Performance Analysis of Alamouti Transmit Diversity with Transmit Antenna Selection for Reduced Feedback Rate

J. M. Chakravarti
Department of E & C
Institute of Technology
Nirma University
Ahmedabad-382481, INDIA
Email: jatin.chakravarti@gmail.com

Y. N. Trivedi
Department of E & C
Institute of Technology
Nirma University
Ahmedabad-382481, INDIA
Email: yogesh.trivedi@nirmauni.ac.in

***Abstract*—We analyze a sub-optimum transmit antenna selection (TAS) scheme in multiple input single output (MISO) systems equipped with N transmit antennas. We keep one antenna fixed and select the best among the remaining $N-1$ antennas. We assume spatially independent flat fading channel with perfect channel state information (CSI) at receiver and an ideal feedback link. We use Alamouti transmit diversity and derive the closed-form expressions of bit error rate (BER) and outage probability for BPSK constellation. We also compare performance of the sub-optimum scheme with performance of the optimum scheme using BER, outage probability expressions and number of feedback bits.**

***Index Terms*—Alamouti transmit diversity (ATD), bit error rate (BER), outage probability, Rayleigh fading channel, transmit antenna selection (TAS).**

I. Introduction

Space-time block codes (STBC) [1] in multiple input multiple output (MIMO) systems have been used to mitigate the effects of fading by providing diversity gain. However, each antenna requires one RF chain, which is not practical to provide in the current scenario of wireless communications. The reason is that, more number of RF chains increase the cost and complexity of the system. Therefore, Antenna selection (AS) [3]-[7] has been proposed in the literature. One benefit of AS is to retain the full diversity order of the MIMO systems with reduced RF chains [6].

The complexity of AS algorithms and number of feedback bits (these bits carry information of the indices of selected antennas) demand additional overheads on the receiver. Most wireless communication systems provide a feedback channel with limited bandwidth. Third generation (3G) cellular system is allocated a feedback rate of 1.5 k bits/sec [4],[8]. Furthermore, the complexity of AS algorithms also increase the cost of the signal processing and the feedback bit requirement. In the above-mentioned papers, optimum AS algorithms have been used which are of full complexity. To alleviate this complexity, some sub-optimum AS algorithms have also been proposed and analyzed in the literature. Three sub-optimum schemes have recently been proposed in [4] to reduce feedback rate compared to the optimum TAS scheme [3]. The exact closed form BER expression for scheme 1 of [4] has been derived in [5].

In this paper, we analyze a new sub-optimum TAS scheme in MISO system employed with N transmit antennas. We keep one antenna fixed and select the best among the remaining $N-1$ antennas. For the Alamouti transmit diversity (ATD) and BPSK constellation, we have derived the closed form expression of received SNR, using which we obtain BER and outage probability expressions. We also compare the considered sub-optimum TAS scheme with the optimum scheme based on the BER and outage probability expressions and feedback bit requirement.

The remainder of the paper is organized as follows. The system and channel models are introduced in Section II. In Section III we present a detailed performance analysis in terms of BER and outage probability. Section IV deals with simulations and results. Finally, we present conclusions in Section V.

II. System and Channel Models

We consider a wireless link in a flat Rayleigh fading environment equipped with N transmit antennas and one receive antenna. A block diagram of the considered system is shown in Fig. 1. The channel fading coefficients h_i between transmit antenna i and the receive antenna is denoted by $1 \leq i \leq N$, are spatially and timely independent identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero mean and unit variance.

We assume that the channel remains constant for at least two consecutive instants. We use Alamouti transmit diversity [2] for which the received symbols can be represented as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_m & h_f \\ h_f^* & -h_m^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \tag{1}$$

where the superscript $*$ denotes the complex conjugate. x_1 and x_2 are transmitted data symbols for BPSK modulation scheme with an average power of $E_s/2$, i.e. $x_1, x_2 \in \{-\sqrt{E_s/2}, \sqrt{E_s/2}\}$ and $n_i \sim \mathcal{C}N(0, N_0)$ for

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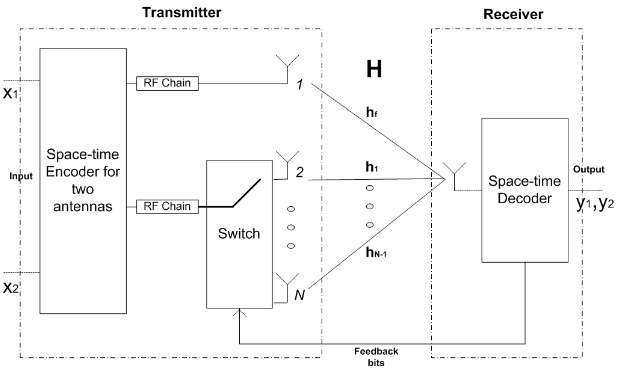

Fig. 1. Block diagram of an Alamouti transmit diversity system with sub-optimum $(N, 2; 1)$ TAS scheme

$1 \leq i \leq 2$ is AWGN noise. y_1 and y_2 are received symbols.

In (1) h_f is the channel between the fixed transmit antenna and receive antenna, whereas h_m is the channel between the selected antenna among the remaining $N-1$ transmit antennas. The selection of antenna is based on the maximization of received SNR, which can be expressed as

$$m = \arg \max_{1 \leq i \leq N-1} \{|h_i|^2\}$$

The receiver feeds back the index (m) via a dedicated ideal link. Here, ideal link denotes zero delay and noiseless.

At the receiver, the resulting decision variables for both symbols are denoted as [2]

$$\widehat{x_1} = [h_m^* \quad h_f] \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$$

$$\widehat{x_2} = [h_f^* \quad -h_m] \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$$

where $\widehat{x_1}$ and $\widehat{x_2}$ are the decision variables for data symbols x_1 and x_2 respectively.

III. Performance analysis

In this section, we derive closed form expression of received SNR and using which we obtain exact expression of BER and outage probability. We assume that both x_1 and x_2 are equally probable. Hence, we can express the instantaneous SNR (γ) per bit as

$$\gamma = \|\mathbf{H}\|^2 \gamma_c \tag{2}$$

where $\mathbf{H} = [h_m \quad h_f]^T$ and $\gamma_c = E_s/2N_o$. Now, we require the pdf of γ which can be determined as follows.

Let us denote the channel power gain $|h_i|^2$ as W_i, where $1 \leq i \leq N$. Then, all W_i are chi-squared distributed variable with two degrees of freedom. As all W_i are equally distributed, we can represent the pdf $p_W(w)$ and the Cumulative Distribution Function (CDF) $F_W(w)$ as [10].

$$p_W(w) = e^{-w}, \quad w \geq 0$$
$$F_W(w) = 1 - e^{-w}$$

Further, since all W_i are independent, the pdf of W_m can be expressed using order statistics [11] as

$$\begin{aligned} p_{W_m}(w_m) &= (N-1)[F_W(w_m)]^{N-2} p_W(w_m), \quad w_m \geq 0 \\ &= (N-1) \sum_{k=0}^{N-2} (-1)^k \binom{N-2}{k} e^{-w_m(1+k)} \\ &= (N-1) \Bigg[e^{-w_m} + \sum_{k=1}^{N-2} (-1)^k \binom{N-2}{k} \\ &\quad \left\{ e^{-w_m(1+k)} \right\} \Bigg] \end{aligned} \tag{3}$$

For the fixed antenna, the channel power gain $|h_f|^2$ is W_f and the pdf $p_{W_f}(w_f) = e^{-w_f}$. Let us denote the resulting channel power gain $\|\mathbf{H}\|^2$ as R. Then

$$R = W_m + W_f \tag{4}$$

Hence, the pdf $p_R(r)$ can be determined by the convolution of pdfs $p_{W_m}(w_m)$ and $p_{W_f}(w_f)$ as [9]

$$\begin{aligned} p_R(r) &= \int_0^r p_{W_m}(w_m) p_{W_f}(r - w_m) dw_m \\ &= (N-1) \int_0^r \Bigg[e^{-w_m} + \sum_{k=1}^{N-2} (-1)^k \binom{N-2}{k} \\ &\quad \left\{ e^{-w_m(1+k)} \right\} \Bigg] (e^{-(r-w_m)}) dw_m \\ p_R(r) &= (N-1) \Bigg[re^{-r} + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-2}{k} \\ &\quad \left\{ e^{-r} - e^{-r(1+k)} \right\} \Bigg] \end{aligned} \tag{5}$$

Finally, since $\gamma = \|\mathbf{H}\|^2 \gamma_c$, the pdf $p_\gamma(\gamma)$ can be represented as

$$\begin{aligned} p_\gamma(\gamma) &= \frac{N-1}{\gamma_c} \Bigg[\frac{\gamma}{\gamma_c} e^{-\frac{\gamma}{\gamma_c}} + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-1}{k} \\ &\quad \left\{ e^{-\frac{\gamma}{\gamma_c}} - e^{-\frac{\gamma(1+k)}{\gamma_c}} \right\} \Bigg] \end{aligned} \tag{6}$$

A. Bit Error Probability

The expression for BER can be derived as [10]

$$P_e = \int_{\gamma=0}^{\infty} P_e(\varepsilon/\gamma) p_\gamma(\gamma) d\gamma \tag{7}$$

where probability of error $P_e(\varepsilon/\gamma)$ is given by the Gaussian tail function

$$P_e(\varepsilon/\gamma) = Q(\sqrt{2\gamma}) = \int_{x=\sqrt{2\gamma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \quad (8)$$

Hence, P_e can be derived by substituting (6) and (8) in (7)

$$P_e = \frac{N-1}{\gamma_c\sqrt{2\pi}} \int_{x=0}^{\infty} e^{-x^2/2} \Bigg[\int_{\gamma=0}^{x^2/2} \Bigg\{ \frac{\gamma}{\gamma_c} e^{-\frac{\gamma}{\gamma_c}} + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-2}{k} \left(e^{-\frac{\gamma}{\gamma_c}} - e^{-\frac{\gamma(1+k)}{\gamma_c}} \right) \Bigg\} d\gamma \Bigg] dx$$

Further, simplifying the equation we get the bit error probability as

$$P_e = \frac{N-1}{\gamma_c} \Bigg[-\frac{\sigma^3}{4} - \frac{\sigma\gamma_c}{2} + \frac{\sigma}{2} + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-2}{k} \Bigg\{ -\frac{\sigma\gamma_c}{2} - \frac{\gamma_c}{2(1+k)} \sqrt{\frac{\gamma_c}{1+k+\gamma_c}} + \frac{\gamma_c}{2} - \frac{\gamma_c}{2(1+k)} \Bigg\} \Bigg] \quad (9)$$

where

$$\sigma = \sqrt{\frac{\gamma_c}{1+\gamma_c}}$$

B. Probability Of Outage

The probability of outage P_{out} is defined as [12]

$$P_{out} = P(\gamma < \gamma_{th}) = \int_{\gamma=0}^{\gamma_{th}} p_\gamma(\gamma) d\gamma \quad (10)$$

where γ_{th} is the minimum required SNR below which the system is in outage. Substituting equation (6) in (10)

$$P_{out} = \frac{N-1}{\gamma_c} \int_{\gamma=0}^{\gamma_{th}} \Bigg[\frac{\gamma}{\gamma_c} e^{-\frac{\gamma}{\gamma_c}} + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-2}{k} \Bigg\{ e^{-\frac{\gamma}{\gamma_c}} - e^{-\frac{\gamma(1+k)}{\gamma_c}} \Bigg\} \Bigg] d\gamma$$

Further, simplifying the equation we get the probability of outage as

$$P_{out} = \frac{N-1}{\gamma_c} \Bigg[-\gamma_{th} e^{-\frac{\gamma_{th}}{\gamma_c}} - \gamma_c e^{-\frac{\gamma_{th}}{\gamma_c}} + \gamma_c + \sum_{k=1}^{N-2} (-1)^k \left(\frac{1}{k}\right) \binom{N-2}{k} \Bigg\{ -\gamma_c e^{-\frac{\gamma_{th}}{\gamma_c}} + \frac{\gamma_c}{1+k} e^{-\frac{\gamma_{th}(1+k)}{\gamma_c}} + \gamma_c - \frac{\gamma_c}{1+k} \Bigg\} \Bigg] \quad (11)$$

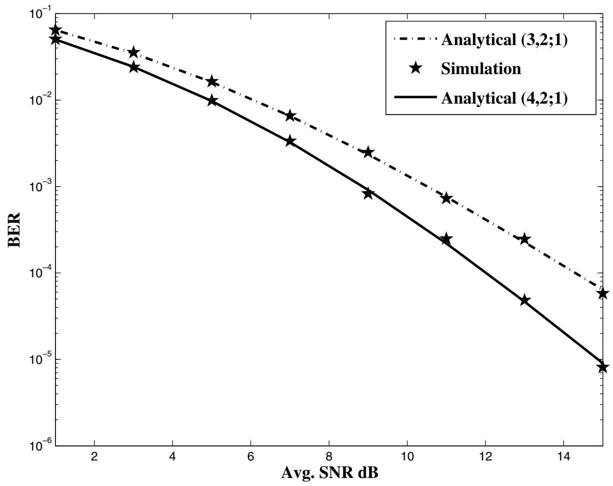

Fig. 2. BER Vs Avg. SNR

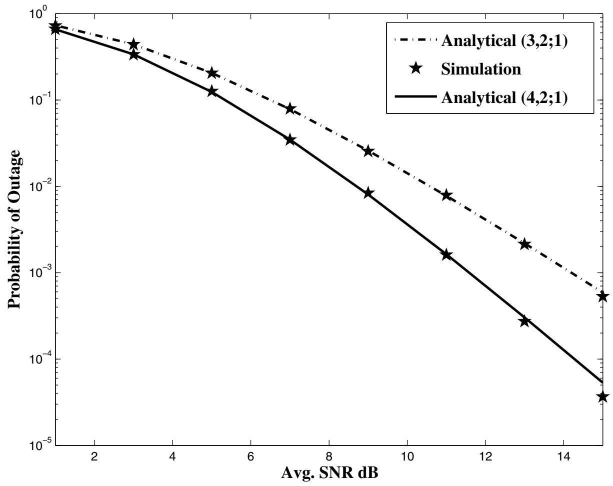

Fig. 3. Probability of Outage Vs Avg. SNR. ($\gamma_{th} = 3\ dB$)

IV. Results

In this section, we show the analytical results obtained in section III and compare them with simulation results. We denote the considered sub-optimum system as $(N, 2; 1)$, which indicates that two transmit antennas are selected from N with one receive antenna. Fig. 2 and Fig. 3 show BER and outage probability (for $\gamma_{th} = 3\ dB$) respectively for $N = 3$ and 4. A close matching between analytical and simulations results validate our analysis.

Fig. 4 shows the BER of our sub-optimum scheme for different values of N ($3 \leq N \leq 7$). We can observe that the performance has been improved for larger values of N as expected.

Now, we compare the performance of our system with (i) two fixed antennas (ii) optimum TAS scheme and (iii) sub-optimum TAS scheme in [4]. Fig. 5 shows the performance of the system with two fixed antennas, optimum TAS scheme [3] and the considered sub-

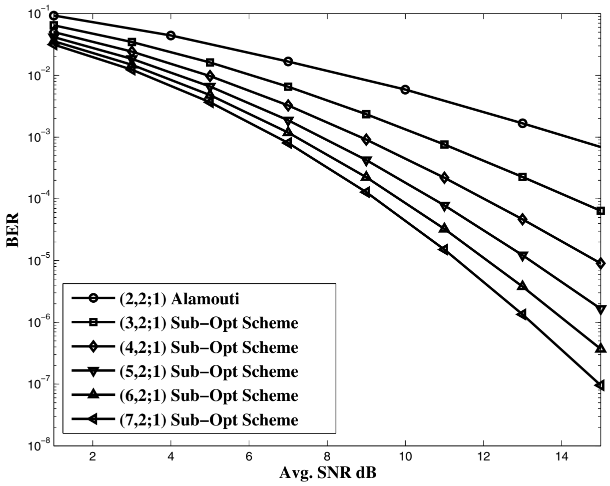

Fig. 4. BER Vs Avg. SNR for $3 \leq N \leq 7$.

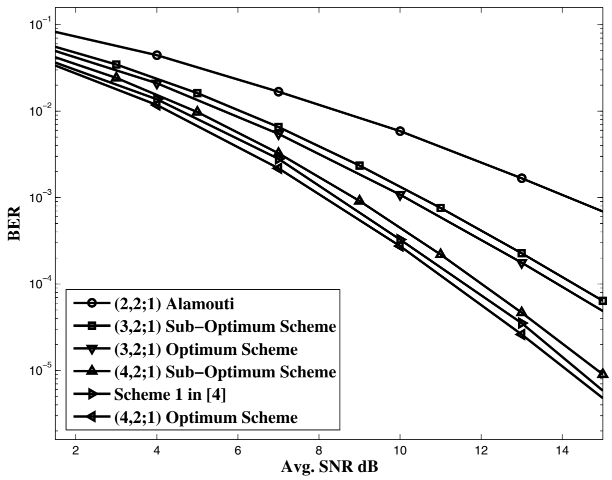

Fig. 5. BER Vs Avg. SNR

optimum TAS scheme for $N = 3$ and 4. We have shown the fixed antenna system and optimum TAS scheme as "$(2, 2; 1)$ Alamouti" and "$(N, 2; 1)$ Optimum scheme" respectively. It can be seen that the performance of our sub-optimum scheme suffers an SNR loss of only 0.4 dB and 0.7 dB for $N = 3$ and $N = 4$ respectively, compared to the optimum scheme. Further, the performance of our scheme is better compared to the performance of $(2, 2; 1)$ fixed antenna scheme due to antenna selection gain. We also compare the performance of our scheme with Scheme 1 in [4] (analyzed in [5]). In Scheme 1, the antennas at the transmitter are divided into two groups, and the best antenna among each group is selected for transmission. The performance of both the schemes is same for $N = 3$ because both the schemes are same for $N = 3$. It can be seen that our sub-optimum scheme suffers an SNR loss of 0.4 dB compared to Scheme 1 for $N = 4$. However, we have reduced the complexity of our scheme with respect to feedback bits as shown in Table I. It shows the feedback bits requirement at the receiver in optimum, sub-optimum, and Scheme 1 in [4] for N transmit antennas. It shows that the considered sub-optimum scheme reduces the feedback bits considerably. For example, optimum, Scheme 1 and sub-optimum schemes require 6, 4 and 3 feedback bits respectively for $N = 8$. Similarly, optimum scheme and Scheme 1 requires 8 and 5 feedback bits respectively for $N = 9$, whereas the considered sub-optimum scheme still requires 3 bits only. Reduction in feedback bits causes reduction in feedback rate. Hence, the considered sub-optimum scheme is favorable for practical applications.

In Fig. 6, we show the comparison between the BER performance of our $(N, 2; 1)$ sub-optimum scheme and the BER performance of $(N, 1; 1)$ scheme [12] for $N = 2$ and 3. In $(N, 1; 1)$ scheme one transmit antenna is selected from N antennas with one receive antenna. The complexity and feedback bit requirement of our

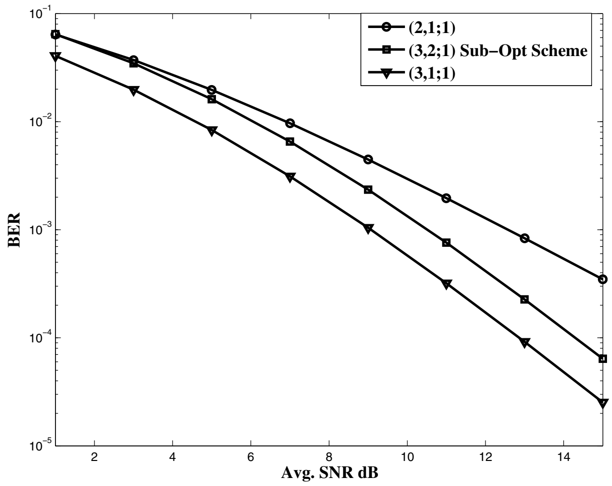

Fig. 6. BER Vs Avg. SNR

$(3, 2; 1)$ and $(2, 1; 1)$ schemes are same. However, it can be seen that the performance of our scheme is better. Now we compare our $(3, 2; 1)$ scheme with $(3, 1; 1)$ scheme. The performance of $(3, 1; 1)$ scheme is better than the performance of $(3, 2; 1)$ scheme, however in the case of $(3, 1; 1)$ the complexity of AS algorithm at the receiver (mobile station) and feedback bit requirement increase.

V. Conclusion

We have considered a sub-optimum transmit antenna selection scheme with N transmit antennas in MISO

TABLE I
NUMBER OF FEEDBACK BITS REQUIREMENT

N	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Optimum (TAS)	4	4	6	6	6	6	8	8
Scheme 1 in [4]	1	2	3	4	4	4	5	5
Sub-Optimum (TAS)	1	2	2	3	3	3	3	4

systems, wherein one antenna is fixed and the other is selected out of the remaining $N-1$ antennas. We have derived the pdf of SNR using which we obtain the closed-form expressions of BER and outage probability for Alamouti transmit diversity. We have also compared our analytical results with simulations and found a close matching between them. We have further compared the performance of the sub-optimum TAS scheme with the optimum one. We have also addressed the issue of feedback bits in TAS. There is a reduction in the number of feedback bits in the considered sub-optimum TAS scheme with a little degradation in the BER performance compared to the optimum AS scheme.

REFERENCES

[1] V Tarokh, H. Jafarkhani and A.R.Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol 45, pp. 1456-1467, July 1999.

[2] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE J. Selected Areas Communications*, vol. 16, pp. 1451-1458, October 1998.

[3] Z. Chen, J. Yuan, B. Vucetic and Z. Zhou, "Performance of Alamouti scheme with transmit antenna selection," *Electronics Letters*, vol. 39, No. 23, pp. 1666-1668, November 2003.

[4] Z. Chen, I. B. Collings, Z. Zhuo and B. Vucetic, "Transmit Antenna Selection Schemes with Reduced Feedback Rate," *IEEE Trans. Wireless Communications*, vol. 8, No. 2, pp 1006-1016, February 2009.

[5] Y N Trivedi , A K Chaturvedi, "Performance analysis of Alamouti Scheme with Transmit Antenna Selection in MISO Systems," *National Conference on Communications (NCC), IIT Madras*,2010.

[6] D. A. Gore and A. Paulraj, "MIMO antenna subset selection with space-time coding," *IEEE Trans. Signal Processing*, vol 50, No. 10, pp. 2580-2588, October 2002.

[7] D. J. Love, "On the Probability of Error of Antenna-Subset Selection With Space-Time Block Codes," *IEEE Trans. Communications*, vol. 53, No. 11, pp. 1799-1803, November 2005.

[8] "Physical layer procedures (FDD)," 3rd Generation Partnership Project (3GPP), Tech. Rep. 25.214, 2005.

[9] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th ed. MH.

[10] J G Proakis, *Digital Communication*, 4th ed. MH, 2001.

[11] H. A. David, *Order statistics*, New York Wiley, 1970.

[12] M K Simon and M S Alouini, *Digital communication over fading channels -A unified approach to performance analysis*, John Wiley & Sons, 2000.