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Integrated watershed modeling using Finite Element Method and GIS approach

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ABSTRACT: Watershed is a hydrological unit that can be used as a physical biological unit and socio-economic-political unit for planning and management of natural resources. For the appropriate management of a watershed, it is essential to calculate the runoff from the particular watershed for the given rainfall. Hence watershed modeling is very important in watershed management. Due to the complexity of the hydrological processes in any of the watersheds, watershed modeling requires advanced computational techniques and data management tools like Finite Element Methods (FEM) and Geographic Information Systems (GIS). In this study, the features of FEM are being added with GIS, which takes into account of the spatial variation of the data of the watershed considered. These two tools have been integrated here for watershed modeling to give more efficient tool in the watershed modeling. The developed model is applied to a watershed for the runoff simulation. The runoff calculated is compared with other models and found to be satisfactory.

1 INTRODUCTION

Watershed is a topographically delineated area that is drained by a stream and a basic hydrological unit with prominent physical and biological features, which can be treated as a unit having interface between hydrology, climatology and ecology. In arid or semi-arid regions, if watersheds are properly managed for water and soil conservation, and preservation of biomass, any climatic failures can be managed and water security can be assured. For the appropriate management of a watershed, it is essential to calculate the runoff from the particular watershed for the given rainfall. Hence watershed modeling is very important in watershed management.

Due to the complexities of the hydrological processes, watershed modeling requires advanced computational techniques and data management tools like Finite Element Method (FEM) and Geographic Information Systems (GIS). With the flexibility in the computer coding and modeling irregular boundaries, FEM is popularly used to solve the complex watershed problems. In the present study, the features of finite elements are being added with GIS, which takes into account of the spatial variation of the data of the watershed considered. These two important tools have been integrated using Graphical User Interface with Visual Basic programming for watershed modeling to give more efficient tool. The GIS package developed at Indian Institute of Technology (IIT), Bombay 'GRAM++' is used as the GIS tool in the present study.

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In the watershed modeling, initially the overland flow is simulated using the mass balance approach combined with GIS. GIS gives the information related to sub-watersheds, length and elevation of the main stream. These data combined with the water balance equation is used in the overland flow calculations. The channel flow is simulated using the continuity and momentum equations. A one-dimensional finite element model is developed for the channel flow based on the kinematic wave model. Finally both the overland flow model and channel flow model are coupled together to find out the runoff. The developed model is applied to a watershed for the runoff simulation.

2 FORMULATION FOR OVERLAND FLOW

The overland flow consists of the precipitated water in excess of the infiltration capacity. This water flows in the direction of the slope on the ground in the form of uniform thin sheet flow. The overland flow can be calculated using the mass balance equation given below:

$$\begin{aligned} \text{Inflow} - \text{Outflow} &= \text{Change in storage} \\ P.A_c - q.L &= \Delta V / \Delta t \end{aligned} \tag{1}$$

where P is the excess rainfall which is the inflow for the catchment i.e., rainfall minus the infiltration, q is the overland flow per unit length of stream from the catchment into the stream element assumed to be perpendicular to the length of the element. ΔV is the change in the detention storage, taken as positive for the increment and negative for the decrement, Δt is the time step and L is the total length of the stream element. Here, equation (1) is solved iteratively in the time domain where outflow from one time step serves as the inflow for the next and written as:

$$P.A_c - q.L = (A_c . \Delta d) / \Delta t \tag{2}$$

$$0.5(P_{t+\Delta t} + P_t) A_c - 0.5 (q_{t+\Delta t} + q_t) L = A_c (d_{t+\Delta t} - d_t) / \Delta t \tag{3}$$

where A_c is the area of the catchment, d is the depth of the overland flow. The variables with subscript shows the value at the time t and time $t + \Delta t$ i.e., at the beginning and end of the time step for one process. Rao and Rao (1988) had given the equation for overland flow as

$$q = (d . d^{2/3} . S^{1/2}) / n \tag{4}$$

where S is the slope of the watershed, which is equal to the friction slope and n is the Manning's roughness coefficient, applicable to the overland flow. If P is expressed in mm/hr, d in mm, L in meters and A_c in sq.m., then equation (3) can be expressed as

$$\frac{(P_{t+\Delta t} + P_t)\Delta t}{72\text{x}10^5} - \frac{\Delta t . L . S^{1/2} (d_{t+\Delta t}^{5/3} + d_t^{5/3})}{A_c . n . 2\text{x}10^5} = \frac{(d_{t+\Delta t} - d_t)}{1000} \tag{5}$$

An iterative type of solution is done for the equation (5) to give the depth at different time steps.

3 FEM FORMULATION FOR CHANNEL FLOW

In the present study kinematic wave analogy (Chow, 1988) is used to solve the channel flow. The present form of equations derived from the continuity and momentum equations (Ross et al., 1979) can be written as,

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial x} KS - q = 0 \tag{6}$$

$$S_o = S_f$$

where $Q=(1/n)\ AR^{2/3}S_f^{1/2}=KS$ is the discharge in the channel (cumecs); n is the Manning's coefficient, S is slope of the channel section, K is conveyance, A is the area of flow in the channel (sq.m); R is the hydraulic radius, q is the lateral inflow (overland flow) per unit length of flow plane (sq.m/s); x is the distance in the direction of flow (m); t is the time (s); S_o is the bed slope and S_f is the friction slope. The area of cross section of the channel A can be written as $W.H$, where W is the width of the channel and H is the depth of the water in the channel where as slope S can be written as dH/dx. Hence the equation (6) can be simplified as: (7)

$$W.\frac{\partial H}{\partial t}+\frac{\partial}{\partial x}.K.\frac{dH}{dx}-q=0 \tag{8}$$

The unknown variable (H) in the equation (8) is approximated using FEM as (Reddy, 1993):

$$H=\sum_{i=1}^{n} H_i(t)\ N_i(x) \tag{9}$$

where n is the number of nodes ($n=2$, for linear element) per element, and N_i are shape functions. For the linear line element, equation (9) can be written as:

$$H=N_1H_1+N_2H_2=[N]\ [H] \tag{10}$$

If L is length of the element and x is the distance from one node, then interpolation functions N_1 and N_2 can be written as: $1-x/L$ and x/L respectively. Here FEM based on Galerkin's formulation is used. Considering a partial differential equation $L(H)$ and applying Glerkin's FEM,

$$\int_0^L [N]^T L(H).dx=0 \tag{11}$$

where L is the differential equation in H, $[N]^T$ is the transpose of the interpolation function. Substituting the equation (11) for $L(H)$, equation (8) can be rewritten as:

$$\int_0^L [N]^T[N].dx\left(\frac{dH}{dt}\right)+\int_0^L [N]^T\frac{K}{W}\frac{\partial}{\partial x}\left(\frac{dH}{dx}\right)dx-\frac{q}{W}\int_0^L [N]^T dx=0 \tag{12}$$

Integration by parts is performed for each term of equation (12) and can be written as:

$$\int_0^L [N]^T[N].dx\{\dot{H}\}-\frac{K}{W}[N]^T\frac{\partial H}{\partial x}\bigg|_{X=0}^{X=L}+\frac{K}{W}\int_0^L\frac{\partial[N]^T}{\partial x}\frac{\partial[N]}{\partial x}dx\{H\}-\frac{q}{w}\int_0^L [N]^T dx=0 \tag{13}$$

where $\dot{H}=dH/dt$. In the present model, the time derivative is approximated using Finite Difference (FD) scheme. The time derivative can be written in the following form:

$$\frac{\partial H}{\partial t}=f(H) \tag{14}$$

where $f=f(H)$ is function of variables H. Equation (14) can be written in the FD form as

$$\frac{H^{t+\Delta t}-H^t}{\Delta t}=f \tag{15}$$

The FD scheme can be explicit or implicit in nature. Here an implicit scheme is used (Reddy, 1993) in which weighted average of f is taken at time t and time $t + \Delta t$.

$$\frac{H^{t+\Delta t} - H^{t}}{\Delta t} = \theta_0 f^{t} + \theta_1 f^{t+\Delta t} \tag{16}$$

where $\theta_0 = 1 - \theta_1$. Here θ_1 is taken as 0.5. For this value, system becomes unconditionally stable (Reddy, 1993). Equation (13) can be written in parts after evaluation of integrals as:

$$\int_0^L [N]^T [N]\{\dot{H}\}.dx = \frac{L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{pmatrix} \dot{H}_1 \\ \dot{H}_2 \end{pmatrix} \quad ; \quad \frac{K}{W}[N]^T \left.\frac{\partial H}{\partial x}\right|_{x=0}^{x=L} = \frac{1}{W}\begin{pmatrix} Q_{in} \\ -Q_{out} \end{pmatrix} \tag{17}$$

$$\int_0^L \frac{\partial [N]^T}{\partial x}\frac{\partial [N]}{\partial x} dx = \frac{1}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad ; \quad \int_0^L [N]^T dx = \frac{L}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{18}$$

Now substituting the above equations (17) and (18) in the equation (13) and rearranging the terms and using $\theta_0, \theta_1 = 0.5$, will give the following element matrix,

$$\begin{bmatrix} \frac{2L}{6\Delta t} + \frac{K}{2LW} & \frac{L}{6\Delta t} - \frac{K}{2LW} \\ \frac{L}{6\Delta t} - \frac{K}{2LW} & \frac{2L}{6\Delta t} + \frac{K}{2LW} \end{bmatrix}\begin{pmatrix} H_1 \\ H_2 \end{pmatrix}^{t+\Delta t} = \begin{bmatrix} \frac{L}{6\Delta t}(2H_1^t + H_2^t) + \frac{qL}{2W} - \frac{K}{2LW}(H_1^t - H_2^t) \\ \frac{L}{6\Delta t}(H_1^t + 2H_2^t) + \frac{qL}{2W} + \frac{K}{2LW}(H_1^t - H_2^t) \end{bmatrix} + \begin{bmatrix} \frac{Q_{in}}{W} \\ \frac{-Q_{out}}{W} \end{bmatrix} \tag{19}$$

The equation (19) gives the element matrix. Now all the element matrices are assembled according to their nodal position into one big matrix, known as global matrix. The boundary conditions are applied and the systems of equations are solved to get the unknowns.

4 CASE STUDY – BADI-KANJAWANI WATERSHED

Badi-Kanjawani watershed lies between 22°50′N 74°20′E and 22°30′N 74°50′E in the Jhabua district, in Madhya Pradesh state, India. It forms an area of approximately 15.5 sq. km and it is a part of larger Jhabua watershed, which forms an area of approximately 1200 sq. km, having an undulating and hilly physiography. Figure 1 shows the watershed boundary and its drainage

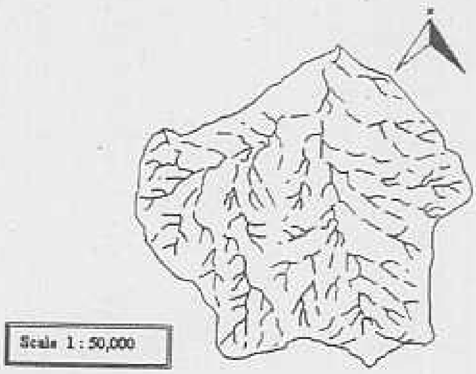

Figure 1. Watershed boundary and drainage for the Badi-Kanjawani watershed.

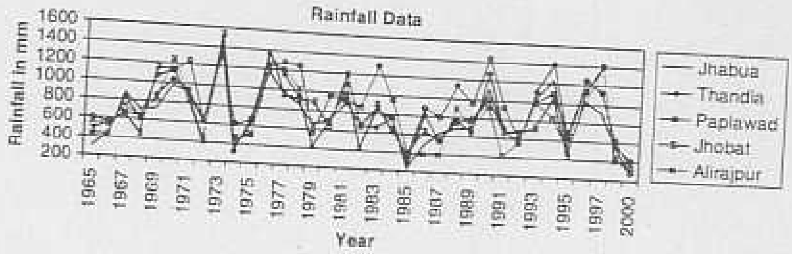

Figure 2. Yearly variation of rainfall for various stations, in and around Jhabua district.

pattern. Most of the watershed area is well drained giving further chance of quick water flow as runoff. The hilly pattern and drainage has adverse impact on soil moisture regime and conditions of the soil. The long-term analysis of rainfall pattern (Figure 2) shows that there occur a large number of dry spells of long duration to affect crop adversely. The average annual rainfall in the area varies from 620 mm to 1310 mm. More than 80% of rain occurs during the monsoon months of June–September with high intensity and short duration. Large portion of the rain goes as runoff casing soil erosion and siltation in the watershed.

5 INTEGRATED GIS AND FEM MODELLING

Here the FEM model described in the earlier section is integrated with the GIS package GRAM++ (developed in IIT Bombay, www.alumni.iitb.ac.in/ news/gisSoftware.htm) for the watershed modeling of the area under study. The topo-sheet of the study area is used for the digitization of the watershed boundary based on the drainage and the contours in the region and around the region. The contours from the topo-sheet at 20 m interval were interpolated to 2 m interval. After cleaning and slivering the contours, streams and watershed boundary, using the options available in the GIS package, a digital elevation model (DEM) for the study area is created. Figure 3 shows the overland boundaries of the watershed. The raster files divide the image file in the small grids/pixels with size of 20 m. These pixels contain the information such as the elevation values of the points on the grounds.

From the drainage lines (Figure 1), the main streams of the watershed have been chosen. Three drainage lines have been identified as the main streams of the Badi-Kanjawani watershed. On the watershed, some points have been identified as the node points for the Finite Element modeling of the watershed. Figure 4 shows the main streams and the node points in the watershed. The numbering of the node points is done on the basis of the stream on which it lies. The streams have been numbered as stream 1, stream 2 and the Main Stream.

The procedures for the watershed modeling are as follows:

1. Each pixel or cell in the raster map of overland boundary contains a particular number, which indicated the overland number of the cell in which it lies. Now all the cells in the watershed can be grouped depending upon their overland number. The total number of cells in a group can be multiplied with the area for each cell to give the total area of each overland region.
2. The raster map of the stream contains the details of the elements in the stream and their number. The length can also be obtained with their respective element numbers for the FEM modeling, by using the same technique used in first step.
3. From the raster map for the stream node, elevation for each node point on the stream is found to calculate slope of the elements in the discretized stream element.
4. After calculating length, slope and area for each overland region and using the formulation for the overland flow, the overland discharge per unit length of the stream can be obtained and stored in a file.

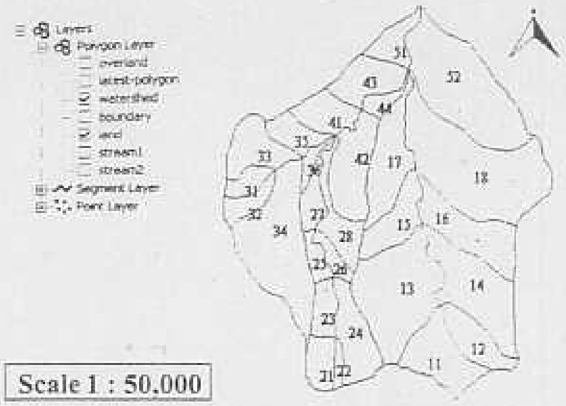

Figure 3. Vector layout (GRAM++ GIS) showing the watersheds and overland boundary.

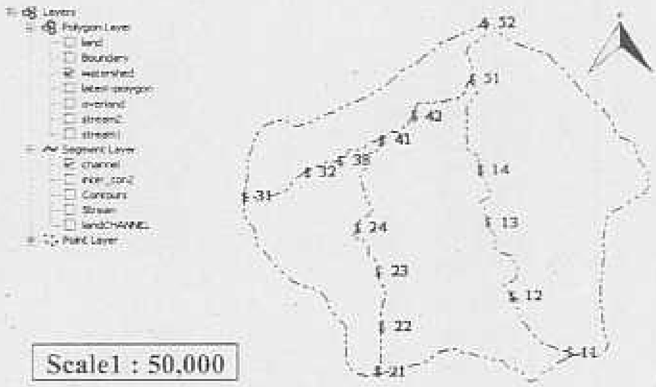

Figure 4. Vector layout showing the channel and node points.

5. For any element of the stream, there lie two overland boundaries on two sides of the element. The programs add these two overland files and generate a single file for overland flow to the element. These files will help during FEM modeling of the watershed.
6. Using the discharge as input parameters for the element of the FEM model and using FEM formulation, the head at each node with the time steps can be calculated.

6 RESULTS AND DISCUSSION

The developed model has been used for the rainfall-runoff simulation of Badi-Kanjawani watershed. For the rainfall-runoff simulation, an hourly rainfall of eight hours for a single storm has been considered from the available rainfall data (due to lack of practical hourly data) as shown in Figure 5. For the modeling, firstly the overland discharge files were generated from the information obtained from the GRAM++ GIS, like the area of the overland, their slope and the length of the stream element adjoining the overland. This gives the discharge of the overland in the units of volume per unit length of the stream element. The overland flows are the inputs to the channel flow and the hydrograph for each node in the watershed is generated. Figure 6 shows the generated runoff hydrograph at various nodes using the model. As can be seen in Figure 6, for the node points down the stream, there is increase in the peak time and peak values. The increase in the peak time is due to the increase in the travel time for the water, whereas the increase in the peak is due to the inclusion of the overland flow. For the main stream at nodes 41 and 51, two different streams are joining which causes increase in peak of the hydrograph at that location. Due to non-availability of

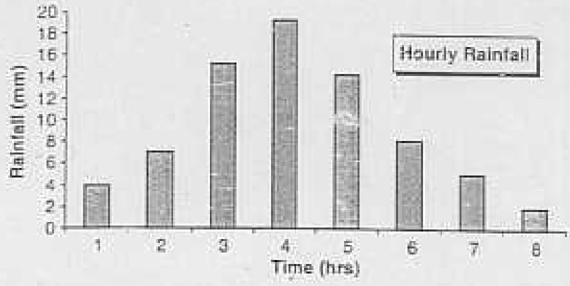

Figure 5. Hourly rainfall data used for the simulation of hydrograph.

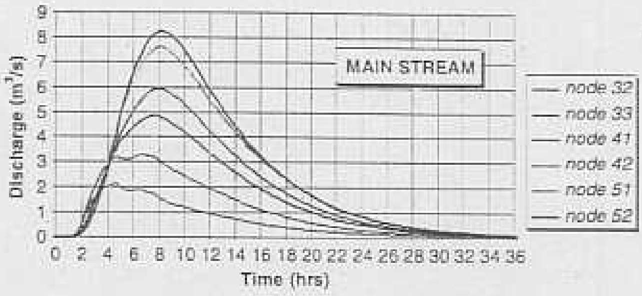

Figure 6. Hydrograph for the nodes on main stream.

the observed runoff data, the developed model is tested with the standard SCS-CN method for the watershed. A comparison between the present model and the SCS-CN model showed satisfactory results.

7 CONCLUDING REMARKS

Due to the complexity of the hydrological processes in any of the watersheds, watershed modeling requires advanced computational techniques and data management tools like FEM and GIS. With the flexibility in the computer coding and modeling irregular boundaries, FEM is popularly used to solve the complex watershed problems. In this paper, the features of FEM are being added with GIS, which takes into account of the spatial variation of the data of the watershed considered. These two important tools have been integrated in this study for watershed modeling to give more efficiency. The developed model is used in the rainfall-runoff simulation of an ungauged watershed. The model predicts satisfactory results with respect to the available data for the watershed.

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