Detection in time-varying wireless channels using partial channel state information

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***Abstract*— Coherent detection requires full Channel State Information (CSI) at the receiver. But if the channel is time-varying, it is difficult to have full CSI at the receiver because of the need to transmit large overheads and the need for frequent channel estimation at the receiver. In this paper we use partial CSI and a simple channel prediction technique to detect data in a time-varying, flat fading channel modeled by a second order Auto Regressive process. The first and second order lag of the process is matched with the Jake's correlation function. The results obtained are quite close to the performance of full CSI.**

I. INTRODUCTION

Mobile communication systems are expected to provide high data rates over time-varying fading channels with little overhead and complexity. Currently used coherent detection techniques assume full channel state information (CSI) is available at the receiver. But in a time-varying channel, obtaining full CSI is difficult because of the need to transmit large overheads and the need for frequent channel estimation. The other option of non-coherent schemes is accompanied with a degradation in performance.

Time-varying, wireless channels are usually characterized by Jakes model [1], where the channel correlation function is represented by $\mathcal{J}_0(\omega_d)$ where $\mathcal{J}_0(.)$ is the zeroth order Bessel function of the first kind and ω_d is the doppler frequency normalized by symbol time.

It is known that the time evolution of such channels can also be characterized by parametric models like ARMA [9][10], AR [8], MA [11] etc. Amongst these three, the AR model has been used more commonly due to two advantages: firstly, AR model can represent time-varying channels over a large range of doppler frequencies, and secondly, it facilitates prediction of future samples in the time domain. Thus use of AR1 [2] and AR2 [4][5] have been reported in the literature. Recently [2] considered partial CSI at the receiver by taking the channel to be AR1. However, the match between the correlation functions of AR1 and Jakes model is not good.

In this paper we begin by using the AR2 model and then use Yule-Walker equations to estimate its parameters as done in [3]. This is done by matching the correlation of the first two lags of AR2 with Jakes correlation. Subsequently, we use this in the context of data detection in a system where the channel cannot be assumed to be same even for two neighboring symbols. Symbols are transmitted in frames of size $N+2$. It is not assumed that full CSI for the whole frame is available at the receiver. Instead we assume that only partial CSI, defined by the doppler frequency and the channel corresponding to only two consecutive symbols out of the whole frame of $N+2$ symbols, is known. We estimate the channel for the remaining N symbols using this partial CSI and then detect the received symbols using these estimates. The performance of the scheme has been compared with the performance corresponding to full CSI and found to be quite close. We have also derived closed form expressions for the Bit Error Rate (BER).

The rest of the paper is organized as follows. Sections II and III present the system model and the proposed partial CSI detection method. Section IV presents the receiver and its performance analysis. Sections V and VI present results and conclusion respectively.

II. SYSTEM MODEL

Consider a communication link consisting of a single-antenna transmitter and receiver that operates in a time-selective and frequency non-selective fading channel. We consider the reception of the symbols in a frame of size $N+2$. The low pass equivalent complex symbol, received at the k^{th} time instant is represented as

$$y_k = h_k x_k + n_k \qquad k = -1, 0, 1, 2....N \tag{1}$$

where $n_k \sim \mathcal{CN}(0, N_0)$, x_k is a symbol from BPSK constellation taking value from $\{-\sqrt{E_s}, \sqrt{E_s}\}$ and $h_k \sim \mathcal{CN}(0,1)$, which is characterized by AR2 model as,

$$h_k = f_1 h_{k-1} + f_2 h_{k-2} + w_k \qquad k = 1, 2....N \tag{2}$$

where the parameters f_1 and f_2 are chosen such that the correlation of the channel for first and second order lags satisfy the Jakes model. Hence $R(1) = \mathcal{J}_0(\omega_d)$ and $R(2) = \mathcal{J}_0(2\omega_d)$, where $R(1) = E[h_k h_{k-1}^*]$, $R(2) = E[h_k h_{k-2}^*]$ and E denotes the expectation operator. $w_k \sim \mathcal{CN}(0, \sigma^2)$

Multiplying (2) by h_{k-1}^* and taking the expected value of both the sides of the resulting equation we get

$$\begin{aligned} E[h_k h_{k-1}^*] &= f_1 E[|h_{k-1}|^2] + f_2 E[h_{k-2} h_{k-1}^*] \\ R(1) &= f_1 R(0) + f_2 R(1) \end{aligned} \tag{3}$$

Now multiplying (2) by h^*_{k-2} and again taking the expected value of both the sides, we get

$$E[h_k h^*_{k-2}] = f_1 E[h_{k-1} h^*_{k-2}] + f_2 E[|h_{k-2}|^2] \quad (4)$$
$$R(2) = f_1 R(1) + f_2 R(0)$$

Equations (3) and (4) are well known Yule-Walker equations [7] and can be expressed in matrix form as

$$\begin{bmatrix} R(0) & R(1) \\ R(1) & R(0) \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \end{bmatrix}$$

Thus f_1 and f_2 can be found from $R(1)$ and $R(2)$ which in turn can be found from the doppler frequency.

In the sequel we assume that partial CSI, defined by doppler frequency ω_d and the the channel state for two consecutive symbols (either in the beginning (preamble) or in the middle (midamble) of the frame), is known to the receiver.

III. Proposed Partial CSI

A. Preamble

We assume that the AR2 parameters have been determined from the doppler frequency. Thus (h_{-1} and h_0) i.e. the channel during the first two symbol durations in a frame of size $(N+2)$ and the parameters of AR2, (f_1, f_2), are known at the receiver. With this information, the channel can be predicted as

$$\begin{aligned} \hat{h}_1 &= f_1 h_0 + f_2 h_{-1} \\ \hat{h}_2 &= f_1 \hat{h}_1 + f_2 h_0 \\ &\ldots\ldots \\ \hat{h}_k &= f_1 \hat{h}_{k-1} + f_2 \hat{h}_{k-2} \\ &\ldots\ldots \\ \hat{h}_N &= f_1 \hat{h}_{N-1} + f_2 \hat{h}_{N-2} \end{aligned}$$

B. Midamble

Here we assume the positions of the two known symbols to be in the middle of the frame. Thus $h_{\frac{N}{2}-1}$ and $h_{\frac{N}{2}}$ are known. In this case, channel estimate $\hat{h}_k$ can be shown as

$$\hat{h}_k = a_{\frac{N}{2}-1-k} h_{\frac{N}{2}-1} + b_{\frac{N}{2}-1-k} h_{\frac{N}{2}} \quad (5)$$

where $k = -1, 0, .., \frac{N}{2} - 2$.

$$\hat{h}_k = a_{k-\frac{N}{2}} h_{\frac{N}{2}} + b_{k-\frac{N}{2}} h_{\frac{N}{2}-1} \quad (6)$$

where $k = \frac{N}{2} + 1, .., N$.

As can be seen from (5) and (6), compared to the case of the preamble, in this case the maximum distance of a time instant for which the channel is not known to the receiver and an instant for which it is known is halved.

IV. Receiver and Performance Analysis

Using the channel estimate $\hat{h}_k$, we consider sub-optimum symbol by symbol detection, instead of the complex Maximum Likelihood Sequence Estimation (MLSE).

A. Preamble

The decision variable v_k, at the k^{th} symbol position, for received symbol y_k can be obtained [6] as

$$v_k = Re\left\{ \frac{\hat{h}^*_k}{|\hat{h}_k|} y_k \right\} \quad (7)$$

where $k = 1, 2, .., N$. $Re\{.\}$ and $*$ denote real part and conjugate respectively. To analyze the performance we need to know the variance σ^2 of w_k in (2), which can be computed as follows.

Multiplying (2) by its conjugate h^*_k and taking the expected value of both the sides of the resulting equation, we get

$$\begin{aligned} E[|h_k|^2] &= f_1^2 E[|h_{k-1}|^2] + f_2^2 E[|h_{k-2}|^2] + E[|w_k|^2] \\ &\quad + f_1 f_2 E[h^*_{k-1} h_{k-2}] + f_1 f_2 E[h_{k-1} h^*_{k-2}] \quad (8) \end{aligned}$$

Now using the fact that $R(0) = E[|h_k|^2] = 1$ and $E[|w_k|^2] = \sigma^2$, we can write (8) as

$$\sigma^2 = 1 - f_1 R(1) - f_2 R(2) \quad (9)$$

Now we can represent $\hat{h}_k$ in terms of h_0 and h_{-1} as under

$$\hat{h}_k = a_k h_0 + b_k h_{-1} \quad (10)$$

where

$$\begin{aligned} a_1 &= f_1 \qquad a_2 = f_1 a_1 + f_2 \\ b_1 &= f_2 \qquad b_2 = f_1 b_1 \\ a_k &= f_1 a_{k-1} + f_2 a_{k-2} \\ b_k &= f_1 b_{k-1} + f_2 b_{k-2} \qquad k = 3, 4 \ldots N \quad (11) \end{aligned}$$

Using the proposed channel estimate $\hat{h}_k$ as in (10), the channel h_k in (2) can be represented as

$$h_k = \hat{h}_k + \delta h_k \quad (12)$$

where

$$\delta h_k = a_{k-1} w_1 + a_{k-2} w_2 + \ldots + w_k \quad (13)$$

is the estimation error at the k^{th} symbol position and $\delta h_k \sim \mathcal{CN}(0, \sigma_k^2)$ where

$$\sigma_k^2 = \left(1 + \sum_{i=1}^{k-1} a_i^2\right) \sigma^2 \quad (14)$$

for $k = 1, 2, .., N$.

Now expanding (7) we get

$$v_k = Re\left\{ |\hat{h}_k| x_k + \delta h_k x_k + n_k \frac{\hat{h}_k}{|\hat{h}_k|} \right\} \quad (15)$$

where Conditioned on $\hat{h}_k x_k$, v_k is a Gaussian variable with mean $|\hat{h}_k| x_k$ and variance $\sigma_k^2 E_s + N_0$.

From (15), the instantaneous SNR (γ_k) for the k^{th} symbol position is

$$\gamma_k = \frac{|\hat{h}_k|^2 E_s}{\sigma_k^2 E_s + N_0}$$

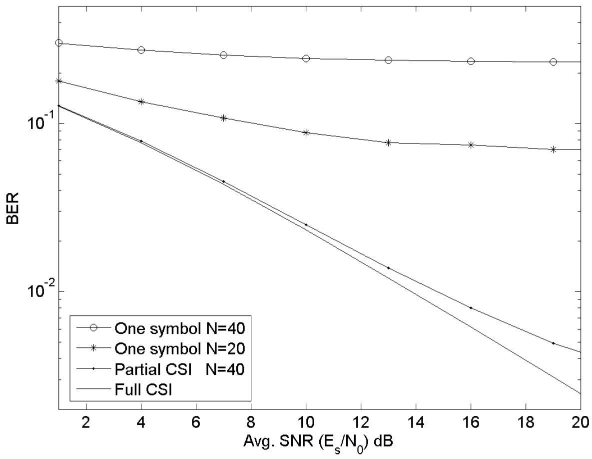

Fig. 1. BER performance of the system with partial CSI (preamble), full CSI and one 'symbol' for $f_d T_s = 0.01$

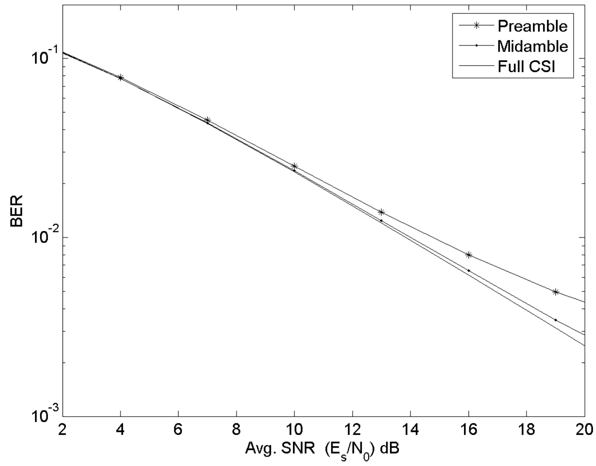

Fig. 2. BER performance of the system with partial CSI using preamble and midamble with $N = 40$ for $f_d T_s = 0.01$

The average SNR (Γ_k), for the k^{th} symbol position is

$$\Gamma_k = \frac{(1-\sigma_k^2)E_s}{\sigma_k^2 E_s + N_0} \tag{16}$$

In case of full CSI, $\sigma_k^2 = 0$ and hence Γ_k is E_s/N_0 which represents coherent detection. Finally, BER at the k^{th} symbol position with average SNR Γ_k is given by [6],

$$P_e(k) = 0.5\left(1 - \sqrt{\frac{\Gamma_k}{1+\Gamma_k}}\right) \tag{17}$$

The average BER P_e for the whole frame is given by

$$P_{e,pre} = \frac{1}{N}\sum_{k=1}^{N} P_e(k) \tag{18}$$

B. Midamble

Similarly for the midamble, the average BER P_e for the whole frame N is given by

$$P_{e,mid} = \frac{2}{N}\sum_{k=1}^{N/2} P_e(k) \tag{19}$$

V. Results

Simulations were carried out for different channel conditions ($f_d T_s$) and different frame sizes (N) and found to be closely matching with (18) and (19). In this section we show the BER performance of the receiver using the proposed partial CSI and full CSI for both the preamble and the midamble cases using analytical expressions.

Fig. 1 presents the performance of the receiver for $f_d T_s = 0.01$ and $N = 40$ for (1) full CSI, (2) partial CSI and (3) 'one symbol' (by simulations) cases. In the 'one symbol' case, the receiver knows the channel corresponding to only the first symbol and it detects all the remaining symbols in the frame with this known state assuming that the channel remains constant through out the frame. We also show the performance of 'one symbol' case for half the frame size i.e. $N/2 = 20$. It can be seen that compared to the full CSI and partial CSI cases, the performance of this receiver is severely degraded even after the frame size has been halved.

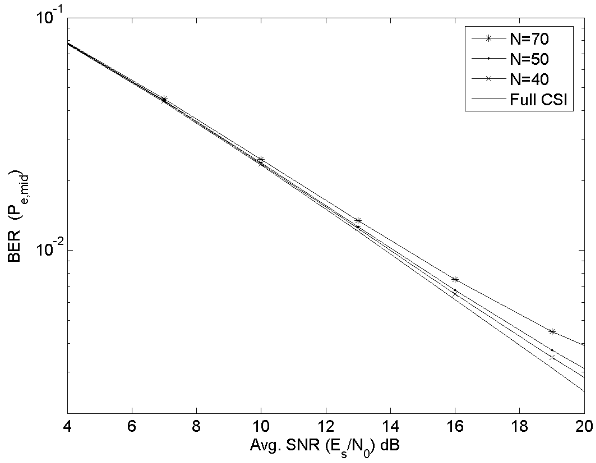

Fig. 3. BER performance of the system with partial CSI (midamble) for $f_d T_s = 0.01$

Fig. 2 shows the performance of the receiver with partial CSI considering the two known channel symbols as preamble and midamble for $f_d T_s = 0.01$ and $N = 40$. For the midamble the performance is better and also close to full CSI.

Fig. 3 shows the performance of the receiver for $f_d T_s = 0.01$ and $N = 50, 60$ and 70. Performance

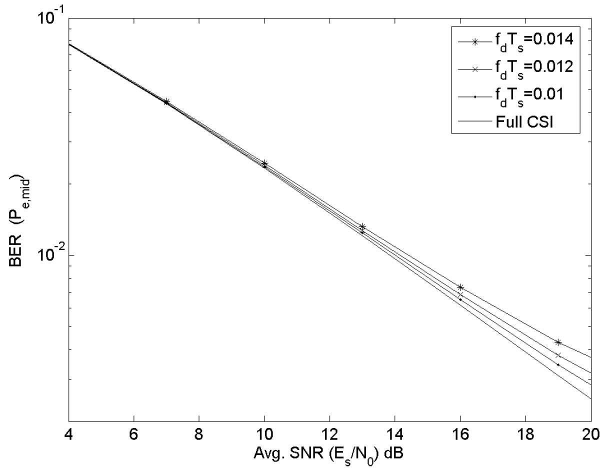

Fig. 4. BER performance of the system with partial CSI (midamble) for $N = 40$

degrades with increasing the value of N because channel estimation error (σ_k^2) will increase as we increase the frame size. To achieve the BER of 10^{-2} with full CSI, the required average SNR is 13.85 dB. Let us consider the required average SNR (X dB) and excess average SNR ($\delta X = X - 13.85$ dB) to recover the loss in performance for the same BER using the partial CSI. For $N = 50$ the values of $X = 14.1$ and $\delta X = 0.25$ dB, which shows that the excess SNR of only 0.25 dB is required.

Fig. 4 shows that the performance of the receiver degrades with increasing the value of $fdTs$ from 0.01 to 0.014 when $N = 40$. Because compared to slow varying channel, fast varying channel introduces larger error (due to larger σ^2 in (14)) in the predicted value at the same symbol position in the frame. In other words, for lower value of doppler, larger increase in frame size is possible without much degradation in performance. For $f_dT_s = 0.01$, the values of $X = 14$ and $\delta X = 0.15$ dB, but for $f_dT_s = 0.012$, the values of $X = 14.15$ and $\delta X = 0.3$ dB. It indicates that penalty of SNR is less in case of slow varying channel.

VI. CONCLUSION

We have used a channel prediction technique along with partial CSI to detect data in a time varying channel modeled by AR2 process. The parameters of the AR2 process are related to the Jakes correlation function. The predicted values of the channel are used as estimates for detecting the received symbols. The performance of the obtained receiver is analyzed for various values of doppler and frame sizes and found to be quite close to that of full CSI.

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