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Exponential synchronization of fractional-order complex chaotic systems and its application

Vijay K. Yadav[a,*], Vijay K. Shukla[b], Subir Das[b]

[a] Department of Mathematics & Humanities, Institute of Technology, Nirma University, Ahmedabad 382481, India
[b] Department of Mathematical Sciences, Indian Institute of Technology (BHU), Varanasi 221005, India

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ABSTRACT

In this article, exponential synchronization between fractional order chaotic systems has been studied by using exponential stability theorem. The stability analysis has been done with help of an existing lemma, which is given for Lyapunov function for fractional order system. The fractional order complex chaotic systems viz., Lorenz and Lu systems are considered to illustrate the exponential synchronization. The numerical simulation results are presented through graphical plots to verify the effectiveness and reliability of exponential synchronization. The application in communication through digital cryptography is also discussed between the sender (transmitter) and receiver using the exponential synchronization. A well secured key system of a message is obtained in a systematic way.

1. Introduction

In non-linear dynamics, chaos is one of the interesting phenomena which occurs in a deterministic dynamical system. It exhibits unpredictable behavior and sensitive dependence on initial conditions. All mechanical systems in nature and technology are vulnerable to chaos. Chaos appears in many fields like physics, chemistry, computer science, medicine, and engineering, etc. It is often used as a tool for encryption of images, information coding, and secure communication even having challenges in practical applications. So the chaotic systems are suitable for cryptography and researchers are showing the applications of non-linear systems with chaos in digital cryptography. A nonlinear system can illustrate a chaotic behavior, if the maximum Lyapunov exponent is positive and therefore, the divergence of nonlinear field is negative. Basically, the maximum entropy is important for a robust secure communication scheme. Hence, an extensive range of chaotic and hyperchaotic systems have been proposed by scientists and researchers in recent years. Chaotic systems are nonlinear, but most of the nonlinear systems we observe can never be chaotic. Their order is either too low or their parameter space does not have any combination of parameters suitable for demonstration of chaos. Nonlinear chaotic systems are bounded when all trajectories are ultimately stayed in a bounded region around an equilibrium point. Control techniques including fuzzy, sliding mode, nonlinear control and robust control are essential features in synchronization schemes of chaotic systems.

The chaotic systems have also been used in some other areas of sciences such as information processing, computing, telecommunications, and electrical engineering [1–4]. In the year 1963, Lorenz [5] was the first to establish a three-dimensional chaotic attractor of autonomous system, when he was studying the atmospheric convection. The chaotic system is a three dimensional or higher dimensional dynamical system that contains one positive Lyapunov exponent having additional complex and rich dynamical behaviors and also have the bounded and unstable behaviors. Chaos is found in different fields of physics, mathematics, engineering, secure communication, data encryption, biology, medicine, etc. Ling et al. [6] introduced a new chaotic attractor in reverse butterfly-shaped type, which was confirmed through an electronic circuit experiment. Fractional calculus has become an active topic among researchers in recent years, and it has potential applications in many areas such as image encryption, secure communication etc. The advantage of the fractional calculus is that the fractional order dynamical accumulates all information about the function in a certain time, which is known as memory property. It also describes the hereditary property of the several processes and it allows the greater flexibility in the model. The fractional-order derivative is a nonlocal operator; it means that the future state depends upon the present state as well as all the history of its previous states. The realistic dynamic models related to real life are also depen-

* Corresponding author.
E-mail address: vijayky999@gmail.com (V.K. Yadav).

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