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Design bounded robust controller using HJB solution for the nonlinear hybrid dynamical systems

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ABSTRACT

This paper deals with the bounded robust controller design for event wise multiple models of the nonlinear hybrid dynamical systems (NHDS) with matched and unmatched uncertainties. The Hamilton-Jacobi-Bellman (HJB) equation based bounded robust control problem is proposed using a suitable non-quadratic term in the objective function of event wise local linearized model. The families of local bounded control laws are designed, to enforce stability using Lyapunov theorem for the individual events with the knowledge of the maximum bound of uncertainty. Using direct method of Lyapunov stability, the local bounded control law is shown to be optimal with respect to the objective function of the system under uncertainty. The new generalized Lyapunov function is proposed with crisp logic variables to establish the stability around the equilibrium points using bounded robust control law. Simulation and experimental results are presented, which validates the proposed bounded robust control approach.

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1. Introduction

Systems which change its continuous dynamics due to interaction of discrete events are called nonlinear hybrid dynamical systems (NHDS) [43,12]. The modeling and control of the NHDS have received more interest, both from researchers and industries due to the interaction of logical discrete dynamics along with continuous dynamics. Applications of the NHDS have been found in mechanical systems [15], air brake system [42,22], chemical processes [29,20,30,21,40] and manufacturing systems. Motivated from the applications, researchers from control and system identification are focused on NHDS related problems, which includes identification [22,20,30,21,35,36,19,37], control [28,29,30,9,10,23,16,6,7], optimization [40,6,7] and the stability [49,11,50,8,13,31] of the NHDS. Different modeling frameworks for the NHDS has been proposed like linear complementarity (LC) systems [43], mixed logical dynamical (MLD) systems [37,9], piecewise affine (PWA) systems [20,35,36,19,37,51,32] and multiple linear model (MLM) [29,30,21,40].

In the present work, MLM framework has been adopted for modeling of the NHDS. It has received a great deal of attention as it converts complex problems into simpler sub-problems. It has been observed from the above mentioned literatures, very limited work has been done to address the robustness of the control sys-

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tem for the NHDS. In addition to this, one has to consider the constraints on the control inputs while practical implementation. The bounded control action may be required due to the physical limitations of control actuators. Looking to these needs, in this paper, the robust bounded control problem is formulated with stability analysis of the NHDS around equilibrium points. To address the constraints, the objective cost function is modified to design a bounded optimal control law. In [24], method to find robust control of nonlinear systems is proposed by assuming the existence of the HJB equation solution. However, it is difficult to find the solution of HJB equation. In the proposed approach, the bounded optimal control law is obtained by using the solution of HJB equation. In [1,3] attempt was made to find constrained optimal control law using non-quadratic objective functional. The results for the robust control of continuous dynamic systems exist and are well developed, but for combined dynamics i.e. for NHDS is limited. The main difficulty in the control of NHDS is that, the model parameters are change due to the occurrence of the discrete event. For the MLM framework, significant results have been developed for stability using multiple Lyapunov functions in [14]. A Lyapunov based bounded control laws was developed in [25,17] that enforce robust stability.

In [51,27,18], model predictive control framework is used for the switched and hybrid systems does not guarantee to provide optimal and robust control. In this paper, the HJB solution based multiple Lyapunov functions are proposed which provides stability in the presence of bounded input. However, the design of a



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