$

Substituting this value in Q-index defined in 4.26,

$$Q - index = \frac{-2VB_L \frac{dV}{dQ_L}}{1 - 2VB_L \frac{dV}{dQ_L}} \quad (4.32)$$

In terms of reactive power,

$$Q - index = \frac{-2\frac{Q_L}{V} \frac{dV}{dQ_L}}{1 - 2\frac{Q_L}{V} \frac{dV}{dQ_L}} \quad (4.33)$$

To calculate Q-index for any load bus j of n-bus system, Q-index can be given as:

$$Q - index = \frac{-2\frac{Q_{Lj}}{V_j} \frac{dV_j}{dQ_{Lj}}}{1 - 2\frac{Q_{Lj}}{V_j} \frac{dV_j}{dQ_{Lj}}} \quad (4.34)$$

It is necessary to find the value of dV_j/dP_{Lj} . This can be calculated from the system Jacobian matrix. If the inverse Jacobian matrix equations are defined as follows (where ΔP_L , ΔQ_L are net bus increments):

$$\begin{bmatrix} \Delta \delta \\ \Delta V \end{bmatrix} = \begin{bmatrix} H, N \\ J, L \end{bmatrix} \begin{bmatrix} \Delta P_L \\ \Delta Q_L \end{bmatrix} \quad (4.35)$$

We may write, for load bus j:

$$\Delta V_j = \sum_{i \in L, G} j_{ji} \Delta P_{Li} + \sum_{i \in L} l_{ji} \Delta Q_{Li} \quad (4.36)$$

or

$$\frac{\Delta V_j}{\Delta Q_{Vj}} \rightarrow \frac{dQ_{Lj}}{dV_j} = \sum_{i \in L, G} j_{ji} \alpha_{ji} + \sum_{i \in L} l_{ji} \alpha_{ji} \beta_i \quad (4.37)$$

Where,

$$\beta_{ji} = \frac{\Delta Q_{Li}}{\Delta Q_{Lj}} = \frac{Q_{Li}}{Q_{Lj}}$$

$$\alpha_{ji} = \frac{\Delta P_{Li}}{\Delta Q_{Li}} = \frac{P_{Li}}{Q_{Li}} = \tan \phi_i, \text{ where } \phi_i \text{ is the power factor angle of the load at bus } i$$

Chapter 5

Results and Discussion

5.1 Two Bus System Results

The two bus system shown in Figure 4.2 with $E = 1.0$ p.u., $Z = 0.01 + j0.2$ p.u., and a load power factor of 0.8, lagging is considered.

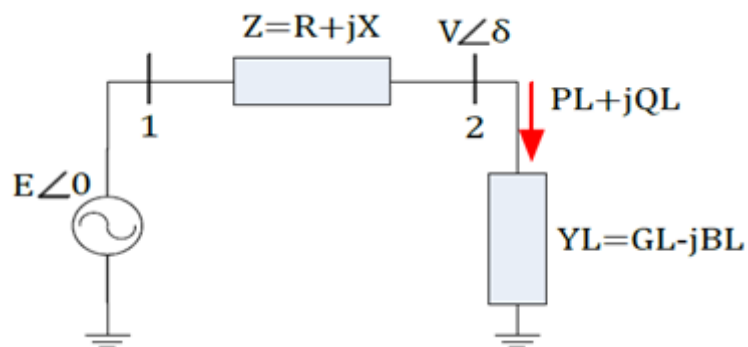


Figure 5.1: Two Bus System

Table 5.1: Comparing Values of P-index and L-index for Two bus System

Active Power Loading	P-index	L-index
0.1	0.0423	0.219
0.2	0.0883	0.2231
0.3	0.1386	0.2297
0.4	0.1938	0.2387
0.5	0.2546	0.2498
0.6	0.3219	0.2627
0.7	0.3969	0.2771
0.8	0.4808	0.2928
0.9	0.5755	0.3097
1.0	0.6832	0.3275
1.1	0.8066	0.3461
1.2	0.9496	0.3653
1.3	1.1171	0.3851
1.4	1.3161	0.4053
1.5	1.5564	0.4259
1.6	1.8524	0.4469
1.7	2.2259	0.4681
1.8	2.7119	0.4895
1.9	3.3703	0.5111
2	4.3127	0.5329

Table 5.2: Q-index Values

Reactive Power Loading	Q-index
0.1	-0.0163
0.2	0.0163
0.3	0.0155
0.4	0.0833
0.5	0.1281
0.6	0.1708
0.7	0.2168
0.8	0.2655
0.9	0.3202
1.0	0.3785
1.1	0.4419
1.2	0.5112
1.3	0.5873
1.4	0.6710
1.5	0.7635
1.6	0.8664
1.7	0.9812
1.8	1.1102
1.9	1.2561
2	1.4220

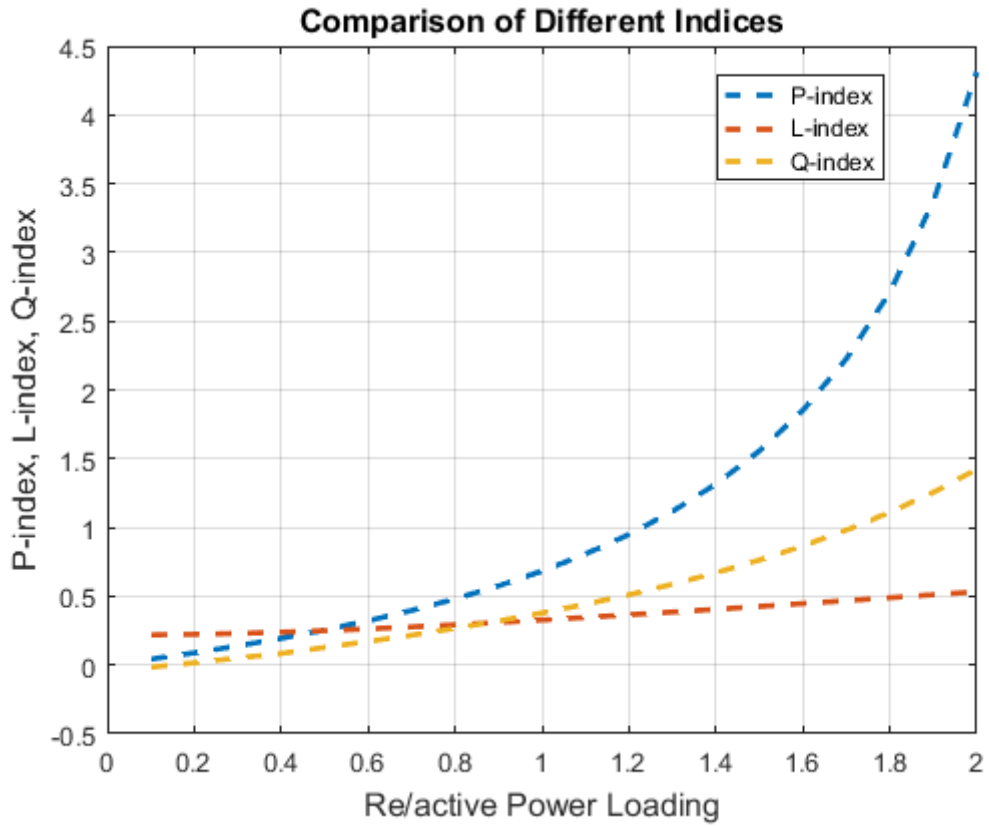


Figure 5.2: Comparison of Different Indices for Two bus system

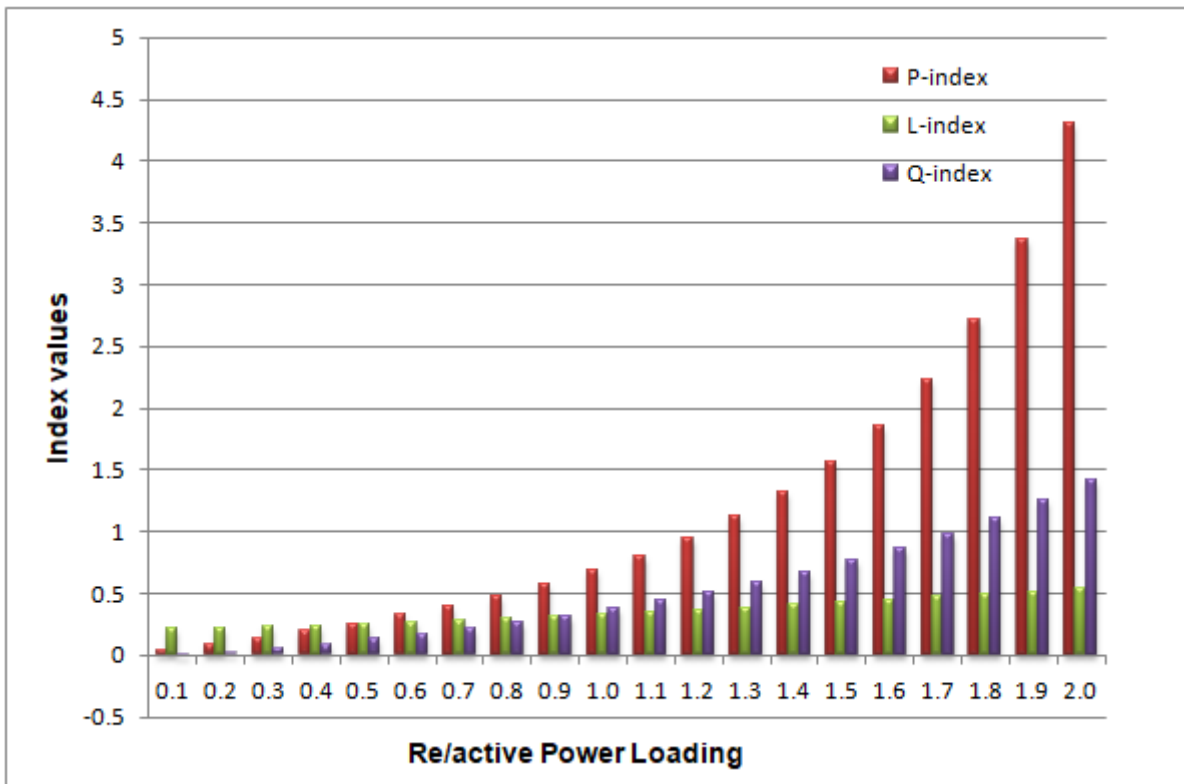


Figure 5.3: Comparison of Different Indices for Two Bus System

5.2 IEEE 14 bus System Results

Performance of P-index, L-index and Q-index is carried out for the IEEE 14 bus test system regarding to power flow solution. Figure 5.4 shows the single line diagram of the IEEE 14 bus test system [22]. Complete data for this test system is given in [29].

Results of P-index, L-index and Q-index at base loading condition is shown in table 5.3. According to obtained results, all three P-index, L-index and Q-index indicates that the bus number 14 is the weakest bus in context with static voltage stability of the system.

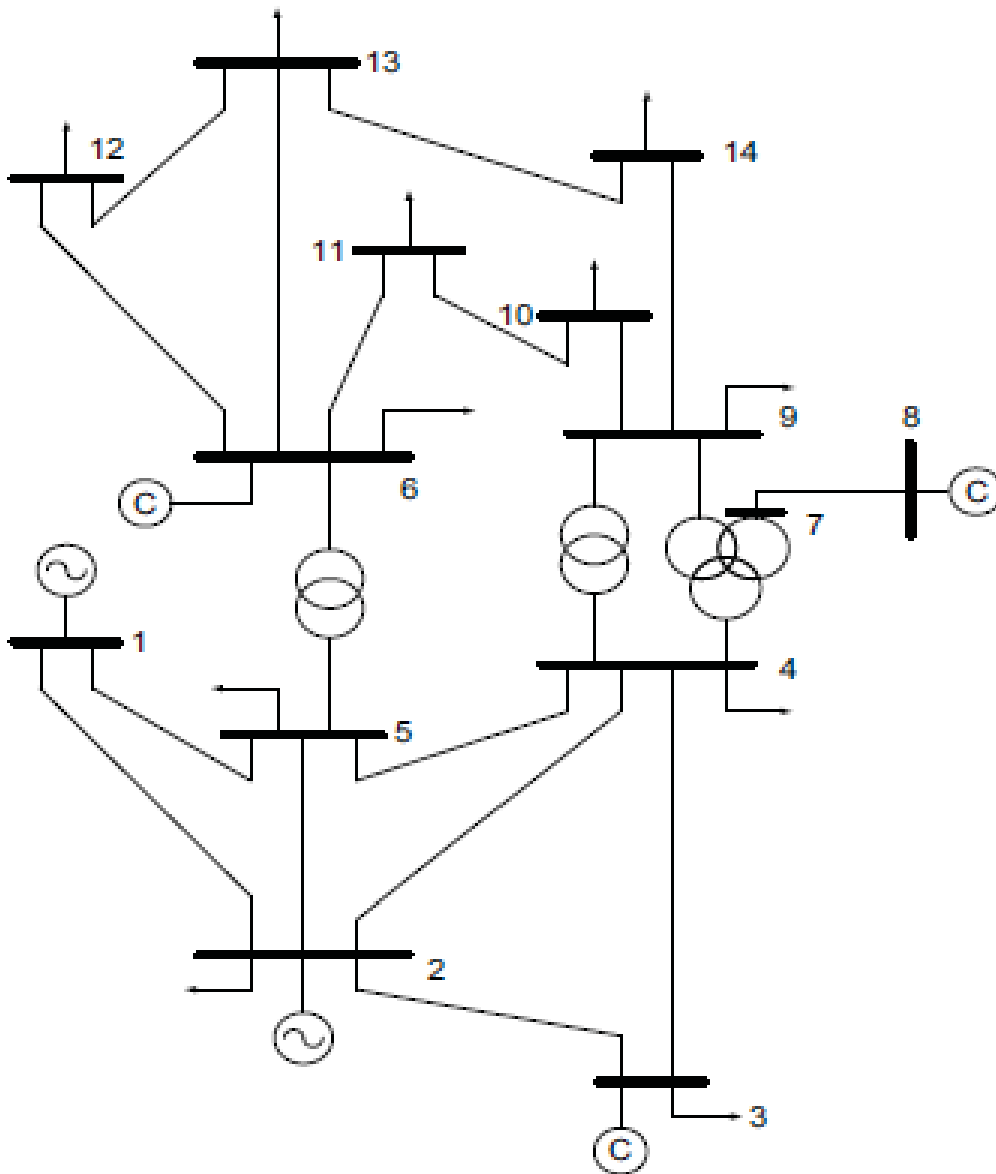


Figure 5.4: Single Line Diagram of IEEE 14 Bus system

Table 5.3: Values of Indices for Loas Buses of IEEE 14 Bus System

Bus No.	P-index	L-index	Q-index
4	0.0990	0.0630	0.1676
5	0.0831	0.0433	0.1615
7	0.0643	0.0771	0.1080
9	0.0775	0.1403	0.1257
10	0.0914	0.1373	0.1420
11	0.0894	0.0767	0.1487
12	0.1131	0.0475	0.1578
13	0.1262	0.0580	0.1958
14	0.1486	0.1117	0.2423

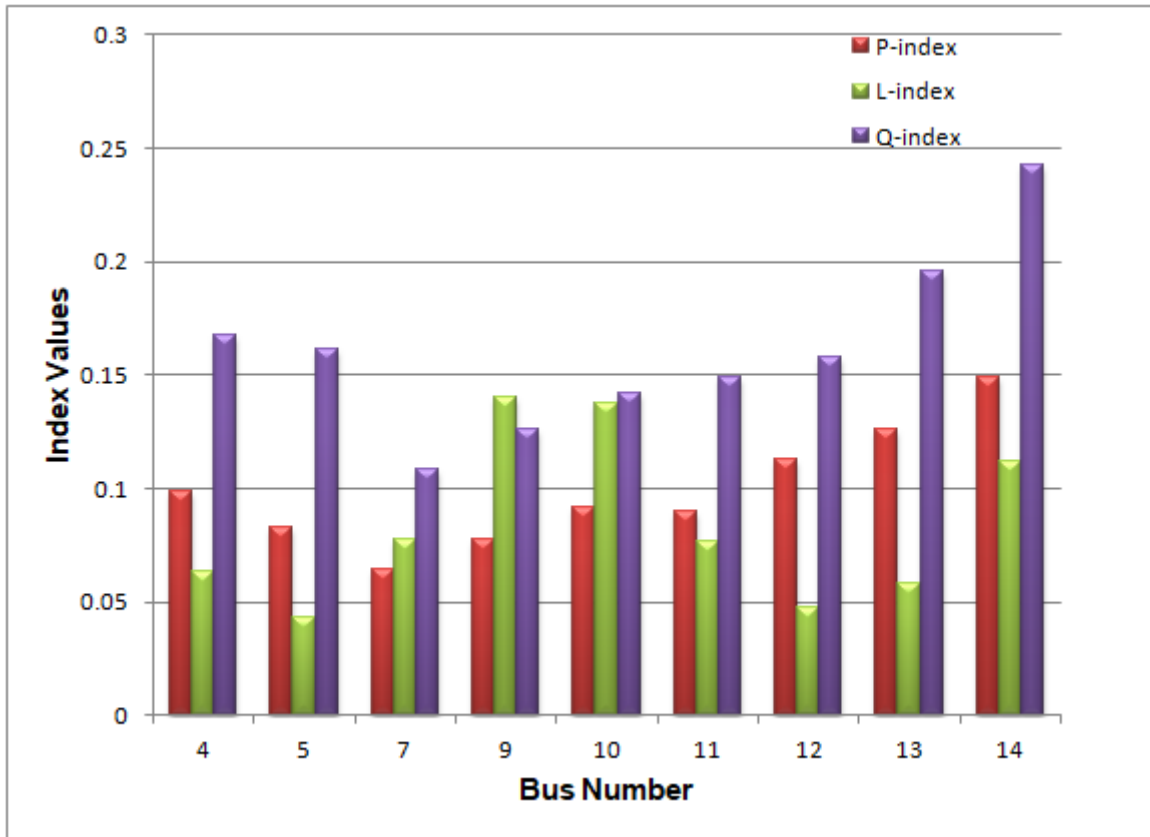


Figure 5.5: Comparison of Different Indices for IEEE 14 bus System

5.3 Results of Generator Reactive Limits for IEEE Bus System

5.3.1 Weakest Buses

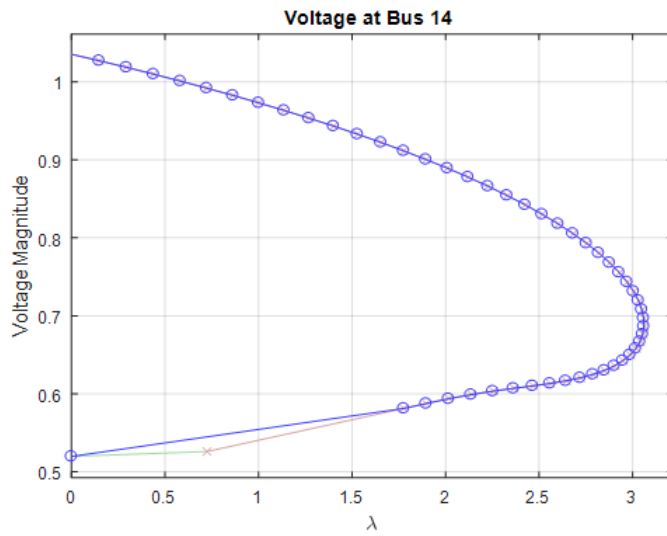


Figure 5.6: Bus 14- Without Generator Q-Limits

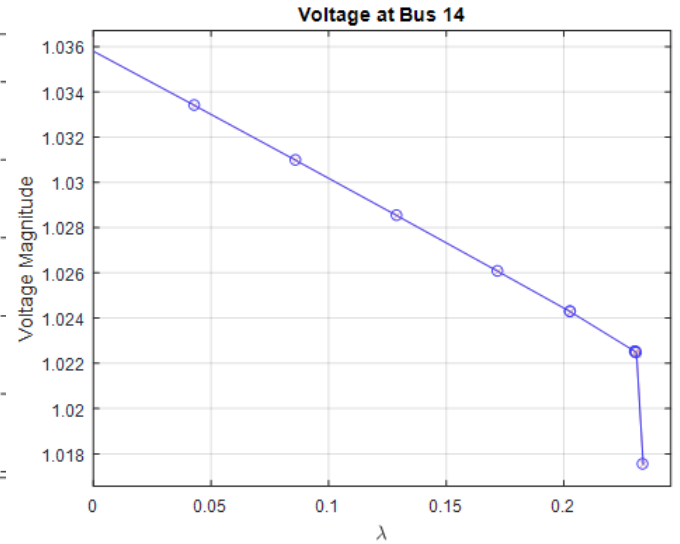


Figure 5.7: Bus 14- With Generator Q-Limits

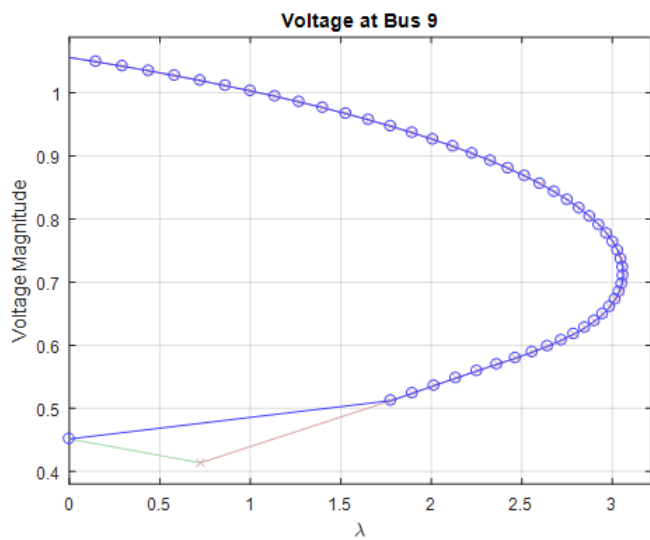


Figure 5.8: Bus 9- Without Generator Q-Limits

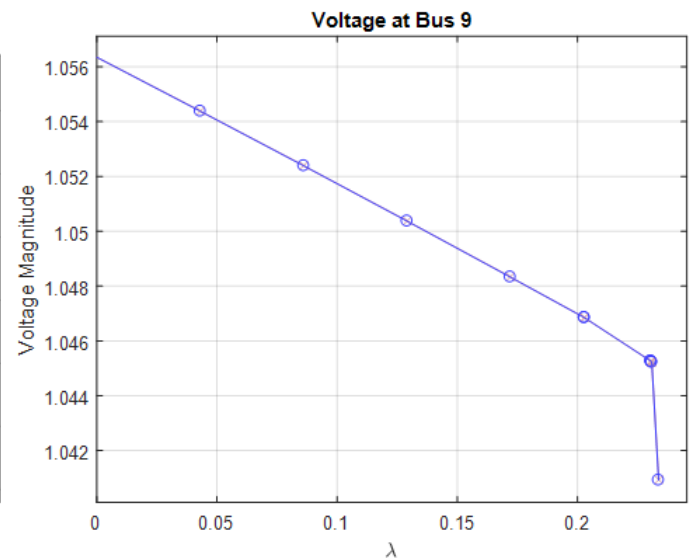


Figure 5.9: Bus 9- With Generator Q-Limits

5.3.2 Strongest Buses

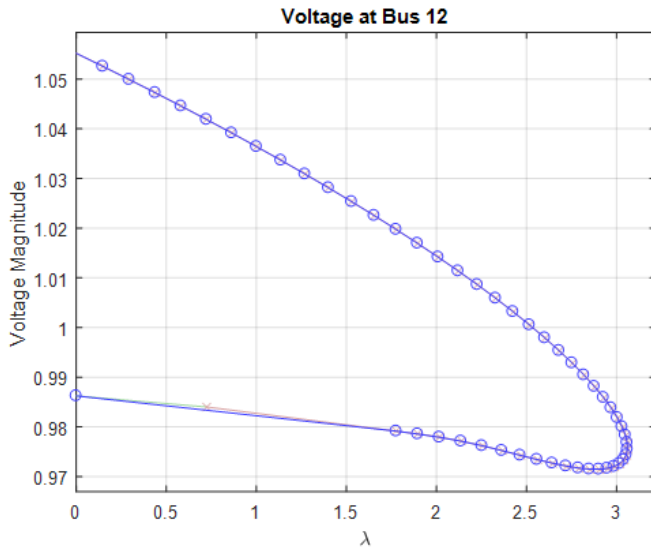


Figure 5.10: Bus 12- Without Generator Q-Limits

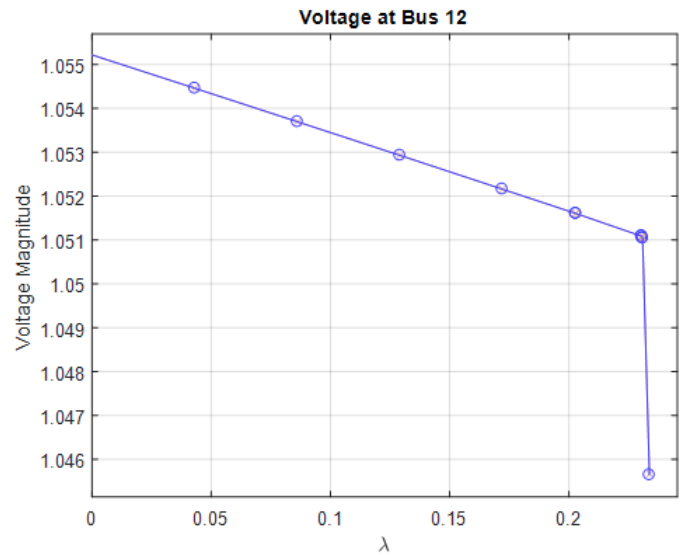


Figure 5.11: Bus 12- With Generator Q-Limits

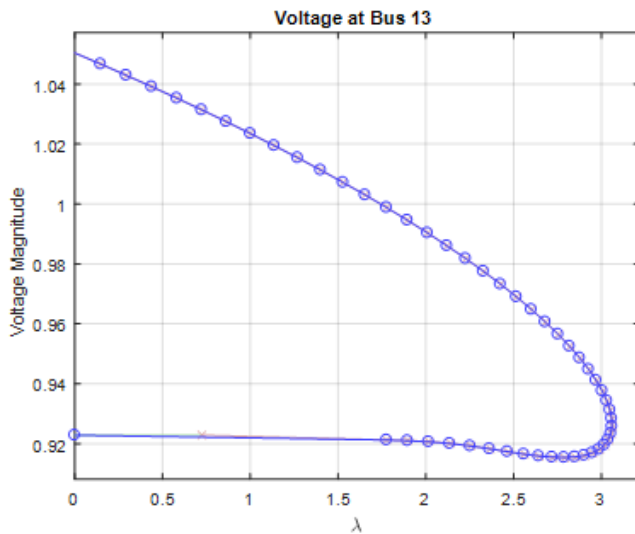


Figure 5.12: Bus 13-Without Generator Q-Limits

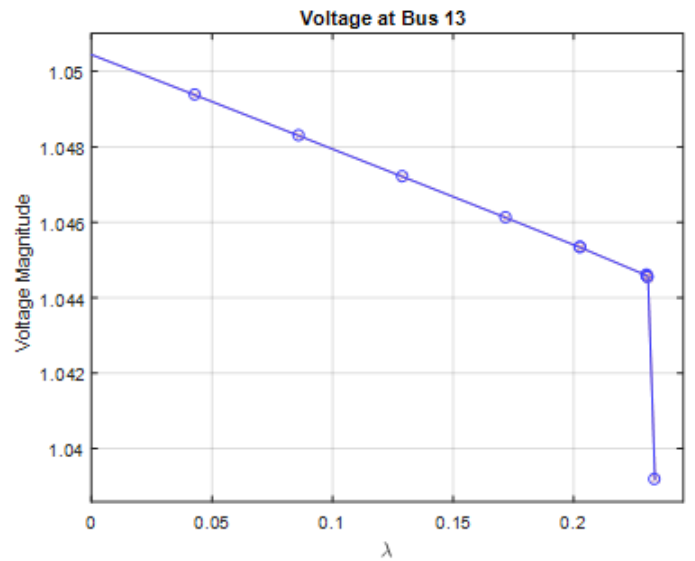


Figure 5.13: Bus 13- With Generator Q-Limits

When the generator field and armature current limiters are activated, the voltage dependency of generator reactive power increases. Also the operation of over exciter limiter causes restricted supply of generator reactive power. The bus voltages remain within limit even when most critical line is out of circuit. But for this to happen, maximum loadability of the system has to be sacrificed to some extent. Therefore an operating point will be more voltage stable

on the V-P curve under blocked operation of overexcited limiters. The end result is that the system moves closer to voltage instability when reactive power limit of generator is activated.

The generators adjust the reactive power within limits by changing the bus voltage. From the results cited above it is imperative that by enforcing the generator reactive power limits the voltages of the load buses are maintained well within limits while sacrificing system maximum loadability.

Conclusion

From the literature, we can see that many indices have been proposed earlier but they suffered from accuracy problem near voltage stability limit, high computational cost and complex structure due to dynamic nature. The Q-index is robust in a sense that it doesn't require any extension moving from two bus to multibus system. The Q-index is also found to be more accurate and superior in estimating the distance to collapse. With the help of Q-index the speed of the calculation is fast as compare to the other indices. The index is robust in a sense that it doesn't require any extension moving from two bus to multi bus system. Q-index was also found to be superior in estimating the distance to collapse.

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