Modification of Kani's Method for One Storey One Bay Portal Frame with Inclined Columns

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***Abstract*—Looking to the preset trend of analysis and design, particularly in analysis, people prefer the analysis method which gives the perfect results with least amount of effort. The rotation contribution method presented by Gasper Kani of Germany is excellent method of analysis. The rotation contribution method is smart extension of the slope deflection method. This is an iterative method. The rotation contribution method has simplicity of the moment distribution method. The rotation contribution method has overcome some disadvantages of the moment distribution method. The rotation contribution method is self corrective in nature. Even if an error committed in the calculation it gets automatically corrected in the course of further work.The conventional rotation contribution method is not applicable for the analysis of the frames with inclined columns. The rotation contribution method is excellent extension of the slope deflection method. To apply the conventional rotation contribution method to analyze the frames with inclined columns, it has to be modified. The conventional rotation contribution method can be modified with help of the slope deflection method.**

***Keywords*—Rotation Contribution Factors, Storey shear, Displacement Contribution Factors, Fixed end moments .**

I. INTRODUCTION

Here modification of the conventional rotation contribution method is done for the one bay one storey portal frame with the inclined columns. The present work can be extended for the multistoried multi-bay frame with inclined columns. The paper includes modification of rotation contribution factor (RCF), Displacement Contribution Factor (DCF) and storey shear. The results obtained by the modified Kani's method are compared with slope deflection method and moment distribution method.

II. DERIVATION

Columns with Fixed Ends

The one storey one bay portal frame is shown in figure 1. The given frame has three degrees of freedom, two rotations, one at joint 2, other at joint 3, and one lateral translation at joint 2 & 3 (neglecting axial deformation) of the frame. The rotations at joint 2 and 3 are denoted as θ2 and θ3 respectively and the horizontal displacement is denoted by Δ. The horizontal translation is constant since the axial deformation of the members is ignored.

The slope-deflection equations of given portal frame are given below. For member 1-2 the slope-deflection equations are as below.

$$M_{12} = MF_{12} + \frac{2EI_1\theta_2}{L_1} + \frac{6EI_1(\Delta/\mathrm{Sin}\alpha)}{L_1^2} \quad \ldots (1)$$

$$M_{21} = MF_{21} + \frac{4EI_1\theta_2}{L_1} + \frac{6EI_1(\Delta/\mathrm{Sin}\alpha)}{L_1^2} \quad \ldots (2)$$

Similarly for member 2-3 & member3-4,

$$M_{23}=MF_{23}+\frac{4\,EI_2\theta_2}{L_2} + \frac{2EI_2\theta_3}{L_2} - \frac{6\,EI_2(\Delta/\mathrm{Tan}\,\alpha +\Delta/\mathrm{Tan}\beta)}{L_2^2} \quad \ldots (3)$$

$$M_{32}=MF_{32}+\frac{4EI_2\theta_3}{L_2}+\frac{2EI_2\theta_2}{L_2}-\frac{6EI_2(\Delta/\mathrm{Tan}\alpha+\Delta/\mathrm{Tan}\beta)}{L_2^2} \quad \ldots (4)$$

$$M_{34}=\frac{4EI_3\theta_3}{L_3}+\frac{6EI_3(\Delta/\sin\beta)}{L_3^2} \quad \ldots (5)$$

$$M_{43} = \frac{2\,EI_3\,\theta_3}{L_3} + \frac{6EI_3\,(\Delta/\mathrm{Sin}\,\beta)}{L_3^2} \quad \ldots (6)$$

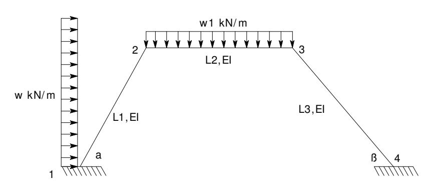


Fig 1: Frame and loading

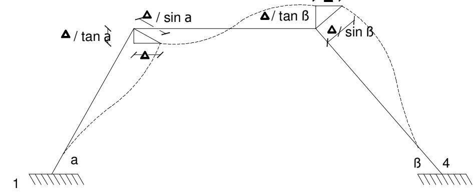


Fig.2: Lateral translation of joints

Taking the summation of the moment at joint 2,

$M_{21} + M_{23} = 0$

Substituting values of M_{21} and M_{23} and taking terms containing θ_2 on the left side of equation and taking θ_2 common,

$$\theta_2 = -\frac{\{MF_{21} + MF_{23} + M'_{32} + M''_{23} + M''_{21}\}}{\{4\,EI_1 / L_1 + 4\,EI_2 / L_2\}}$$

Where,

The term $M'_{32} = \dfrac{2\,EI_2\,\theta_3}{L_2}$,

will be referred as rotation contribution at joint 3 in member 2-3

The term $M''_{21} = \dfrac{6\,EI_1\,(\Delta / \sin\alpha)}{L_1^2}$

will be referred as displacement contribution at 2 in member 1-2

The term $M''_{23} = -\dfrac{6\,EI_2\,(\Delta / \tan\alpha + \Delta / \tan\beta)}{L_2^2}$

will be referred as displacement contribution at 2 in member 2-3

Putting $I_1 / L_1 = K_1$, $I_2 / L_2 = K_2$ and $K_1 + K_2 = \sum K$

$$\theta_2 = -\frac{\{MF_{21} + MF_{23} + M'_{32} + M''_{23} + M''_{21}\}}{4\,E\sum K} \qquad \text{... (7)}$$

Similarly at joint 3,

$$\theta_3 = -\frac{\{MF_{32} + MF_{34} + M'_{23} + M''_{23} + M''_{34}\}}{4\,E\sum K} \qquad \text{... (8)}$$

One more independent equation is necessary to solve for the three unknowns θ_2, θ_3 and Δ.The third conditional equation can be obtained by considering that the summation of all the forces in the horizontal direction on the entire structure is zero.

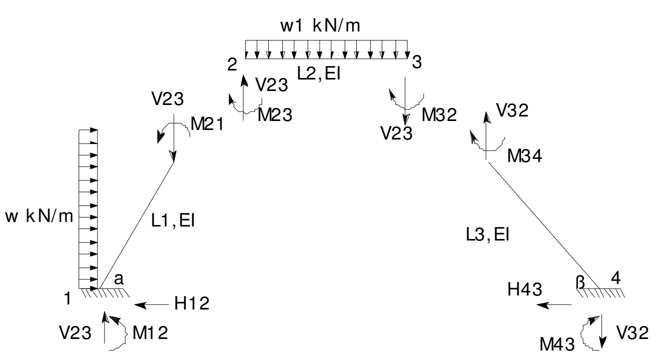


Fig.3: free body diagram of the frame members

The forces involved in this equilibrium equation are horizontal UDL of W kN/m and the shears in the columns at the bases.

$$\sum F_H = H_{12} + H_{43} - W \times L_1 \sin\alpha = 0. \qquad \text{... (9)}$$

Expression for shear H_{12} can be obtained by taking summation of moments about joint 2 on the left hand side column.

$$-H_{12} \times L_1 \sin\alpha - V_{23} \times L_1 \cos\alpha + M_{12} + M_{21} + \frac{W(L_1 \sin\alpha)^2}{2} = 0 \qquad \text{... (10)}$$

Putting values of M_{12} and M_{21} from equations 1 and 2

$$-H_{12} \times L_1 \sin\alpha - V_{23} \times L_1 \cos\alpha + MF_{12} + MF_{21} + \frac{6\,EI_1\,\theta_2}{L_1} + \frac{12\,EI_1(\Delta/\sin\alpha)}{L_1^2} + \frac{W\,(L_1 \sin\alpha)^2}{2} = 0$$

Similarly for shear H_{43} can be obtained by taking summation of moments about joint 3 on the right hand side column.

$$-H_{43} \times L_3 \sin\beta - V_{32} \times L_3 \cos\beta + M_{34} + M_{43} = 0. \qquad \text{... (11)}$$

Putting values of M_{34} and M_{43} from equations 5 and 6

$$-H_{43} \times L_3 \sin\beta - V_{23} \times L_3 \cos\beta + \frac{6EI_3\theta_3}{L_3} + \frac{12EI_3(\Delta/\sin\beta)}{L_3^2} = 0.$$

Vertical reaction V_{23} can be obtained by taking moment of all forces on member 2-3 about joint 3.

$$-V_{23} \times L_2 + M_{23} + M_{32} + \frac{W_1 \times L_2^2}{2} + \frac{P \times b}{L_2} = 0$$

Putting values of M_{23} and M_{32} from equations 3 and 4 respectively and simplifying,

$$\uparrow \therefore V_{23} = R_{F23} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12EI_2\Delta(\cot\alpha + \cot\beta)}{L_2^3} \qquad \text{... (12)}$$

Where R_{F23} is upward fixed end reaction at joint 2 in member 2-3.

$$\uparrow R_{F23} = \frac{MF_{23} + MF_{32}}{L_2} + \frac{W_1 \times L_2}{2} + \frac{P \times b}{L_2}$$

Similarly vertical reaction V_{32} can be obtained by taking moment of all forces on member about joint 2.

$$-V_{32} \times L_2 + M_{23} + M_{32} - \frac{W_1 \times L_2^2}{2} - \frac{P \times a}{L_2} = 0$$

Putting values of M_{23} and M_{32} from equations 3 and 4 respectively and simplifying,

$$\downarrow V_{32} = -RF_{32} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12EI_2\Delta}{L_2^3}(\text{Cot}\alpha + \text{Cot}\beta) \quad \ldots (13)$$

Where R_{F32} is fixed end reaction at joint 3 in member 2-3 acting upward.

$$\uparrow R_{F32} = \frac{MF_{23} + MF_{32}}{L_2} - \frac{W_1 \times L_2}{2} - \frac{P \times b}{L_2}$$

From equation 10

$$-H_{12} \times L_1 \text{Sin}\alpha - V_{23} \times L_1 \text{Cos}\alpha + M_{12} + M_{21} + \frac{W (L_1 \text{ Sin } \alpha)^2}{2} = 0$$

$$\therefore H_{12} = -V_{23} \times \text{Cot } \alpha + \frac{M_{12} + M_{21}}{L_1 \text{ Sin } \alpha} + \frac{W (L_1 \text{ Sin } \alpha)}{2}$$

Putting values of V23 from equation 12 and values of M12 and M21 from equations 1 & 2 in above equation,

$$H_{12} = \frac{MF_{12} + MF_{21}}{L_1 \text{ Sin } \alpha} + \frac{W (L_1 \text{ Sin } \alpha)}{2} + \frac{3 M'_{21}}{L_1^2 \text{ Sin } \alpha} + \frac{12 EI_1 \Delta}{L_1^3 (\text{Sin } \alpha)^2}$$

$$- \left\{ R_{F23} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12EI_2\Delta}{L_2^3}(\text{Cot}\alpha + \text{Cot}\beta) \right\} \text{Cot } \alpha$$

From equation 11

$$H_{43} = \frac{M_{34} + M_{43}}{L_3 \text{ Sin } \beta} - V_{32} \times \text{Cot } \beta$$

Putting values of V_{32} from equation 13 and values of M_{34} and M_{43} from equations 5 and 6 in above equation,

$$H_{43} = \frac{3 M'_{34}}{L_1^2 \text{ Sin } \alpha} + \frac{12 EI_1 \Delta}{L_1^3 (\text{Sin } \alpha)^2}$$

$$- \left\{ - R_{F32} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12EI_2\Delta}{L_2^3}(\text{Cot}\alpha + \text{Cot}\beta) \right\} \text{Cot } \beta$$

Form equation 9

$$\sum F_H = H_{12} + H_{43} - W L_1 \text{ Sin } \alpha = 0.$$

Putting values of H_{12} and H_{43} and simplifying we get,

$$\therefore \Delta = -3/2 \frac{-R_{F21} + \frac{3M'_{21}}{L_1 \text{Sin}\alpha} + \frac{3M'_{34}}{L_3 \text{Sin}\beta} - \frac{(3M'_{23} + 3M'_{32})}{L_2}(\text{Cot}\alpha + \text{Cot}\beta) - \frac{R_{F32}}{\text{Tan}\alpha} + \frac{R_{F32}}{\tan\beta}}{6E \left\{ \frac{I_1}{L_1^3 (\text{Sin } \alpha)^2} + \frac{I_2 \times (\text{Cot } \alpha + \text{Cot } \beta)^2}{L_2} + \frac{I_3}{L_3^3 (\text{Sin}\beta)^2} \right\}}$$

Now in equation 1

$$M_{12} = MF_{12} + \frac{2 EI_1 \theta_2}{L_1} + \frac{6 EI_1 (\Delta / \text{Sin } \alpha)}{L_1^2}$$

As defined earlier

$$\frac{2EI_1\theta_2}{L_1} = M'_{21}$$

Putting value of θ_2 from equation 7 in above M'_{21},

$$\therefore M'_{21} = -1/2 \times \frac{K_1}{\sum K} \times \{ MF_{21} + MF_{23} + M'_{32} + M''_{23} + M''_{21} \}$$

The $\{-1/2 \times K_1/\sum K\}$ factor will be referred as RF_{12} (Rotation Factor at joint 1 in member 1-2).

The rotation factor meeting at any joint is obtained by distributing the value (-1/2) among the members in the proportion of their K-values. The sum of the rotation factor at any joint is (–1/2).Also

$$\frac{6 EI_1 (\Delta / \text{Sin } \alpha)}{L_1^2} = M''_{21}$$

Putting value of Δ in above equation and simplifying,

$$M''_{21} = DF_{12} \times \frac{Qr \times L_1}{3} + M'_{21} \times C_{12} + (M'_{23} + M'_{32}) \times C_{23} + M'_{34} \times C_{34}$$

Where the terms C_{12}, C_{23}, C_{34} are called height modification factor for member 1-2, 2-3and 3-4 respectively and Qr is the storey shear.

$$Qr = - \left\{ RF_{21} - \frac{RF_{32}}{\text{Tan } \alpha} + \frac{RF_{23}}{\text{Tan } \beta} \right\}$$

$$C_{12} = \frac{L_1 \times \text{Cosec } \alpha}{L_1}$$

$$C_{23} = - \frac{L_1 \times (\text{Cot } \alpha + \text{Cot } \beta)}{L_2}$$

$$C_{34} = \frac{L_1 \times \text{Cosec } \beta}{L_3}$$

$$DF_{12} = -3/2 \times \frac{K_{12} \times C_{12}}{\{K_{12} \times C_{12}^2 + K_{23} \times C_{23}^2 + K_{34} \times C_{34}^2\}}$$

where DF_{12} is Displacement Contribution in member 1-2.

The sum of displacement factors for any storey is (-3/2). The displacement factors are obtained by distributing the constant (-3/2) among the members of frame in their respective K-values.

The above result for the individual rotation contribution may be rewritten as:

$$M'_{21} = RF_{12} \times \{MF_{21} + MF_{23} + M'_{32} + M''_{23} + M''_{21}\}$$

Where, "RF_{12}" denotes the rotation factor as above.

From equation 1,

$$M_{12} = MF_{12} + \frac{2 EI_1 \theta_2}{L_1} + \frac{6 EI_1 (\Delta / \text{Sin } \alpha)}{L_1^2}$$

The equation for M_{12} may now be written as

$$M_{12} = MF_{12} + M'_{21} + M''_{21}$$

Similarly from equation 2, the equation for M_{21} may now be written as

$$M_{21}=MF_{21} + 2M'_{21} + M''_{21}$$

Similarly from equation 3

$$M_{23} = MF_{23}+\frac{4EI_2\,\theta_2}{L_2} + \frac{2\,EI_2\,\theta_3}{L_2} + \frac{6\,EI_2\,(\Delta / \mathrm{Tan}\,\alpha + \Delta / \mathrm{Tan}\,\beta)}{L_2^2}$$

Where the term

$$\frac{2\,EI_2\,\theta_2}{L_2} = M'_{23}$$

Putting values of θ_2 from equation 8 in above M'_{23},

$$M'_{23} = \frac{-2EI_2}{L_2} \times \frac{\{MF_{21} + MF_{23} + M'_{32} + M''_{23}\}}{4E \sum K}$$

$$\therefore M'_{23} = -1/2 \times K_2 \times \frac{\{MF_{21} + MF_{23} + M'_{32} + M''_{23}\}}{\sum K}$$

Similarly for all member moments may be written as,

$$M_{32}=MF_{32}+M'_{23}+2M'_{32}+M''_{23}$$

$$M_{34}=MF_{34}+2M'_{34}+M''_{34}$$

$$M_{43}=MF_{43}+M'_{34}+M''_{34}$$

Columns with Hinged End

A column with a hinged end may be replaced, for purpose of analysis, by fixed end, provided the real column and the substitute column are structurally similar at the top of the column. For the structural similarity to exist, the following two conditions must be satisfied:

1. A moment M applied at the top of the column must give the equal rotations at the top in both cases (real column and substitute column).
2. A lateral translation occurring at the top of the column must give equal fixed end moments at the top in both the cases.

If these two conditions are satisfied, it is immaterial for the remaining part of the frame weather the column is the hinged or fixed at the base. Simple algebraically relation ship between the properties of real and substitute columns implied by the above two conditions will be derived.

Let I and h be the moment of inertia and the actual height of the real (hinged) column, and let I' and h' be those of substitute (fixed) column.

Condition 1

If a moment M is applied at the top of the real column the rotation at the top, (fig.4 (a))

$$= \frac{M \times h}{3EI}$$

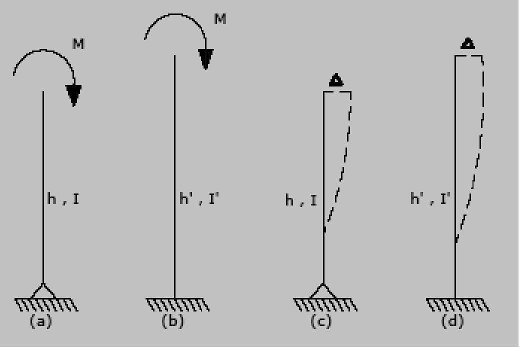


Fig. 4

If the moment M is applied at the top of the substitute column, one know that this induces a moment M / 2 at the base in the same direction, so that the rotation at the top

$$= \frac{M \times h'}{3\ EI'} - \frac{M \times h'}{2 \times 6\ EI'}$$

$$= \frac{M \times h'}{4\ EI'}$$

For condition 1 to be satisfied:

$$\frac{M \times h'}{3\ EI'} = \frac{M \times h'}{4\ EI'}$$

$$\therefore\quad I'/\,h' = (3/4)\ I\,/\,h$$

$$\therefore\quad K' = (3/4)\ K$$

Condition 2

If there is a lateral sway Δ at the top of the real column (fig.4a) fixed end moment at the top

$$= \frac{3EI\ \Delta}{h^2}$$

If the sway Δ at the top of the substitute column (fig.4b) fixed end moment at the top

$$= \frac{6EI\ \Delta}{h^2}$$

For the condition 2 to be satisfied,

$$\frac{3EI\ \Delta}{h^2} = \frac{3EI'\ \Delta}{h'^2}$$

$$\therefore K / h = 2\ K' / h'$$

But for condition 1 to be satisfied,

$$\therefore\quad K' = (3/4)\ K$$

$$\therefore K / h = (3/2)\ K / h'$$

$$\therefore\ h' = 3/2\ h$$

Thus if substitute column is provided with the K- value equal to 3/4 K and height equal to 3/2 h, the replacement does not affect the rest of the structure and is therefore justified.

Necessity for the modification to the displacement contribution equation

The displacement of the real column by the substitute column must be taken in to account while deriving the displacement contribution equation. The equilibrium equation for the column, which is always the first step in this derivation, involves the actual height. But since the calculations are going to be done with the substitute column (fixed end) the displacement contribution equation must be expressed with reference to the substitute heights. It is possible that the some of the columns in the storey may be hinged at the base while others may be fixed. The displacement contribution equation involves the summation of certain values for all the columns of the storey. The form of the displacement contribution equation must be in general, applying to hinged as well as the fixed columns of the storey.

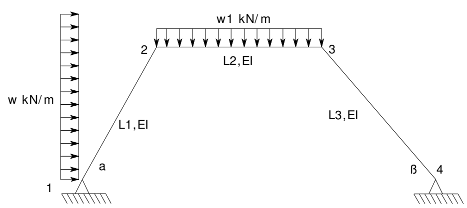


Fig. 5: Frame with hinged support and loading

The slope deflection equations for member 1-2 are,

$$M_{12} = MF_{12} + \frac{2\,EI_1\,\theta_2}{L_1} + \frac{4\,EI_1\,\theta_1}{L_1} + \frac{6\,EI_1\,(\Delta / \text{Sin }\alpha)}{L_1^2}$$

$$M_{21} = MF_{21} + \frac{4\,EI_1\,\theta_2}{L_1} + \frac{2\,EI_1\,\theta_1}{L_1} + \frac{6\,EI_1\,(\Delta / \text{Sin }\alpha)}{L_1^2}$$

Similarly for member 2-3,

$$M_{23} = MF_{23} + \frac{4\,EI_2\theta_2}{L_2} + \frac{2EI_2\,\theta_3}{L_2} - \frac{6\,EI_2\,(\Delta / \text{Tan }\alpha + \Delta / \text{Tan }\alpha)}{L_2^2}$$

$$M_{32} = MF_{32} + \frac{4EI_2\,\theta_3}{L_2} + \frac{2EI_2\,\theta_2}{L_2} - \frac{6\,EI_2(\Delta / \text{Tan }\alpha + \Delta / \text{Tan }\beta)}{L_2^2}$$

Similarly for member 3-4

$$M_{34} = \frac{4\,EI_3\,\theta_3}{L_3} + \frac{2\,EI_3\,\theta_4}{L_3} + \frac{6\,EI_3\,(\Delta / \text{Sin }\beta)}{L_3^2}$$

$$M_{43} = \frac{2\,EI_3\,\theta_3}{L_3} + \frac{4\,EI_3\,\theta_4}{L_3} + \frac{6EI_3\,(\Delta / \text{Sin }\beta)}{L_3^2}$$

Here at the hinge support the moments are zero.

$$M_{12} = 0 \text{ and } M_{43} = 0$$

Applying $M_{12} = 0$ and simplifying, we get,

$$\therefore\ \theta_1 = \frac{-L_1}{4\,EI_1}\left\{MF_{12} + \frac{2\,EI_1\,\theta_2}{L_1} + \frac{6\,EI_1\,\Delta}{L_1^2\,\text{Sin }\alpha}\right\}$$

And similarly for member 3-4 applying $M_{43} = 0$,

$$\therefore\ \theta_4 = \frac{-L_3}{4\,EI_3}\left\{MF_{43} + \frac{2\,EI_3\,\theta_3}{L_3} + \frac{6\,EI_3\,\Delta}{L_3^2\,\text{Sin }\beta}\right\}$$

Putting the values of θ_1 & θ_4 in the slope deflection equations and simplifying,

$$M_{12} = 0$$

$$M_{21} = MF_{21} - \frac{MF_{12}}{2} + \frac{3\,EI_1\,\theta_2}{L_1} + \frac{3\,EI_1\,(\Delta / \text{Sin }\alpha)}{L_1^2}$$

$$M_{23} = MF_{23} + \frac{4EI_2\,\theta_2}{L_2} + \frac{2EI_2\,\theta_3}{L_2} - \frac{6EI_2\,(\Delta / \text{Tan }\alpha + \Delta / \text{Tan }\alpha)}{L_2^2}$$

$$M_{32} = MF_{32} + \frac{4EI_2\theta_3}{L_2} + \frac{2EI_2\,\theta_2}{L_2} - \frac{6\,EI_2\,(\Delta / \text{Tan }\alpha + \Delta / \text{Tan }\beta)}{L_2^2}$$

$$M_{34} = \frac{3\,EI_3\,\theta_3}{L_3} + \frac{3\,EI_3\,(\Delta / \text{Sin }\beta)}{L_3^2}$$

$$M_{43} = 0$$

Applying $M_{23} + M_{21} = 0$ and simplifying one get value of θ_2 as,

$$\theta_2 = -\frac{\{MF_{21} + MF_{23} + M'_{32} + M''_{23} + M''_{21}\}}{4E\,\{K'_{12} + K_{23}\}} \qquad \text{... (14)}$$

Applying the condition $M_{32} + M_{34} = 0$ and simplifying one get value of θ_3 as,

$$\theta_3 = -\frac{\{MF_{34} + MF_{32} + M'_{23} + M''_{23} + M''_{34}\}}{4E\,\{K'_{34} + K_{23}\}} \qquad \text{... (15)}$$

Taking moment about joint 2 in member 1-2

$$H_{12} = \frac{1}{L_1\,\text{Sin }\alpha}\left(M_{12} + M_{21} + \frac{W_1(L_1\,\text{Sin }\alpha)^2}{2} - V_{23} \times L_1\,\text{Cos }\alpha\right)$$

Putting the values of $M_{12,}$ M_{21} & V_{23} and simplifying,

$$H_{12} = \frac{MF'_{21}}{L_1\,\text{Sin }\alpha} + \frac{W\,(L_1\,\text{Sin }\alpha)}{2} + \frac{3\,EI_1\,\theta 2}{L_1^2\,\text{Sin }\alpha} + \frac{3\,EI_1\,\Delta}{L_1^3\,(\text{Sin }\alpha)^2}$$

$$-\left\{R_{F23} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12\,EI_2\Delta(\text{Cot}\alpha + \text{Cot}\beta)}{L_2^3}\right\}\text{Cot }\alpha$$

Where $MF'_{21} = MF_{21} - \frac{MF_{12}}{2}$

Similarly taking moment about joint 3 in member 3-4,

$$H_{43} = \frac{MF'_{34}}{L_3\,\text{Sin }\beta} + \frac{3\,EI_3\,\theta_3}{L_3^2\,\text{Sin }\beta} + \frac{3\,EI_3\,\Delta}{L_3^3\,(\text{Sin }\beta)^2}$$

$$-\left\{-RF_{32} + \frac{3M'_{23} + 3M'_{32}}{L_2} - \frac{12\,EI_2\,\Delta\,(\text{Cot}\alpha + \text{Cot}\beta)}{L_2^3}\right\}\text{Cot }\beta$$

Where $MF'_{34} = MF_{34} - \frac{MF_{43}}{2}$

Applying the third condition $\sum H = 0$

$$H_{12} + H_{43} = W_1 \times L_1 \sin\alpha$$

Putting values of H_{12} and H_{43} and simplifying,

$$\Delta = \frac{-Qr + \dfrac{3\,EI_1\,\theta_2}{L_1^2 \sin\alpha} - \dfrac{3\,M'_{23} + 3M'_{32}}{L_2}(\cot\alpha + \cot\beta)}{\left\{ \dfrac{3\,EI_1}{L_1^3 (\sin\alpha)^2} + \dfrac{3\,EI_3}{L_3^3 (\sin\beta)^2} - \dfrac{12\,EI_2 (\cot\alpha + \cot\beta)^2}{L_2} \right\}}$$

Now

$$M_{21} = MF_{21} - \frac{MF_{12}}{2} + \frac{3\,EI_1\,\theta_2}{L_1} + \frac{3\,EI_1\,(\Delta / \sin\alpha)}{L_1^2}$$

By definition

$$M''_{21} = \frac{3\,EI_1\,(\Delta / \sin\alpha)}{L_1^2}$$

Putting the value of Δ and simplifying,

$$M''_{21} = DF_{12} \left\{ \frac{Qr \times L_1'}{3} + M'_{21} \times C_{12} + (M'_{23} + M'_{32}) + M'_{34} \right\}$$

Where,

DF_{12} = displacement contribution factor for member 1-2

$$= \left\{ \frac{-3\ K_{12} \times C_{12}}{m \times K_{12} \times (C_{12})^2 + K_{23}\,(C_{23})^2 + m \times K_{34} \times (C_{34})^2} \right\}$$

Where,

C_{12} = height modification factor

$$C_{12} = \frac{L_1' \times \operatorname{Cosec}\alpha}{L_1'}$$

$$C_{23} = -\frac{L_1' \times (\cot\alpha + \cot\beta)}{L_2}$$

and

$$C_{34} = \frac{L_1' \times \operatorname{Cosec}\beta}{L_3}$$

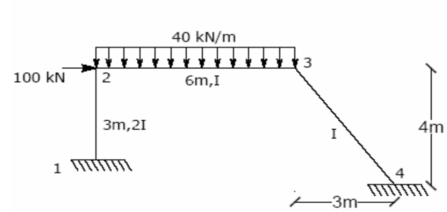
$$L_1' = \frac{3}{2} \times L_1$$

m = end condition factor

= 1 for fixed column

= 3/4 for hinged column

III. COMPARISON



End moments	*Modified Kani's method*	*Slope-deflection method*	*Moment distribution method*
M12	5.55315	5.554	5.568
M21	-91.55135	-91.551	-91.555
M23	91.55136	91.551	91.555
M32	-94.44472	-94.445	-94.433
M34	94.44472	94.445	94.436
M43	58.77136	58.772	58.765

IV. CONCLUSION

Here the modified Kani's equations are applicable for the one bay one storey portal frame with inclined columns. The modified Kani's equations are also applicable to the one storey one bay portal frame with vertical columns. For the frames with the rectangular joints, the unknown joint translations are usually horizontal in direction; therefore it may be called unknown side sways. The number of unknown side sways should be equal to the number of the stories in the rigid frame. Using this side sways the conventional rotation contribution method can be modified, using the slope deflection equations for multi storey multi bay portal frame with inclined columns.

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