Modelling, Simulation and Altitude-Range-Analysis of Quad-copterUAV

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Abstract: In the recent years UAV (Unmanned Aerial Vehicles) having quad-rotor helicopter or quad-copter configuration have been receiving increasing attention amongst the global researchers due to its wide-range of applications such assurveillance formilitary, civilian and disaster management. This paper presents our work on the mathematical modeling, simulation and a novel study hereby called as the *Altitude-Range-Analysis* (ARA). The ARA covers an interestingand useful study of finding the feasible altitude-ranges or limits for the given quad-copter machine considering different parameters like payload-weight,rotor-thrust,reference altitude positions etc. This ARA study is carried out using a matlab-based simulator for the quad-rotor helicopter. The simulator is developed based on the popularly used momentum and blade-theory mathematical models, available in the published literature, with certain modifications to account for air-density variations. These results are also presented and discussed.

Key words: UAV, Quad-copter, Quad-rotor, Modeling, Simulation

1. INTRODUCTION

Unmanned Aerial Vehicles (UAV) have been receiving an increasing attention amongst the global researchers, particularly due to its wide range of applications such as surveillance in military, civilian and disaster management, see Sarris(2001).The UAV can be broadly classified in two category i.e. autonomous aero-plane and autonomous helicopter. The helicopters have clear advantages over the aeroplanes due to their specific capabilities like better hovering operation, ability of landing/take-off in limited space, etc. Hence, they are more suitable for a class of surveillance applications, such as, inspection inside factory/residential buildings, reconnaissance within an urban environment, observation of a structurally unsafe building, etc.

A quad-copter or quad-rotor is a four propeller helicopter given in described in Canetta et al (2007) and described here for a quick reference. As shown in Fig.1, the quad-copter consists of two pairs of counter rotating rotors situated at the ends of a cross-frame that is symmetric about the centre of gravity coinciding with the origin of the reference system used.The basic motions of a quad-copter are generated by varying the rotational speed of all four rotors, and thereby changing the absolute and differential lift (or thrust) forces. The helicopter tilts towards the direction of low lift rotor, which enables acceleration along that direction. Spinning directions of the rotors are set to balance the moments, therefore eliminating the need for a tail rotor.

This flying robot presents the main advantage of having quite simple dynamic features. One of the advantages of quad-rotors is the payload augmentation. They have more lift thrusts than conventional helicopters therefore they offer better payload capacity. Moreover, they are potentially simpler-to-build and highly manoeuvrable. Quad rotor aerial robot can generate 6-DOF movement in the inertial frame through changing rotational speed of the motors. The 6-Degree of Freedom (DOF) motion include three translational motions along three coordinate axes (i.e. surge, sway and heave) and three rotational motion around three rotary axes (i.e. roll, pitch and yaw).

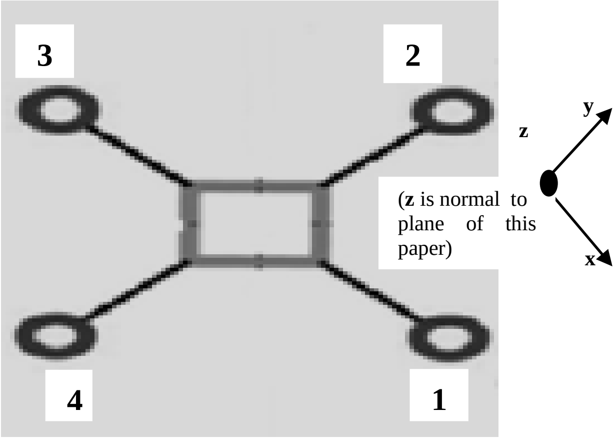

Fig.1.Schematic view of a quad-copter& axis configuration

When the total lift or thrust from all four motors equal the total weight of the quad-copter (including body and payloads), the quad-rotor aerial robot becomes a hoverable robot. An altitude rise (or fall) along the vertical z-axis is achieved by simultaneously increasing (or decreasing) the speed of all four motors by the same quantity. A pitch-angle can be changed by providing a differential thrust between the front and the rear rotors and simultaneously maintaining the total thrust. A roll-angle can be changed by providing a differential thrust between the left and right rotor. Similarly, a Yaw-angle can be achieved by the difference in the counter-thrust between each pair (1, 3 and 2, 4) of rotors. The total thrust should remain unchanged to avoid the up-down motion while manoeuvring.

A full-scale four rotor helicopter was first built by De Bothezat in 1921. Other examples are the Mesicopter in Kroo et al (1967) and Hoverbot in Borenstein(1967).Recently, there has been a rapid increase of literature covering study of different aspects of the quad-rotor. For example, Gomes et al (1998),Pounds et al (2006),Mahonav et al (2012),Patel et al (2012)covers mathematical modelling, simulation and controls;whereas Kumar (1997) and Sarris (2001) describes several interesting surveillance applications.However, we found there is a lack of published literature that covers systematic investigation of the feasible or attainable altitude-range (herein called as ARA) by quad-copter machines against various pay-loads, motor-thrust, and reference altitude. Such investigation could be of significant interest in practical applications of quad-copter.

In this paper, we present our preliminary work to address this gap using a Matlab-based quad-copter simulator. The simulator is based on popular momentum and blade-theory based mathematical models for the quad-copter after minor modifications accounting for the effects of air-density variations. Section 2 covers the mathematical model of a quad-rotor UAV.Section-3 covers simulation result and discussions including the validation and/or comparison with the published literature. It also presents the results of ARA investigation for the specific quad-copter machine available in our laboratory, for two different geographical locations.Section-4 covers the conclusion and future work.

2. MODELLING

There is a significant number of publications describing the mathematical modelling, simulation and control of quad-copter UAV such as Gomes et al(1998),Pounds et al(2006),Mahonav and Kumar(2012),Patel et al (2012). They modelled a quad-copter by incorporating the airframe and motor dynamics as well as aerodynamics and gyroscopic effects. In order to derive a complete dynamic model, the rigid body dynamics and the effects of aerodynamics are studied in this section based on the reference paperDong et al (2013), which is modified in our work to account for variations in air-density variations with change in the altitude.

2.1. Rigid body dynamics

Rigid body dynamics of the quad rotor UAV governs the response of attitude control. The expressions are derived in two coordinate systems: an inertial coordinates and a body fixed coordinates.

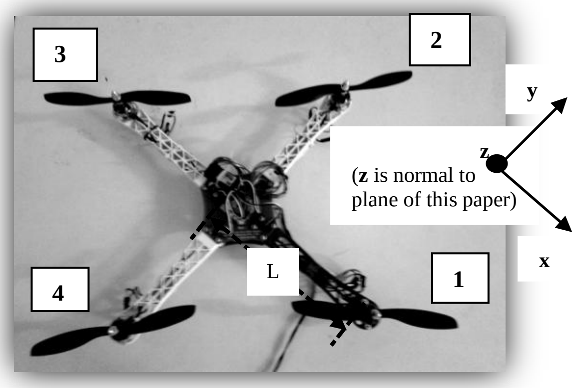

Fig. 2.Photograph of our quad-rotor machine

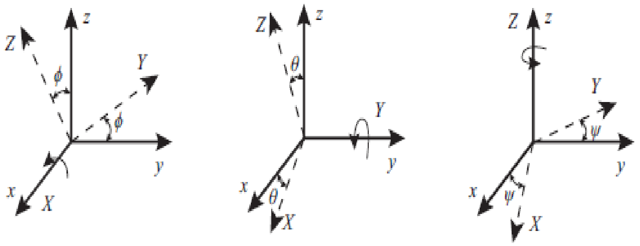

Fig. 3. Attitude Definition

The body fixed coordinates is defined as follows. As indicated in Fig. 2, the lever marked with black strip is chosen as the **x** axis, and the perpendicular lever is the **x** axis. Then the **z** axis is defined by the right hand rule. For inertial coordinates, the point where the quad rotor starts its flight is set as the origin, and an east-north-up orthogonal coordinate system is established by the right hand rule.

With attitude angles defined as in Fig. 3, the transformation matrix R from inertial coordinates to body fixed coordinates as described in Dong et al(2013)is given by equation (1).

$$R\,(\varphi, \theta, \psi) =$$

$$\begin{bmatrix} c(\psi)c(\theta) & c(\psi)s(\theta)s(\varphi) - s(\psi)c(\varphi) & c(\psi)s(\theta)c(\varphi) + s(\psi)s(\varphi) \\ s(\psi)c(\theta) & s(\psi)s(\theta)s(\varphi) + c(\psi)c(\varphi) & s(\psi)s(\theta)s(\varphi) - s(\varphi)c(\psi) \\ -s(\theta) & c(\theta)s(\varphi) & c(\theta)c(\varphi) \end{bmatrix} \quad (1)$$

Where s *and* $c$stands for trigonometric operators '*sin*' and'*cos*' respectively.Whereas, the operands φ, θ, ψ represent attitude angles roll, pitch, and yaw respectively.

In the body fixed coordinates, the direct inputs are RPM (revolutions per minute) commands for the motors. The resultant outputs are **z** directional thrusts in body fixed coordinates. However, the concerned outputs are altitude(i.e. position) and attitude. To eliminate this gap, four control variables are defined as ---

$$
\begin{aligned}
U_H &= F_1 + F_2 + F_3 + F_4\\
U_\theta &= (F_3 - F_1)\,L\\
U_\phi &= (F_2 - F_4)\,L\\
U_\psi &= F_1 + F_3 - F_2 - F_4
\end{aligned}
\qquad (2)
$$

Where F_1,F_2, F_3 andF_4 are thrusts and *L* is the lever length as illustrated in Fig. 3. The subscripts correspond to the ordinal numbers in Fig. 1.Thus U_H, U_ϕ, U_θ and U_ψ are the input variables used to manipulate altitude, roll, pitch, and yaw movement of the quad-copter respectively.

According to the Newton-Euler formalism, the rigid body dynamics is governed by

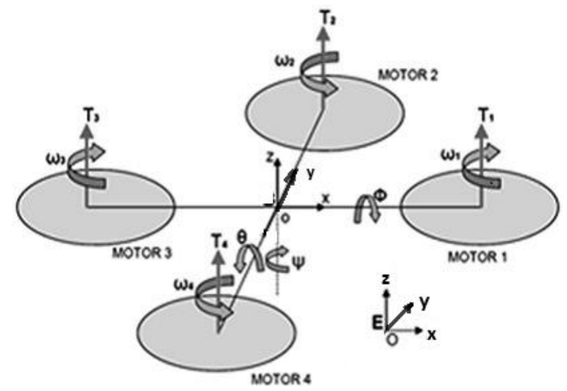

Fig.4. Thrust diagram for Quad rotor

$$
m\ddot{r}=R\begin{bmatrix}0\\0\\U_H\end{bmatrix}-\begin{bmatrix}0\\0\\mg\end{bmatrix}-\dot{q}\times m\dot{r}
$$

$$
I\ddot{q}=\begin{bmatrix}U_\theta\\U_\emptyset\\U_\varphi\end{bmatrix}-\dot{q}\times I\dot{q} \qquad (3)
$$

Where,

'm' ismass of the quad-rotor machine,'g'is local gravity constant, 'r'position in inertial frame,'q'is attitude in body fixed frame and 'I'is the rotary inertia.

Since the rotary inertia is small and the quad rotor UAV is symmetric, estimation for (3) can be expressed in the form of (4), which is also adopted by other researchers

$$
\begin{aligned}
\ddot{x}&=U_H(\sin\psi\sin\varphi+\cos\psi\sin\theta\cos\varphi)/m\\
\ddot{y}&=U_H(\sin\psi\sin\theta\cos\varphi-\cos\psi\sin\varphi)/m\\
\ddot{z}&=U_H\cos\varphi\cos\theta/m-g\\
\ddot{\varphi}&=U_\emptyset/Ixx\\
\ddot{\theta}&=U_\theta/Iyy\\
\ddot{\psi}&=U_\varphi/Izz
\end{aligned}
\qquad (4)
$$

Where,Ixx, Iyy, and Izz are rotary inertia around **x**, **y**, and **z** axes respectively.

2.2Effect of aerodynamics

Thrust diagram for quad-rotor shown in Fig 4can be used to explain the aerodynamic effects as described in Naidoo et al (2011). Two main effects are taken into consideration. One concerns how thrust is generated while the other deals with the drag force.

The thrust *T* produced by each motor is calculated as --

$$
T=\rho(H)C_TA\omega_m^2R^2 \qquad (5)
$$

Where

C_T: thrust coefficient ρ: air density
A: rotor disk area R: blade radius
H:Altitude (Height) of the Quad-copter

According to (4) and (5),$\ddot{\varphi}$and $\ddot{\theta}$are related to motors speeds in the form of

$$
\ddot{\varphi}=\rho(H)C_TAR^2(\omega_3^2-\omega_1^2) \qquad (6)
$$

$$
\ddot{\theta}=\rho(H)C_TAR^2(\omega_2^2-\omega_4^2) \qquad (7)
$$

Where,

ω_1, ω_2, ω_3, ω_4 are rotary speeds of four rotors, $\omega_3=\omega_1+\delta\omega\varphi$, and $\omega_2=\omega_4+\delta\omega\theta$.

3. SIMULATION RESULTS AND DISCUSSION

We simulated the model described in previous section using MATLAB as a simulation platform and considering thrust-forces (or control voltage to motor driver) of all the four motors as four control inputs to the quad-rotor model.

3.1 Simulation results for quad-copter machine in literature

We first configured our simulator for the physical parameters of the quad-copter machine used by Canetta et al (2007) to validate our model and simulator by considering 2 different cases – case-I) fixed air-density case (as used in this reference), and case-II) our proposed variable air-density case. The results are shown in Fig.5.

When all motors supply the same thrust then the quad-rotor rises(i.e. altitude increases),if the total thrust produced is larger than the weight of quad-copter. It is observed that the results of case-I exactly match with the results in Canetta et al (2007).Whereas, the simulation results of case-II correctly show quad-copter attaining slightly lower altitude as compared to case-I, because our proposed variable density model simulator correctly captures the effect of lower thrust caused due to lower air-density along increase in the altitude.

3.2 Simulation results for our quad-copter machine

Next, we configured our simulator for the physical parameters of the quad-copter machine available in our laboratory using our modified approach of using variable air-density model. Table-1 shows various physical parameters of our quad-copter machine. Fig.6 shows the altitude simulation for our quad-copter and found satisfactory.

Again, when all the motors supply the same thrust, the quad rotor rises until the total thrust produced exactly balances with the weight of quad rotor (including pay-load).

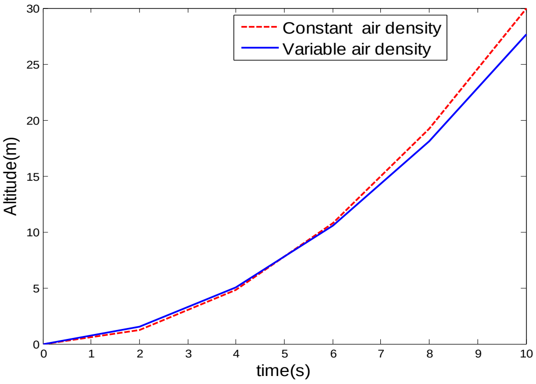

Fig.5. Simulation results for quad-copter machine of Canetta et al (2007) for 2 cases:Case-I) fixed air-density model and Case-II) our modified variable air-density model.

Table-1.Parameters of our quad-copter machine.

Parameter	Value	Description
W[m]	0.1050	Width of hub
D [m]	0.1050	Depth of hub
h [m]	0.0390	Height of hub
Q [kg]	0.0400	Mass of motor
P [kg]	0.1000	Mass of hub
R [m]	0.0130	Radius of motor-body
Hm [m]	0.0150	Height of motor-body
R [m]	0.2475	Motor to hub distance
C_T	0.5000	Thrust Coefficient
M [kg]	1.2000	Total mass

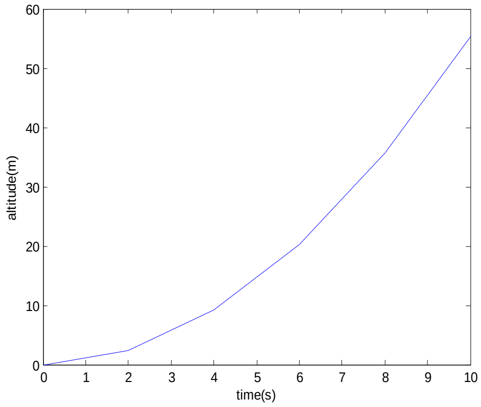

Fig.6.Altitude simulation for our quad-copter machine(variable density model)

3.3 Altitude-Range-Analysis for our quad-copter machine

Next, we performed Altitude-Range-Analysis (ARA) for our quad-copter machine to find maximum attainable altitude for different voltage (control-voltage to quad-rotor motor driver) with different pay-load weight.

Table-2 shows results of ARA for the geographical location-1 having lower reference altitude so higher air-density values. Whereas, Table-3 shows results of ARA for the geographical location-2 having higher reference altitude i.e. lower air-density values.

It is observed that for a given pay-load weight, the feasible altitude increases with the control-voltage (i.e. motor thrust). But, for a particular control-voltage, the feasible altitude-limit reduces with an increase in pay-load weight, as expected.

Further, if the control-voltage and pay-load are kept same, the altitude-limit is lower in case of the geographical location-1 (low reference altitude) as compared to geographical location-2 (higher reference altitude), which is as expected.

Also note that in case of the geographical location-2 (which is at higher reference altitude), for control voltage<1 volt, simulation of our quad-copter machine does not provide any value for the attainable height. This is because the thrust generated is not sufficient to lift the quad-copter when control voltage < 1 V.Fig.7 provides a clear graphical representation of these investigations for two different geographical locations.

Table 2. Altitude vs. Control-Voltage for location-1

V	Pay load(kg)			Altitude (m)%		
	M1	M2	M3	H1	H2	H3
1	0.5	0.75	1	26.12	15.38	4.35
2	0.5	0.75	1	81.86	78.66	75.99
3	0.5	0.75	1	92.19	90.49	89.26
4	0.5	0.75	1	95.82	94.63	96.09
5	0.5	0.75	1	97.47	97.04	93.93

Table-3. Altitude vs. Control-Voltage for location-2

V	Pay load(kg)			Altitude (m)%		
	M1	M2	M3	H1	H2	H3
1	0.5	0.75	1	------	------	-----
2	0.5	0.75	1	52.33	49.14	46.96
3	0.5	0.75	1	62.67	60.97	60.26
4	0.5	0.75	1	66.29	65.11	64.93
5	0.5	0.75	1	67.95	67.52	67.08

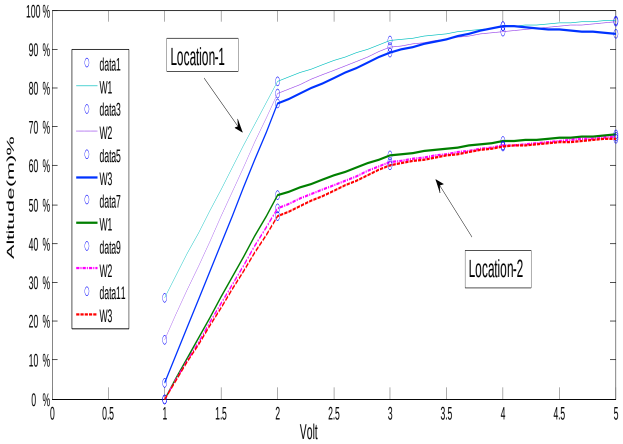

Fig.7. Graphical representation of Altitude-Range-Analysis for Location-1 and Location-2(Note: Altitudes are uniformly normalized for comparison, and W1, W2 and W3 represent 3 different payload mass.)

4. CONCLUSION AND FUTURE WORK

The Matlab-based simulators are developed based on the mathematical models in literature and our proposed modified model that accounts for the variable air-density effects.The simulation results are validated with the literature and are found in agreement and satisfactory. A concept of Altitude-Range-Analysis is proposed and shown for two different geographical locations having different reference altitude values. Our investigations are found to be logical and interesting from application perspective. For example, it correctly indicates that attainable altitude is lower if one accounts for the variable density. It also suggests that the use of the same capacity quad-copter machine will provide relatively a lower altitude-range if used at the higher reference altitude. Next, this work will be extended to perform validation and tuning of our quad-copter model simulator; ARA study with the help of actual flight data using our quad-copter machine; and then further extend to more rigorous analysis.

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Appendix A.

List of symbol

Φ	Roll angle
θ	Pitch angle
ψ	Yaw angle
ρ	Air density (kg/m^3)
T	Thrust (Kg)
M	Mass (kg)
H	Height or Altitude (m)
V	Control Voltage