Application of Transmit Diversity to Rapidly Time Varying Channels with Partial Channel Knowledge at the Receiver

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***Abstract*— For coherent detection, accurate channel knowledge is required at the receiver. But in rapidly time varying channels this leads to poor spectral efficiency. A possible compromise is to use partial channel knowledge at the receiver. However as can be seen from [3], partial channel knowledge degrades the BER performance over rapidly time varying channels. In this paper, we consider Alamouti's [1] transmit diversity scheme over rapidly time varying channel with partial knowledge, derive BER expression and match it with Monte Carlo simulations, show that it is possible to significantly improve the BER performance along with simultaneous reduction in channel estimation rate and also show that by staggering the partial channel information at the receive antenna, a performance can be even further improved.**

***Index Terms*— Transmit diversity, Auto Regressive (AR) model, partial channel knowledge**

I. Introduction

Future wireless communication systems must efficiently use the limited bandwidth and power resources to enable high data rate and reliable transmission over rapidly time varying channels. In coherent detection accurate channel knowledge is required at receiver, which increases the channel estimation rate over rapidly time varying channel. This frequent channel estimation leads to poor spectral efficiency.

However in rapidly time varying channel, to prohibit frequent channel estimate, [3] considers Single Input Single Output (SISO) system with partial channel knowledge, where the accurate channel knowledge is available only at the start of the frame. This (partial) channel knowledge is used to detect the whole frame. But the performance of [3] degrades with increasing the frame size due to using more outdated information, or increasing the rate of variation, of channel.

We propose Alamouti's [1] transmit diversity scheme with partial channel knowledge over rapidly varying channel and derive the closed form expression for Bit Error Rate (BER). We compare performance of the proposed system with [3] and show that a performance gain of 7 dB, along with a reduction in channel estimation rate by a factor of 5, is achieved. We also compare the proposed system for BER of 5 x 10^{-4} with partial and full channel information and show that for a small performance loss of 2 dB, channel estimation rate is reduced by a factor of ten with partial channel knowledge. Further we consider staggered channel information, instead of simultaneous channel information, for the proposed system and show that a gain of 2 dB is achieved. The rest of this paper is organized as follows. Section II presents system model experiencing rapidly time varying, frequency non selective and spatially uncorrelated Rayleigh fading and Alamouti's transmit diversity scheme. Section III presents staggered channel information and expressions for BER. Section IV presents results and discussions for the proposed system. Section V concludes this paper.

II. System model

Consider a communication link consisting of two transmit antennas and one receive antenna, with BPSK constellation, that operates in a time selective and frequency non selective Rayleigh fading environment modeled by a first-order AR process [2].

$$\begin{aligned} h_k &= ah_{k-1} + \sqrt{1-a^2}w_k \\ h_k &= a^k h_0 + \frac{\sqrt{1-a^2}}{a}\sum_{i=1}^{k} a^i w_i \\ h_k &\sim \mathcal{CN}(a^k h_0, 1-a^{2k}) \end{aligned} \tag{1}$$

where $k = 1, 2, ..N$, N is the number of data symbols in a frame, a is the correlation coefficient, $0 < a \leq 1$, and w_k is an independent and identically distributed random process with complex normal density $\mathcal{CN}(0, \sigma_h^2)$, where σ_h^2 is the variance of h_0, and h_0 represents the partial channel knowledge for the whole block of length N. We assume that h_0 and a are known at the receiver. We also assume that the temporal variation of the fading channels follows the Jakes' correlation model [4] and the relation between the Doppler frequency and a can be approximated by

$$a = \mathcal{J}_0(2\pi f_d T_s)$$

where $\mathcal{J}_0(x)$ is the zeroth-order Bessel function of the first kind, $f_d = f_c v/\lambda$ is the Doppler shift, and T_s is the symbol duration.

A. Alamouti's scheme

The received symbols, at the k^{th} symbol position, are given by [1]

$$[y_{1,k} \quad y_{2,k}] = [h_{1,k} \quad h_{2,k}] \begin{bmatrix} x_{1,k} & -x^*_{2,k} \\ x_{2,k} & x^*_{1,k} \end{bmatrix} + [n_{1,k} \quad n_{2,k}] \tag{2}$$

Here k is the position of the symbol in a frame of length N and l is the index of transmit antenna, where ($k = 1, 2...N$) and ($l = 1, 2$). $x_{l,k}$ is a symbol from BPSK constellation and taking value from $\{-\sqrt{E_s}, \sqrt{E_s}\}$. $n_{l,k}$ is a zero mean complex gaussian variable with power spectral density $\mathcal{N}_0$. $y_{l,k}$ is a received signal and $h_{l,k}$ is, a channel coefficient, derived from AR process and $h_{l,k} \sim \mathcal{CN}(a^k h_{0,l}, 1 - a^{2k})$.

$h_{0,l}$ is, used to generate $h_{l,k}$ by AR model as shown in (1) for time varying channel and, the available partial channel knowledge at the receiver for detection of the received signal $y_{l,k}$ for the whole frame of length N.

Equivalently, by conjugating $y_{2,k}$, (2) can be written as

$$\begin{bmatrix} y_{1,k} \\ y^*_{2,k} \end{bmatrix} = \begin{bmatrix} h_{1,k} & h_{2,k} \\ h^*_{2,k} & -h^*_{1,k} \end{bmatrix} \begin{bmatrix} x_{1,k} \\ x^*_{2,k} \end{bmatrix} + \begin{bmatrix} n_{1,k} \\ n^*_{2,k} \end{bmatrix} \tag{3}$$

With obvious notation (3) can be written as

$$Y_k = H_k X_k + N_k$$

III. Performance Analysis

Using the partial channel information $H_0 = [h_{0,1} \quad h_{0,2}]$ in classical Maximal Ratio Receive Combining (MRRC) given in [1], decision variables for two symbols, at k^{th} symbol position, are

$$s_{1,k} = [h^*_{0,1} \quad h_{0,2}] \begin{bmatrix} y_{1,k} \\ y^*_{2,k} \end{bmatrix} \tag{4}$$

$$s_{2,k} = [h^*_{0,2} \quad -h_{0,1}] \begin{bmatrix} y_{1,k} \\ y^*_{2,k} \end{bmatrix}$$

As both the symbols are equally likely, we consider $s_{1,k}$ to calculate the BER. For real BPSK constellation, decision variable z_k is $Re\{s_{1,k}\}$. Simplifying (4), we get

$$\begin{aligned} z_k = Re\{&a^k|H_0|^2 x_{1,k} + h^*_{0,1} n_{1,k} + h_{0,2} n^*_{2,k} \\ &+ \frac{\sqrt{1-a^2}}{a} \sum_{i=1}^{k} a^i (h^*_{0,1} w_{1,i} + h_{0,2} w^*_{2,i}) x_{1,k} \\ &+ \frac{\sqrt{1-a^2}}{a} \sum_{i=1}^{k} a^i (h^*_{0,1} w_{2,i} - h_{0,2} w^*_{1,i}) x_{2,k}\} \end{aligned} \tag{5}$$

For given $x_{1,k} = \sqrt{E_s}$ and H_0, $s_{1,k}$ is a normal gaussian variable with mean and variance of

$$E[s_{1,k}/H_0] = a^k |H_0|^2 \sqrt{E_s}$$
$$Var[s_{1,k}/H_0] = \frac{(N_0 + 2(1-a^{2k})E_s)|H_0|^2}{2}$$

The probability of error conditioned H_0 [5] is

$$P(\epsilon_k/H_0) = Q\left(\sqrt{\frac{2a^{2k}E_s|H_0|^2}{N_0 + 2(1-a^{2k})E_s}}\right) \tag{6}$$

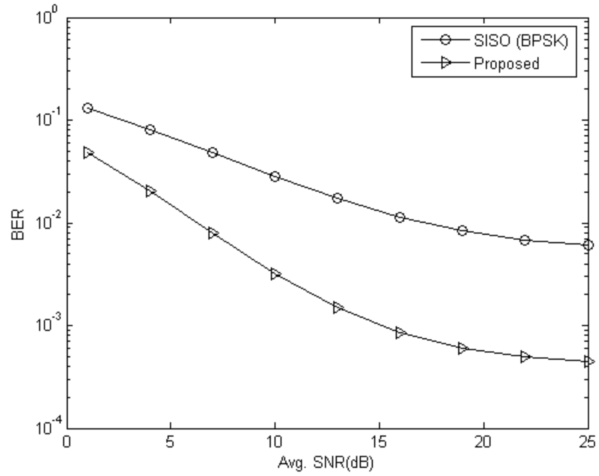

Fig. 1. BER performance of proposed scheme and SISO for $a = 0.999$ and $N = 20$

From (6) the instantaneous effective SNR (γ_k), for the k^{th} symbol position, is

$$\gamma_k = \frac{a^{2k}E_s|H_0|^2}{2(1-a^{2k})E_s + N_0}$$

The average effective SNR (Γ_k), for the k^{th} symbol position [3], is

$$\Gamma_k = \frac{a^{2k}E_s}{2(1-a^{2k})E_s + N_0}$$

The average error probability $P_e(k)$, at the k^{th} symbol position, given by [1], is

$$P_e(k) = \frac{(1-\mu_k)^2(2+\mu_k)}{4},$$

where $\mu_k = \sqrt{\frac{\Gamma_k}{\Gamma_k + 1}}$

The overall average BER, for a frame of N symbols, given by [3], is to be

$$P_e = \frac{1}{N}\sum_{k=1}^{N} Pe(k)$$

Staggered channel information: Till now we considered partial channel knowledge, for both the channels, which is available at the receiver at the same instant. We refer to it as 'Simultaneous Channel Information'. We propose an alternative scheme where partial channel information of the two channels is available with a time separation equivalent to half of the frame size. This 'Staggered Channel Information' reduces the outdatedness of the channel knowledge, alternately, in the signals from the two transmit antennas and consequently a performance gain is achieved. We simulate the proposed system with staggered partial channel information.

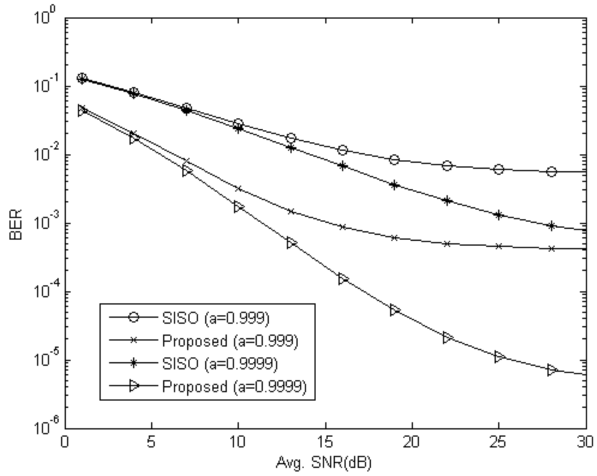

Fig. 2. BER performance of proposed scheme and SISO for $a = 0.999$ and $a = 0.9999$ and for $N = 20$

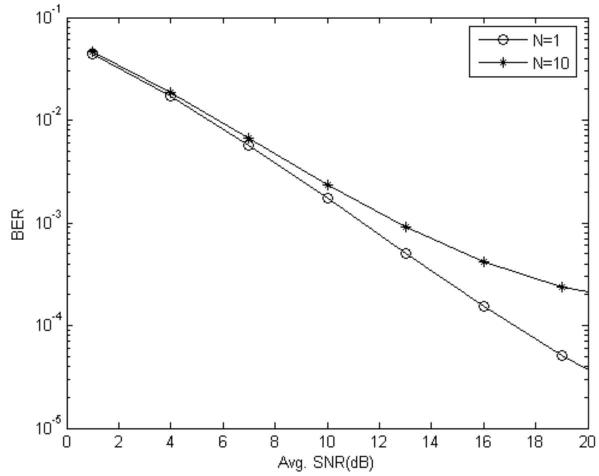

Fig. 5. BER performance of proposed scheme for $a = 0.999$

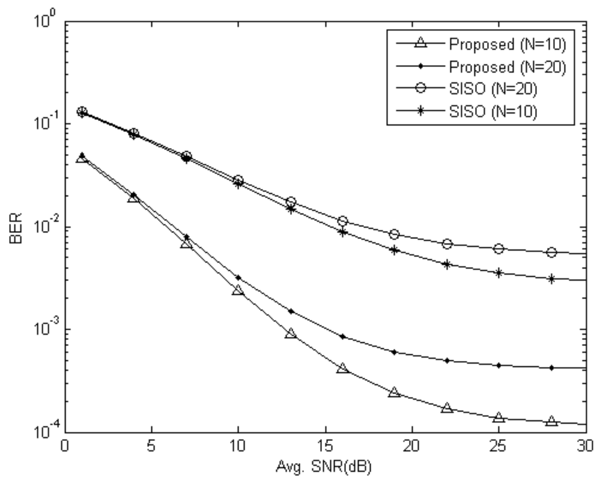

Fig. 3. BER performance of proposed scheme and SISO for $a = 0.999$ and for $N = 10$ and $N = 20$

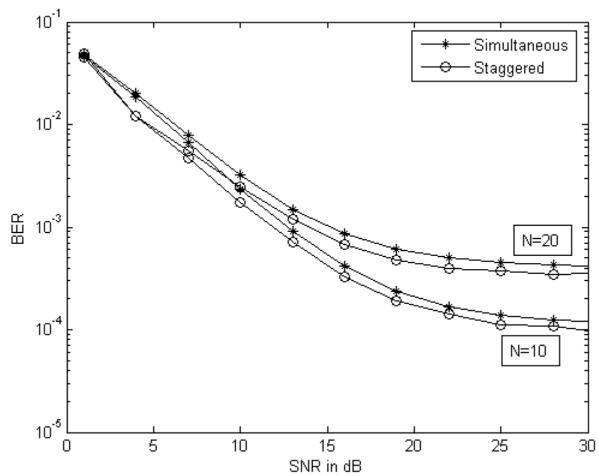

Fig. 6. BER performance of proposed and staggered proposed systems for $a = 0.999$

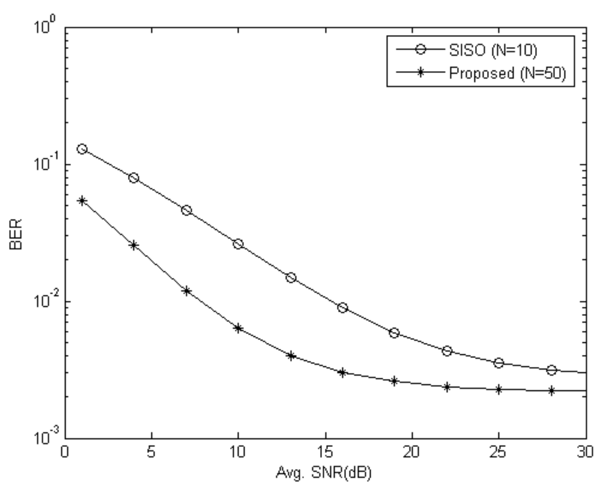

Fig. 4. BER performance of proposed scheme for $N = 50$ and SISO for $N = 10$ for $a = 0.999$

IV. Results

Using the Monte Carlo simulation, we verified the theoretical expression for BER. In particular, with two independent parameters (1) channel frame size N of AR model and (2) time correlation coefficient a, we compare the BER performance of proposed system with [3] SISO.

Fig. 1 shows the BER performance for $N = 20$. Due to rapidly time varying channel, error floor still exists in the proposed system as in SISO [3], but it is drastically reduced. The diversity gain in BER performance is achieved through partial channel knowledge also. Fig. 2 shows that, for two channel conditions $a = 0.999$ and $a = 0.9999$, the performance gain is achieved with partial channel information. For $a = 0.9999$, the gain is higher because of less outdated channel information. Fig. 3 shows that, for two different frame sizes $N = 10$ and $N = 20$, the improvement in the performance gain

TABLE I

COMPARISON OF ERROR FLOOR REDUCTION FACTOR FOR $a = 0.999$

	$N = 100$	$N = 50$	$N = 10$
SISO	1	2	8
Proposed	3	12	218

for $N = 10$ is more compared to $N = 20$ due to less outdated channel information for smaller frame size $N = 10$.

Fig. 4 shows the performance of the proposed system for $N = 50$ and SISO for $N = 10$. It clearly shows that a performance gain of 7 dB, with reduction in channel estimation rate by a factor of five, is achieved.

Fig. 5 compares the proposed system for full channel ($N = 1$) and partial channel ($N = 10$) knowledge. It can be seen that for a BER of 5 x 10^{-4}, the performance loss due to partial channel knowledge is only 2 dB but the channel estimate rate is reduced by a factor of 10. Fig. 6 compares, simultaneous channel information to staggered channel information in the proposed system and, shows a gain of approximately 2 dB.

Table I shows the error floor reduction factor for the proposed system with reference to SISO for different values of N. When N is reduced from 100 to 50, in SISO, the error floor is reduced by a factor of 2. But with proposed system, it is reduced by a factor of $12/3 = 4$. For $N = 100$, with the proposed system the error floor is reduced by a factor of 3 compared to SISO.

V. CONCLUSION

We derived the closed form expression for BER, for Alamouti's transmit diversity scheme, for BPSK constellation, with partial channel information over rapidly time varying channel and compare our results with SISO [3]. We show that by using transmit diversity, it is possible to significantly improve the BER performance, along with simultaneous reduction in channel estimation rate, with partial channel knowledge. Compare to full channel knowledge, with partial knowledge in the proposed system, we show a significant reduction in channel estimation rate with a little loss in performance. With staggered partial channel information, even further performance gain is achieved.

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