Design and implementation of Robust Multirate Output Feedback based Sliding Mode Controller for Induction Motors using FPGA

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***Abstract*—This paper demonstrates design of Multirate Output feedback(MROF) based Discrete-time Sliding Mode Controller(DSMC) for sensorless control of Induction Motor(IM) implemented using System Generator (SysGen). A multirate output technique is used to estimate the state variables of Induction Motor. For designing the observer and controller, a linearised model is used. The efficacy of the scheme is shown in the simulation result.**

Keywords-Multirate output feedback; Sliding Mode Controller (SMC); Field Programmable Gate Array (FPGA); Sensorless IM Drives; System Generator.

I. INTRODUCTION

Induction motors are the horsepower of industry and they need to be controlled. There are various techniques available in literature for the control of sensorless Induction drives[1],[2],[3].Variable speed drive for induction motor is widely used drive in the industry. Generally, variable speed drives (VSD) for induction motor require both wide range of operating speed and fast torque response, regardless of any disturbances and uncertainties. This demands more advanced control techniques for speed control of Induction Motor. Further, in the formulation of any control problem there will typically be discrepancies between the actual plant and the mathematical model developed for controller design. This mismatch may be due to un modeled dynamics, variation in parameters or the approximation of complex plant behavior by straightforward model. The designer has to ensure that the resulting controller has the ability to produce required performance levels in practice despite such plant/model mismatches. One particular approach to robust controller design is Sliding Mode Control methodology [4],[5],[6],[7].

A detailed survey on development and applications of SMC is presented in [8]. SMC is useful for linear and certain class of nonlinear systems and making it a leading contender for IM drives. It gives order reduction, invariance to both torque variation and parameter uncertainties along with fast dynamic response. The real implementation issues can be addressed with DSMC only. Discrete-time counterpart of SMC is taken up by various research groups across the globe[7],[9].

One of the popular and widely used control technique for the IM is Vector Control strategy which requires flux position and speed measurement as input variables. Speed sensors reduce the reliability of the drive along with increasing the price. Also, direct measurement of magnetic field using hall effect sensors need the mounting of sensors in the air gap of the machine increases the complexity. The flux and speed estimator or another technique may be used to solve the said problems[2],[3]. Recently developed MROF based SMC is discussed in [10],[11], but the issues of implementation are not discussed yet.

Also, most of the AC drives nowadays are implemented using either fully Digital Signal Processor(DSP) based control strategy or with both FPGA and DSP together, as DSP based circuits are simple and also flexible in adapting various applications. But these circuits are sluggish and allow limited computation resources due to the sequential computation feature, complicated design process and long development time cycle. Multiple DSPs can solve the problem but with increased cost. FPGA provide an economic solution and fast circuit response due to its simultaneous execution property. FPGA have high processing rate and consume less power compared to DSPs [12],[13],[14],[15].

In this paper, we propose an MROF based DSMC for sensorless IM using FPGA. The simulations have been carried out using SysGen-the industry's leading high-level DSP design tool from XILINX. With SysGen, people with little knowledge of VHDL/Verilog, can create production quality FPGA implementations using MATLAB-Simulink. SysGen's Hardware Co-simulation feature allows one to validate working hardware as it supports JTAG communication between FPGA hardware platform and Simulink [16]. It is a universally accepted simulation tool for modeling and analysis of FPGA based concepts[17],[18],[19],[20].The paper is organized as follows: The basic concept of DSMC and MROF is discussed in section II. Section III discusses

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the design. Section IV encapsulates the simulation results of MROF based SMC for sensorless Induction Motor followed by the conclusion in section V.

II. REVIEW OF MROF BASED DISCRETE-TIME SMC TECHNIQUE

Consider the discrete-time plant

$$x(k+1) = A_d x(k) + B_d u(k) \tag{1}$$

$$y(k) = C_d x(k) \tag{2}$$

where, $x \epsilon R^n$, $u \epsilon R^m$, $y \epsilon R^p$, $A_d \epsilon R^{n\times n}$, $B_d \epsilon R^{n\times m}$ and $C_d \epsilon R^{p\times n}$ such that $C_d B_d$ is non-singular. We also assume that $(A_{d\tau}, B_{d\tau})$ is completely controllable and $m < n$.

The design of sliding mode control includes designing a switching surface $s(x(k),k) = \{x(k)/s(k) = Cx(k) = 0\}$ and design of a suitable control law $u(x(k),k)$ such that any state of the system outside the said switching surface is driven to reach the surface in finite time [21],[22].

A. DSMC design

1) Switching surface design: The switching function for DSMC is given as

$$s(k) = c^T x(k). \tag{3}$$

Let the system in Eqn. 1 be transformed to a regular form by a transformation $\bar{x}(k) = T_r x(k)$, where T_r is the transformation matrix, resulting in the following dynamics:

$$\bar{x}(k+1) = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix} \bar{x}(k) + \begin{bmatrix} 0 \\ B_{d2} \end{bmatrix} u. \tag{4}$$

The sliding surface for the transformed system in Eqn. 4 is given by $\bar{c}^T \bar{x} = 0$, where $\bar{c}^T = \begin{bmatrix} K & I_m \end{bmatrix}$

On the sliding surface, the system will follow the relation $\bar{x}_2 = -K\bar{x}_1$, where $\bar{x}_2$ comprises of last (m) states of $x(k)$. Thus, the $\bar{x}_1$ dynamics for the transformed equation becomes

$$\bar{x}_1(k+1) = (A_{d11} - A_{d12}K)\bar{x}_1(k). \tag{5}$$

From Eqn.5, it can be said that if K is chosen such that the eigenvalues of $(A_{d11} - A_{d12}K)$ are placed at the desired locations, then $\bar{x}_1$ is stabilised and as $\bar{x}_2 = -K\bar{x}_1$, $\bar{x}_2$ also becomes stable on the switching surface.

Thus,the sliding surface in terms of original state coordinates is given by

$$s(k) = \bar{c}^T \bar{x}(k) = \bar{c}^T T_r x(k). \tag{6}$$

2) Control law design: The Gao's reaching law [22] for DSMC is given by

$$s(k+1) - s(k) = -q\tau s(k) - \epsilon\tau sgn(s(k)) \tag{7}$$

where,
τ is the sampling period
$q, \epsilon > 0$ and
$1 - q\tau > 0$ should hold to guarantee the reaching phase stability. Using Eqn.1, Eqn.3 & Eqn.7, the control law can be defined as

$$u = Fx(k) + \gamma sgn(s(k)) \tag{8}$$

where,
$F = -(c^T B_d)^{-1} c^T [A_d - I + q\tau]$
$\gamma = -(c^T B_d)^{-1} \epsilon\tau.$

In DSMC, the measurement and the control signal are updated only at regular intervals ,i.e. the sampling period and the control input is considered to remain constant for a sampling period. In DSMC, unlike the continuous SMC, the states move about the sliding surface but are unable to stay on it. Thus, DSMC is said to exhibit Quasi Sliding mode.

As discussed above, the DSMC design is based on state feedback. However, in many systems it is not possible to get or measure all the state variables and so resort us to use output measurements.

It has been recently shown [23] that using multi-rate output feedback technique, it is always possible to obtain state vector for all controllable and observable systems within one sampling period. Moreover,it is also shown in [23],that MROF guarantees closed loop stability which is not the case in static output feedback.

MROF is the concept of sampling the control input and sensor output at different rates. Here the output is sampled faster than the input[10],[11],[24],[25].

It was found that the state feedback control law may be realised by the use of MROF, by representing the system states in terms of past control input and multirate sampled system outputs[23].

Consider the continuous system

$$\dot{x} = Ax + Bu \tag{9}$$

$$y = Cx \tag{10}$$

where, $x \epsilon R^n$, $u \epsilon R^m$, $y \epsilon R^p$, $A \epsilon R^{n\times n}$, $B \epsilon R^{n\times m}$ and $C \epsilon R^{p\times n}$. Let the above system be sampled at a sampling interval τsec and given as

$$x(k+1)\tau = A_{d\tau} x(k) + B_{d\tau} u(k) \tag{11}$$

$$y(k\tau) = C_{d\tau} x(k). \tag{12}$$

Consider the input to be sampled every τsec and the output be sampled faster at a period Δsec such that $\Delta = \tau/N$, where N is an integer greater than or equal to the observability index of the system.[23]

The system sampled at Δ period be given by

$$x(k+1)\Delta = A_{d\Delta} x(k) + B_{d\Delta} u(k) \tag{13}$$

$$y(k\Delta) = C_{d\Delta} x(k). \tag{14}$$

If the past N sampled outputs are represented as

$$y_k = \begin{bmatrix} y((k-1)\tau) \\ y((k-1)\tau) + \Delta \\ . \\ . \\ y(k\tau - \Delta) \end{bmatrix}, \tag{15}$$

and if $k\tau$ is replaced by k, then the multirate output sampled system can be written as

$$x(k+1) = A_{d\tau}x(k) + B_{d\tau}u(k) \tag{16}$$

$$y(k+1) = C_0x(k) + D_0u(k). \tag{17}$$

where,

$$C_0 = \begin{bmatrix} C \\ CA_{d\Delta} \\ CA_{d\Delta}^2 \\ \cdot \\ \cdot \\ CA_{d\Delta}^{N-1} \end{bmatrix}, D_0 = \begin{bmatrix} 0 \\ CB_{d\Delta} \\ C(B_{d\Delta}+I) \\ \cdot \\ \cdot \\ C\Sigma_{j=0}^{N-2}{A_{d\Delta}}^{j}B_{d\Delta} \end{bmatrix}. \tag{18}$$

Solving the MROF system equations, the estimated states $x_o(k)$can be given by

$$x_o(k) = L_1y(k) + L_2u(k-1) \tag{19}$$

where,
$L_1 = A_{d\tau}(C_o^TC_o)^{-1}C_o^T$
$L_2 = B_{d\tau} - A_{d\tau}(C_o^TC_o)^{-1}C_o^TD_o$.

Substituting the value of the estimated states from Eqn.19 into Eqn. 8, the control law for the MROF can be written as,

$$u_{mrof}(k) = Fx_o(k) + Gsgn(s_{mrof}(k)) \tag{20}$$

where,
$s_{mrof}(k) = c^Tx_o(k)$
$F = -(c^TB_{d\tau})^{-1}c^T(q\tau - I + B_{d\tau})$
$G = -(c^TB_{d\tau})^{-1}\varepsilon\tau$.

III. DESIGN OF DSMC CONTROLLER FOR INDUCTION MOTOR

The behaviour of a three-phase, four pole, induction motor in the synchronous reference frame can be given by the equations given in [10].

The parameters used for the induction motor model are:

Power: 3 HP/2.4 KW
Voltage :460 Volts (L-L,RMS)
Frequency : 60 Hz
Phases:3
Full-load current:4 A
Full-Load efficiency: 80.0%
Full-load speed:88.5%
Power factor:80.0%
No.of poles: 4
R_s = 1.77Ω R_r=1.34Ω
X_{ls}=5.25Ω (at 60 Hz)
X_m=139 Ω (at 60 Hz)
X_{lr}=4.57Ω (at 60 Hz)

The system given by the equations are linearised at a given operating point to give the state space model where

$$A = \begin{bmatrix} -69 & 5359 & -39 & 51 & 5145 \\ -5359 & -69 & 12 & -5146 & 51 \\ -270 & -828 & 0 & -438 & -803 \\ 67 & -5170 & 39 & -53 & -4963 \\ 5170 & 67 & -13 & 4963 & -53 \end{bmatrix}$$

$$B = \begin{bmatrix} 38.96 & 0 \\ 0 & 38.96 \\ 0 & 0 \\ -37.72 & 0 \\ 0 & -37.72 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

where,

$$\Delta x(k) = [\Delta i_{ds} \quad \Delta i_{qs} \quad \Delta\omega_r \quad \Delta i_{dr} \quad \Delta i_{qr}]^T$$

$$\Delta u(k) = [\Delta V_{ds} \quad \Delta V_{qs}]^T$$

$$\Delta y(k) = [\Delta i_{ds} \quad \Delta i_{qs}]^T$$

For N=3, this system is discretised at a sampling interval of $\tau = 0.09sec$ to obtain the discretised state space model as

$$A_{d\tau} = \begin{bmatrix} -0.58 & -0.07 & -0.01 & -0.60 & -0.07 \\ 0.20 & -0.30 & -0.01 & 0.20 & -0.30 \\ -0.27 & 0.36 & 0.04 & -0.28 & 0.35 \\ 0.60 & 0.08 & 0.01 & 0.61 & 0.08 \\ -0.21 & 0.31 & 0.01 & -0.22 & 0.31 \end{bmatrix}$$

$$B_{d\tau} = \begin{bmatrix} -0.04 & 0.02 \\ 0.05 & 0.07 \\ 0.14 & 0.01 \\ 0.04 & -0.01 \\ -0.06 & -0.08 \end{bmatrix}$$

$$C_{d\tau} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The system is also discretised at an interval of $\Delta = 0.03sec$ to obtain the discretised state space model as

$$A_{d\Delta} = \begin{bmatrix} 5.11 & -2.70 & -0.15 & 5.24 & -2.63 \\ -0.60 & -1.86 & 0.16 & -0.82 & -2.00 \\ -13.34 & -4.10 & -0.01 & -13.74 & -4.11 \\ -5.25 & 2.68 & 0.16 & -5.40 & 2.61 \\ 0.83 & 2.00 & -0.17 & 1.05 & 2.06 \end{bmatrix}$$

$$B_{d\Delta} = \begin{bmatrix} -0.05 & -0.02 \\ -0.01 & 0.06 \\ 0.13 & 0.06 \\ 0.05 & 0.02 \\ -0.02 & -0.06 \end{bmatrix}$$

$$C_{d\Delta} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

The sliding surface coefficient matrix c is obtained using the method given in [26]

$$c = \begin{bmatrix} 379.9954 & -83.9690 \\ -144.9981 & 49.8898 \\ 212.0087 & -46.0841 \\ -94.0999 & 19.5327 \\ -70.1751 & -3.2512 \end{bmatrix}.$$

The states are estimated using Eqn.19 with

$$L_1 = \begin{bmatrix} 0.01 & -0.01 & -0.07 & -0.07 & -0.11 & 0.18 \\ -0.01 & 0.01 & 0.06 & 0.05 & -0.01 & 0.03 \\ -0.01 & -0.02 & -0.09 & 0.14 & 0.14 & -0.18 \\ -0.01 & 0.01 & 0.07 & 0.07 & 0.11 & -0.19 \\ 0.01 & -0.01 & -0.06 & -0.05 & 0.01 & -0.03 \end{bmatrix}$$

and $L_2 = \begin{bmatrix} -0.05 & 0.01 \\ 0.05 & 0.07 \\ 0.14 & -0.01 \\ 0.05 & 0.01 \\ -0.06 & -0.07 \end{bmatrix}.$

The control input is found using Eqn.20 with gains

$$F = \begin{bmatrix} -97.1 & 36.13 & -24.74 & -33.50 & 21.01 \\ 71.64 & -36.14 & 17.34 & 26.73 & -12.27 \end{bmatrix}$$

$$G = \begin{bmatrix} 4.82 & -6.24 \\ 0.14 & 20.67 \end{bmatrix}.$$

IV. SIMULATION RESULTS USING SYSTEM GENERATOR

The linear system with error states has to track reference of zero, for the nonlinear system to track a constant reference speed. MROF based SMC is simulated using System Generator and the system model as shown in the Fig.1:

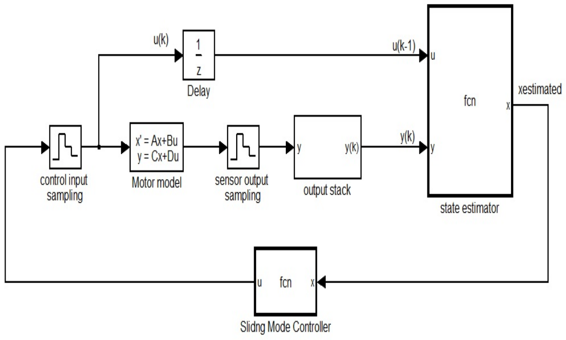

Fig.1. Simulink block diagram with System Generator for control law implementation

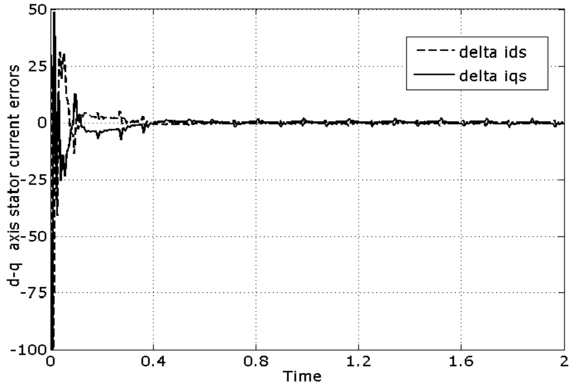

Fig.2. Output currents Δi_{ds} and Δi_{qs}.

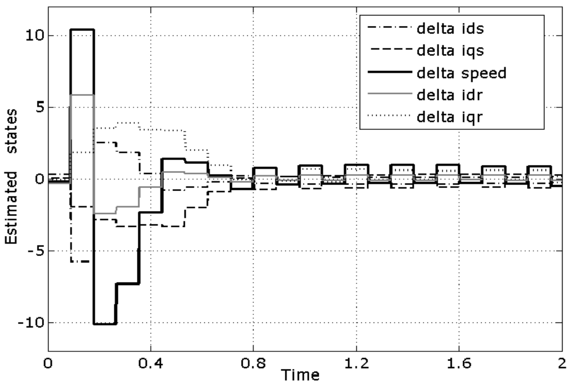

Fig.3. Estimated states $\hat{x_1}$,$\hat{x_2}$,$\hat{x_3}$,$\hat{x_4}$,$\hat{x_5}$.

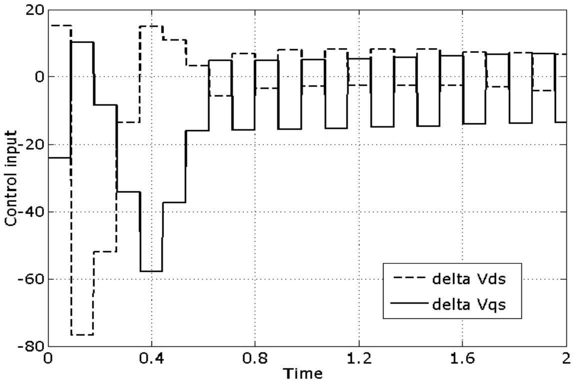

Fig.4. Control inputs ΔV_{ds} and ΔV_{qs}.

It is observed from Fig. 2 that the designed MROF based SMC drives the d-q axis current errors to zero. Fig.3 shows that all the states estimated using Multirate output feedback converges quickly. The control inputs and the sliding surfaces are shown in Fig.4 and Fig.5 respectively.

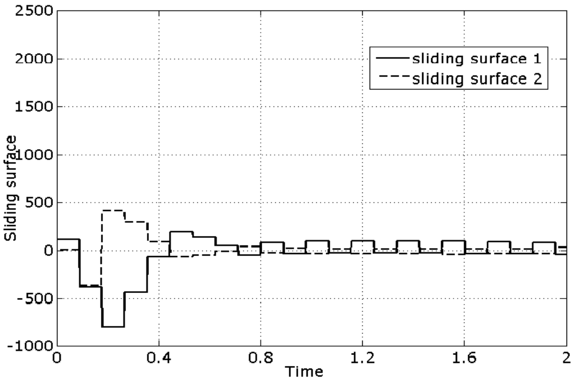

Fig.5. Sliding surfaces

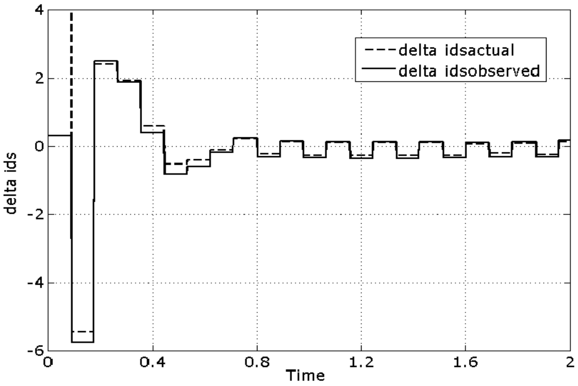

Fig.6. Observed and actual Δi_{ds}

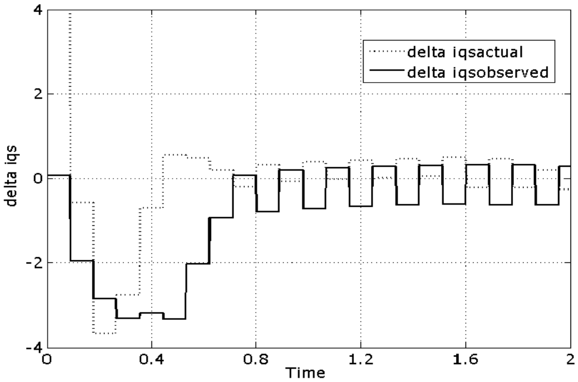

Fig.7. Observed and actual Δi_{qs}

Fig.6 and Fig.7 show that the estimated and actual output states Δi_{ds} and Δi_{qs} track the reference zero as needed.

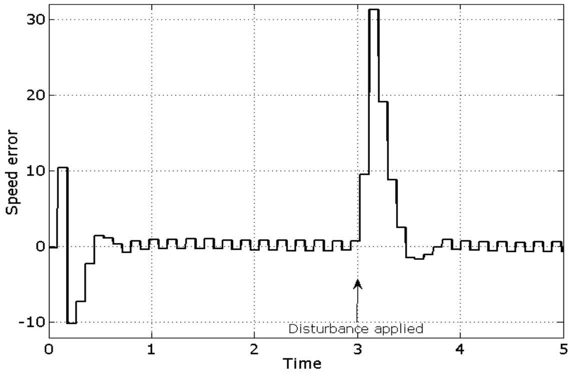

Fig.8. System response for external disturbance

Fig.8 shows that the designed MROF based SMC law brings the speed error to zero, when external disturbance is applied at 3 sec. and also it is inferred that the system remains stable.

V. CONCLUSION

In this paper, MROF based DSMC is designed for a sensorless induction motor. The speed of the motor is controlled by measuring the stator current only. The multirate output feedback approach drives the error states to zero swiftly. The controller is simulated with System Generator for implementation on FPGA. The scheme also has the merits of robustness of sliding mode control and the fast processing rate of FPGA. As seen from the plots, the MROF based DSMC simulated using System Generator is able to drive all the states to zero rapidly and rejects the disturbance also.

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