# DESIGN OF INTEGRAL SLIDING MODE CONTROLLER FOR MIMO NONLINEAR SYSTEM

**Project Report**

*Submitted in partial fulfillment of the requirements*

*For the degree of*

**MASTER OF TECHNOLOGY**

**IN**

**INSTRUMENTATION AND CONTROL ENGINEERING**

**(Control and Automation)**

By

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**INSTITUTE OF TECHNOLOGY**

**NIRMA UNIVERSITY**

**AHMEDABAD-382 481**

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Under the guidance of

**Prof. Rajesh L. Zadfiya**


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**Department of Electrical Engineering**

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# Declaration

This is to certify that

i) The thesis comprises my original work towards the degree of Master of Technology in Instrumentation and Control Engineering at Nirma University and has not been submitted elsewhere for a degree.

ii) Due acknowledgement has been made in the text to all other material used.

**Rikin C. Hingrajiya**

## Certificate

This is to certify that the Major Project report entitled **"Design of Integral Sliding Mode Controller for MIMO Nonlinear System"** submitted by Mr. Rikin C. Hingrajiya (12MICC04), towards the partial fulfillment of the requirements for the award of degree in Master of Technology in the field of Instrumentation and Control Engineering drives of Nirma University is the record of work carried out by him under our supervision and guidance. The work submitted has in our opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other university or institution for award of any degree or diploma.

**Date:** **Place**:Ahmedabad

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**Rikin C. Hingrajiya**

## Abstract

In this project, the controller is designed and simulated using sliding mode controller with integrator for uncertain non-linear Multi Input Multi Output (MIMO) three tank system. Robustness against unknown disturbances are considered and improved using SMC with integrator. A sliding mode controller is a robust controller against the external disturbance. Consequently, robustness against uncertainties increases from the very beginning of the process. Furthermore, the control laws considerably alleviate chattering along the switching manifold. In addition, the performance of the controller boost up in the presence of uncertainties. An ideal sliding mode controller has a discontinuous switching function like saturation, hyperbolic, signum and tangent. Such a switching functions produce a chattering phenomenon that is highly undesirable. This chattering phenomenon is reduced by integral sliding mode controller. For the above mention reason, the robust controller is designed for MIMO non-linear three tank system. A comprehensive comparative analysis carried out with sliding mode control and integral sliding mode control which demonstrates superiority of the newly designed control law. A reduced chattering regulate uncertain non-linear system with improved performance in the presence of external disturbances, which ensures the robustness of the proposed integral sliding mode controller.

# List of Symbols

1. $h_1$ = Height of tank 1
2. $h_2$ = Height of tank 2
3. $h_3$ = Height of tank 3
4. $h_{2d}$ = Desired value of tank 2
5. $h_{3d}$= Desired value of tank 3
6. $\dot{h}_2$ = Derivative value of tank 2
7. $\dot{h}_3$ =Derivative value of tank 3
8. $\dot{h}_{2d}$ = Derivative value of desired value of tank 2
9. $\dot{h}_{3d}$ =Derivative value of desired value of tank 3
10. a = Tank section
11. $s_n$= Cross section of valve
12. $a_{zi}$= Flow correction term(i=1,2,3)
13. $b_{zi}$= leakage flow correction term(i=1,2,3)
14. $h_{max}$= Maximum water level in each tank(i=1,2,3)
15. $Q_{imax}$= Maximum inflow through pump(i=1,2)
16. g =Gravitational constant
17. $C_1, C_3$ =Parameter of opening and closing of valve
18. $Q_1, Q_2$ =System Inputs

# List of Abbreviations

1. SMC = Sliding mode control
2. ISMC = Integral sliding mode control
3. VSC = variable structure control
4. MIMO = Multi input multi output
5. Sign = Signum function

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# Chapter 1

# Introduction

The sliding mode control theory is a plays important role in variable structure control(VSC). This sliding mode controller theory is robust control method. This sliding mode control concept is related to systems state space. The SMC is consist of two parts one is sliding manifold(sliding surface) and another is controller design. The sliding mode controller is a robust controller against the uncertainties and parameter variation[1]. In the sliding mode controller is generate the chattering phenomenon. This switching phenomenon is continuously switching between two sates on sliding surface. A highly switching is affect or damage the system itself[2].

In this project, literature survey is carried out on design and application of sliding mode controller and integral sliding mode controller for non-linear multi input multi outout(MIMO) three tank system. The sliding mode control is important for both linear and non-linear system. This robust controller gives better accuracy and robustness against disturbances. It's also provide the minimum chattering. It is a leading contender for Three tank multi input multi output system. This three tank MIMO system is designed based on SMC and integral sliding mode controller(ISMC).

# Chapter 2

# Design of Sliding Mode Controller

## 2.1 Variable Structure Control

A Variable structure system is a dynamical system whose structure changes with the current value of the state. It is made up of two independent structures along with a switching logic to switch over between the structures[4].

### 2.1.1 Example of Variable structure system

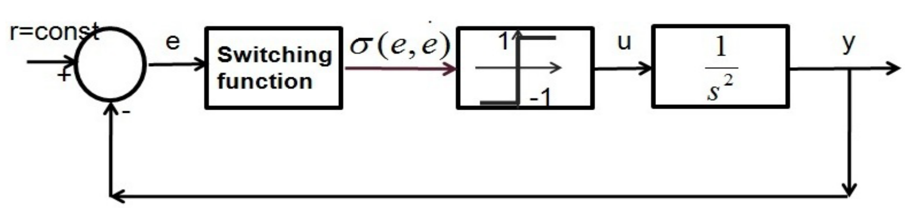



Figure 2.1: Variable Structure System

- Let

$$X1 = e \tag{2.1}$$

$$X2 = \dot{e} \tag{2.2}$$

- The System model can be given as:

$$\dot{x1} = x2 \tag{2.3}$$

$$\dot{x1} = -u \tag{2.4}$$

- The trajectories are given by the following equations:

$$x1(t) = -1/2x2^2(t) + x1(0) + 1/2x2^2(0) \tag{2.5}$$

$$x1(t) = 1/2x2^2(t) + x1(0) - 1/2x2^2(0) \tag{2.6}$$

- Phase portrait of the two structures are as shown in Fig. Each structure

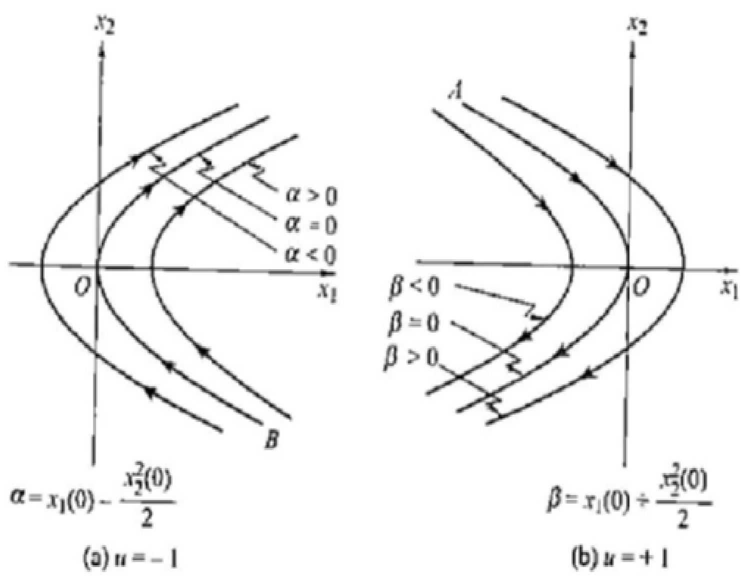



Figure 2.2: Phase portrait

is a family of hyperbola and they are not asymptotically stable. By choosing a suitable switching logic, we can improve system stability[10,11].

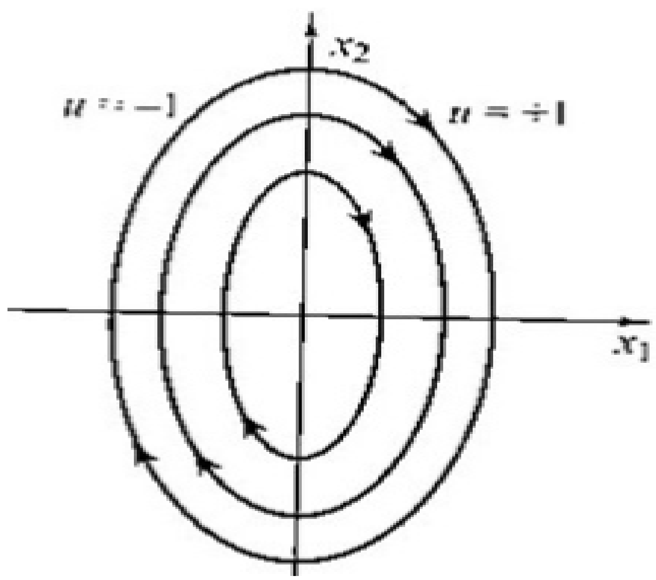
### 2.1.2 Effect of switching logic

If the switching of control law (u) is done as:

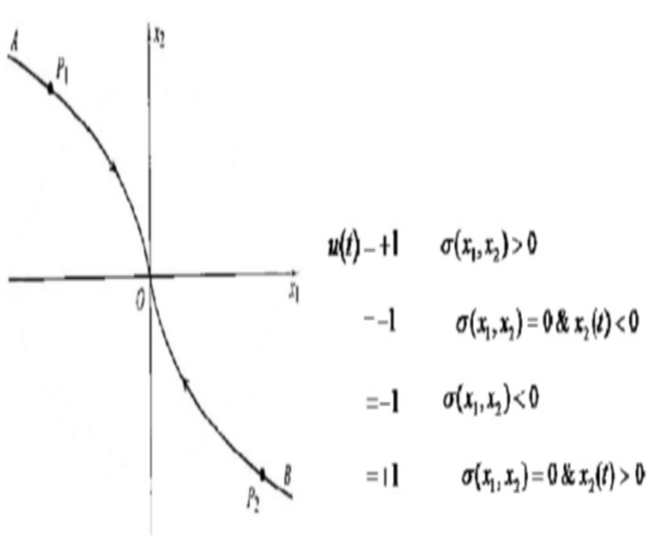
$$u(t) = +1; \sigma(x1, x2) > 0 \tag{2.7}$$

$$u(t) = -1; \sigma(x1, x2) < 0 \tag{2.8}$$

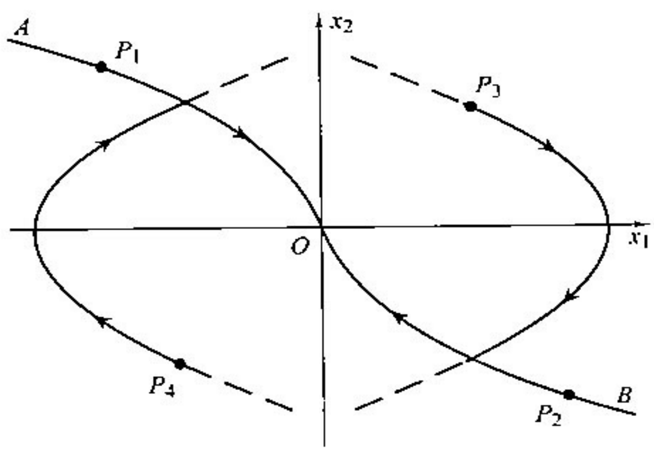
Following phase portrait is obtained which shows the system being marginally stable.




If the switching surface($\sigma$) and control law are as shown below.




It is seen from the resulting phase portrait below that the system is asymptotically stable.



**For the switching logic in example:**

- Equation of optimal switching curve AOB is obtained as:

$$x_1(t) = -\frac{1}{2}x_2(t)\,|x_2(t)| \tag{2.9}$$

by setting

$$x_1(0) + \frac{1}{2}x_2^2(0) = x_1(0) - \frac{1}{2}x_2^2(0) = 0 \tag{2.10}$$

- Switching function is

$$\sigma(x_1, x_2) = x_1(t) + \frac{1}{2}x_2(t)\,|x_2(t)| = 0 \tag{2.11}$$

**The given switching logic works as under:**

- If the initial state point lies at $(P_1)$ on the segment A-O, the state point $(x_1(t); x_2(t))$ is driven to the origin along a segment of a parabola corresponding to u=+1.
- If the initial point lies at $(P_2)$ on the segment B-O, the state point $(x_1(t);x_2(t))$ is driven to the origin along a segment of a parabola corresponding to u=-1.
- If the initial state point lies above or below the curve A-O-B, then only one switching is required to drive the state point to the origin. Consider the initial point at $(P_3)$, which is above the curve A-O-B. The state point $(x_1(t); x_2(t))$ follows a parabola corresponding to u=+1 till it reaches the segment B-O. This is followed by switching of the control to -1, and driving of the state point to the origin along B-O with u=-1.

- Consider the initial point at ($P_4$), which is below the curve A-O-B. The state point $(x_1(t); x_2(t))$ follows a parabola corresponding to u=-1 till it reaches the segment A-O. This is followed by switching of the control to +1, and driving of the state point to the origin along A-O with u=+1.

# 2.2 SMC Design

## 2.2.1 Hyperplane Design

In the hyper plane design stage, a sliding hyper plane is computed on the basis of the nominal model, a hyper plane design is fully presented for both eigenvalue allocation and optimal sliding-mode approaches in a unified manner for linear and nonlinear systems. On the basis of the partition transformation method for linear systems in Utkin et al. 1979, a direct allocation method that may be the simplest way to see what the invariance property really means and is extendable into nonlinear systems.

A hyper plane normalization is proposed to greatly simplify a control function in the controller design stage. There are 2 types of eigenvalues to be proposed: sliding-eigenvalue and hyperplane-eigenvalue. Sliding-eigenvalues determine the dynamical model of three tank systems. Hyperplane-eigenvalue is determines the desired values of sliding-eigenvalues that is related to desired dynamic of the three tank system. In addition, the concept of sliding-eigenvalue is conveniently applied in the stability problem and the invariance property[8].

$$\dot{x} = Ax + Bu \tag{2.12}$$

$$s = Hx \tag{2.13}$$

$$\dot{s} = H\dot{x} \tag{2.14}$$

Here,

$$\dot{x} = Ax + Bu \tag{2.15}$$

$$H\dot{x} = H(Ax + Bu) \tag{2.16}$$

### 2.2.2 Controller Design

A VSS controller design will be fully developed in a unified manner that is extendable into robust control and MIMO nonlinear systems where a VSS control is a discontinuous sliding mode control (SMC). Based on the concept of sliding-eigenvalues introduced, we propose a stability criterion which is much simpler than the current approach.

By the sliding condition, a control function is first computed in a standard form as in Utkin 1977. The sliding condition is a necessary condition to guarantee the existence of the sliding mode[10].

Lyapunov's direct stability criterion equation

$$V = 1/2 * s^2 \tag{2.17}$$

$$\dot{v} = s\dot{s} \tag{2.18}$$

sliding condition is

$$s.\dot{s} \ < 0 \tag{2.19}$$

## 2.3 Requirements of SMC Design

The SMC design approach consists of two components. The first involves the design of a switching function so that the system motion on the sliding manifold (termed the sliding motion) satisfies the design specifications. The second is concerned with the selection of a control law, which will make the sliding manifold attractive to the system state in the presence of external and internal disturbances/uncertainties.

The main features of the sliding mode include its insensitivity to external and internal disturbances matched by the control, ultimate accuracy, and finite-time reaching of the transient.

The requirements for the SMC design are:

- The sliding surface should attract the trajectories when they are in the vicinity and once the trajectory intersects the sliding surface, it should stay on it thereafter.
- The sliding mode exists only if, in the vicinity of the sliding surface, the state velocity vectors are directed towards the surface.
- The trajectories should reach the sliding surface in finite time.

## 2.4 Properties of Sliding Mode Control

- Sliding mode control improves the stability.
- Entire dynamics of the system is governed by the sliding line/surface parameters only, which is reduced order function.
- During sliding mode, dynamics are independent of system parameters.

# Chapter 3

# Literature Review

## 3.1 Sliding Mode Control

The sliding mode control is mainly use for non-linear system. In this method dynamic model of non-linear system is forces to the system to sliding surface. The control law are defined for continuous function of time. The sliding mode controller is gives the continuously switching between one state to another state. This control method is also known as variable structure control method. The multiple control structures are designed so that trajectories always move toward an adjacent region with a different control structure and so the ultimate trajectory will not exist entirely within one control structure.

The switching logic is slide along the control structure. The switching of the system is slide towards the surface is called sliding mode and the switching area is known as sliding surface. Switching logic means chattering phenomenon. This fast switching can damage the system. So, less switching is better for system. In the context of modern control theory, any variable structure system, like a system under SMC, may be viewed as a special case of a hybrid dynamical system as the system both flows through a continuous state space but also moves through different discrete control modes[17].

In the theory of sliding mode control, it's consist of two modes. The first one is the reaching mode and another one is sliding mode. In the reaching mode, system starts from the non zero initial condition and reaches at the sliding surface. after some it's reaches at the origin through chattering phenomenon. A SMC design is composed of 2 phases, hyperplane design and controller design. In the sliding mode control theory Lyapunov stability Criterion Theorem is applied for the stability purpose.

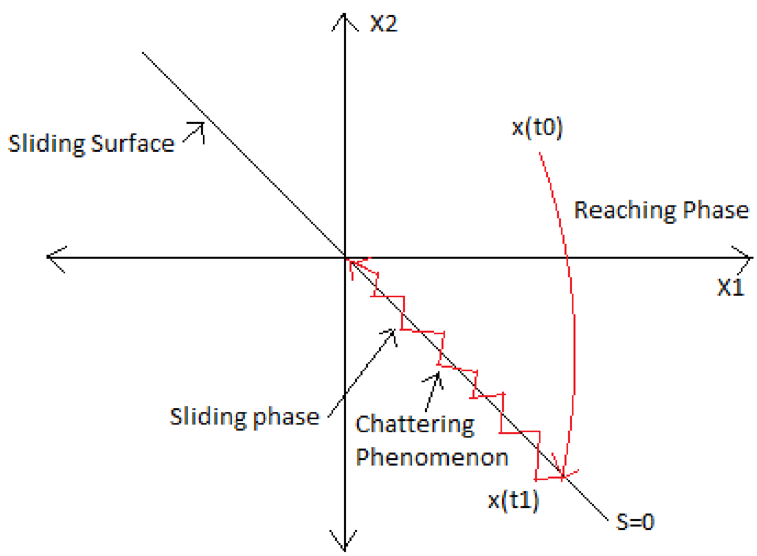



Figure 3.1: Sliding surface and Reaching Phase

In the reaching mode, the control dynamics depend on system parameters, but in the sliding mode they depend on the hyper plane; this is the invariance property of the sliding mode.

The chattering phenomenon is the main disadvantage of these types of controllers and all efforts are concentrated on decreasing this singularity. This chattering phenomenon is reduced by the robust sliding mode controller[15].

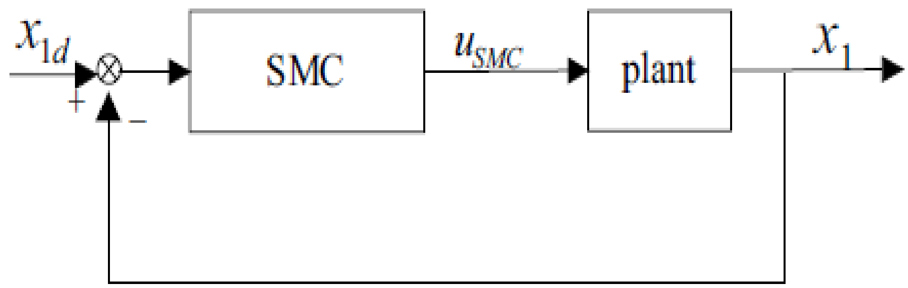



Figure 3.2: Structure of the used control system

Here, a shown in fig.(3.2) is a block diagram of close loop control of sliding mode controller. it's consist of controller and plant. plant means dynamic model of three tank MIMO system is put on this block. Plant gives actual output $X_1$. This $X_1$ is compared with $X_{1d}$ and it's generate the error. Controller can take a action to minimize this error. In the SMC block different control law are defined to control the external disturbances and error.

**SMC with Saturation function**

The major inconvenient of classic sliding mode controller is the existence of chattering phenomenon. To avoid this problem, saturation function are used.

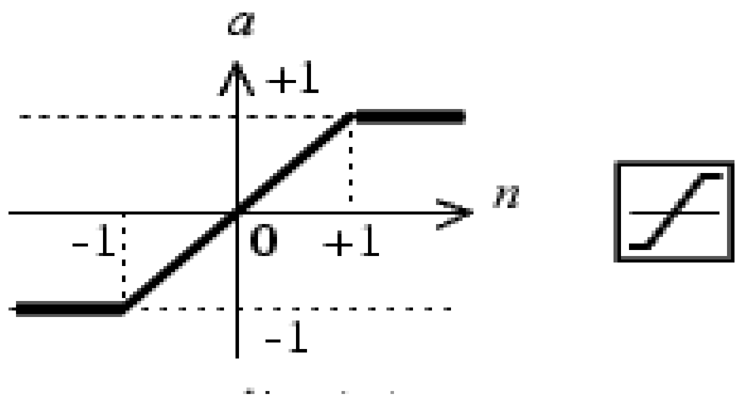



Figure 3.3: Saturation function

**SMC with Signum function**

In mathematics, the sign function or signum function is extracts the sign of a real number. In mathematical expressions the sign function is often represented as sgn.

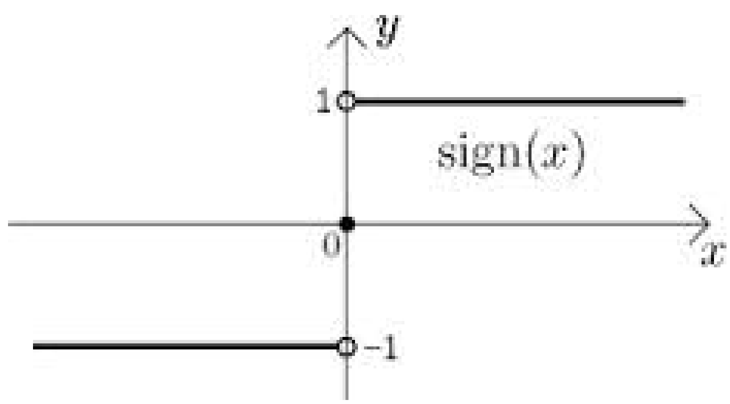



Figure 3.4: Signum function

$$sgn(x) = -1 \;\; if x < 0$$

$$sgn(x) = 0 \;\; if x = 0$$

$$sgn(x) = +1 \quad if x > 0$$

## 3.2 Integral Sliding Mode Control

The concept of integral sliding mode control (ISMC) has been proposed and defined by Utkin and Shi in 1996. An integral sliding mode controller is constructed by incorporating an integral term in the switching surface. The main feature of the ISMC is that, when in sliding mode, the sliding manifold spans the entire state space. The main advantage of the ISMC, in the sequel, is the ability to directly design and specify the sliding manifold in the entire state space. This is achieved through the integral term, which provides a direct means for us to shape the sliding manifold based on the knowledge of the system nominal part.

In integral sliding mode controller is applied for minimize the steady state error[5].This development is introduced and justified only by tests on specific systems. Our idea consists on reconstituting a control law to eliminate steady-state error created by disturbance. To do so we added an integral action when the trajectories of states approach their references[6].

An immediate consequence is that we can easily design a non-linear ISMC to stabilize the sliding manifold in the ideal closed-loop. This project proposes a dynamic integral sliding mode controller for uncertain MIMO nonlinear systems. The new controller design incorporates an integral sliding manifold which results in the elimination of reaching phase. Consequently, sliding starts from the beginning of the process and the system becomes robust against external disturbance and parametric uncertainties with improved performance and considerable attenuation in chattering[4].

# Chapter 4

# Three Tank MIMO System

## 4.1 System Description

A shown in fig.(4.1) is a three tank multi input multi output system, this system having two inputs and three outputs. A coupled tank system consist on three cylindrical pipes of identical section $S_n$. This pipes is communication between two tanks $T_1$ and $T_2$ . This $T_1$ and $T_2$ are valves; these valve can control manually. flow rate of connection pipes is controlled by ball valves $a_{z1}$ and $a_{z2}$ and system having one outlet pipe located at the bottom of tank $T_3$[6].

In this system three others pipes each one is connected at the bottom part of the each tank; they gives the direct connection (out flow rate) to the storage with ball valves $b_{z1}$, $b_{z2}$ and $b_{z3}$, respectively, this ball valve are manipulated manually. in this system pumps 1 and 2 are supplied the water from the storage tank with flow rates $Q_1(t)$ and $Q_2(t)$, respectively. The level of $h_1(t)$, $h_2(t)$ and $h_3(t)$ tanks carried out by the piezo resistive differential pressure sensors[9].

The state equations are obtained by writing that the variation of the water volume in a tank is equal to the difference between the incoming flow and the outgoing flows, that means, the water of the tanks 1 and 2 can flow toward the tank 3.

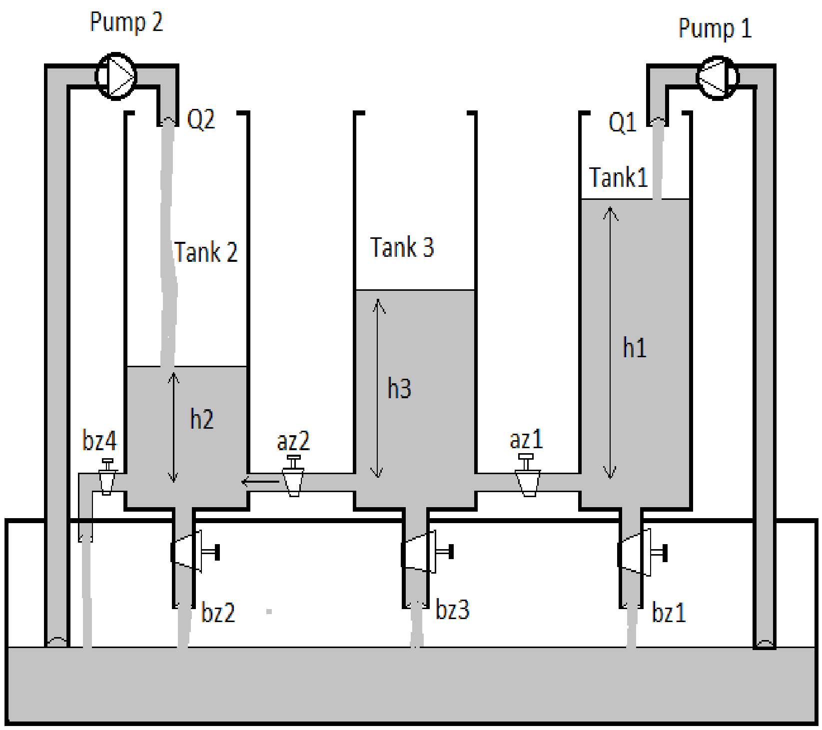



Figure 4.1: Schematic diagram of three tank MIMO system

Then the system can be represented by the following equation[14]:

$$\dot{h}_i(t) = \frac{1}{a}(Q_i^{in}(t) - Q_{ij}^{out1}(t) - Q_{ij}^{out2}(t) \qquad i = 1, 2, 3 \tag{4.1}$$

Where $\dot{h}_i(t)$, $Q_i^{in}(t)$ and $Q_{ij}^{out1}(t)$ are respectively the levels of water, the input flow and the output flow rates. The parameters of three tank system are defined in the following table: The controlled signals are the water levels $(h_2, h_3)$ of tank 2 and tank 3. These levels are controlled by two pumps. The system can be considered as a multi inputs multi outputs system (MIMO) where the inputs are in flow rate $Q_1$, $Q_2$ and the outputs are liquid levels $h_2$,

$h_3$ using the law of Torricelli[14].

Torricelli's law can be demonstrated in the spouting can experiment, which is designed to show that in a liquid with an open surface, pressure increases with depth. It consists of a tube with three separate holes and an open surface. The three holes are blocked, then the tube is filled with water. When it is full, the holes are unblocked. The jets become more powerful, the fluid exit's velocity is greater the further down the tube they are.

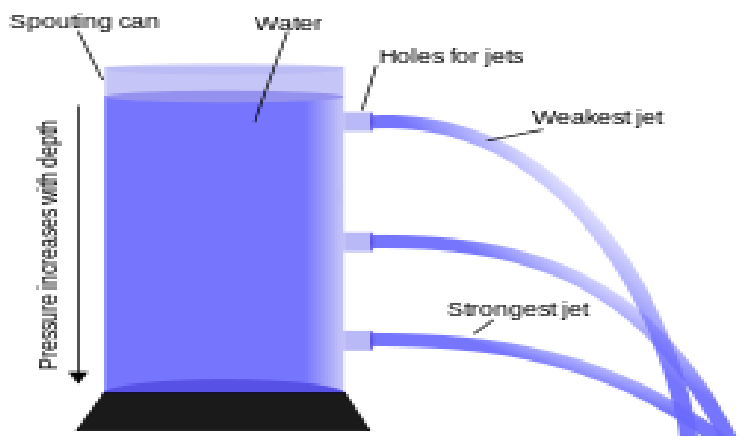



Figure 4.2: Fluid system

Torricelli's law, also known as Torricelli's theorem, is a theorem in fluid dynamics relating the speed of fluid flowing out of an opening to the height of fluid above the opening. Torricelli's law states that the speed of efflux, v, of a fluid through a sharp-edged hole at the bottom of a tank filled to a depth h is the same as the speed that a body (in this case a drop of water) would acquire in falling freely from a height h, i.e. $v = \sqrt{2gh}$, where g is the acceleration due to gravity (9.81 N/kg). This last expression comes from equating the kinetic energy gained, $\frac{1}{2}mv^2$, with the potential energy lost, mgh , and solving for v[18-19].

## 4.2 System Modelling

The constant parameters of non-linear MIMO three tank system are defined in the table: in this system two controlled variables are the water levels($h_2, h_3$) of height of tank 2 and height tank 3 respectively. These levels are controlled by two pumps. The system is considered as a multi inputs multi outputs system (MIMO) where the inputs are inflow rate Q1, Q2 and the outputs are liquid levels h2, h3. Then the three tanks system can be modelled by the following three differential equations[16]:

$$\frac{dh1}{dt} = -c1sign(h1-h3)\sqrt{|h1-h3|} + \frac{Q1}{a} \tag{4.2}$$

$$\frac{dh2}{dt} = c_3 sign(h3-h2)\sqrt{|h3-h2|} - B_4\sqrt{h_2}\frac{Q2}{a} \tag{4.3}$$

$$\frac{dh3}{dt} = c_1 sign(h1-h3)\sqrt{|h1-h3|} - c_3 sign(h3-h2)\sqrt{|h3-h2|} \tag{4.4}$$

Here,$C_1$ and $C_3$ is calculated by;

$$C_i = \frac{1}{a} a_{zi} s_n \sqrt{2g} \qquad i = 1, 3 \tag{4.5}$$

$B_4$ is calculated by;

$$B_j = \frac{1}{a} b_{zi} s_n \sqrt{2g} \qquad i = 1, 2, 3, 4 \tag{4.6}$$

Sliding Surface is given by[3]:

$$S_1 = \lambda(x_2 - x_{2d}) + (\dot{x}_2 - \dot{x}_{2d}) \tag{4.7}$$

$$S_2 = \lambda(x_3 - x_{3d}) + (\dot{x}_3 - \dot{x}_{3d}) \tag{4.8}$$

Sliding surface is designed with the help of error and rate of change of error. In this system, as per eq. no.(4.7) $(x_2 - x_{2d})$ and $(\dot{x}_2 - \dot{x}_{2d})$ is a $e$ and $\dot{e}$ respectively. similarly eq. no.(4.8) is designed.

In this system two controlled signals ($h_2$,$h_3$) of tank 2 and tank 3. So, two sliding surface are required $S_1$ and $S_2$ for tank 2 and tank 3.

## 4.3 System Controller Design

with the help of $l_1, l_2, u_1$ and $u_2$ developed a controller.

$$l_1 = c_3\sqrt{\,|h3 - h2\,|} - B_4\sqrt{h_2} - \dot{x}_{2d} \tag{4.9}$$

$$l_2 = \lambda_2(c_1\sqrt{|h1 - h3|} - c_3\sqrt{|h3 - h2|} - \dot{x}_{3d}) + c_1\frac{-2c_1\sqrt{|h1 - h3|} + c_3\sqrt{\,|h3 - h2|}}{2\sqrt{|h1 - h3|}} \tag{4.10}$$

$$-c_3\frac{c_1\sqrt{|h1 - h3|} + 2c_3\sqrt{\,|h3 - h2|} + B_4\sqrt{x_2}}{2\sqrt{|h3 - h2|}} - \dot{x}_{3d} \tag{4.11}$$

The control SMC with ISMC is defined by:

$$u = -b^{-1}\left(l + \begin{bmatrix} k1sign(s1)) \\ k2sign(s2)) \end{bmatrix}\right) + K\begin{bmatrix} \int (h_2 - h_{2d})\,dt \\ \int (h_3 - h_{3d})\,dt \end{bmatrix} \tag{4.12}$$

Here,

$$b^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{4.13}$$

$$b^{-1} = \begin{bmatrix} -\frac{ac_3\sqrt{|x1-x3|}}{\lambda_1 c_1\sqrt{|x3-x2|}} & \frac{2a\sqrt{|x1-x3|}}{c_1} \\ \frac{a}{\lambda_1} & 0 \end{bmatrix} \tag{4.14}$$

Here, value of K matrix is given in table no.(5.1)

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \tag{4.15}$$

Put the eq.(4.14) and (4.15) in eq.(4.12). finally we get controller output in the form of u(1)and u(2). this u(1) and u(2) is inputs of a tank 1 and tank 2.

$$u(1) = -b_{11}l_1 - b_{12}l_2 - b_{11}k_1 signs_1 - b_{12}k_2 signs_2 \tag{4.16}$$

$$u(2) = -b_{21}l_1 - b_{21}k_1 signs_1 \tag{4.17}$$

| Symbol | Value | Meaning |
|---|---|---|
| a | $0.0154m^2$ | Tank section |
| $S_n$ | $2.5.10^-5m^2$ | cross section of valve |
| $a_{zi}$ | $0 \leq a_{zi} \leq$ | Flow correction term |
| $b_{zi}$ | $0 \leq b_{zi} \leq$ | Leakage flow correction term |
| g | $9.81\text{m}/s^2$ | Gravity Constant |
| $h_{max}$ | 0.6m | Maximum water level in each tank |
| $Q_{imax}$ | $1.17.10^-4m^3/s$ | Maximum inflow through pump i |
| $C_1$,$C_3$ | 0.03595 | Parameter of opening and closing of valve |
| $K_1$ | 0.699 | |
| $K_2$ | 0.53 | |
| $K_{11}$ | $10^-4$ | |
| $K_{12}$ | $18.10^-3$ | |
| $K_{21}$ | $7.10^-4$ | |
| $K_{22}$ | $10^-4$ | |
| $\lambda$ | 0.6 | |
| sign | Signum function | |
| SMC | Sliding mode control | |
| ISMC | integral sliding mode control | |

Table 4.1: Numerical value for the physical parameters of the three tank system

## 4.4 Simulink Diagram

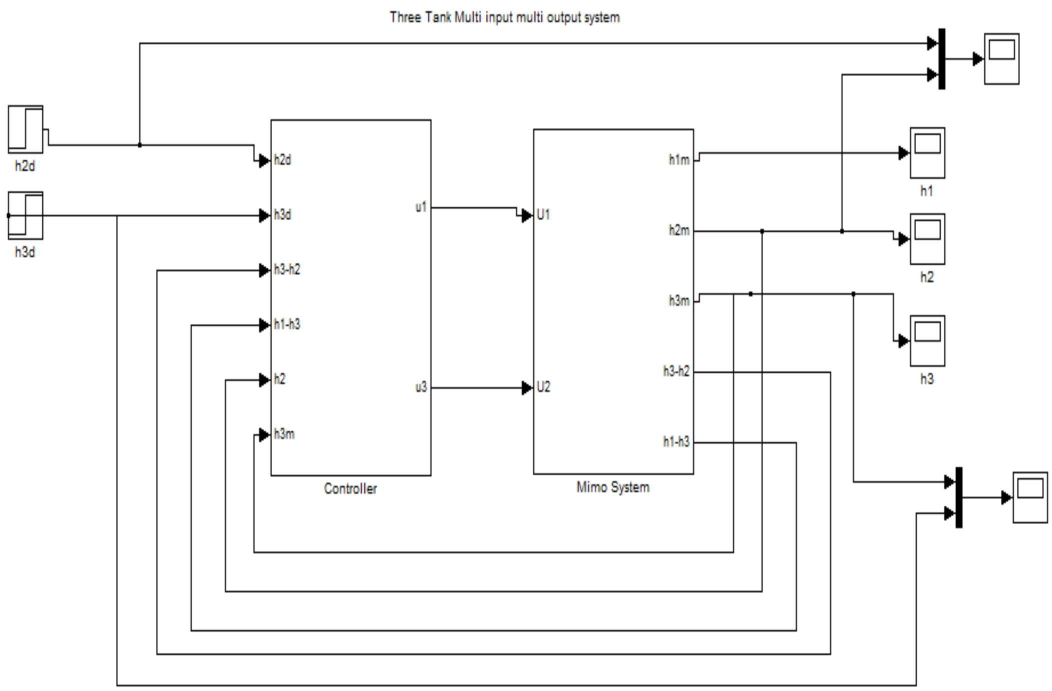



Figure 4.3: Simulink Digram of Three Tank MIMO System

Integral SMC based Three tank MIMO system is simulated in MATLAB/Simulink. The model diagram for the scheme is as shown in Fig.(4.3).

# Chapter 5

# Simulation Results

## 5.1 Simulation Results of SMC

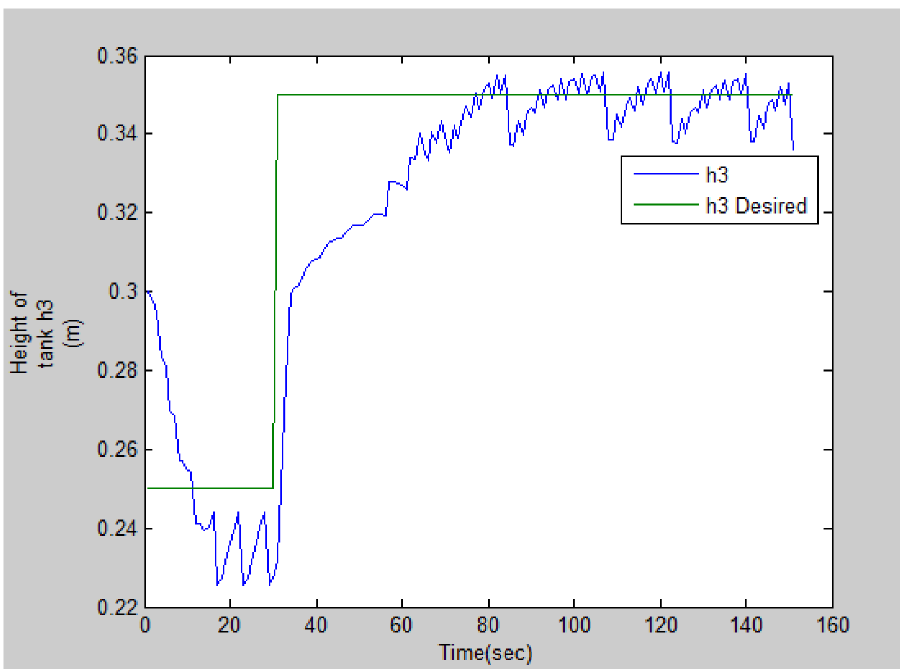



Figure 5.1: Height of tank 3 ($h_3$) with SMC

A shown in fig.(5.1) is a height of tank 3. I have set the initial height of tank 3 is 0.30m and desired value is 0.35m. Output of $h_3$ is follows their desired reference $h_{2d}$. With sliding mode controller it's gives the more chattering around the desired reference. $h_3$ is controlled variable of this system.

**Sliding surface for $h_3$**

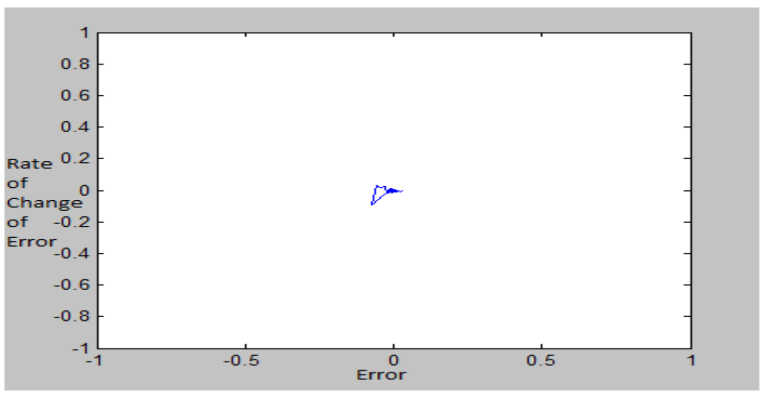



Figure 5.2: Sliding Surface for Height ($h_3$)

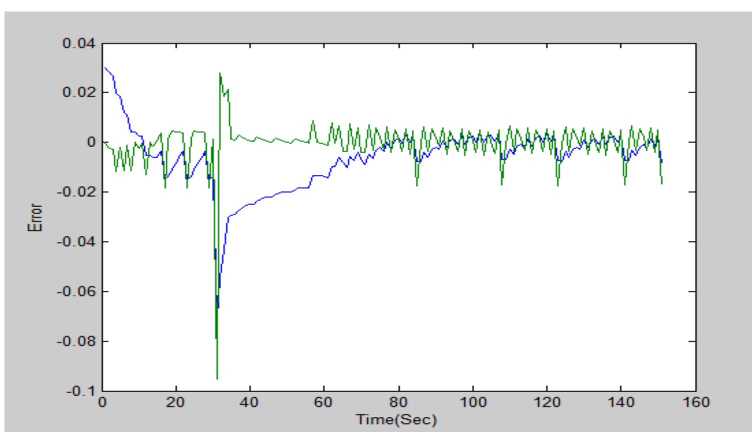



Figure 5.3: Sliding Surface for Tank ($h_3$)

A shown in fig.(5.2) and (5.3) is sliding surface for tank 3 with sliding mode controller. It's gives more switching between one state to another state. This surface is design based on $e$ and $\dot{e}$. Blue line indicate the error and is developed by $(h_3 - h_{3d})$ and green line indicate the rate of change of error it's developed by $(\dot{h}_3 - \dot{h}_{3d})$. System start from non zero initial condition and after some times it's reaches at the sliding surface through chattering phenomenon.

**Height of tank 2**

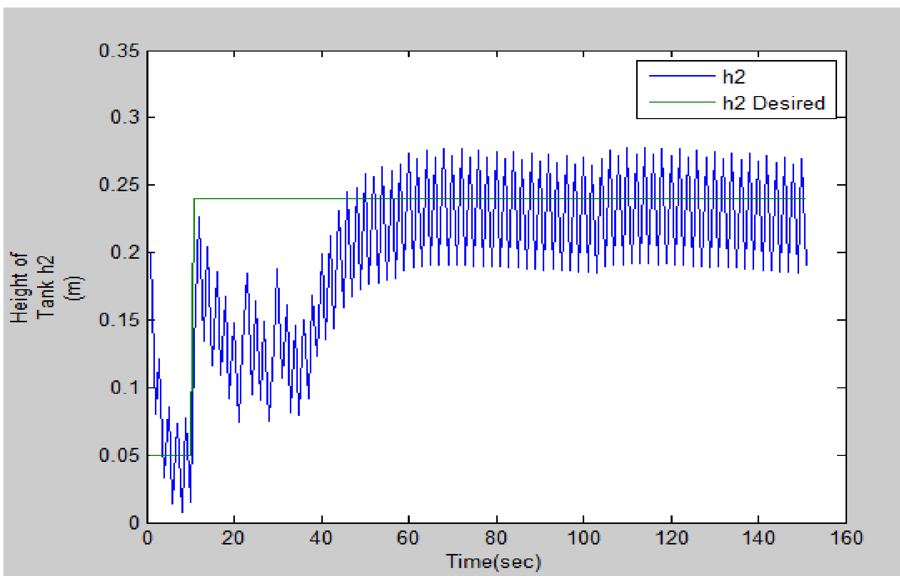



Figure 5.4: Height of Tank 3 ($h_2$) with SMC

A shown in fig.(5.4) is a height of tank 2. I have set the initial height of tank 2 is 0.20m and desired value is 0.25m. Output of $h_2$ is follows their desired reference $h_{2d}$. With sliding mode controller it's gives the more chattering around the desired reference because it's continuously switching between one sate to another. Height of tank h2 is not gives the smooth response compared to height of tank h3.

**Sliding surface for $h_2$**

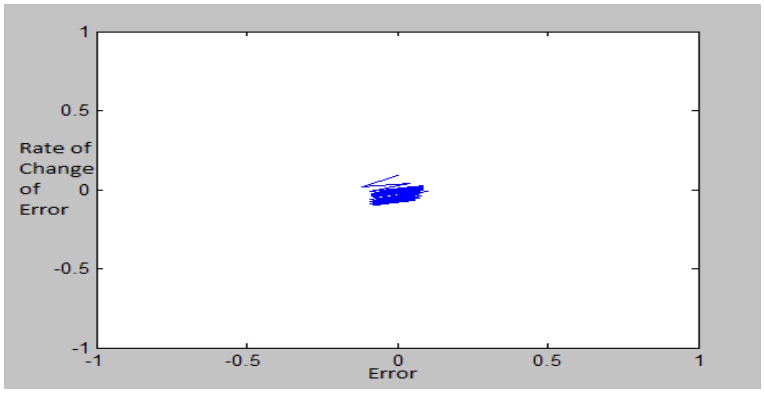



Figure 5.5: Sliding Surface for Height ($h_2$)

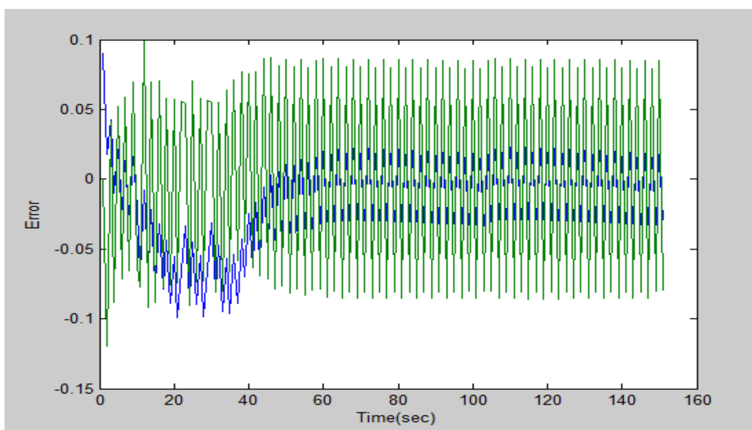



Figure 5.6: Sliding Surface for Height ($h_2$)

A shown in fig.(5.5) and (5.6)is sliding surface for tank 2 (height of tank 2). This surface is design based on $e$ and $\dot{e}$. Blue line indicate the error and is developed by $(h_2 - h_{2d})$ and green line indicate the rate of change of error it's developed by $(\dot{h}_2 - \dot{h}_{2d})$. System start from non zero initial condition and after some times it's reaches at the sliding surface through chattering phenomenon. It's gives more switching compare to $h_3$ is shown in fig.(5.2) and (5.3).

## 5.2 Simulation Results of Integral Sliding Mode Controller

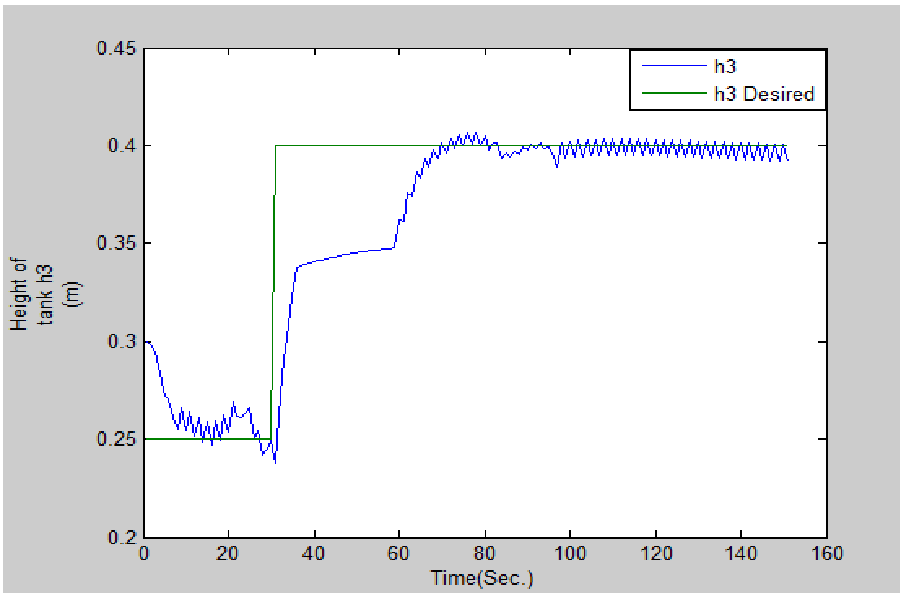



Figure 5.7: Height of Tank 3 ($h_3$) with ISMC

A shown in fig.(5.7) is a height of tank 3. I have set the initial height of tank 3 is 0.30m and desired value is 0.40m.Output of $h_3$ is follows their desired reference $h_{2d}$ and Integral sliding mode controller gives the less chattering compare to sliding mode controller. $h_3$ is controlled variable of this system.

**Sliding surface of $h_3$**

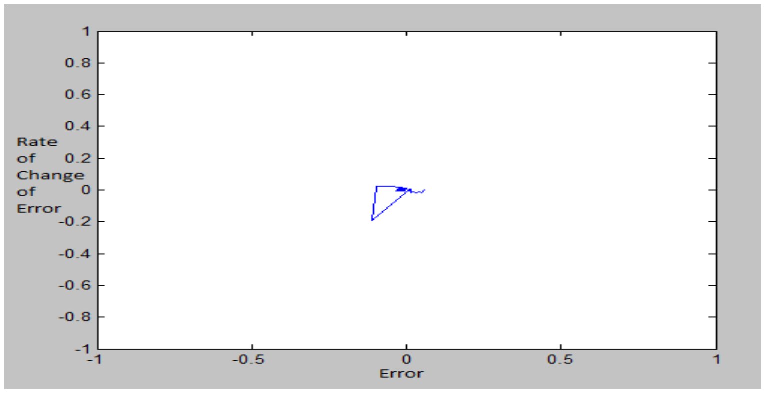



Figure 5.8: Sliding Surface for Height ($h_3$)

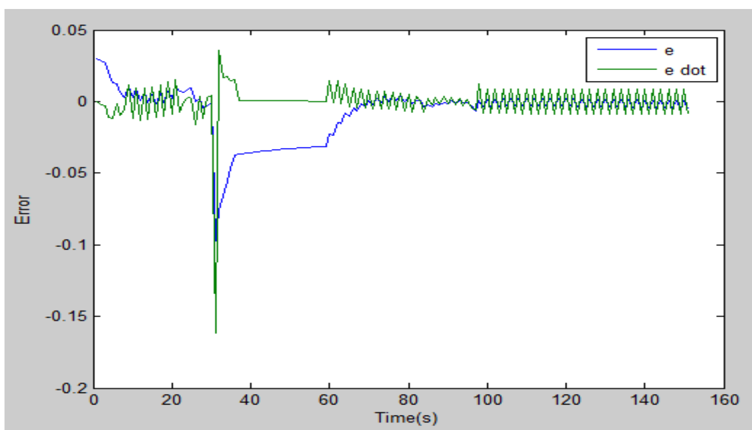



Figure 5.9: Sliding Surface for Tank ($h_3$)

A shown in fig.(5.8) and (5.9)is sliding surface for tank 3. This surface is design based on error and rate of change of error. Blue line indicate the error and is developed by $(h_3 - h_{3d})$ and green line indicate the rate of change of error it's developed by $(\dot{h}_3 - \dot{h}_{3d})$. System start from non zero initial condition and after some times it's reaches at the sliding surface through chattering phenomenon.

### Height of Tank 2

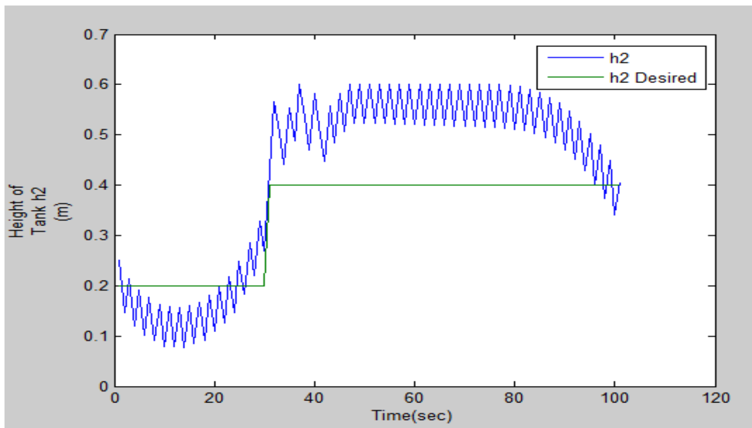



Figure 5.10: Height of Tank $h_2$

A shown in fig.(5.10) is a height of tank 2. I have set the initial height of tank 2 is 0.25m and desired value is 0.40m. Output of $h_2$ is follows their desired reference $h_{2d}$ but more chattering are produce and not get the smooth response With integral sliding mode controller.

### Sliding surface of $h_2$

A shown in fig.(5.11) and (5.12)is sliding surface for tank 3. This surface is design based on $e$ and $\dot{e}$. Blue line indicate the error and is developed by $(h_3 - h_{3d})$ and green line indicate the rate of change of error it's developed by $(\dot{h}_3 - \dot{h}_{3d})$. System start from non zero initial condition and after some times it's reaches at the sliding surface through chattering phenomenon.

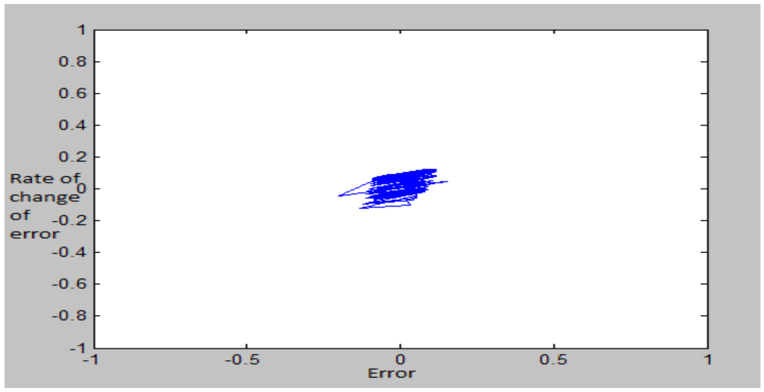



Figure 5.11: Sliding surface for height $h_2$

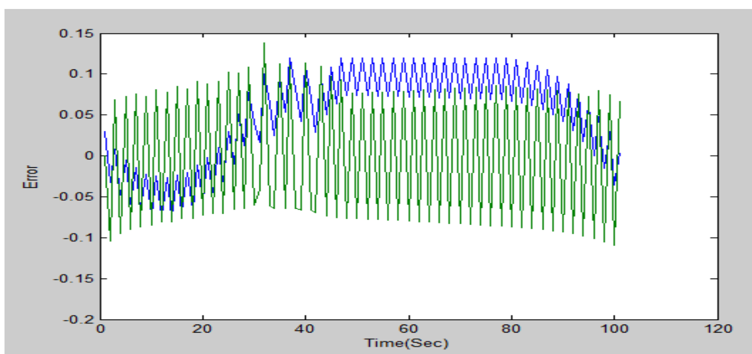



Figure 5.12: Sliding surface for height $h_2$

**Height of Tank $h_1$**

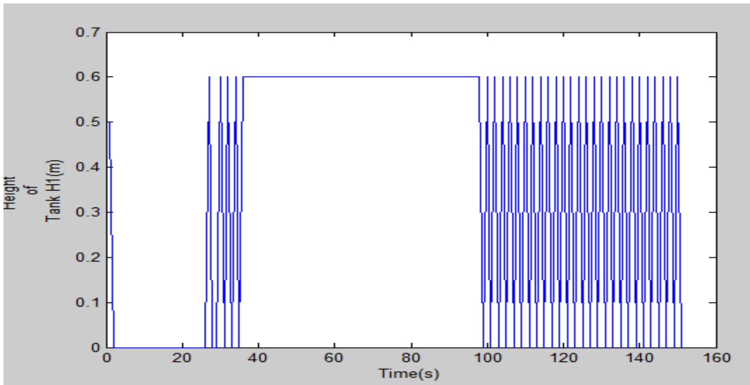



Figure 5.13: Height of Tank $h_1$

A shown in fig.(5.13) is a height of tank 1($h_1$). I have set the initial height of tank 1 is 0.5m. It's completely follows the set point and it is continuously varied. Tank 1 is not a controlled variable.

**Input flow of tank 1 and tank 2**

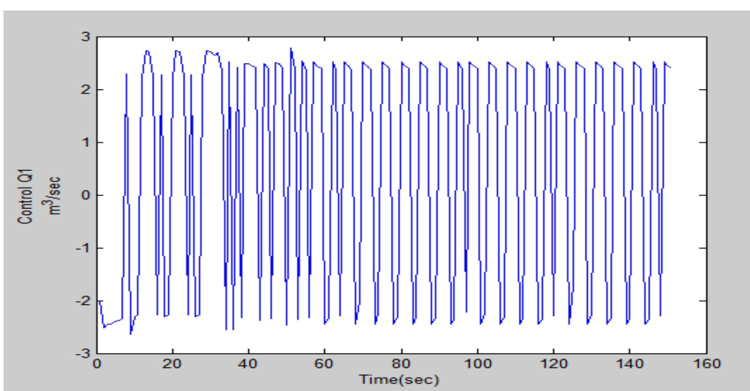



Figure 5.14: input flow of $Q_1$

A shown in fig.(5.14) is a input flow of tank 1. Pump 1 is supplied the water from reservoir with flow rate $Q_1(t)$.

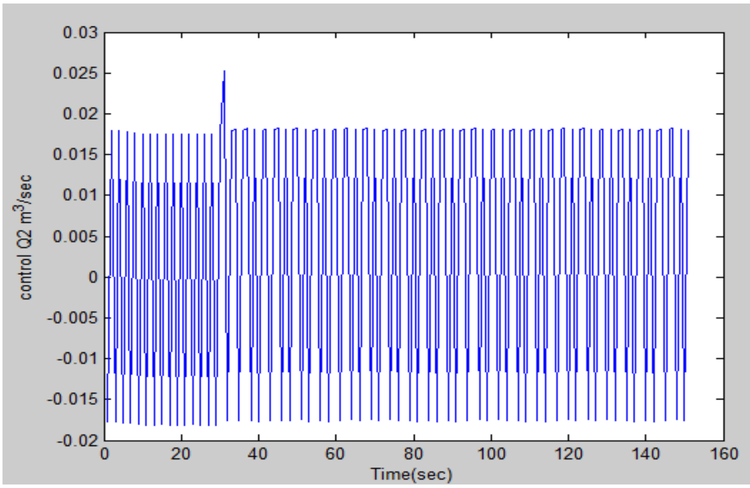



Figure 5.15: Input flow of $Q_2$

A shown in fig.(5.15) is a input flow of tank 2. Pump 2 is supplied the water from reservoir with flow rate $Q_2(t)$.

# Chapter 6

# Conclusion and Future Scope

## 6.1 Conclusion

A robust controller is designed for class of non-linear MIMO three tank system using SMC with integrator. A designed controller with SMC has a discontinuous switching function like saturation, signum and tangent switched from one value to another value continuously and fast. Such a switching function produces chattering phenomenon that is highly undesirable. Results shows that sliding mode controller without integrator produces more chattering, which is reduced using SMC with integrator for the same system. Simulated results shows that designed controller is robust against external disturbances and uncertainties.

## 6.2 Future Scope

- Implementation of integrated SMC based designed controller for the non-linear three tank MIMO system.

# Bibliography


[1] V.Utkin,“Sliding Mode in Control and Optimization,” Springer-Verlag,Berlin,1992.

[2] L Fridman,“An averaging approach to chattering”, IEEE Transactions of Automatic Control., 46(8)(1260- 10 1265), 2001.

[3] C. Edwards and S. Spurgeon,“Sliding Mode Control: Theory and Applications,” Taylor-Francis, London,1998.

[4] R. A. DeCarlo, S. H. Zak, and G. P. Mathews,“Variable Structure Control of Non-linear Multi variable Systems: A Tutorial,” Proceedings IEEE, Vol. 76, No.3, March 1988.

[5] J.J.E. Slotine and W. Li, “Applied nonlinear Control,” Prentice-Hall International, New Jersey,1991.

[6] R. Benayache, L. Chrifi-Alaoui, P. Bussy and J. M. Castelain, “Sliding Mode Control with Integral Corrector: Design and Experimental Application to an Interconnected System,” Mediterranean Conference on Control and Automation, Thessaloniki, 24-26 June 2009, pp. 831-836. doi:10.1109/MED.2009.5164647

[7] K. Ogata,“System Dynamics,” Englewood Cliffs, New Jersey, 1978.

[8] K.D.Young,V.I. Utkin and U. Ozguner, "A Control engineer's guide to sliding mode control," IEEE transactions on Control System Technology, vol.7(3), May1999, pp.328-342.

[9] M. Iqbal, A. I. Bhatti, "Dynamic Sliding Modes Control of Uncertain Nonlinear MIMO Three Tank System,"

[10] DUY-KY NGUYEN,M.E.,"SMC P.hd Thesis on Sliding-Mode Control: Advanced design Techniques,"

[11] M. sami fadali,university of nevada,Reno Ph.d thesis,"sliding mode control,"

[12] M.Gopal,"Digital control and state variable method,"

[13] G. Wheeler, C. Y. Su, and Y. Stepanenko, A "sliding mode controller with improved adaptation laws for the upper bounds on the norm of uncertainties," IEEE Proc. Amer. Control Conf., pp. 2133-2137, Jun.1997.

[14] Bartolini G. Ferrar A. 1995 IEEE,on "MIMO sliding mode control of uncertain nonlinear system,". New orleans.

[15] A. Levant, Chattering analysis, European Control Conference, 2-5 July, Kos, Greece 2007.

[16] Anouar Benamor, Larbi Chrifi-alaui, Hassani Messaoud,Mohamed Chaabane,"Sliding Mode Control, With Integrator, for a class of MIMO Nonlinear System," Published Online May 2011 (http://www.SciRP.org/journal/eng)doi:10.4236/eng.2011.35050.

[17] S. Laghrouche, F. Plestan, and A Glumineau, "Multivariable practical higher order sliding mode control," Proceeding of the 44th IEEE Conference on Decision and Control,and the European Control Conference, 2005.

[18] T. E. Faber (1995). Fluid Dynamics for Physicists. Cambridge University Press. ISBN 0-521-42969-2.

[19] Dennis G. Zill, A First Course in Differential Equations (2005)

# Undertaking for Originality of the Work

I, **Rikin C. Hingrajiya, Roll.No.12micc04**, give undertaking that the Major Project entitled **"Design Integral Sliding Mode Controller for MIMO Non-linear System"** submitted by me, towards the partial fulfillment of the requirements for the degree of **Master of Technology in Control and Automation** of Nirma University, Ahmedabad, is the original work carried out by me and I give assurance that no attempt of plagiarism has been made. I understand that in the event of any similarity found subsequently with any published work or any dissertation work elsewhere; it will result in severe disciplinary action.

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Signature of Student

Date:_______________

Place:_______________

Endorsed by

______________________

(Project Guide)

**Prof. Rajesh L. Zadfiya**

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**Publication on the work**

- Rikin C. Hingrajiya, Rajesh L. Zadfiya, Vijay J. Ramani " **Comparative study of different optimization techniques for sliding mode controller for uncertain non-linear MIMO system"** International Journal of Electrical and Electronics Engineering Research,APRIL 2014.

- Vijay J. Ramani, Rajesh L. Zadfiya, Rikin C. Hingrajiya " **Integrated Sliding Mode Controller Design for Semi Active Vehicle Suspension System using hyperbolic tangent"** International Journal of Electrical and Electronics Engineering Research,APRIL 2014.