Software Development for Calculation of Stress Intensity Factor (SIF)

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***Abstract*—The Stress Intensity Factor (SIF) is a measure of the severity of a crack in an elastic solid and is closely related to the stress field in the vicinity of the crack tip. There is a direct relationship between the SIF and the energy release rate which governs the criticality of a crack. Since the range of SIF during a fatigue loading cycle governs the crack growth rate, knowledge of the SIF for a given crack geometry is essential in any fatigue crack growth computation. Software is being developed for calculation of SIF.**

***Index Terms*—The Stress Intensity Factor, Fatigue Crack Growth Computation.**

I. Introduction

Sress Intensity Factor, $K = \sigma\ (\pi a)^{1/2}$ K is a means of characterising the elastic stress distribution near the crack tip. Units of K are $MN/m^{-3/2}$ or $MPam^{1/2}$. Basis of the LEFM design approach is, All materials contain flaws or cracks. The stress intensity factor, K, may be calculated for the particular loading and crack configuration. Failure is predicted if the calculated value of K exceeds the critical value, K_c for the material. Critical value of K is referred to as the *Fracture Toughness*. To extend application beyond a central crack in an infinite plate, K is usually expressed in a general form $K = f(\beta)\sigma\ (\pi a)^{1/2}$.

II. Effect of Plate Thickness

In a thin Plate, out of plane stress components σ_{33}, σ_{31}, σ_{32} are able to relax to zero and material deforms easily within the plastic zone. In the vicinity of the crack tip, both the free surfaces of the plate move in forming depression sowing to in-plane tensile stress and Poison's effects. On the contrary the material in a thick late is constrained giving rise to tensile stress σ_{33}.

There must be limit on plate thickness under which a material is able to flow easily and plate is deformed under plane stress. Similarly there must be a limit on plate thickness over which the material is taken to be deformed under plane strain. Between upper and lower limit on plate thickness, the cases are known to have transitional behavior ; that is near both the free surfaces the materials flows easily and deformed in plane stress and, in the interior, the material is constrained and is subjected to plane strain.

Typical nature of critical SIF dependence on the thickness is shown in Fig.1. For $B \geq 2.5(K_{IC}/\ \sigma_{YS})^{1/2}$, critical SIF remains constant and then we can regard the critical stress intensity factor as the material property. Value of K_{IC} of commonly used materials are available.

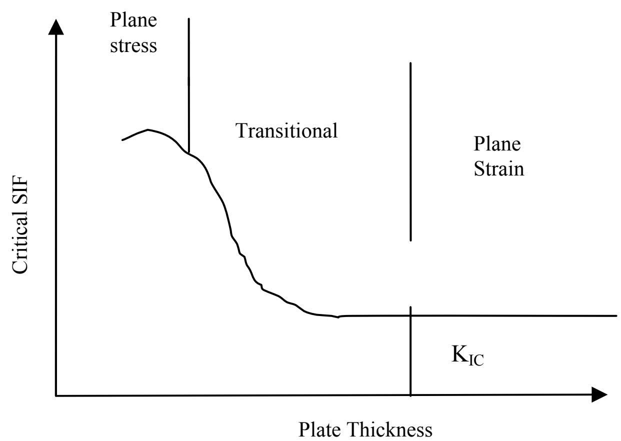

Fig. 1: Variation of Critical SIF with Plate Thickness

For $B < 2.5\ (K_{IC}/\ \sigma_{YS})^{1/2}$, critical stress intensity factor depends on the thickness B. The relation between critical SIF and thickness may be regarded as a behavior of material and be provided.

A plot of log da/dN versus log ΔK, a sigmoidal curve, is shown in Fig. 2.

This curve may be divided into three regions. At low stress intensities, **Region I**, cracking behavior is associated with threshold, ΔK_{th}, effects. In the mid-region, Region II, the curve is essentially linear. Many structures operate in this region. Finally, in the Region III, at high ΔK values, crack growth rates are extremely high and little fatigue life is involved.

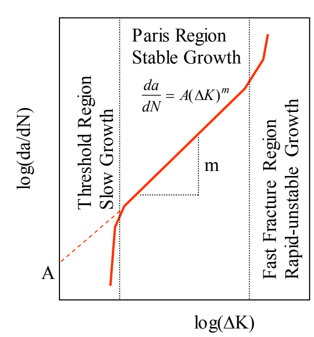

Fig. 2: Three regions of crack growth rate curve

Region II

Most of the current applications of LEFM concepts to describe crack growth behavior are associated with Region II. In this region the slope of the log **da/dN** versus log **ΔK** curve is approximately linear and lies roughly between 10^{-6} and 10^{-3} in/cycle. Many curve fits to this region have been suggested. The Paris equation, which was proposed in the early 1960s, is the most widely accepted. In this equation

$$\frac{da}{dN} = C(\Delta K)^m$$

where **C** and **m** are material constants

Values of the exponent, **m**, are usually between 3 and 4. These range from 2, 3 to 6, 7 with a sample average of m = 3,5. In addition, tests may be performed. ASTM E647 sets guidelines for these tests.

The crack growth life, in terms of cycles to failure, may be calculated using Equation

The relation may be generally described by

$$\frac{da}{dN} = f(k)$$

Thus, cycles to failure, **Nf**, may be calculated as

$$N_f = \int_{ao}^{af} \frac{da}{f(k)}$$

Using the Paris formulation,

$$\frac{da}{dN} = C(\Delta K)^m$$

$$N_f = \int_{ao}^{af} \frac{da}{C(\Delta K)^m}$$

Because **ΔK** is a function of the crack length and a correction factor that is dependent on crack length. The integration above must often be solved numerically. As a first approximation, the correction factor can be calculated at the initial crack length and can be evaluated in closed form.

As an example of closed form integration, fatigue life calculations for a small edge-crack in a large plate are performed below. In this case the correction factor, f(g) does not vary with crack length. The stress intensity factor range is

$$\Delta K = 1.12\Delta\sigma\sqrt{\pi a}$$

Substituting into the Paris equation yields

$$\frac{da}{dN} = C(1.12\Delta\sigma\sqrt{\pi a})^m$$

Separating variables and integrating (for m < >2) [14]

$$N_f = \int_{a_i}^{a_f} \frac{da}{C(1.12\Delta\sigma\sqrt{\pi a})^m}$$

$$= \frac{2}{(m-2)C(1.12\Delta\sigma\sqrt{\pi a})^m}\left(\frac{1}{a_i^{(m-2)/2}} - \frac{1}{a_f^{(m-2)/2}}\right)$$

Before this equation is solved, the final crack size, a_f, must be evaluated. This may be done using as follows [14].

$$K = f(\beta)\sigma\sqrt{\pi a}$$

$$a_f = \frac{1}{\pi}\left[\frac{K_c}{\sigma f(\beta)}\right]^2 = \frac{1}{\pi}\left[\frac{K_c}{1.12\sigma_{\max}}\right]^2$$

For more complicated formulations of **ΔK**, where the correction factor varies with the crack length, **a**, iterative procedures may be required to solve for $\mathbf{a_f}$ in equation. It is important to note that the fatigue-life estimation is strongly dependent on $\mathbf{a_i}$, and generally not sensitive to af (when $\mathbf{a_i \ll a_f}$). Large changes in af results are very small. Changes of $\mathbf{N_f}$ as shown schematically in Fig. 3.

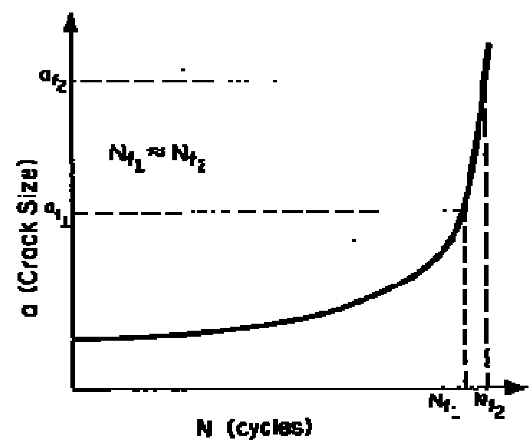

Fig. 3: Effect of final crack size on life

III. Stress Intensity Factor Calculation

The Stress Intensity Factor (SIF) is a measure of the severity of a crack in an elastic solid and is closely related to the stress field in the vicinity of the crack tip. There is a direct relationship between the SIF and the energy release rate which governs the criticality of a crack. Since the range of SIF during a fatigue loading cycle governs the crack growth rate, knowledge of the SIF for a given crack geometry is essential in any fatigue crack growth computation.

Preprocessing is performed after the entry of geometric dimensions to derive a two dimensional table for a specific problem.

- Select Residual strength as a function of crack size. σ = f(a).
- Maximum permissible crack size a_c.
- Minimum detectable crack size by NDT is a.

- For given value of σ one can determine the critical value of crack size a_c.
- For given value of crack size one can determine the critical value of stress σ.
- $K_{IC} = f(\beta)\, \sigma \sqrt{\pi a}$ where $\beta = a/W$, W = width.
- For plane stress conditions to reach $B \geq 2.5 (K_{IC}/\sigma ys)^2$
- Compute critical SIF.

IV. Choosing Crack Geometry

- The user selects a suitable crack geometry from among the many configurations built into the library by clicking on the "Select Geometry" tab and then using drop-down boxes.
- In addition, several surface-crack cases have been extended to include general loading.
- As per the Geometry of Crack geometric factor function f(β) is already included with the each geometry. Also the crack location effects are considered in this tab.
- For Internal And External Cracks in Cylinder The valid range of geometric parameters for this case is extended to cover the full range of *R i* /R 0, the ratio of internal to external radius. The stress intensity factor is defined as usual by

$$K_I = f(\beta)\sigma\sqrt{\pi a}$$

where σ is the applied uniform tensile stress. The correction factors for Crack location Internal are,

$$f_{\text{int}} = f(\beta)\sqrt{1-a/t}$$

similar results for the external crack location. Here, the stress intensity correction factors are defined as,

$$f_{ext} = f(\beta)\sqrt{1-a/t}\,/(1+R_o/R_i)$$

$$f_{ext} = f(\beta)\sqrt{1-a/t}\,/(1+R_o/R_i)$$

V. Entering Dimensions of the Component

- After selecting the geometry of the component, user is required to enter the dimensions of the component.
- All the dimensions entered by the user must be in mm. Pressure and Stress entry must be in Mpa. Software converts the entered values into the desired out put format. It is giving the stress induced in Mpa and Stress intensity factor in Mpa√m.

VI. Entering Initial Flow Size

Nondestructive Evaluation (NDE) crack size has a high probability of detection when inspection is performed in accordance with the proper specifications. From damage tolerance point of view, the standard NDE flaw size is one that may be just missed by the inspector and hence the structure should be able to withstand it for certain number of fatigue load cycles.

- For a surface crack in a solid circular section, the crack depth, c, is a function of *a* and *D* and can be calculated from:

$$c = r(1+\tan\theta - \sec\theta),\ \theta = \frac{a}{r} = \frac{2a}{D}$$

where D=2 r is the major diameter. Alternately, the surface crack length, a, may be calculated from the following expression:

$$a = r\tan^{-1}\left[\frac{c(2r-a)}{2r(r-a)}\right]$$

VII. Yielding Checks

- Stress exerted in the component should not be greater then yield Strength of the material. Also to be on safer side, factor of safety should be kept 2-3.
- At the designing stage software gives the checking weather the induced stress is greater then yield we can go for the different material option. Or either stress can be reduced or also crack size & location can be adjusted to go for the same material.
- Also similar check is available for stress intensity factor. This displayed the message if induced stresses intensity value exceeds critical stress intensity factor value.

VIII. How to Run Software

Step 1: To compute stress intensity factors, choose the relevant option from the main menu.

Step 2: Form the file menu select New Project. This enables the all buttons in the software. Also select the Elasticity type, Pressure type, & Calculation mode from the options menu.

Step 3: Select the Geometry of cracked component from the library available. When you select the option for Geometry, the GUI can be used to input geometry and dimensions of the crack case to be used. Select the desired option for crack location internal or and external and enter pressure & Stresses.

Step 4: Select material from the library, or user can enter the new choice for the material and enter the material's yield strength as well as Fracture Toughness of the newly entered material.

Step 5: To get Output push run button will give the Output of the critical crack size of the component and also the printable format of the output is displayed on the screen.

Fig. 4: Out put Showing Stress Intensity Factor

IX. Results and Discussion

Crack analysis of Cylindrical Pressure Vessels subjected to internal and external pressures is carried out using fundamentals of Linear Elastic Fracture Mechanics and Fatigue. To help this developing software is the basic aim of this project. Through the cross section of the literature it is found that there are defects in the cylindrical components of pressure vessels. Modules for each crack type contained in the cylindrical pressure vessel component is prepared in Visual Basic and analysis is carried out. Result gives us analytical solution for the life of the component. Thus behavior of the crack is analyzed.

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