ANALYSIS AND DESIGN OF CABLE ROOFS

By

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DEPARTMENT OF CIVIL ENGINEERING Ahmedabad 382481 May 2008

ANALYSIS AND DESIGN OF CABLE ROOFS

Major Project

Submitted in partial fulfillment of the requirements

For the degree of

Master of Technology in Civil Engineering (Computer Aided Structural Analysis & Design)

By

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CERTIFICATE

This is to certify that the Major Project entitled "Analysis and Design of Cable Roofs" submitted by Miss Kshatriya Mamta A. (06MCL006), towards the partial fulfillment of the requirements for the degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) of Nirma University of Science and Technology, Ahmedabad is the record of work carried out by her under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination. The results embodied in this major project, to the best of my knowledge, haven't been submitted to any other university or institution for award of any degree or diploma.

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ABSTRACT

Applications of Cable structure in building have gained high popularity during twentieth century. Advancement in form finding and high speed computer applications, as well as rapid growth in construction technology has led its way to find economical and simpler design using cables for large span structures.

Present study is carried out with an objective to understand the various forms, geometry and structural behaviour of cable systems. Various types of cables, connections, anchorages, stabilizers and fittings for cable roof systems are studied.

Approximate methods used in the numerical analysis of single, double cable roof truss and saddle shaped cable net structures that provide preliminary design is described.

These structures are usually very light and flexible and require analysis methods, which takes their non-linear behaviour into account. Exact methods of nonlinear analysis on which the software's are based are presented.

Parametric study of change in cable forces for different sag/span ratios is presented. Study of effect of change in sag, span, pre-tension, method of analysis i.e. approximate, linear or nonlinear method, static and dynamic wind is discussed.

It is observed that iterative procedure is required to find optimum solution for minimum breaking strength, area and pre-tension value of cable force. In order to prevent slack of cables, pre-tensioning is essential in cable structures.

Design and detailing of saddle shaped cable net roof is carried out to understand and explain the various parameters that are critical in the design and selection of shape and material of cable roofs.

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NOMENCLATURE

Т	Tension in Cable
W	Load/unit length radial to cable
R	Radius of cable
L	Span of cable
f	Cable sag- single cable
Н	Horizontal reaction at support
V	Vertical reaction at support
T _{max}	Maximum Tension in Cable at support
W	Vertical load/unit length
Δf	Change in Sag
A	Cross-sectional area of Cable
E	Young's modulus of Elasticity of cable
ΔL	Elastic elongation of the cable
f _n	Natural Frequency of Cable
g	Acceleration due to gravity
n	any integer
ω	Circular frequency of vibration
m	Lumped mass of cable structure
T _b	Initial Tension in bottom Cable of cable truss
T _u	Initial Tension in top Cable of cable truss
f _u	Initial sag in bottom Cable of cable truss
f _b	Initial sag in top Cable of cable truss
ΔT_u	Change in Tension in top cable
ΔT_{b}	Change in Tension in bottom cable
f _{nb}	Natural Frequency of bottom cable
f _{nu}	Natural Frequency of top cable
q	Uniformly distributed load on cable- diaphragm load
q _i	Initial Diaphragm load due to pretension in cable
q u	Load on / Mass of top cable
q _b	Load on / Mass of bottom cable
р	Superimposed load
h _u	Horizontal component of tension increment in upper cable
h _i	Horizontal component of tension increment in lower cable

Z	initial ordinate of the cable
W	vertical deflection
H _u	Horizontal component of tension in upper cable
HI	Horizontal component of tension increment in upper cable
m	mass per unit length or area
t	time variation
x	distance from support in X-direction
κ	Ratio of upper and lower cable stiffness for cable truss
M _p	Moment in Simply supported beam
Δq	Change in Diaphragm force
A _u	Area of top Cable of cable truss
A _b	Area of bottom cable of cable truss
К	Stiffness Matrix
U	Unknown deformation matrix
Р	Total applied load
R	Residual Tension
X', Y'	Cable ordinates to define a surface
X, Y, Z	Ordinate of cable node in space
a, b and k'	Constants/functions of the curvature.
Θ and $\overline{\Theta}$	Curvature constants
k	Stiffness of cable
β_{u}	Function of cable stiffness k, curvature constant $\Theta\;$ and Cable span
ν	Ratio of upper and lower cable rise and sag
α	Ratio of upper and lower cable stiffness for cable net
po	load due to initial tension in cable
$\overline{H}_X / \overline{H}_y$	Horizontal components of tension per unit width of strip
\overline{h}_{X} / \overline{h}_{y}	Horizontal component of tension increment per unit width
α	Angle of the anchor cable at the anchorage
β	Angle of suspended cable at anchorage

Note: Subscripts u and I are for upper and lower cable of cable truss and x and y are for cable in X and Y direction of cable net For exact analysis equations -

W	Total potential energy of a system
S	Step length
V	Descent vector
L _{jn0}	Initial Length of link j _n
X _{ni}	Ordinate of the element i at node n
$\mathbf{Y}_{\mathbf{j}\mathbf{i}}$	Ordinate of the element i at node j
e _{jn}	Change in length of element i
g _s	Gradient vector at step length s
t _{jn}	Tension coefficient of force in member j _n
T _{jn0}	Initial tension in member j_n due to pretension
a1, a2, a3, C0	Constants (as defined wherever used)
C1, C2, C3, C4	4 Constants (as defined wherever used)
Fs	External force on element
V ₀	Conjugate vector which defines the path of minimization at step 0
k _{sr}	Stiffness vector of r th iteration at step length s
x _r	Displacement vector of r th iteration at step length s
δx _k	Descent/Displacement vector for Newton Raphson method
S _k	Step for Newton Raphson method
K _k	Stiffness matrix of individual element for pin joint pretension link at
	k th iteration
I	Unit matrix
T _k	Tension in element at k th iteration
Gk	Matrix of ratio of nodal ordinate at k^{th} iteration to initial length
[K]	Stiffness matrix of element
{δ}	Nodal Displacement matrix
{P}	Load vector
$\{P'\}_k$	Out of balance force
L _o	Initial length of 2D element
X, Y, Z	Nodal ordinate (Subscript A, B, C indicates node name)
[L]	Load vector for 2D/3D planar element
[X]	Displacement vector 2D/3D planar element
е	Element extension of a 2D or 3D element

$l_p, m_p, n_p,$ $l_q, m_q, n_q,$ l_r, m_r, n_r	Direction cosines along P, Q and R axis
Р	Element force in terms of initial pre-tension and displacement
Po	Initial Pretension force
F _{xa} , F _{ya} , F _{za} F _{xb} , F _{yb} , F _{zb} F _{xc} , F _{yc} , F _{zc}	Force component along respective axis for nodes A, B and C
[Kg]	Geometric Stiffness matrix
u, v and w	Element displacement in local co-ordinate system
^x _a , ^y _a , ^z _a ^x _b , ^y _b , ^z _b	Displacement in global co-ordinate along respective axis at node
x_{c}, y_{c}, z_{c}	
С	Connectivity matrix at free node
C _f	Connectivity matrix at fixed node
Cs	Combined connectivity matrix of the cable net
L _{mx} , L _{my} , L _{mz}	Projected length of members in X, Y and Z direction
Q_{mx}, Q_{my}, Q_{mz}	Component of internal forces
$P_{x^0} P_{y^0} P_{z^0}$	External load vector
x, y, z,	Vectors containing the coordinates of the n free nodes
X _f , Y _f , Z _f	Vectors containing the coordinates of the n_f fixed nodes
q	Force densities
$D_{x_i} D_{y_i} D_z$	Displacement vector for force density method

For Parametric Study -

S	Static wind
D	Dynamic wind
Р	Pressure
S	Suction
DL	Dead load
LL	Live load
WL	Wind load

1.1 GENERAL

Space structures being economical and aesthetically pleasing in appearance have always fascinated engineers for their use in providing solution for column free areas. A structural system in form of 3D assembly, which resists loads applied at any point, inclined at any angle to the surface of structure and acting in any direction having integrated load sharing defines the 3D space structures [1].

Space structures are mainly classified as Skeleton (Braced) frameworks/ grid structures, Stressed skin systems – grids and domes and Suspended (Cable or membrane) structures [2]. This work aims to understand the various forms of tension structures and thereby in details analysis and design of cable roofs.

1.2 TENSION STRUCTURES

Tensile structures have always fascinated architects and engineers, mainly because of the aesthetic shapes they produce. Despite this, very few tensile structures have been built. The reason might be that tent-like structures have always been thought of as temporary. Although, a probably more correct answer is that they are more difficult to analyze and construct than traditional buildings. From a structural viewpoint, tension structures have several special features, such as light weight and flexibility. These features require special care in the design; for example, an error in the distribution of the pre-tensioning forces may lead to damage of the cladding under large loads.

Tension structures mainly include roofs, guyed towers and suspended bridges. Tensile-resistant systems include stayed, suspended, cable truss, Synclastic and anticlastic as well as pneumatic structures depending upon the geometry and type of supporting system. Tensile stress is prominent in such structures, although beams and joists resisting shear and bending are required at certain locations. Tensile elements are most efficient, as they utilize their material capacity to its full [3].

Chapter 1 Introduction



Fig.1.1 (a) Cable Roofs (b) Guyed Tower (c) Cable Bridges

Chapter 1 Introduction

1.2.1 Need of Tension Structures

The use of suspended cables is perhaps suggested by nature. In the tropical countries of Southeast Asia and Africa, abundance of ropes and vines and creepers lead to the development of constructing small suspension bridges, using the natural ropes [4].

Tensile architecture represents the new trend in design and construction with the minimum amount of material. The primary advantage of tensile members over compression members is that they can be as light as the tensile strength permits. With new materials, such as high strength steel cables and silicone-coated glass fiber membranes, larger distances can be spanned using the same amount of material as before.

Elements that carry bending as well as tension and compression are able to withstand reversals of loads. Cables carry only tension working upto full tensile stress of materials, thus the forces during reversal of loads are different when applied to the same structural arrangement. High Strength Steel cables are efficient for long span roofs. Creating variety of shapes it gives aesthetic appearance by using the suspension systems, along with possibility of having column free space.

1.2.2 Classification of Tension Structures

Tension structures come in a wide range of forms, which can also be broadly categorized as follows [2]:

- (1) Two-dimensional Suspension bridges
 - Draped cables
 - Cable-stayed beams
 - Cable trusses
 - Straight Tensioned Cables
- (2) Three-dimensional Bicycle wheel
 - 3D cable trusses
 - Tensegrity structures
- (3) Surface-stressed Pneumatically-stressed membranes
 - Pre-stressed membranes

1.2.2.1 Suspension bridges

A suspension bridge is essentially a Catenary cable pre-stressed by dead weight only. Early suspension bridges with flexible decks suffered from large deflections and sometimes from unstable oscillation under wind. A system of inclined hangers was proposed to reduce deflection under live load and also to counteract wind effects. The suspension cable is taken over support towers to ground anchors. The stiffened deck is supported primarily by the vertical or inclined hangers. The system is ideally suited to resisting uniform downward loads.



Fig.1.2 Suspension Bridges

1.2.2.2 Draped cable

The same principle of suspension bridges has been extended for use in buildings, which is termed as draped cable structures.

1.2.2.3 Cable-stayed beams

Cables assist the deck beam by supporting its self-weight. Compression is taken in the deck beam so that ground anchors are not required. The cable-stayed principle has also been developed for single-storey buildings. The cable system is designed for and primarily resists gravity loads; in buildings with lightweight roof construction the uplift forces, which are of similar magnitude, are resisted by bending of the stiffening girders. The system is suitable for spans of 30–90 m and has recently been widely used for industrial and sports buildings.



Fig.1.3 Cable Stayed Beam

1.2.2.4 2D Cable trusses

The hanging cable resists downloads and the hogging cable resists upload. If diagonal bracing is used, non-uniform load can be resisted without large deflections but with larger fluctuation of force in the cables.



Fig.1.4 Cable Truss

1.2.2.5 Three-dimensional cable truss

The classic form of this structure is the bicycle wheel roof, in which a circular ring beam is braced against buckling by a radial cable system. These radial cables are divided into an upper and lower set, providing support to the central hub.



Fig.1.5 Bi-Cycle Wheel Roof

The system is suitable for spans of 20–60 m diameter. This system has been developed (by David Geiger) into a cable dome, having two or three rings of masts. The radial forces at the bases of the masts are resisted by circumferential cables. The masts are also cross-cabled circumferentially to maintain their stability. These structures can span up to 200m.



Fig.1.6 Cable Dome

1.2.2.6 Tensegrity Structures

The term Tensegrity is derived from the words 'Tension-integrity'. A system is established when a set of discontinuous compression components interact with a set of continuous tensile components to create a stable volume in space. Tensegrity structures are composed of bars and cable nets. The bars are arranges in such a way that no bar is connected to another. Tensegrity shells obtain a stable configuration by pre-stressing the bars against cable net. [1]



Fig.1.7 Tensegrity Structures

1.2.2.7 Surface-stressed structures

A cable network can be arranged to have a doubly-curved surface either by giving it a boundary geometry which is out-of-plane or by inflating it with air pressure. The cable net must be pre-stressed either by tensioning the cables to the boundary points or by the inflation pressure. The effect of the double curvature and the pre-stress is to stiffen the structure to prevent undue deflection and oscillation under loads. Cable net structures can create dramatic wide-span roofs very economically. They can be clad with fabric, transparent foil, metal decking or timber boarding, insulation and tiles. Low-profile air-supported roofs can provide the most economical structure for covering very large areas. In designing these structures the aerodynamic profile must be taken into consideration, as must snow loading and the methods of installation and maintenance [1].



Fig.1.8 Surface Stressed Structures

Chapter 1 Introduction

1.2.3 History and Development

A historical survey leads to some other simple conclusions. One that suspended bridges and roofs has existed in some form or another for over two thousand years, the other that the pace of development in the twentieth century has been very rapid. [5]

Cable-stayed bridges can be dated back to the 1784 design of a timber bridge by German carpenter C.T. Loescher. Many early suspension bridges were of hybrid suspension and cable-stayed construction, including the 1817 footbridge at Dryburgh Abbey, and the later Albert Bridge (1872) and Brooklyn Bridge (1883). Their designers found that the combination of technologies created a stiffer bridge, and John A. Roebling took particular advantage of this to limit deformations due to railway loads in the Niagara Falls Suspension Bridge.

John Roeblin 1806-1869 and his son Washington Roeblin 1837-1929 are the pioneers in the construction of suspension bridges. They are best known for building of Brooklyn Bridge. In 1841 Roebling invented the twisted wire-rope cable, an invention which foreshadowed the use of wire cable supports for the decks of suspension bridges.

The development of cable bridges lead the way to development of large span cable stayed and suspended roof structures. Although, tensile structures have long been used in tents, the real development in regards to roof began by Russian engineer Vladimir Shukhov.



Fig.1.9 Brooklyn Bridge

Vladimir Shukhov was the first person to develop practical calculations of stresses and deformations of tensile structures, shells and membranes. Shukhov designed eight tensile structures and thin-shell structures exhibition pavilions for the Nizhny Novgorod Fair of 1896, covering the area of 27,000 square meters.



Fig.1.10 Nizhny Novgorod Fair

A big step in the development of suspended roofs came in 1950 when Matthew Nowicki designed the State Fair Arena, at Raleigh, North Carolina, USA. Architect William Henry Deitrick and civil engineer Fred Severud continued this work, after Nowicki's death and in 1953 the arena was completed [5].



Fig.1.11 The State Fair Arena at Raleigh, North Coralina, U.S.A.

A very early large-scale use of a membrane-covered tensile structure is the Sidney Myer Music Bowl. The project design was by Yuncken Freeman Architects and Griffiths and Simpson during 1956. The project architect was Barry Patten. Construction commenced in 1958 with an innovative system of cables laced together. The main cable at the edge of the canopy are anchored deep into the ground in concrete blocks while the longitudinal cables hold up the roof and transverse cables hold it down.



Fig.1.12 The Sidney Myer Music Bowl

The concept was later pioneered by German architect and engineer Frei Otto and in 1957 he formed the Development Centre for Lightweight Construction in Berlin. In 1964 he incorporated the centre into the Institute of Light Surface Structuress at University of Stuttgart. Massive research work of the two institutes was undertaken and finally he published the Tensile Structures in two volumes. [6]

The idea of tensile structure was put forward by Otto in the construction of the German pavilion at Expo 67 in Montreal.

Another pioneering structure during 1970's was the large low-profile super elliptic air-supported roof, with a membrane attached to a diagonal cable net. This structure was designed by David Geiger for the United States pavilion at the World's fair in Osaka 1970.

An incredible 100,000 square foot inflatable roof made of a special vinyl membrane, with stiffening steel cables anchored to a concrete compression ring

around the perimeter. The roof was designed to resist wind forces, as well as the air pressure inside the pavilion. This building was a pioneering effort by engineer David Geiger in the field of pneumatic structure.

After success of Montreal design, Otto next used the idea for the roof of the Olympic Stadium for the 1972 Summer Olympics in Munich.



Fig.1.13 The Olympic Stadium at Munich

After the Osaka dome, several air-supported domes were built around the world, because they provided the economically best alternative to span large distances. However, several of them deflated due to heavy snow loads or compressor failure.

To overcome the deflation problems, David Geiger invented another structure 1986— the cable dome. The cable dome concept was inspired by the tensegrity principle by Kenneth Snelson and Richard Buckminster Fuller. The first two domes were built for the 1988 Seoul Olympics. The latest and biggest, the Georgia Dome, was built in Atlanta 1994. The dome has a height of 82.5 meters

and spans over a 227 meters with width of 185 meters. The dome is the largest cable-supported dome in the world.

The Georgia Dome was completed in 1992 at a cost of \$214 million which came from the Georgia General Assembly making it one of the largest state funded construction projects in state history. From its completion until the December 31, 1999 opening of the 20-acre Millennium Dome in London, it was the largest domed structure of any type in the world, but still remains the largest indoor sporting facility in the United States.

In the year 2000, the Millennium Experience of exhibition was held in Greenwich, London, close to the Greenwich meridian. A largest dome of diameter 364 m and the height is 50 m was formed giving an overview of advantages that Tensile structure provide.

Since the 1960s, tensile structures have been championed by designers and engineers such as Ove Arup, Buro Happold, Frei Otto, Eero Saarinen, Horst Berger, Matthew Nowicki, Jorg Schlaich, the duo of Nicholas Goldsmith & Todd Dalland at FTL Design & Engineering Studio and David Geiger.

The development and increasing use of cables has led its way to construction of towers and glass supporting elevations. J. Schlaich (former partner of Leonhardt, Andra and Partners) developed cable-net cooling tower at Schmehausen, which with increasing size, become more economic than concrete towers.

Steady technological progress has increased the popularity of fabric-roofed structures. The low weight of the materials makes construction easier and cheaper than standard designs, especially when vast open spaces have to be covered. Although the nonlinear behaviour of such structures need to be understood for its form finding and proper analysis and design.

1.3 OBJECTIVES OF STUDY

- Solution Study space structural system formed by cables Cable supported and cable suspended systems and different types of tension structures.
- Study details of cable roofs, its forms, analysis and design.

- So understand behavior of cable suspension system with single curved cable,double curved cable and cable net under different loading combinations for static as well as dynamic – linear and nonlinear behaviour.
- To find the proper modeling aspects for cable suspended roof structures in SAP.
- To calculate and compare approximate analytical results of single cable, double layer cable truss and cable net with exact results of SAP software.
- Solution To study various types of connections like cable to strut (Tension member to compression member), cable to column (Hinge connection) and cable to Ground (Gravity+ ground anchor).
- Design and detailing of cable net roof

1.4 SCOPE OF WORK

- ✤ Analysis of single cable roof using approximate and exact analysis.
- Analysis of double cable truss using approximate and exact analysis.
- Analysis of saddle shaped cable net using exact and approximate method of analysis.
- ✤ Design of saddle shaped cable net roof based on exact method of analysis.
- Study of wind and seismic effects on the cable roofs.
- Parametric study of effect of change in sag and span for static and dynamic loading condition of single cable, double cable truss and cable net on tensile force, frequency and time period.
- Parametric study of forces, frequency and displacement for single cable, cable truss and cable net for linear, nonlinear and approximate method.

1.5 ORGANIZATION OF MAJOR PROJECT

The report on "Analysis and Design of Cable Roofs" has been divided in chapters as follows:

Chapter 1 describes introductory part of the space structures. Basic information of tension structures and historical development for cable stayed and suspended

roof structure is discussed. The chapter also emphasizes on the objective of study and scope of present work.

Chapter 2 presents the literature review. It provides an overview of the available books on the subject of cable roofs. Overviews of papers from various journals are discussed to provide the understanding and development of different features of such structures.

Chapter 3 covers the introduction to cable roof structure and its classification. Basics of cable structure behavior under applied loading, components, material and connections are mentioned. Other considerations such as serviceability, corrosion and fatigue are described along with protection methods for cable and anchorages.

Chapter 4 describes approximate method of analysis for single cable, double cable roof truss and saddle shaped cable net roof. Static and dynamic behaviour of cables are discussed. Various loads and load combinations that are critical for design are explained.

Chapter 5 presents the exact methods of analysis for cables, which is based on geometric nonlinearity. Modeling aspects of cable structures in SAP software is also mentioned.

Chapter 6 presents design examples for single cable roof; double curved roof truss based on approximate method of analysis. Design of Saddle cable net roof systems is based on exact method of analysis. Comparative study carried out is explained for the results of approximate methods and exact method (using SAP). Parametric study for effect of change in span and sag, column spacing and analysis method is based on the sag/span ratio.

Chapter 7 includes summary and conclusion of the Major Project and future scope of work.

2.1 GENERAL

Tensile structures have always fascinated architects and engineers, mainly because of the aesthetic shapes they produce. Despite this, very few tensile structures have been built. The reason behind may be due to the fact that tensile structures are more difficult to analyze and construct than traditional buildings.

From a structural viewpoint, tension structures have several special features, such as light weight and flexibility. These features require special care in the design; for example, an error in the distribution of the pre-tensioning forces may lead to damage of the cladding under large loads. Extensive research and development has taken place in this field. Various papers describing analysis, design, construction and material aspects are studied to get in depth view of the development. The review also considers the research and studies for nonlinearity arising due to wind that have undertaken in past few decades.

2.2 LITERATURE REVIEW

N Subramanian [1] describes in detail various types of space structures and approximate methods of analysis in his book on Principle of Space structures. Historical development and details of Single and Multilayer grids, braced domes, folded structures, stressed skin structures, cable roofs, tensile membrane structures, and tensegritic structures are presented with various examples.

Approximate methods of analysis for double layer grids, curved space frames, folded plate roofs and design of suspended roof are presented with examples. The book covers various types of connectors and a few important aspects as construction, support conditions, cladding and aesthetic, with some details about current and future trends in space structures. A computer program for buckling analysis of skeletal space frame is given, with complete input and output file details.

Buick Davison and Graham W. Owens [2] describe complete analysis and design background of various Steel structures. The manual includes chapters for single storey, multi-storey, industrial, bridges and special structures. The manual presents excellent views of different authors in regards to standards and design

of tension, compression, bending, analysis of beams and frames, connections, joints, decks, trusses, plate girders etc.

Properties of various sections as equal angle, unequal angles, hot and cold finished hollow sections, castellated beams, joists, tee sections, bolts and their capacities as well as sheet pile sections are specified for easy and quick design. Discussion of fire protection and corrosion is also included in the manual. Standards of BS, European codes and ISO are presented where ever required. Plastic design is also a part of the manual.

G.G. Schierle [3] introduces the principles of foundation of creative design and demonstrates successful application on many case studies from around the world. The book clarifies concepts without calculus yet provides a profound understanding. It includes structural details in wood, steel, masonry, concrete and fabric to facilitate design of structures that are effective and elegant.

Prem Krishna [4] introduces various forms and classification of cable roofs. General design consideration for freely suspended cables and pre-tensioned cable systems are discussed in different chapters. Single cable, cable trusses and cable nets are discussed for initial geometry determination and forces.

Dynamic analysis being important aspect of light weight structures is discussed using lumped mass model for double cable truss and cable nets. Experimental study is given due importance and its various aspects such as dimensional analysis, choice of scale; wire models and instrumentation are presented in brief. Design of existing cable structures is described for better understanding along with construction materials and detailing.

Frederick and Otto [5] discussed the Tensegrity structures and cable roofs in two Volumes. Detail description of various ground anchors systems such as Screw, Expanding, Folding, Anchor needles, Piles and anchor plates and clamped rods is explained in Volume I.

Volume II Otto states the non-pre stressed and pre-stressed cable systems. Single cable, cable trusses and cable nets with symmetrical, radial and various other forms are presented to provide an overview of the cable systems. Fedrrick presents the analytical description of single cables, cable truss and cable nets orthogonal in plan.

P.Krishna [6] in the paper on Tension roofs and bridges covers various aspects as materials, analytical approach, illustration of the various form and latest information and trends on cable technology and aerodynamics. Emphasis of laid upon the material development and increase in strength-weight ratio, which substantially increases the capacity of the structure. The author supports use of an alternative material i.e. Carbon Reinforced Plastic fibers, which exhibits favorable physical attributes as cables.

A brief view for analytical approach which has evolved on basis of geometrical form, structural response and progress of matrix methods is presented. The paper describes reasons for development of the approximate methods of analysis. Detail description of cables, wires, ropes and their uses in accordance with size and breaking strength are indicated. Lastly other aspects as corrosion protection, fatigue, wind effects and end fittings are accounted for.

David E.Eckmann, Stephanie J. Hautzinger and Thomas R. Meyer [7] based on ASCE 19-96 presents the design consideration of cable stayed roof structures and its configuration with various examples. The details of structural system, tie back and masts, wind tunnel testing and roof erection procedure is presented for The University of Chicago Gerald Ratner Athletics Center

Lev Zetlin [8] presents single and double cable truss system in detail. An example for both the systems is discussed with a view to describe its behaviour in static and dynamic condition. Application of the systems is presented with a view to describe its use in practical work.

Craig G. Huntington [9] presents the tensioned fabric structural systems.

Its elements for form, methods of form-finding, material and their properties, connections, design and detailing aspects are given in detail for every element of these structures. Various existing structure are taken as example that describes the increasing use of tensioned fabric roof systems.

H.A.Buchholdt [10] provides structural engineers with a concise introduction to the architectural, structural and technological aspects of cable roofs. Information for single cable static and dynamic behaviour is discussed along with tension anchoring systems. It also provides an overview to the cable net system and design consideration.

J.Lenoard [11] presents the numerical background for tension structures. It describes static as well as dynamic behaviour of cable systems and membrane structures for linear, pre-stress and nonlinear behaviour.

Roger L. Brockenbrough and Federick S. Merritt [12] describe various structural steel members, their properties and various structures. The designer Handbook is based on ASTM standards. Methods of analysis – both continuous and discrete for cable structure and its aspects as forms, saddles, anchorages and connections are described with examples.

W. J. Lewis [13] describes pre-stressed cable nets and beams as well as membrane structures. The most critical design issue of form-finding is explained with respect of geometric non-linearity. Various methods of form-finding are explained with simple small examples. The advantages of dynamic relaxation method for saddle cable nets over transient stiffness matrix, wherein the results didn't converge is presented. Case studies for cable roof structures are highlighted in regards of form-finding and patterning.

E.Hernandez-Montes, R.Jurado-Pina and E.Bayo [14] present a new mapping method for easier form-finding. Use of linear method i.e. force density is taken as the knowledge of initial non fixed nodes is not required to describe the geometry and only the connectivity of nodes is necessary.

M. Mollart [15] based on force density method develops a form-finding solution for mixed structures. The force density method is described in detail and additional constraints are discussed for form-finding.

M. R. Barnes [16] presents a review of the various methods – Matrix and vector that are usually employed for solution of nonlinear iterative analysis.

Advantages of each of the methods i.e. conjugate gradient, dynamic relaxation and stiffness method are discussed. Form finding of uniform mesh, geodesic and principal curvature nets is also discussed.

W.H. Melbourne [17] presents review of the data for Aero- elastic model tests carried out on cantilevered, enclosed, free edged, arched and suspended roof systems. The paper aims at obtaining generalized design data and thereby verifying the equation developed for Australian Wind Loading code AS1170.2.-1989 for the dynamic response of cantilevered roof systems. This equation was formed in view of the results of various tests carried out at Monash University.

Zhi-Hong Zhang and Yukio Tamura [18] presents the preliminary results of wind-tunnel tests on cable dome of Geiger type which was fabricated in Wind Engineering Research Center of Tokyo Polytechnic University. Series of tests including construction analysis, ambient model test, structural test subject to harmonic load and earthquake load and wind tunnel test have been carried out.

Geometric parameters are defined for cable and bars of the dome, which further defines the membrane cutting pattern and equilibrium matrix for force finding. Turbulent and smooth wind tests suggest that the turbulence intensity is most important parameter for wind tunnel tests on flexible roofs. It presents that no aero elastic instabilities were observed, which may be due to high damping of model and limitation of wind speed at the time of test.

M.A. Chrisfield [23] discussed various nonlinear aspects using finite element analysis for all types of solid and structural elements. Numerical methods used for iterative purpose are described well in its 1st Volume named Essentials.

Peter Broughton and Paul Ndumbaro describes the nonlinearity importance for cable structures and thereby explains in details the 2D and 3D cable element. The geometric stiffness matrix required for both the elements are well explained for increase in stiffness due to nonlinearity.

Peter Broughton and Paul Ndumbaro [24] describe in detail the analysis of cable and catenary structures. As these structures behave nonlinearly the
sources of nonlinearity are described. Analysis of 2D and 3D cable elements subjected to large deformation is discussed. The additional geometric stiffness matrix that adds to the stiffness of the structure are described for both 2D planar element and 3D spatial element.

Ivar Talvik [25] in his paper on Finite Element Modeling of Cable Networks with flexible supports presents a mathematical model for analysis of saddle shaped pre-stressed cable network with flexible beam. The aim of establishing a reliable computer model is undertaken by considering square and elliptical plan, for both static and dynamic analysis. To account for non-linearity the models stiffness properties are confined to different predetermined localities using sub structuring concept. Compatibility between both sub structures is achieved by iteration scheme.

It is observed that the theoretical evaluation results overestimate the experimental data. It is noted that cable net, with relatively stiff boundary structure may lose its stiffness, if pretension is not sufficient. Deformation of the contour beam, caused by loading of network enables the pre-stress in the net cables to be retained.

Harry H. West and Anil K. Kar [26] describe a method of analysis for single layer cable nets based on discrete mathematical model. Forces that result from live loads and temperature are of main concern; however the initial dead load configuration can also to obtain from the described equilibrium equations.

Zhang Limei et al. [27] has considered the important effect on initial Prestress of Geiger cable dome due to error that arises at time of manufacturing. The proposed Blue Sea cable dome which was built in Beijing before 2008 Olympic Game is compared to the classical cable dome and rib-hoop cable dome.

It is summarized that the manufacture errors of cable length in practical projects is mainly due to cable elongation, cable shortening or cable elongation and shortening together. Percentage variation in pre-stress due to each of this is compared to maximum value formulated as maximum value due to variable probability function. Maximum error is observed in the outer ridge cable, while the minimum is for outer diagonal cables.

Six combinations are formed for manufacturing errors in outer ridge cable, inner cable and inner diagonal cables to find the most disadvantageous effects. Finally study of variation of cable stiffness (3% to 8%) suggests that it is not important as far as cable length manufacture error is considered. The paper thus suggests importance of manufacture error and installation precision.

3.1 GENERAL

Nature has always served as the best teacher to the engineering concept. Cable structures are one such example which is based on the creepers. With many advantages in hand as architectonic expression, translucency, light weight, high strength material efficiency, reduction in construction time, easy transportation, good seismic resistance, cheaper fire protection and low maintenance, tensile structures have gained attraction to be used as roofs for large span column free space.

3.2 CLASSIFICATION OF CABLE ROOFS

Cable roofs are classified into three basic structures [4] as:

- 1. Cable Stayed or supported roofs
- 2. Cable Suspended roofs
- 3. Cable and Air-supported roofs

3.2.1 Cable Stayed Roof

The principle of Cable supported roof is similar to cable-stayed bridges. Cablestayed structures support horizontal planes (bridge decks, roofs, floors) with inclined cables that are attached to, or run over, tower(s). The deck is carried by girders or trusses, which are supported by cables. It is advantageous for long cantilever spans.

3.2.1.1 Cable Configurations

There are two primary cable configurations: **radial patterns** (or fan) **and parallel systems** (or harp). In the radial system, the upper end of all of the stays attach to a single point at the top of the tower. The advantage to this system is that the maximum degree of inclination of the cables is achieved which creates nearly vertical forces exerted at the top of the tower. This minimizes the bending moment in the tower. In the parallel system of stays, each stay is parallel and thus connects to the tower at a different height. This creates large bending moments in the tower because the forces from the cables have larger horizontal components [7].

Hybrid profiles are developed as a combination of the two systems, known as Radiating and Star systems. The axial compressive members that support the cables are designed as tied-column elements. These members take different profiles and are oriented as vertical or sloped masts.

The symmetry of structural system, architectural preferences, as well as the cable loads applied to the masts, govern the final shape and orientation of the supporting axial compressive element.



(a)

(b)



(c)

Fig.3.1 (a) Radial System (b) Parallel System (c) Hybrid System

3.2.1.2 Structural Behaviour of Cable Stayed System

The cables of a cable-stayed structure work solely in tension. The horizontal surface is made sufficiently stiff to transfer and/or resist the lateral and torsional stresses induced by wind, unbalanced live loads, and the normal force created by the upward pull of the stays. The stays are usually attached symmetrically to the column or tower with an equal number of stays on both sides. This is so that the horizontal force component of the inclined cables will cancel each other out and minimize the moment at the top of the tower/mast/column.

Case Studies

The University of Chicago Gerald Ratner Athletics Center, is the first asymmetrical supported cable stayed building with multiple levels of splaying cables in Chicago [7].



Fig.3.2 The University of Chicago Gerald Ratner Athletics Center

3.2.2 Cable Suspended Roof

In cable suspended roofs, the roof deck and other loads are directly carried by the cables. The cables need to be pretension for resisting wind. Suspended roof systems classified on the basis of number of cables used and the Gaussian curvature of imaginary are as follows [1]:

3.2.2.1 According to Number of cables

1. **Single cable:** This system has no stiffness and is extremely lightweight. It is susceptible to wind uplift and wind-induced oscillations unless heavy roof deck, as precast panel is utilized. It is also used for forming hung roofs in radial pattern. The cables are stretched between two peripheral rings and pre-cast panels are hung from the radial cables.

Structural Behaviour of Single cable suspended roof

Prefabricated roof deck panels are hung from parallel cables (supported on masts). The maximum spacing of cables adopted is 3 m. Pre-stressing the cables increases the flexural rigidity of the system.



Fig.3.3 Single Cable Suspended Roof

Case Studies

- a. Nijny-Novgorod Pavilion, constructed in 1896: 30 m x 70 m, form two-span suspended roof
- b. Dulles International Airport in Chantilly, Virginia- 61 m span between two towers.

2. **Cable Networks** - Two or three cables forming cable trusses or nets form the cable network systems. It consists of either cable trusses or nets.

Cable Trusses -Cable Trusses consists of pair of cables, one concave downward and other upward. Such system eliminates uplift and oscillations.



(c)

Fig.3.4 (a) Convex Cable Truss, (b) Convex-Concave Cable Truss (c) Concave Cable Truss

Structural Behaviour of Cable truss

The cable which is concave downward carries Gravity load, while the one having concavity upwards, resists upward load and damping. The vertical spreaders / Diagonals do not increase stiffness and are under compression, used only to support the cables and give the required geometrical shape. Both the cables are initially tensioned. The pre-stress is large enough that any compression induced is such as to reduce the tension in cable and no compression stresses occur [8].

Case Studies

- a. State Fair Arena, Raleigh, North Carolina
- b. The Palasport, Genova, Italy

Cable Nets- Cables are arranged in parallel, radial or mesh pattern to form double, triple, quadruple and hexagonal threaded nets.



Fig.3.5 Cable Net structure - Scandinavium Arena in Gothenburg, Sweden

Structural Behaviour of Cable nets

A set of primary and secondary cables, where secondary cables and supported by the primary cable can be made to form a net covering large areas. The size of primary cables is large so as to support the secondary cables. This configuration enables to form small mesh for supporting light and flexible roofing material, without causing large deflections.

Case Studies

- a. Olympic Stadium at Munich
- b. German Pavilion at Montreal Exposition

3.2.2.2 According Gaussian curvature

Any surface can be formed by translating a curve that lies in one plane (Generator) along a curve in another plane (Geometrix) or by rotating the

generator about a line. Gaussian curvature is the product of the curvatures of the generator and of a line on the surface perpendicular to the generator [1]. Based on this there are three possibilities:

1. Positive Gaussian curvature (Synclastic) – if the curvatures are on the same side.



Fig.3.6 Synclastic and Anticlastic roof

- Negative Gaussian curvature (Anticlastic) if the curvatures are on opposite side. Saddle shaped anticlastic surface; provide an advantage as the intersecting cables are pre-stressed against each other to produce the required structural rigidity without use of temporary gravity loading.
- 3. Zero Gaussian curvature (Grids) no curvatures are made

3.2.3 Cable and Air supported roof

It is a hybrid roof system formed by membrane stabilized by a system of cables or ropes. As the air pressure stretches the membrane which is attached to the cable, the cables are tensioned. Major elements of air supported roof include the membrane and inflation equipments along with the cables and anchorage systems. The unique feature of this form is that it is possible to adjust the rigidity of the system by increasing or decreasing the internal air pressure in accordance with variation of external loads [4].

3.2.3.1 Air Inflated Structure

It is of tubular or cellular construction which is capable of transmitting applied loads to the points of support. Constant pumping is not required if leakage of air is prevented.

3.2.3.2 Air supported Structure

It provides a single wall enclosure and the membrane is attached to the support along the periphery. The membrane is stretched and elevated by slight increase in the internal air pressure so that it can support applied loads.

Case Studies

- a. Pontiac Metropolitan Stadium, Michigan
- b. U.S. Pavilion at International Exposition, Osaka, Japan in 1970

3.3 COMPONENTS OF CABLE SYSTEM

It consists of the cables, anchorages, fittings and supporting stabilizers and masts.

3.3.1 Cables

The strength of cables under static and fatigue loading depends on its composition, clamps, fittings and changes in curvature. Cables can be of mild steel, high strength steel (drawn carbon steel), stainless steel or polyester or aramid fibres.

The basic component of a cable is wire drawn from high-strength steel rods. The wire is then galvanized by the hot-dip or eletrolytic process [2]. Mainly available structural cables are Galvanized and stainless steel cables. Number of wires spun together form a **strand** and **wire ropes** are formed by assembling a number of strands. Wire rope cables are spun from high tensile wire. For structural work the cables are multi-strand, with independent wire rope core and galvanized [2].



Fig.3.7 Cable Components

3.3.1.1 Classification Of Strands

Structural cables are made of a series of small strands twisted or bound together to form a much larger cable.

- a. Spiral Strand: Steel cables here circular rods are twisted together and "glued" using a polymer form a Spiral Strand. Spiral strand is slightly weaker than locked coil strand. Steel spiral strand cables have a Young's modulus, E of 150±10 kN/mm² (or 150±10 GPa) and come in sizes from 3 to 90 mm diameter. Spiral strand suffers from construction stretch, where the strands compact when the cable is loaded. This is normally removed by pre-stretching the cable and cycling the load up and down to 45% of the ultimate tensile load.
- b. Locked Coil Strand: where individual interlocking steel strands form the cable (often with a spiral strand core). Locked coil strand typically has a Young's Modulus of 160±10 kN/mm² and comes in sizes from 20 mm to 160 mm diameter.
- c. Parallel-Wire Strand: This strand consists of a set of wires assembled parallel to each other. The advantage is the greater length of strand for the same material and also greater value of Young's Modulus (193 kN/mm²). Parallel strand cable having lesser bending stiffness are also used instead of Prallel Wire Cable.
- d. **Pre-stressing Strand:** It is obtained by grouping together concrete prestressing wire, and has advantage of being a readily available standard material, along with standard terminal fittings.







Locked Coiled Strand Fig.3.8 Type of cables



Bridge Strand

3.3.1.2 ROPE vs. STRAND

- a. Ropes are more flexible than strands and thus are easier to handle where cables have to pass over saddles. i.e. smaller radii can be used for ropes as compared to strands.
- b. Ropes are easier to grip than strands.
- c. Strands develop bending stresses at clamps and terminal fittings.
- d. Strands have greater modulus of elasticity and so cable roofs of strands deflect less. Thus, it requires greater accuracy of length as extension is lesser for strand.
- e. Strands have better strength/weight ratio.

Material properties	Material E	Ultimate tensile strength	
	(kN/mm²)	(N/mm²)	
Solid steel	210.0	400-2000	
Strand	150.0	2000	
Wire rope	112.0	2000	
Polyester fibers	7.5	910	
Aramid fibers	112.0	2800	

Table 3.1 Material properties for Cables

3.3.2 Roof Cladding Material and Fabrics

Components of roof cladding include roofing, deck, and insulation. Corrugated sheeting from metals—galvanized iron, aluminium alloys, stainless steel—plain or corrugated, and sheets from non-metals such as fiber reinforced glass or plastic, timber planks, concrete slabs, and fabrics of different type, are produced to a high degree of sophistication are available [6].

Lightweight metallic roofs are preferred for pre-tensioned cable structures, while concrete and timber is advantageous for simply suspended systems. Corrugated decking is largely employed and plastic or glass is used for temporary and semi-permanent constructions.

Galvanized-Steel Sheeting is available as 600 to 900 mm wide X 1.8 to 3 m long X 0.46 to 3.5 mm thick. Weight varies from 37 to 277 N/m². Glass is available as corrugated sheets (10 mm thick) with maximum size of 1.25×36

m and glass sheets reinforced with wire mesh (6 mm thick) with maximum size of 1.5 x 3.3 m. Corrugated sheets of glass-fiber-reinforced plastic having 850 mm width and 2.4 to 3.6 m length is used either in translucent or coloured form. Opaque vinyl plastic is useful for curved surfaces. It has high resistance to deterioration and prolonged exposure to sunlight [1].

Insulating layer is used to prevent the transfer of hear. Loose fill, blankets, batts, structural insulation board, slab or block insulators, reflective insulators, and foamed-in-place, sprayed on and corrugated insulation sheets are basic kinds of thermal insulators.

Various forms that are achieved by the cable net structures require use of fabrics that can effectively carry tension. The advances of nonlinear analysis and demanding long free spans and architectural applications have evolved the fiberglass fabrics coated with polytetrafluoroethylene (PTFE).

PTFE fabrics are durable, light weight, low cost and provides excellent day lighting, good fire resistance as well as easy to take dramatic forms. Silicone coated fiberglass and PVC fabrics are also widely used.

Selection of Fabrics is done on the basis of its strength, required lighting, fire resistance and seam spacing. Polyesters coated with Polyvinyl chloride are available in 1.5 to 2 m width, whereas PTFE fabrics are available up to 4 m width [9].

3.3.3 Vertical Supports

It provides vertical clearance within the structure, as cable sags between the supports. Either tower or posts of walls are used as vertical supports. Most tension structure building forms consist of either central support or perimeter support, or a mixture of the two [2].

In all cases it is advantageous but not essential that forces are balanced about the mast. Out-of-balance loads will obviously generate variable horizontal and vertical forces, which require resolution in the assessment of suitable structural sections.



Fig.3.9 (a) Types of Perimeter Supports (b) Types of Central Supports

3.3.4 Anchorages

It resists tension in the cable. Heavy foundations, pile foundation or perimeter compression and interior tension rings are basic forms of anchorages. Tension anchors for ground anchoring includes: Gravity, Rock, Plate anchors and tension piles [5].

The cables are anchored into the boundary structures, which resist the cable forces due to either geometry or self-weight. These structures are usually rings, arches and masts made of concrete or steel. In open systems the cable forces are resisted by tension anchors in the ground. Selection of alternatives that will be most economical, if both are architecturally accepted, depends upon the ground conditions, cost of material, and availability of expertise and skilled labour.

3.3.5 Fittings

A critical feature of any tension member is the means of attachment to the structure or anchorage. Most factory-made stays are supplied cut to length and

with the end fittings already attached. The stay manufacturer therefore designs, fits and warrants the cable end attachments. To be totally effective, the end fittings must

1. Withstand the full breaking force of the stay without significant yielding.

2. It shall be completely reliable under all serving conditions (loading and environmental) for the full life of the stay.

3. Endure dynamic load cycling without risk of fatigue failure of the fitting, and without inducing fatigue failure of the stay (local to the termination) [6].

Fittings are classified according to the type of application- as friction or clamp type, the pressed or swaged types or socketed type.

3.3.5.1 End Fittings

Pressed fittings are used at ends or along length of small-size strands or ropes. Sockets are used for larger size cables. These are usually made from forged or cast steel. These are standard appliances and required strength is provided by manufacturer specifications [10].



Socketed Type





The most reliable, but also the most expensive, of the end fittings is the socketed type. It is manufactured by splaying the end of the cable a prescribed length and cleaning the individual wires. When the wires are cleaned and dried the conical socket of machined or casted steel is positioned on the splayed cable section. Then molten socketing material is poured into the socket, hardens and forms a cone. As tension is applied to the cable the cone is drawn into the socket and wedging forces are developed which grip the wires. As socketing material either of zinc or resin is used. Pure zinc has been used for over a century and it offers a

cathodic protection for the cable, but it is sometimes criticized for impairing the fatigue resistance of the cable in this region. Another, more important, disadvantage with sockets filled with pure zinc is that they are prone to creeping effect under high stresses. Therefore zinc alloy, with improved creep resistance, is often used. Polyester or epoxy resin has better creep resistance. As the resin is casted at low temperature, the fatigue resistance of the cable will not be impaired.

The simplest and cheapest type of termination is a swaged Talurit Eye made round a thimble and connected into a clevis type connection or on to the pin of a shackle.



Fig.3.11 Swaged Talurit Eye

The neatest and most streamlined fitting is a swaged eye or jaw end termination. Hot-poured zinc terminations have to be used for very heavy cables of greater diameter than 50mm. Epoxy resin with steel balls can be used as filler in place of zinc, offering an improvement in fatigue life at the termination.



Fig.3.12 Swaged Eye or Jaw End Terminator and Forged Steel Clamp

Forging is expensive for small numbers and so for smaller structures machined aluminium components may be preferred. Double cables can have a swaged aluminium extrusion prefixed to each pair of cables, which can then be connected with a single bolt. For the attachment of net cables to edge cables, forged steel clamps are generally used. Lower cost alternatives are bent plate or machined aluminium clamps.

When the cables have to run continuously over supports like columns and masts, they have to be supported by saddles. When designing a saddle one has to take the bending stiffness of the cable into account. Two factors have to be checked:

- The tensile stress in the outer wires, and
- The pressure between the cable and the saddle.

If the pressure between the cable and the saddles is too high the fatigue resistance of the cable will be affected. The common rule is that the diameter of the saddle should not be less than 30d, where d is the diameter of the cable



Fig.3.13 Saddle

3.3.5.2 Intermediate Fittings

Clamps are used as intermediate fittings. These are to be designed so as to have uniform pressure on the cable. Clamping is done by mild and high-tension blots.

Intermediate fittings are used to connect cables to other cables. These fittings are usually not standard appliances and their behaviour depends on the frictional force between the cable and the clamp. To prevent sliding of the clamp, the clamping force must be large and thereby high radial stresses are induced. Cables are more prone to fatigue when the pressure between adjacent wires is high and it is, therefore, important to use fittings where the clamping force is evenly distributed over the cable. The resistance of a spiral strand and a locked coil strand to clamping forces, where the latter has the higher resistance. When the cable is tensioned the diameter will decrease and consequently the clamping force. It can therefore be necessary to re-tension the clamp bolts to prevent sliding. To avoid abrasion between the clamp and cable under cable movements, which results in fatigue failure, the ends of the fittings must be radiuses.





Clamp Connection



Single U Bolt Connection

Swaged Clamp Connection





Fig.3.14 Intermediate Fittings

In the search for the best economical solution one key is to use few types of structural details, as the number of fittings in, for example, a cable net can be quite large. A way to achieve this is to use a fitting which can be adjusted for different angles between cables.

On-site connections can be made with bulldog clips but they are ugly and damage the rope. For cable net construction the standard detail is a three-part forged steel clamp, of which the two outer parts are identical.



Fig.3.15 Bull Dog Clip

3.3.6 Stabilizers

These are required to stabilize the structural geometry, as cables change shape when loaded. These are used to form cable trusses of different shapes. The pretensioning of cables is required to give the structure required form and geometry. These are then supported on the strut members (which are in compression), that help in maintaining the original shape of the member, although deformation and movement of whole assembly takes place. Hogging/Upper



Fig.3.16 Struts / Stabilizers of cable truss

3.4. OTHER CONSIDERATIONS

3.4.1 Serviceability

Ropes and bars are not normally used in building construction because they lack Stiffness, but they have been used in some cases as hangers in suspended buildings. Very light, thin tension members are susceptible to excessive elongation under direct load as well as lateral deflection under self-weight and lateral loads [2].

3.4.2 Fatigue

Special problems may arise where the members are subjected to vibration or conditions leading to failure by fatigue, such as can occur in bridge deck hangers. [2]. Structural cables are primarily subjected to axial tensile stresses, and flexure at the junctions or saddle points. Much of the stress arises from dead loads, pretension or other permanent live loading, while the fluctuating component comes from moving load (more specifically in bridges rather than in roofs) and vibrations due to wind. As such cables are subject to repeated dynamic stresses, which can cause fatigue. The issue is of comparatively greater concern in bridges than in roofs.

3.4.3 Corrosion

Wires in the cables should be protected from corrosion. The most effective protection is obtained by hot galvanizing by steeping or immersing the wires in a bath of melted zinc, automatically controlled to avoid overheating. A wire is described as terminally galvanized or galvanized re-drawn depending on whether the operation has taken place after drawing or in between two wire drawings prior to the wire being brought to the required diameter. For reinforcing bars and cables, the first method is generally adopted. A quantity of zinc in the range of 250–330g/m² is deposited, providing a protective coating 25–45 mm thick. [2]

Damage through corrosion is undesirable; adequate protective measures must be adopted. One such process is coating the cables. The coating process, used currently for locked-coil cables, consists of coating the bare wires with an anticorrosion product with a good bond and long service life. The various substances used generally have a high dropping point so as not to run back towards the lower anchorages. They are usually high viscosity resins or oil-based grease, paraffin's or chemical compounds.

For increased corrosion-resistance, the largest diameter wire should be used, and cables can be filled with zinc powder in a slow-setting polyurethane varnish during the spinning process.

For even greater corrosion resistance, filled strand or locked-coil strand can be used to which a shrunk-on polyurethane or polypropylene sleeve can be fitted. Stainless steel, although apparently highly corrosion-resistant, is affected by some aggressive atmospheres if air is excluded; the resulting corrosion can be more severe than with mild steel.

Cable life is reduced by corrosion and fatigue. Galvanized cables under cover suffer very little corrosion; external cables properly protected should have a life of 50 years. Plastic sheathing has the great disadvantage of making inspection of the cable impossible. Fatigue investigations have shown that it is wise to limit the maximum tension in a cable to 40% of its ultimate strength for long-life structures. For structures with a design life of up to ten years a limit of 50% is

acceptable. Flexing of the cables at clamps or end termination will cause rapid fatigue damage.

3.4.4 Drainage and Water-tightness

Drainage from roof surfaces if not properly considered cause excessive accumulation of water, increasing the dead weight on the structural system. Also, water-tight surfaces preventing leakage are necessary to prevent damage of material as well as proper usage of structure. Sealing the joints between decking material with pitch, synthetic plastic is a usual practice.

3.4.5 Protection of anchorages

The details of the connections between the ducts and the anchorages must prevent any inflow or accumulation of water. The actual details depend on the type of anchorages used, on the protective systems for the cables, and on their slope. There are different arrangements intended to ensure water tightness of vital zones.

3.5 APPLICATION OF TENSION STRUCTURES

Land based: Temporary shelters, warehouses, tents, hanging roofs and suspension systems [11].

In addition to many public buildings, such as swimming pools, exhibition halls, stadia etc., suspended roof structures have also been used for many industrial buildings, airport hangars etc [1].

Sea based: Moored Vessels, trawl lines and nets, towed arrays, floating hospitals and support facilities, floating or submerged breakwater or storage tanks and tension leg or catenary's leg platform [11].

4.1 ANALYSIS OF CABLE SYSTEM

The type of analysis to be carried out depends on goal of analysis and errors in the system's response that can be tolerated [12].



Fig.4.1 Methods of Analysis

4.1.1 Single Cable

Exact analysis being tedious and time consuming, requires that initial parameters to be decided using approximate methods. The results of approximate and exact methods can be compared by comparing the manual and software results, only if all the parameters of software are understood correctly. Cables are assumed to be flexible i.e. resistance to bending is small and hence neglected [2].

4.1.1.1 Elementary Cable Mathematics

(a) Circular arc loaded radially

Tension in cable

$$T = WR$$
 ... (4.1)



Fig.4.2 Circular arc loaded radially

Radius of circular arc,

$$R = \frac{L^2}{8f} + \frac{f}{2} \qquad ... (4.2)$$

(b) Catenary loaded vertically



Fig.4.3 Catenary Loaded Vertically

Horizontal force,

$$H = \frac{WL^2}{8f} \qquad \dots (4.3)$$

Vertical force,

$$V = \frac{WL}{2} \qquad \dots (4.4)$$

Maximum tension,

$$T_{max} = \sqrt{H^2 + V^2}$$
 ... (4.5)

(c) Load-extension relationship

Extension or change in sag,

$$\Delta f = \frac{TL}{AE} \qquad \dots (4.6)$$

4.1.1.2 Design Of Single Cable For Static Loads

Statically one layer of suspension cables is adequate to support any commonly prescribed dead and live loads. Indeed, many designs of suspension roofs were and are of this type.

The first step of the analysis is to determine sag of cables, as erected i.e. before application of dead and live loads. The magnitude of the sag has both architectural and structural implications. In suspension bridges, the sag-to span ratio is about 1:8. Architecturally, smaller sag is always desirable. Elimination of flutter requires cable tensions in excess of those induced by superimposed dead and live loads. This makes it possible within the economical range of construction to meet architectural requirements and to provide relatively small sag-to-span ratios. If q is the applied dead and live load per linear foot of horizontal projection, the maximum tension in cable at support is given by

$$T = \frac{ql^2}{8f}\sqrt{1 + 16(f/l)^2} \qquad ... (4.7)$$

Angle of the cable at the anchorage point is given by

$$\tan\beta = \frac{4f}{I} \qquad \dots (4.8)$$

Vertical reaction,

$$V = T \sin\beta = \frac{qI}{2} \qquad \dots (4.9)$$

Horizontal reaction,

$$H = T\cos\beta = \frac{ql^2}{8f}$$
 ... (4.10)

Tension at mid-span = horizontal component (H) at the anchorage point

The sag of the cable after the application of q will be larger than the initial f. However, the difference is very small for practical purposes, and hence in equation (4.7) above, the initial value of sag f for approximate analysis is taken. In accordance with equation (4.7), as the sag increases, the tension decreases

The actual tension along the cable varies from maximum value T_i at the anchorage point to the minimum at mid-span.

For small sag-to-span ratio, the approximate initial length of the cable (before the application of q) is given by

$$L = 1 \Big[1 + 8/3(f/l)^2 \Big] \qquad ... (4.11)$$

Elastic elongation of the cable is

$$\Delta L = \frac{T_i L}{AE} \qquad \dots (4.12)$$

Increase in sag, Δf , due to the cable elongation of ΔL is

$$\Delta f = \frac{\Delta L}{(16/15)(f/I)[5-24(f/I)^2]} \qquad ... (4.13)$$

The tension in the cable depends solely on the magnitude of q and the geometry of the cable, i.e., its shape, span and sag.

For initial sag tension is slightly higher than due to change in sag that occurs on loading. At equilibrium, the tension in the cable lies between initial value of sag and tension (without loading) and the final value of sag and tension (completely loaded).

Thus the decrease of the tension in the cable illustrates an important characteristic of suspension roofs. Destructive internal forces and reactions on abutments are reduced when deflections are increased. This characteristic should be utilized in investigating the actual factor of safety of suspension roofs. In general, at incipient collapse, the forces which tend to destroy a suspension roof are being gradually reduced, and this stabilizes the structure.

4.1.1.3 Dynamic Behavior of Single Cable

A description of the phenomenon of flutter and of self-exciting vibrations in a suspension roof would entail a lengthy mathematical treatise. However, elimination of flutter could be explained through consideration of natural frequencies of the individual cables [8].

Natural frequency of a suspended cable depends on the load attached to it, and the tension in the cable. The natural frequencies of a tight cable for swinging motion out of plane is given by

$$f_n = n(\pi/I)\sqrt{T/(q/g)}$$
 ... (4.14)

The difference between I and L is small.

Since T is proportional to the applied load q, the natural frequencies do not depend on the magnitude of load applied to the cable. This independence of natural frequencies from the load occurs only if: (a) Tension T is computed on the basis of the initial sag f or (b) there is no other tension in the cable except that due to the uniformly distributed load only.

Condition (a) is of minor significance and is accurate enough for practical purposes. Condition (b), on the other hand, is of major significance. If the tension T in the cable is due to a combination of uniformly distributed loads and

a series of concentrated loads, the natural frequencies of the cable would depend on the locations and relative magnitudes of the concentrated loads.

There are an infinite number of natural frequencies of this cable, and an infinite number of modes of vibration corresponding to each integer value of n.



Fig.4.4 Mode Shapes and Frequency of single cable

Resonance in the roof can occur when the externally applied dynamic load has a frequency f_e equal natural frequency of the roof i.e. when the ratio of f_e/f falls within the approximate range a–b. The accurate value of range of values could not be ascertained and the value of W_e is indeterminate.

As pretension adds to the stiffness of the structure, the frequency of cable is given in terms of the tension component and mass. n being the integer for different modes of vibration.

$$f_n = 2n\pi \sqrt{\frac{H}{qL^2}}$$
 ... (4.15)

A.G.Pugsley presented a semi-empirical approach for lower frequencies of free vibration of an inextensible suspended chain of span length L and sag ratio f for $1/10 \le f \le 1/4$.For the first three modes it is given as:

$$f_1 = \frac{\pi}{\sqrt{2}} \sqrt{\frac{g}{L}} \sqrt{\frac{1 - 3f^2}{f}} \qquad ... (4.16)$$

$$f_2 = \pi \sqrt{\frac{g}{L}} \sqrt{\frac{1 - 1.5f^2}{f}}$$
 ... (4.17)

$$f_3 = \sqrt{2\pi} \sqrt{\frac{g}{L}} \sqrt{\frac{1 - 0.7f^2}{f}}$$
 ... (4.18)

In a number of suspension roofs built throughout the world, attempts have been made to combat flutter by addition of a continuous mass—such as cast-in-place concrete or pre-cast concrete plank over the cables or by provision of anchor guy cables tying the suspension roof to the ground. Since criteria used on one suspension roof could not be applied to another suspension roof with different dimensions and characteristics, it is difficult to conclude that such systems will remain out of fluttering effect. Hence, analysis of each system individually is very important for cable suspended structures.

4.1.2 Cable Trusses

As it is relatively easy and economical to achieve full dampening of a suspension roof by constructing the suspension roof with interconnected cables in such a manner that the entire assembly would comprise an internally self-dampening system, cable trusses have gained importance in cable structural systems [8].

The suspension roof is composed of two layers of cables. Cables b in the lower layer is similar to the primary cables of single cable system. Each cable b in the lower layer is connected to the corresponding cable u in the upper layer by struts.

Cables b and u are erected with initial tension (pre- stress) T_b and T_u . The magnitude of these tensions depends on the spread ($f_u + f_b$), number and location of struts and the size and weight of cables b and u. When erected, the only vertical load that the cables carry is their own weight and the weight of the struts.

As dead load (e.g. roof deck) and live load are applied, the assembly of the two cables (with the struts) acts essentially as a beam with a span I. Consequently, the tension on the top cable u is decreased by amount ΔT_u , while the tension in the bottom cable b is increased by amount ΔT_b . Magnitudes of ΔT_u and ΔT_b depend on the magnitude and distribution of the applied dead and live loads and sizes of cables. If, under the most critical combination of dead and live loads, the value of ΔT_u is less than T_u , while $T_b + \Delta T_b$ is less than the design capacity of the lower cable b, both cables b and u will remain under tension without overstress.

Also, the following should be noted:

a. Values of ΔT_u and ΔT_b could vary within a wide range during the service life of the roof.

b. The value of $T_u - \Delta T_u$, i.e., the residual tension in the upper cable at any time should not be too small to cause undesirable sag of the upper cable between struts (the upper cables may be supporting the roof deck).

c. Residual tension $T_u - \Delta T_u$ and $T_u + \Delta T_b$ in the upper and lower cables respectively should be such that the deflection of the assembly is not excessive.

4.1.2.1 Assumption for Analysis

Both the lower and the upper cables have parabolic shapes, i.e., that all applied loads are uniformly distributed on horizontal projection along both the lower and the upper cables. This also means that all the struts together are equivalent to a continuous diaphragm having the necessary properties to satisfy the assumption.

This will not be so in reality, since most of the load will be distributed through the struts at concentrated points; therefore, shapes of the cables will not be parabolic and actual tensions will be different than in parabolic cables subjected to the same total load. This approximation, however, would not detract from the illustration or from the principle of the design approach.

4.1.2.2 Dynamic Analysis Of Double Cable Truss

Under any superimposed load there is decrease ΔT_u in tension of the upper cable and increase ΔT_b in tension of the lower cable.

Natural frequency of the bottom cable is:

$$f_{nb} = n \frac{\pi}{l} \sqrt{\frac{T_b + \Delta T_b}{q_{b/g}}} \qquad \dots (4.19)$$

Natural frequency of the upper cable is:

$$f_{nu} = n \frac{\pi}{l} \sqrt{\frac{T_u - \Delta T_u}{q_{u/g}}}$$
 ... (4.20)

where, q_b and q_u are the weights per linear foot of the bottom and upper cables

With applied load, $T_b + \Delta T_b$ increase while $T_u - \Delta T_u$ decreases. When the complete dead load has been applied, natural frequencies are:

For the lower cable

$$f_{nb} = n \frac{\pi}{l} \sqrt{\frac{T_b + \Delta T_{bd}}{q_{b/g}}}$$
 ... (4.21)

For the upper cable

$$f_{nu} = n \frac{\pi}{l} \sqrt{\frac{T_u + \Delta T_{ud}}{q_{u/g}}}$$
 ... (4.22)

Therefore, if under dead load, T_b is always made to be greater than T_u , it is seen that the natural frequencies of the lower cable, f_{nb} , and the upper cable, f_{nu} , corresponding to a particular value on the integer n will always have different values at any magnitude of the live load.

When a cable vibrates, its actual geometry is a superposition of several of its fundamental modes, as well as of the modes due to forced vibration. It thus follows that under a given externally applied dynamic force, the geometry of vibration of the lower cable will always tend to be different from that of the upper cable i.e. the two cables are in an imaginary situation without the struts.



Fig.4.5 Mode Shapes of Cable Truss

The equilibrium equation at any point of a truss is

$$h_{u}\left(\frac{d^{2}z}{dx^{2}}\right)_{u} + h_{l}\left(\frac{d^{2}z}{dx^{2}}\right)_{l} + \left(H_{u} + H_{l}\right)\frac{d^{2}w}{dx^{2}} = -m\frac{d^{2}w}{dt^{2}} \qquad ... (4.23)$$

If L is the span of the cable truss and $(EA)_l$ and $(EA)_u$ Cable rigidities of lower and upper cables, the ratio of cable rigidity is given as

$$\kappa = \frac{(EA)_1}{(EA)_u} \dots (4.24)$$

$$v = \frac{\Theta_1}{\Theta_1} \qquad \dots (4.25)$$

 Θ_l and Θ_u being curvatures of the lower and upper cable, the frequency of dual cable system is given as

$$\omega^{2} = \frac{\pi^{2}}{mL^{2}} \left[(H_{u} + H_{1}) + \frac{8}{\pi^{2}} \left(\frac{\Theta_{u}^{2}(E\overline{A})_{u}L^{2}}{\pi^{2}} (1 + v^{2} \kappa) \right) \right] \qquad \dots (4.26)$$

4.1.2.3 Important structural characteristics of the assembly

a. If under some load the bottom cable deflects Δf , the upper cable would deflect the same amount. (This holds true even if f_{nb} is unequal to f_{nu} .)

b. When the assembly deflects, the gain in tension of the bottom cable ΔT_{b} is not generally equal to the loss in tension ΔT_{u} of the upper cable.

c. In general, the assembly could not be considered as a simply-supported beam in which the bending moment M_p due to some superimposed load p (applied either to the top or the bottom cable) would be resisted by a couple consisting of changes in cable tensions. Thus, the below relationship is incorrect.

$$\Delta T_{b} (f_{nb} + f_{nu}) = \Delta T_{u} (f_{nb} + f_{nu}) = M_{p} \qquad ... (4.27)$$

d. When the superimposed load p is applied to the upper or the lower cable, the tension in the upper or lower cable would be less than the combined tension due to q_i and p.

e. Under a superimposed uniformly distributed load p, the force q_i exerted by the "equivalent diaphragm" will be reduced by Δq_i . When $f_b = f_u$, the value of Δq_i is given by the following equation:

$$\Delta q = p \frac{A_u}{A_u + A_b} \qquad \dots (4.28)$$

Thus, the lower cable has to be designed for a tension caused by a uniformly distributed load

$$q_w + q_i + (p - \Delta q_i)$$
 ... (4.29)

The upper cable has to be designed for a tension caused by a uniformly distributed load

$$q_i - \Delta q_i$$
 ... (4.30)

f. In practice, there must be enough residual tension left in the upper cable under the most critical superimposed load, to keep its sag between consecutive struts to permissible maximum. If $A_u = A_b$, the lower cable will carry a tension due to q_i plus $\frac{1}{2}p$; if A_u is considerably greater than A_b , the additional tension in the lower cable will be that due to but a fraction of p; if A_b is considerably greater than A_u , the additional tension in the lower cable will be due to almost complete intensity of p.

4.1.2.4 Important observations from natural frequencies

- a. In a single suspended cable, the tension is always proportional to the load.
 Hence, the natural frequency of a suspended cable under its own weight or in conjunction with a superimposed load stays the same.
- b. Natural frequency of either the bottom of the upper cable in double cable truss, where they are separated by struts and where most of the load is exerted and transmitted through the struts, could be calculated only after a lengthy mathematical procedure. The actual natural frequency of each of the cables is not of practical importance. As long as the natural frequencies of each of the cables are computed using the same common condition, and as long as the frequencies thus computed are different from each other, there is sufficient assurance that the two cables would tend to vibrate in different modes (under one specific externally applied dynamic force), and thus flutter would be eliminated.
- c. To satisfy requirement above, it is adequate to compute the natural frequencies using the total span of the cable, the actual existing tension, and only the weight of the cable itself.
- d. In choosing the initial tensions and geometrical configuration the designer should ascertain that at the application of the entire dead load the natural

frequencies of the upper and lower cables are different and that as the load increases, the natural frequencies diverge further. This will assure that at no time and under no live load would the natural frequencies of the two cables coincide.

4.1.3 Cable nets

Analysis of cable nets consists of finding the shape of the structure, which is in equilibrium under applied pre-tension load. The structure is then analyzed for the other superimposed loads.

4.1.3.1 Shape of Cable Nets

Pre-tensioned cable net roofs can be given infinite number of shapes by varying the geometries of their boundaries, by introducing internal columns and by applying varying ratio of pretension forces in the cables.

4.1.3.1.1 Shape Finding

Cable nets and fabric membrane structures adopt unique shapes under tension. Such shapes are not known a priori and thus require a process called as formfinding.

The process should yield optimal structural shapes satisfying the functional requirements and attending to durability as well as strength at minimum cost – so as the follow the lightweight principle [13].

For easy node/intersection point's location, it is preferred to consider the length of the unstressed links as equal. The co-ordinates of the joints of a net can be determined by:

 Building a model and measuring the joint co-ordinates – The shape finding of cable nets with edge cables and those which cannot be described by mathematical model is difficult. Thus, use of model serves some purpose. Models that are sufficiently accurate are expensive. The most efficient way for measuring joint co-ordinates of a model is use of photo-grammetry.

Soap film models are also popular in which a boundary of the wire is created similar to the real boundary and recording the shape of the soap films. This concept needs expertise although is the very suitable from the concept of minimal surfaces. Models of nylon nets used for gardening or fishing can be created if the geometry permits. The geometry being a function of the pretension force it is required to modify the assumed cable tensions and the co-ordinates with respect to it. The adjustments are carried out by an iterative process. This will lead to different link lengths of the net even at unstressed state [10].

A physical model can be used for visualization purposes, but detailed description entails problem of error magnification. Measurement of actual tension field in the surface and cables is tedious and inaccurate. Thus, iterative computations that amount to gradual adjustment of surface geometry till it becomes compatible with state of static equilibrium and prestress level define more accurate form-finding of tension structures. Development of computational models to represent a set of numerical and graphical data describing structures shape, stresses and deformation thus got stimulated [13].

 Defining the roof shape by means of a mathematical function – The simplest configuration is a Hyperbolic Paraboloid. Vertical co-ordinate z is calculated as:

$$Z = aX^2 - bY^2$$
 ... (4.31)
 $Z = k'X'Y'$... (4.32)

The magnitude of a, b and k are functions of the curvature. [9] A program for one such net using mathematical model is developed in C for saddle shape cable net with rigid boundary conditions. (Refer Appendix B)

3. Jacking up the numerical model of a flat net on the computer until satisfactory geometrical shape is achieved.

Experience indicates that a combination of computational and physical modeling is a more promising strategy than exclusive use of either of the methods. Various researchers have used the computational method discussed in next chapter for analysis and shape finding of the structures under pretension load [14-16].

4.1.3.1.2 Patterning

It includes translating and relaxing of a three-dimensional shape of the tensioned surface into a two-dimensional cutting pattern. This enables the manufacturing of the membrane. It is more common for fabric-tensioned structures.

4.1.3.2 Approximate Analysis for Cable Nets

For estimating the deflections and forces in cables under idealized static loading of cable nets Mollmann have contributed to easier methods. The elements of cable net resemble the cable trusses that are not connected at centre. The upper and lower cables of the truss are assumed to be equivalent to the two orthogonal cables, one sagging and other hogging. This in turn is equivalent to the membrane unit.



Fig.4.6 Cable Truss not connected at center = Cable net

It thus represents a plane system and assumptions made are:

- 1. Principle of superposition is admitted.
- 2. Horizontal deflection is neglected as compared to vertical deflections.
- 3. Only vertical uniformly distributed loads are considered.

The tension increment h, due to applied vertical load p is given as:

$$h = -k \delta \frac{d^2 z}{dx^2} w(x) dx$$
 ... (4.33)

z is the initial ordinate of the cable and w is the vertical deflection.

$$\frac{d^2 z}{dx^2} = -\Theta \,\phi(x)$$
 ... (4.34)

$$\frac{d^2 w}{dx^2} = -\overline{\Theta} \,\phi(x) \qquad \qquad \dots (4.35)$$

 $\Theta \, and \, \overline{\Theta} \,$ are constants that depend on curvature and k is the stiffness of cable

$$k = \frac{EA}{L(1 + 8f^2/L^2)} \qquad ... (4.36)$$

Horizontal component of tension increment in upper/hogging cable and lower/sagging cable is given as

$$h_{u} = \frac{\overline{\Theta}}{\Theta_{u}} \beta_{u} \qquad \dots (4.37)$$

$$h_1 = -\alpha v \frac{\overline{\Theta}}{\Theta_u} \beta_u$$
 ... (4.38)

$$\alpha = \frac{k_1}{k_u}$$
 and $\nu = \frac{\Theta_1}{\Theta_u}$... (4.39)

$$\overline{\Theta} = \frac{P_0}{\beta_u (1 + \alpha v^2) + H_u (1 + 1/v)}$$
... (4.40)

$$\Theta_{\rm u} = \frac{8f_{\rm u}}{L^2}$$
 ... (4.41)

Table 4.1 gives values deflection at distance x, maximum deflection and constant β for two load cases of a cable net, which is considered equivalent to the cable truss not connected at centre as indicated in Fig.4.7.



Fig.4.7 Load Cases - Cable Truss not connected at center - Cable net

Table 4.1 Approximate	analysis of cable net	- Value of deflection	and constant β
11	2		

Load Case	Deflection w(x)	Max. Def. at x	Max. Def. ^w max	β _u
Case I –	$\frac{\overline{\Theta}L^2}{16}(1+2x)$			
0≥x≥-L/2	$\frac{10}{-2(2)}$	L/8	$\frac{9\overline{\Theta}L^2}{122}$	$\frac{5}{102}\kappa_{\mu}\Theta^2L^3$
0≤x≤L/2	$\frac{\Theta L^2}{16} \left(1 + 2x - \frac{8x^2}{L^2} \right)$		128	192 u
Case II	$\frac{\overline{\Theta}L^2}{8} \left(1 - \frac{4x^2}{L^2} \right)$	0	$\frac{\overline{\Theta}L^2}{8}$	$\frac{1}{12}\kappa_u\Theta^2L^3$

4.1.3.3 Dynamic Analysis of Cable Nets

The analysis neglects edge displacements and horizontal loads as well as horizontal displacements. Only vertical loads and displacements are considered. For a surface with mass m per unit area and horizontal tensile force components of H_x+h_x and H_y+h_y the equilibrium equation is given as

$$\overline{h}_{x} \frac{d^{2}z}{dx^{2}} + \overline{h}_{y} \frac{d^{2}z}{dy^{2}} + \overline{H}_{x} \frac{d^{2}w}{dx^{2}} + \overline{H}_{y} \frac{d^{2}w}{dy^{2}} = -m \frac{d^{2}w}{dt^{2}} \qquad \dots (4.42)$$

The frequency of vibration is thus given by

$$\omega^{2} = \frac{4\pi^{2}}{m} \left[\frac{\overline{H}_{x}}{L_{x}^{2}} + \frac{\overline{H}_{y}}{L_{y}^{2}} + \frac{8}{\pi^{2}} \left(\frac{\Theta_{x}^{2} E \overline{A}_{x}}{\pi^{2}} + \frac{\Theta_{y}^{2} E \overline{A}_{y}}{\pi^{2}} \right) \right] \qquad \dots (4.43)$$

 \overline{H}_x and \overline{H}_y are the horizontal components of tension in x and y directions per unit width of strip i.e. H_x/a and H_y/a where a is spacing of cables.

4.1.4 Anchor cables

The suspension cables are usually supported on either side by the supporting piers using anchor cables. The anchor cables transfer the tension of he suspended cables / trusses to the anchorage which consists of huge mass of concrete.

The two arrangements for suspension cable are: (a) Guide Pulley support and (b) Saddle mounted on roller. When cable is passed over pulley support, tension in anchor cable and suspended assembly is the same.

Vertical Force on top of pier

$$V = T \sin \beta + T \sin \alpha \qquad \dots (4.44)$$

Horizontal Force on top of pier

$$H = T \cos \beta + T \cos \alpha \qquad \dots (4.45)$$



Fig.4.8 Types of Anchor cable supports - Guide pulley and Roller Supports

When cable is passed over roller support, the horizontal components of tension in the suspended cable / truss and anchor cable will be equal, since the rollers do not have any horizontal reaction.

$$T \cos \beta = T_A \sin \alpha = H \qquad \dots (4.46)$$
Vertical Force on top of pier

$$V = T \sin \beta + T \sin \alpha \qquad \dots (4.47)$$

4.2 LOAD AND LOAD COMBINATIONS

This is another area of prime concern and development related to tension structures. Apart from the pre-stress, roofs have to be designed for other conventional loads, such as live load, impact, seismic loads, but are particularly sensitive to wind. However non-uniformly distributed loads are more dangerous to cable structures than uniform loads. Therefore, it is important to determine the 'true' load distribution on the structure. The unusual shape of these structures, together with their low weight and large scale, make this a difficult task. A further complication is that practically no guidance is available from codes of practice.

4.2.1 Loads

4.2.1.1 Dead Load

It consists of the weight of cladding, insulating material, cables and fittings etc. The value of dead weight may be as low as 240 to 720 N/m² if cloth, plastic, or corrugated metal sheets are used and varies form 720 to 1440 N/m² if concrete or timber is used. [4]

4.2.1.2 Live Load

Specifications for intensity of uniformly distributed live load are taken into account as per the standards. The cable roofs have large spans and curved surfaces thus can be considered as inaccessible to people except for maintenance purposes. Hence, design of cladding for lighter design live loads for cable system and supporting structure is preferable.

4.2.1.3 Wind Load

Due to the low weight of cable roofs with membrane cladding, wind pressure is one of the most important forms of loading. The variability and large number parameters involved in the determination of wind effects on structures make it a very complex problem. Some undesirable effects and partial collapses have been caused by wind on tension structures. The issues related to tension roof aerodynamics are briefly addressed below: Cable supported roofs or air-inflated membranes both are inherently flexible structures and are thus prone to large displacements under wind loading. Whereas smaller spans can be adequately assessed by a quasi-static approach, it is best to analyze larger spans with methods suitable for dynamic analysis. Even if one were to consider quasi-static effects wind comprises a very important load for such roofs for more than one reason [17].

1. Tension roofs are often light weight and as such wind suction can cause large displacements.

2. The fluctuating nature of wind load causes vibrations in the roof structure and its various elements.

3. Since cable roofs have non-conventional geometric shapes, wind pressures for these are commonly not covered in design loading codes or existing literature on the subject.

Importance of wind loading on cable roofs was not fully comprehended till the 1960s. This is evidenced by the fact that designers considered it adequate to design these for uniform pressure, which indeed is greatly erroneous. Not only is it true that most curved roofs experience large suctions due to wind, rather than pressures, the distribution is far from uniform. Furthermore, it is non-uniform loading which is more crucial far determining maximum deformations. The only reasonable way to determine wind loading on such roofs (as indeed for other kinds of roofs) is to make measurements on models in a wind tunnel or on prototypes-the former being more practical [18].

4.2.1.3.1 Flutter – It is the unstable oscillatory motion of a structure due to coupling between aerodynamic force and elastic deformation of the structure. The most common form is the oscillatory motion due to combined bending and torsion. Although oscillatory motions in each degree of freedom may be damped, instability can set in due to energy transfer from one mode of oscillation to another, and the structure is seen to execute sustained or divergent oscillations with a type of motion which is combination of the individual modes of motion. Such energy transfer takes place when the natural frequencies of modes, taken individually are close to each other ($f_{ni} / f_{nj} < 2$).

Flutter can set at wind speeds much less than those required for exciting the individual modes of motion. Members with large value of d/t, where d is the depth of member parallel to wind stream and t is least lateral dimension of the member are prone to low speed flutter. Wind tunnel testing is required to determine critical flutter speeds and the structural response [19].

4.2.1.4 Earthquake Load

Another important form of loading, which has to be considered in certain parts of the world, is earthquake ground motion. Even smaller earthquakes may lead to collapse of stiff structures. Many studies have been concerned with the earthquake response of building structures but, like the wind and snow load studies, very few have included cable roof structures.

4.2.1.5 Blast Load

As most of large span structures are used as public buildings, the resistant to blast is a major factor. Not many researches have taken this aspect into consideration. An approach to this is presented in behaviour of suspended roofs under blast loading.

4.2.2 Load Combinations

Load and load combinations shall be based on the type of structure – Steel or concrete. As this report considers cable roof supported on the steel structure load combinations selected are based on IS 800:1984. (Clause 3.4.2.1) [20].

- 1. Dead Load + Imposed Load
- 2. Dead Load + Imposed Load + Wind or Earthquake Load
- 3. Dead Load + Wind or Earthquake Load

Loads applied to the structure are based on available code of practices for dead and live loads [21-22].

In the previous chapter approximate methods of analysis have been discussed. For full and accurate analysis it is necessary to use a non-linear computer analysis which takes into account the change of curvature caused by stretch. For well-curved cables the hand analysis is accurate enough and gives a useful guide to the forces involved and hence the sizes of cables and fittings.

Tension structures are usually characterized by non-linear geometric hardening which results in a less proportional increase of stress in elements in relation to increased external loads.

The analytical approach for cable roof systems has evolved on the basis of geometrical form, structural response and the progress in the use of matrix method and large high speed computers. From the 1960s, the use of matrix methods and digital electronic computer became common-place and the methods of analyzing these highly indeterminate non-linear systems fell in line—following the 'stiffness' or the 'flexibility' approach [6].

The method of approximate analysis is time-tested and based on continuous system. For more appropriate results the discrete system of analysis is suitable, although it is time consuming and tedious.

5.1 NONLINEAR ANALYSIS

The equation of equilibrium shall not be directly applied to the cable roof, as the system is flexible. The change in geometry and material property along with the displacement of contact surfaces plays a critical part in design of such structures. This effect is the nonlinearity due to geometry, material and contact [23].

5.1.1 Features of Nonlinear Analysis

Various features that make linear analysis simpler are not applicable to nonlinear state. It includes:

- 1. The principle of superposition does not hold
- 2. Analysis can be carried out for one load case at a time
- 3. The history (sequence) of loading influences the response

5.

4. The initial state of system (Pre-stress) may be important

5.1.2 Sources of Nonlinearity

The major source of nonlinearity in any structure includes:

- 1. Geometric arises from nonlinear strain-displacement relations
- 2. Material nonlinear constitutive behavior (Stress-Strain) of material
- 3. Changing initial or boundary conditions

5.1.2.1 Geometric Nonlinearity

It is basically classified as:

- 1. Hardening Type: due to change in geometry and deflection of nonlinear is less than linear deflection
- 2. Softening Type: due to change in geometry and deflection of nonlinear is greater than linear deflection

The stiffness matrix due to geometry nonlinearity is added to the actual stiffness matrix.

5.1.2.2 Material Nonlinearity

Nonlinear stress strain behaviour of material is observed experimentally for any material. The stiffness matrix due to material nonlinearity is subtracted from the original stiffness matrix.

Usually geometric nonlinearity is taken into consideration, while the material nonlinearity is neglected in the research works / papers. This may be due to the fact that usually geometric nonlinearity is dominating.

The method of solving equation for consideration of nonlinearity is iterative. To converge to required results, Modified Newton-Raphson method, Newton-Raphson method or Incremental loading method is adopted. Usually Newton-Raphson method serves as the best numerical solving tool.

5.2 SOLUTION OF NONLINEAR EQUATIONS BY ITERATIVE METHODS

Pre-tensioned cable assemblies belong to the category of geometrically nonlinear structures. The linear method overestimates the displacements when it is

stiffened and underestimates when it is softening. Thus, nonlinear analysis of these structures is a must. The determination of the equilibrium is an iterative process based on tension coefficient approach or minimizing of potential energy.

5.2.1 Methods based on Minimization of Potential Energy

The total potential energy of the structure in displacement space is given as: W = U (stored strain energy of structure) + V (potential energy of loading) [10]. For minimum potential energy

$$\frac{\partial W}{\partial S} = 0 \qquad \dots (5.1)$$

The location of position for minimum W is achieved by moving down the energy surface along the descent vector v by a distance of S_v till the potential energy in that direction reduces to 0.

For each step length S, descent vector v is calculated and a new point for descent vector is calculated. This process is then repeated till the residual force is within acceptable limit.

At x in displacement space, length of link j_n is given as:

$$L_{jn0}^{2} = \sum_{i=1}^{3} (X_{ni} - Y_{ji})^{2} \qquad \dots (5.2)$$

The length of element on application of pre-tensioned force is given by:

$$(L_{jn0} + e_{jn})^2 = \sum_{i=1}^{3} (X_{ni} + X_{ni} - X_{ji} - X_{ji})^2 \qquad \dots (5.3)$$

Thus, since $L_{in0} >> e_{in}$

$$e_{jn} = \frac{1}{L_{jn0}} \sum_{i=1}^{3} ((X_{ni} - X_{ji})(x_{ni} - x_{ji}) + \frac{1}{2}(x_{ni} - x_{ji})^2) \qquad \dots (5.4)$$

The gradient vector is defined as:

$$g_{s} = \sum_{n=1}^{f_{j}} \sum_{r=1}^{12} (k_{sr} x_{r})_{n} - \sum_{n=1}^{p_{j}} (t_{jn} (X_{ni} + x_{ni} - X_{ji} - x_{ji})) - F_{s} \qquad \dots (5.5)$$

Where, the tension coefficient of force in member j_n is given as:

$$t_{jn} = \frac{(T_{jn0} + \frac{EA}{L_{jn0}} e_{jn})}{L_{jn0}} \dots (5.6)$$

Different approaches and methods for minimizing the potential energy/ iterative methods solution are used by researchers. A few of them are discussed below:

- 1. Method of steepest descent
- 2. Method of Conjugate Gradient
- 3. Newton-Raphson Method

5.2.1.1 Method I: Method of steepest Descent

Minimizing the potential energy at k_{th} iteration, it required to move in direction of $v_k = -g_k$ till the minimum value of W is reached.

If initial displacement vector is assumed to be zero and for full convergence of the equation:

$$x_{k+1} = \sum_{i=0}^{k} hS_{iVi}$$
 ... (5.7)

Smaller the value of h, more closely is the real steepest descent direction. At x_{k+1} in displacement space

$$x_{k+1} = x_k + s_k v_k$$
 ... (5.8)

$$e_n = e_{jn} = (a_1 + a_2 S + a_3 S^2) / L_{jn0}$$
 ... (5.9)

where,

$$\mathbf{a}_{1} = \sum_{i=1}^{3} ((\mathbf{X}_{ni} - \mathbf{X}_{ji}) + \frac{1}{2} (\mathbf{x}_{ni} - \mathbf{x}_{ji}))^{\mathsf{T}} (\mathbf{x}_{ni} - \mathbf{x}_{ji}) \qquad \dots (5.10)$$

$$a_{2} = \sum_{i=1}^{3} ((X_{ni} - X_{ji}) + (x_{ni} - x_{ji}))^{T} (v_{ni} - v_{ji}) \qquad \dots (5.11)$$

$$a_{3} = \sum_{i=1}^{3} \frac{1}{2} (v_{ni} - v_{ji}))^{T} (v_{ni} - v_{ji}) \qquad \dots (5.12)$$

The total potential energy is given as

$$W = C_4 S^4 + C_3 S^3 + C_2 S^2 + C_1 S + C_0 \qquad \dots (5.13)$$

where,

$$C_{1} = \sum_{n=1}^{p} (t_{0}a_{3} + EA(a_{2}^{2} + 2a_{1}a_{3})/2L_{0}^{3}) + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} (x_{s}k_{sr}u_{r})_{n} - \sum_{n=1}^{N} F_{n}u_{n} \qquad \dots (5.14)$$

$$C_{2} = \sum_{n=1}^{p} (t_{0}a_{3} + EA(a_{2}^{2} + 2a_{1}a_{3})/2L_{0}^{3}) + \sum_{n=1}^{f} \sum_{s=1}^{12} \sum_{r=1}^{12} (\frac{1}{2}v_{s}k_{sr}v_{r})_{n} \qquad \dots (5.15)$$

$$C_3 = \sum_{n=1}^{p} (EAa_2a_3 / L_0^3)_n$$
 ... (5.16)

$$C_4 = \sum_{n=1}^{p} (EAa_3^2 / 2L_0^3)_n \qquad \dots (5.17)$$

5.2.1.2 Method II: Method of Conjugate Gradient

This method is similar to the gradient vector method except that the conjugate vector v which defines the path of minimization is given by:

$$v_0 = -g_0$$
 ... (5.18)

$$v_{k} = -g_{k} + \beta_{k-1}v_{k-1} \qquad ... (5.19)$$

$$\beta_{k-1} = g_k^T (g_k - g_{k-1}) / v_{k-1}^T (g_k - g_{k-1}) \qquad \dots (5.20)$$

The above two methods provide compact methods of analyzing structures with large number of joints.

5.2.1.3 Method III: Newton Raphson Method

A typical load deflection characteristics nonlinear curve of a cable is as shown in Fig.5.1. The tangent to this curve at any point represents the stiffness K of the structure for the state of equilibrium at that point. The tangent at the origin represents the initial stiffness K of the system, while at any other point K' is representative of the instantaneous stiffness for equilibrium at O'.

For applied load P and deflection U, the point C on the curve is required. The intercept CD between the tangent at origin and the curve represents the value of nonlinear terms contained in residual force R.



Fig.5.1 Netwon Raphson Method

The descent vector δx_k is taken along step $S_k \delta x_k$ for Newton Raphson method. The stiffness matrix of individual element for pin joint pretension link at kth iteration is given as:

$$K_{k} = \frac{EA - T_{k}}{L_{0}} \begin{bmatrix} G_{k}G_{k}^{\mathsf{T}} & -G_{k}G_{k}^{\mathsf{T}} \\ -G_{k}G_{k}^{\mathsf{T}} & G_{k}G_{k}^{\mathsf{T}} \end{bmatrix} + \frac{T_{k}}{L_{0}} \begin{bmatrix} I & -I \\ -I & I \end{bmatrix} \qquad \dots (5.21)$$

where I is unit matrix (3x3) and

$$G_{k} = \frac{1}{L_{0}} \left[X + x_{K,} Y + y_{K,} Z + z_{K,} \right]^{T}$$
 ... (5.22)

5.2.1.4 Iterative Steps for method I, II and III

- 1. Calculate Tension coefficient
- 2. Initial displacement x_j - $x_i = y_j y_i = z_j z_i = 0$
- 3. Initial length of member L_m
- 4. Gradient vector g_s
- 5. Euclidean Norm R
- 6. For Newton Raphson method or Instantaneous stiffness matrix, evaluate K_k and Change in displacement δx_k = vector gradient
- 7. Step length polynomials coefficients are evaluated and replaced in equation to solve for S
- 8. Update tension coefficients $t_{\mbox{\tiny jn}}$ and displacement vector
- 9. Repeat all the steps till value of Euclidean $R_k = 0.001\%$ or 0.1% of R_0 .

5.2.2 Methods based on Tension-Coefficient Method

- 1. Instantaneous Stiffness method
- 2. The force-density Method
- 3. The dynamic relaxation method

All of these methods employ the above discussed iterative methods for solution to the final tension force in the cables.

5.2.2.1 Transient Stiffness Method

It is based on certain assumptions as [4] below:

- 1. The cable is treated as completely flexible
- 2. The cable is incapable of taking any compressive forces
- 3. The cladding does not contribute to the stiffness of the roof structure.
- 4. The intersection of two or more cable is treated as a joint for cable nets.
- 5. The cable elements lie along straight lines between joints

The assembled equation of equilibrium is given by:

K. U =
$$-P + R$$
 ... (5.23)

This method is evolved from small displacement theory of conventional structures. The linear dependence of deflection on forces is given as

$$[K] \{\delta\} = \{P\} \qquad ... (5.24)$$

For structures that exhibit large displacements the above equation does not directly apply. The problem of geometric non-linearity arises and thus it is required to proceed stepwise.

$$\{\delta\} = [K]^{-1} \{P\}$$
 ... (5.25)

Displacement being large changes the initial configuration of the structure. The stiffness calculated is at first based on initial configuration and so need to be modified due to change in geometry. A new stiffness matrix is thus evaluated based on new geometrical configuration and the steps repeated till the residual force is reduced to almost zero.

If at iterative step k, $\{X\}_k$ is the current geometry of structure and $\{K\}_k$ stiffness matrix, then displacement vector is given as:

$$\{\delta\}_{k+1} = [K]_k^{-1} \{P\}$$
 ... (5.26)

The new geometry is then calculated as:

$$\{X\}_{k+1} = \{X\}_{k} + \{\delta\}_{k+1} \qquad \dots (5.27)$$

For large displacement the new stiffness matrix is $[K]_{k+1}$

$$[K]_{k+1}{\delta}_{k+1} = {P'}_k \qquad \dots (5.28)$$

 $\{P'\}_k \, is the out of balance force$

Thus, the residual force is

$$\{R\}_{k+1} = [\{P\} - \{P'\}_{k+1}]$$
 ... (5.29)

The residual force vector yields a correction or increment for displacement vector $\{\delta\}$, say $\{\Delta\delta\}$. The increment in displacement vector, which updates the geometry at each iterative step is given as

$$\{\Delta\delta\}_{k+1} = [K]_{k+1}^{-1} \{R\}_{k+1} \qquad \dots (5.30)$$

As tension structures are subjected to pre-stressing forces, the geometric stiffness matrix plays a very important role. Newton Raphson Method is the widely used for modifying the stiffness matrix due to pre-tension to account for the geometric nonlinearity. Cable is discretized as line element or as parabolic element. The line element is widely adopted for its easier understanding.

... (5.31)

For planar element A-B within global system of co-ordinate (X-Y) and (P-Q) as local co-ordinates, as shown in figure below, the length of the element is [24]:



Fig.5.2 2D planar element

The direction cosines for rotation transformation from local to global system is -

$$l_p = \frac{(X_B - X_A)}{L_0} = m_q$$
 and $m_p = \frac{(Y_B - Y_A)}{L_0} = -l_q$... (5.32)

In global system of coordinate- the load vector [L] is connected to the Displacement vector [X] by [L] = [K][X], where

$$[L] = [F_{xa}, F_{ya}, F_{xb}, F_{yb}] \qquad \dots (5.33)$$

$$[X] = [X_a, Y_a, X_b, Y_b]$$
 ... (5.34)

In local system the load vector is given by [R] = [R, S] and displacement vector [U] = [u, v] are obtained by rotation transformation of global systems The element extension is given as

$$e = \sqrt{(L_0 + u)^2 + v^2} - L_0$$
 ... (5.35)

Element force in terms of initial pre-tension and displacement is given as

$$P = P_0 + \frac{(EA)}{L_0}e \qquad ... (5.36)$$

Thus, the global forces as obtained due to initial pre-tension and displacement is

$$\begin{bmatrix} F_{xa} \\ F_{ya} \\ F_{xb} \\ F_{yb} \end{bmatrix} = \begin{bmatrix} -l_p & -l_q \\ -m_p & -m_q \\ l_p & l_q \\ m_p & m_q \end{bmatrix} x \begin{bmatrix} \frac{L_0 + u}{L_0 + e} \\ \frac{v}{L_0 + e} \end{bmatrix} x \begin{bmatrix} P_0 + EAx \left[\sqrt{(L_0 + u)^2 + v^2} - L_0 \right] x \begin{bmatrix} -l_p & -m_p & l_p & m_p \\ -l_q & -m_q & l_q & m_q \end{bmatrix} x \begin{bmatrix} x_a \\ y_a \\ x_b \\ y_b \end{bmatrix}$$
... (5.37)

The geometric stiffness matrix of the 2D element that adds up to the elastic stiffness matrix at each stage of calculations is given as:

$$\begin{bmatrix} K_{g} \end{bmatrix} = \begin{bmatrix} -l_{p} & -l_{q} \\ -m_{p} & -m_{q} \\ l_{p} & q \\ m_{p} & m_{q} \end{bmatrix} x \begin{bmatrix} \frac{L_{0} + u}{L_{0} + e} \\ \frac{v}{L_{0} + e} \end{bmatrix} x \begin{bmatrix} \frac{EA}{L_{0}} x \begin{bmatrix} \frac{L_{0} + u}{L_{0} + e} & \frac{v}{L_{0} + e} \end{bmatrix} + \begin{bmatrix} \frac{Pv^{2}}{(L_{0} + e)^{3}} & \frac{-Pv(L_{0} + u)}{(L_{0} + e)^{3}} \\ \frac{-Pv(L_{0} + u)}{(L_{0} + e)^{3}} & \frac{P(L_{0} + u)^{2}}{(L_{0} + e)^{3}} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -m_{p} & l_{p} & m_{q} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{p} & l_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{p} & l_{p} \\ -l_{q} & -m_{q} & l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{q} & m_{q} \\ -l_{q} & -m_{q} & -l_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{q} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{q} & m_{q} \\ -l_{p} & -l_{p} & -l_{p} & m_{q} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{q} & m_{q} \\ -l_{p} & -l_{p} & -l_{p} & -l_{p} & -l_{p} & -l_{p} \end{bmatrix} x \begin{bmatrix} -l_{p} & -l_{p} & m_{q} & m_{q} \\ -l_{p} & -l_{p}$$

For a 3D element A-B within global system of co-ordinate (X-Y-Z) and (P-Q-R) as local co-ordinates, as shown in figure below, the length of the element is:



Fig.5.3 3D planar elements

The direction cosines for rotation transformation from local to global system is -

$$l_{p} = \frac{(X_{B} - X_{A})}{L_{0}} ; m_{p} = \frac{(Y_{B} - Y_{A})}{L_{0}} ; n_{p} = \frac{(Z_{B} - Z_{A})}{L_{0}} \qquad \dots (5.40)$$

$$l_{q} = \frac{(X_{C} - X_{A})}{L_{0}}; m_{q} = \frac{(Y_{C} - Y_{A})}{L_{0}}; n_{q} = \frac{(Z_{C} - Z_{A})}{L_{0}} \dots (5.41)$$

$$l_{r} = (m_{p} n_{q} - n_{p} m_{q}); m_{r} = (n_{p} l_{q} - l_{p} n_{q}); n_{r} = (l_{p} m_{q} - m_{p} l_{q}) \qquad ... (5.42)$$

In global system of coordinate- the load vector [L] is connected to the displacement vector [X] by [L] = [K] [X], where

$$[L] = [F_{xa}, F_{ya}, F_{za}, F_{xb}, F_{yb}, F_{zb}] \qquad \dots (5.43)$$

$$[X] = [X_a, Y_a, Z_a, X_b, Y_b, Z_b]$$
 ... (5.44)

In local system the load vector is given by [R] = [R, S, T] and displacement vector [U] = [u, v, w] are obtained by rotation transformation of global systems. The element extension is given as

$$e = \sqrt{(L_0 + u)^2 + v^2 + w^2} - L_0 \qquad \dots (5.45)$$

Element force in terms of initial pre-tension and displacement is given as

$$P = P_0 + \frac{(EA)}{L_0}e$$
 ... (5.46)

Thus, the global forces as obtained due to initial pre-tension and displacement is

$$\begin{bmatrix} F_{xa} \\ F_{ya} \\ F_{ya} \\ F_{za} \\ F_{zb} \end{bmatrix} = \begin{bmatrix} -l_{p} & -l_{q} & -l_{r} \\ -m_{p} & -m_{q} & -m_{r} \\ -n_{p} & -n_{q} & -m_{r} \\ l_{p} & l_{q} & l_{r} \\ F_{zb} \end{bmatrix} \begin{bmatrix} \frac{L_{0} + u}{L_{0} + e} \\ \frac{v}{L_{0} + e} \\ \frac{w}{L_{0} + e} \end{bmatrix} x \begin{bmatrix} \frac{l_{0} + u}{L_{0} + e} \\ \frac{v}{L_{0} + e} \\ \frac{w}{L_{0} + e} \end{bmatrix} x \begin{bmatrix} r_{p} - m_{p} & -n_{p} & l_{p} & m_{p} & n_{p} \\ -l_{q} & -m_{q} & -n_{q} & l_{q} & m_{q} & n_{q} \\ -l_{r} & -m_{r} & -n_{r} & l_{r} & m_{r} & n_{r} \end{bmatrix} x \begin{bmatrix} x_{a} \\ y_{a} \\ z_{a} \\ x_{b} \\ y_{b} \\ z_{b} \end{bmatrix}$$
 ... (5.47)

The geometric stiffness matrix of the 2D element that adds up to the elastic stiffness matrix at each stage of calculations is given as:

$$\begin{bmatrix} K_{g} \end{bmatrix} = \begin{bmatrix} -l_{p} & -l_{q} & -l_{r} \\ -m_{p} & -m_{q} & -m_{r} \\ -n_{p} & -n_{q} & -n_{r} \\ l_{p} & l_{q} & l_{r} \\ n_{p} & n_{q} & m_{r} \\ n_{p} & n_{q} & n_{r} \end{bmatrix} x \begin{bmatrix} \frac{L_{0} + u}{L_{0} + e} \\ \frac{v}{L_{0} + e} \end{bmatrix} x \begin{bmatrix} \frac{P(v^{2} + w^{2})}{(L_{0} + e)^{3}} & \frac{-Pv(L_{0} + u)}{(L_{0} + e)^{3}} \\ \frac{-Pv(L_{0} + u)^{2} + w^{2}}{(L_{0} + e)^{3}} & \frac{-Pvw}{(L_{0} + e)^{3}} \end{bmatrix} \begin{bmatrix} \frac{P(v^{2} + w^{2})}{(L_{0} + e)^{3}} & \frac{-Pvw}{(L_{0} + e)^{3}} \\ \frac{-Pv(L_{0} + u)}{(L_{0} + e)^{3}} & \frac{-Pvw}{(L_{0} + e)^{3}} \\ \frac{-Pw(L_{0} + u)}{(L_{0} + e)^{3}} & \frac{-Pvw}{(L_{0} + e)^{3}} \\ \frac{-Pw(L_{0} + u)}{(L_{0} + e)^{3}} & \frac{-P((L_{0} + u)^{2} + w^{2})}{(L_{0} + e)^{3}} \\ \frac{-Pw(L_{0} + u)}{(L_{0} + e)^{3}} & \frac{-P((L_{0} + u)^{2} + w^{2})}{(L_{0} + e)^{3}} \end{bmatrix} \end{bmatrix} x \begin{bmatrix} \frac{l_{1}}{p} & -m_{p} & -n_{p} & l_{p} & m_{p} & n_{p} \\ -l_{q} & -m_{q} & -n_{q} & l_{q} & m_{q} & n_{q} \\ -l_{r} & -m_{r} & -n_{r} & l_{r} & m_{r} & n_{r} \end{bmatrix} \\ x \begin{bmatrix} -l_{p} & -m_{p} & -l_{p} & l_{p} & m_{p} & n_{p} \\ -l_{q} & -m_{q} & -l_{q} & l_{q} & m_{q} & n_{q} \\ -l_{r} & -m_{r} & -n_{r} & l_{r} & m_{r} & n_{r} \end{bmatrix} \\ \dots (5.48)$$

5.2.2.2 Force Density Method

This method was proposed by Lintwitz which relies on the mathematical assumption that the ratio of tension force to length of each cable is a constant. This enables the transformation of number of nonlinear equations to a set of linear equations. However the large size matrices require iterative methods of solutions [13].

In the force density method it is assumed that the cables are straight and pinjointed to each other or to the supporting structure. First, a graph of a network is drawn and all nodes are numbered from 1 to n_s , all elements from 1 to m. The n_f nodes which are to be fixed points are taken at the end of the sequence. All the other n nodes are free. Thus, the total number of nodes is $n_s = n + n_f$.

Then the connectivity matrix C_s is constructed with the aid of the graph. Each element m has the node numbers i and k (from i to k). The connectivity matrix C_s for the structure is defined by (i = 1, 2... n_s):

$$C_{s}(m, j) = \begin{cases} +1 \text{ for } j = i, \\ -1 \text{ for } j = k, \\ 0 \text{ in the other cases.} \end{cases} \qquad \dots (5.49)$$

This connectivity matrix can be divided into two matrices which contains free nodes C and fixed nodes $C_{\rm f}$

$$C_s = [C C_f]$$
 ... (5.50)

The projected length of members L_{mx} , L_{my} and L_{mz} can be expressed using the connectivity matrix [C] as

$$L_{mx} = x_i^0 - x_k^0 = \left[1 - 1\right] \begin{cases} x_i^0 \\ x_k^0 \end{cases} = \left[C\right] \{X\} \qquad ... (5.51)$$

$$L_{my} = y_i^0 - y_k^0 = \begin{bmatrix} 1 - 1 \end{bmatrix} \begin{cases} y_i^0 \\ y_k^0 \end{cases} = \begin{bmatrix} C \end{bmatrix} \{Y\} \qquad ... (5.52)$$

$$L_{mz} = z_i^0 - z_k^0 = \begin{bmatrix} 1 - 1 \end{bmatrix} \begin{cases} z_i^0 \\ z_k^0 \end{cases} = \begin{bmatrix} C \end{bmatrix} \{Z\} \qquad ... (5.53)$$

The component of internal forces Q_{mx} , Q_{my} , and Q_{mz} in cable members are expressed as product of force density Q and projected member length.

$$[Q_{mx}] = [Q] \{L_{mx}\} \qquad \dots (5.54)$$

$$\left[Q_{my} \right] = \left[Q \right] \left\{ L_{my} \right\} \qquad \dots (5.55)$$

$$[Q_{mz}] = [Q] \{L_{mz}\}$$
 ... (5.56)

$$\begin{bmatrix} Q \end{bmatrix} = \begin{bmatrix} q_1 & & & \\ & q_2 & & \\ & & q_3 & \\ & & & q_4 \end{bmatrix} \qquad \dots (5.57)$$

The equilibrium force at each node given as sum of component of internal force in members must balance the external load vector at each node. Thus,

$$\sum \left[\mathbf{Q}_{\mathsf{mx}} \right] = \mathbf{P}_{\mathsf{x}^0} \qquad \dots \tag{5.58}$$

$$\sum \left[Q_{my} \right] = P_{y^0} \qquad \dots (5.59)$$

$$\sum \left[\mathbf{Q}_{\mathrm{mz}} \right] = \mathbf{P}_{\mathbf{z}^0} \qquad \qquad \dots \tag{5.60}$$

Multiplying the internal force components \boldsymbol{Q}_{mx} with $[\boldsymbol{C}]^{^{\mathsf{T}}}$ gives:

$$[C]^{\mathsf{T}}[Q][C]{X} = \mathsf{P}_{x^0} \qquad \dots (5.61)$$

$$[C]^{\mathsf{T}}[Q][C]{Y} = \mathsf{P}_{y^0} \qquad ... (5.62)$$

$$[C]^{T}[Q][C]{z} = P_{z^{0}} \qquad ... (5.63)$$

Denoting the vectors containing the coordinates of the n free nodes x, y, z, and n_f fixed nodes x_f , y_f , z_f , the coordinate differences for each element can be written as:

$$u = C_s x_s = C_x + C_f x_f$$
 ... (5.64)

$$v = C_s y_s = C_y + C_f y_f$$
 ... (5.65)

$$w = C_s z_s = C_z + C_f z_f$$
 ... (5.66)

The equilibrium equations for the free nodes for the x, y and z-directions are written as:

$$C^{T}UL^{-1}t = f_{x}$$
 ... (5.67)

$$C^{T}V L^{-1}t = f_{y}$$
 ... (5.68)

$$C^{T}W L^{-1}t = f_{z}$$
 ... (5.69)

By using the force-to-length ratios for the elements, i.e. the force densities $q = L^{-1}t$, as description parameters, Equations (5.67)–(5.69) are written as:

$$C^{T}Uq = f_{x}$$
 ... (5.70)

$$C^{T}Vq = f_{y}$$
 ... (5.71)

$$C^{T}Wq = f_{z}$$
 ... (5.72)

Using Equations (5.64)–(5.66) and the following identities

$$Wq = Qw$$
 ... (5.75)

Equations (5.70)–(5.72) are written as:

$$C^{T}QCx + C^{T}QC_{f}x_{f} = f_{x}$$
 ... (5.76)

$$C^{T}QCy + C^{T}QC_{f}y_{f} = f_{y}$$
 ... (5.77)

$$C^{T}QCz + C^{T}QC_{f}z_{f} = f_{z} \qquad \dots (5.78)$$

If $D = C^{T}QC$ and $D_{f} = C^{T}QC_{f}$, equations (5.76)–(5.78) can be written as:

$$D_x = f_x - D_f x_f$$
 ... (5.79)

$$D_y = f_y - D_f y_f$$
 ... (5.80)

$$D_z = f_z - D_f z_f$$
 ... (5.81)

Equations (5.79)–(5.81) are solved using elementary algebra:

$$x = D^{-1} (f_x - D_f x_f) \qquad ... (5.82)$$

$$y = D^{-1} (f_y - D_f y_f)$$
 ... (5.83)

$$z = D^{-1} (f_z - D_f z_f)$$
 ... (5.84)

5.2.2.3 Dynamic Relaxation Method

This method was perceived as a numerical finite difference technique. It relies on a discretized continuum in which mass of the structure is assumed to be concentrated at given nodes (points) on the surface. This system oscillates about the equilibrium position under influence of out-of-balance forces. The system comes to rest under influence of damping. This method makes use of viscous damping or kinetic damping.

The relaxation of out of balance force at node of the structures is carried out at each successive step. The method employs lumped stiffness at each node of the structure, which scales the size of the iterative step. The main advantage being small number of arithmetic operations needed at any one time [13].

5.2.2.4 Discrete Analysis- Finite element analysis

If the numerical analysis of building structures is concerned, the finite element method is the dominating tool. In this method, the structural characteristics and external loads are described by matrices and vectors. The sought parameters, e.g. displacements and internal forces, are found by matrix operations [4].

The first step in the analysis process is the definition of the geometry of the structure. However, this is not the case for tensile structures. Due to the negligible flexural stiffness of cables and membranes, the initial configuration of these structures must be stressed, even if the self-weight is disregarded. Thus, before the analysis of the behavior of the structure to external loads can be performed, the initial equilibrium configuration must be found.

The shape of a tensile structure, which very much depends on the internal forces, also governs the load-bearing capacity of the structure. Therefore, the process of determining the initial equilibrium configuration calls for the designer's ability to find an optimum compromise between shape, load capacity and constructional requirements.

Several numerical methods, applicable to the initial equilibrium problem, can be found in literature. Most of these methods are not included in general finite element programs

After the initial reference configuration has been determined, the structural members have to be described by stiffness matrices and force vectors. Special elements for cables or chains are often not available in commercial finite element programs.

The single cable is instead modeled by one or several other elements depending on the sag-to-span ratio. Nevertheless, this approach has problems such as numerical instability of the solution. Since cable structures in general are very flexible, a geometrically nonlinear solution method has to be used. The most common is the Newton-Raphson method, with iterations carried out for each load increment and modifying the stiffness matrix at the end of each step.

The final step in the analysis process is to define the external loads on the structure. For civil engineering structures there are a number of loads that must be considered: self-weight vehicles, wind, rain, snow, ice, earthquakes, temperature, etc. The magnitude and distribution of these loads is a constant source of research.

Tensile structures often have irregular shapes and low self-weights which may give rise to unforeseen effects such as very high snow loads and flutter instability due to wind. To ensure the safety of the structure, experimental tests have to be undertaken together with statistical analyses to find the magnitudes of the snow and wind loads.

Even with the right tools, the design of tensile structures will not be straightforward. Each new roof type has its own features. Various approaches for flexible and rigid supported roofs using finite element tool are made [25-26].

5.3 NONLINEAR ANALYSIS USING SAP-2000 - MODELING, ANALYSIS AND DESIGN

As already stated, nonlinear analysis being very important aspect for cable roofs – it is carried out to compare the results with the linear analysis using approximate method. Also, as discussed use of computer software for full and accurate nonlinear analysis is an important tool. Emphasis is laid on the important changes required in cable analysis and only those are discussed below.

5.3.1 Modeling Aspects

As modulus of elasticity for the cable is different from that of steel, defining material as cable and its properties is required. This option is available under Define – Material.

Under the Main Menu bar Define option consists of Cable Section. The following window appears for defining the cable properties.

	Cable Section Data
Cable Sections	Cable Section Name CAB1
Click to: CAB1 CAB1 Click to: Add New Section	Cable Material Material Property STEEL
Add Copy of Section Modify/Show Section Delete Section	Specify Cable Diameter O.0287 Specify Cable Area G.452E-04
OK Cancel	Moment of Inertia 3.312E-08 Shear Area 5.806E-04
	Modify/Show Cable Property Modifiers
	Units Display Color
	Cancel

Fig.5.4 Cable section assigning data box

Either the cable diameter can be specified or the area of the cable.

For drawing the cable element it is the same as for any frame element. Draw – Frame/Cable/Tendon option is used for this. Under this the type of line object to draw is selected as Cable and the respective section as defined above is selected.

Properties of Object	×
Line Object Type	Cable
Section	LABI
XY Plane Offset Normal	0.
Drawing Control Type	None <space bar=""></space>
1	

Fig.5.5 Drawing Cable

The selection of two points on the defined axis gives the parameters to be selected. Any one of the following options can be selected for defining the cable

1. Cable – Undeformed length

- 2. Cable Minimum Tension at I-End
- 3. Cable-Minimum Tension at J-End
- 4. Cable Tension at I-End
- 5. Cable Tension at J-End
- 6. Cable Horizontal Tension Component
- 7. Cable Maximum Vertical Sag or
- 8. Cable Low point vertical sag

P <mark>arameters For Curve</mark> File Edit	ed Frames And Cables For Line Object 1	J		
Line Object Parameters Line Object Type Curve/Cable Type Section Property Start End Line Object Meshing © Keep as Single Obj © Break into Multiple	Cable Cable Cable - Undeformed Length Cable - Minimum Tension At J-End Cable - Minimum Tension At J-End Cable - Horizontal Tension Component Cable - Horizontal Tension Component Cable - Low-Point Vertical Sag Cable - Undeformed Length ect Equal Length Objects Disaste with Envel Preisested Length on Chard	Cable Parameters Number of Linear Segments Added Weight Per Unit Length Projected Uniform Gravity Load Tension At I-End Tension At J-End Horizontal Tension Component Maximum Vertical Sag Low-Point Vertical Sag Length Coordinate System	10 Refres 0. T/C Limit 0. Undeform Deformed Undeform 0. 0. Units KN, m, C	ned
Computed Point Coordin	nates for Linear Segments (Undeformed Cable Geo eometry for Cable Object Use Deforme	metry) d Geometry for Cable Object	Planar View OK Cance	

Fig.5.6 Parameters for curved cables as line objects

5.3.2 Analysis

With increase in use of finite element analysis due to discrete analysis results being more accurate – it is possible to either keep the cable element as a single component or break it into multiple objects.

As discussed earlier, due to large displacement of flexible cables the behaviour is mainly non-linear and geometric non-linearity is governing factor.

Under the Define Menu – Analysis case can be selected as linear or non-linear. To define the pre-tension in the cable, temperature loads are used. The reference

temperature for the element is specified as zero and a negative temperature defined as it is required to develop tension in the element.

Analysis Case Data - Nonlinear Static	
Analysis Case Name TEMP Set Def Name	Analysis Case Type Static
Initial Conditions	Analysis Type
Zero Initial Conditions - Start from Unstressed State	C Linear
C Continue from State at End of Nonlinear Case	Nonlinear
Important Note: Loads from this previous case are included in the current case	Nonlinear Staged Construction
Modal Analysis Case All Modal Loads Applied Use Modes from Case MODAL	
Loads Applied	
Load Type Load Name Scale Factor	
Load TEMP 1.	
Load TEMP 1. Add	
Modifu	
Modily	
Delete	
Other Parameters	
Load Application Full Load Modify/Show	
Results Saved Final State Only Modify/Show	Cancel
Nonlinear Parameters User Defined Modify/Show	

Fig.5.7 Nonlinear analysis case data – Pretension

This is done using the Assign menu – Frame/Cable/Tendon load – Temperature Load option.

For the temperature load the zero Initial condition – Unstressed state is selected, along with nonlinear. For the dead load, the state of analysis is continued from the temperature load defined. It is required to modify the nonlinear parameters depending upon the type of structure. For cable structures – P-Delta with large displacement is preferred.

Analysis Case Name DL Set Def Name	Analysis Case Type Static
Initial Conditions C Zero Initial Conditions - Start from Unstressed State Continue from State at End of Nonlinear Case Important Note: Loads from this previous case are included in the current case Modal Analysis Case All Modal Loads Applied Use Modes from Case	Analysis Type C Linear O Nonlinear O Nonlinear Staged Construction
Loads Applied Load Type Load Name Scale Factor Load DL I. Add Modify Delete	
Other Parameters Load Application Results Saved Multiple States Nonlinear Parameters	Cancel

Fig.5.8 Nonlinear analysis case data - other load cases

Material Nonlinearity Parameters	Solution Control		
Frame Element Tension/Compression Only	Maximum Total Steps per Stage	200	
🔽 Frame Element Hinge	Maximum Null (Zero) Steps per Stage	50	
🔽 Cable Element Tension Only	Maximum Iterations per Step	10	
🔽 Link Gap/Hook/Spring Nonlinear Properties	Iteration Convergence Tolerance (Relative)	1.000E-04	
🔽 Link Other Nonlinear Properties	Event Lumping Tolerance (Relative)	0.01	
Time Dependent Material Properties			
Geometric Nonlinearity Parameters	Hinge Unloading Method		
C None	Unload Entire Structure		
C P-Delta	C Apply Local Redistribution		
P-Delta plus Large Displacements	Restart Using Secant Stiffness		
Reset To Defaults OK Cancel			

Fig.5.9 Nonlinear parameters

The load application method selected is either the force control or displacement control. For cable structures – as the loading is known the force control method is selected. The results can be obtained at various steps by modify the data of results saved. Finally, under Analyze – Run analysis is performed to obtain the required results of tension in cable and reactions. The axial force diagram gives the maximum/minimum tension in the cable element in stepped and envelope format.

5.3.3 Load and Load Combination

It is not required to form any load combination, as each load case is specified to follow the other.

5.3.4 Design

Design parameter is not considered, as standard breaking load capacity values of cable as manufacturer specification are used. Analysis results can be used for design purpose. Manufacturer errors also affect the initial pre-stress values [27].

SAP hence provides nonlinear static analysis results for assigned static and dynamic values of load. It is not possible to perform dynamic wind analysis directly in the software to obtain the mode shapes, frequency and time period. Although Modal calculations give the different mode shapes, frequency and time period for any structure.

6.1 SINGLE CABLE

To find the optimum results for span and sag, it is very critical to understand the behaviour of cables under wind static and dynamic loading condition, as well as seismic loading condition. The structural configuration as shown below is selected for the purpose of parametric study.

6.1.1 Analysis and Design Based On Approximate Method

Length – 51 m
Width - 50 m
Height – 13 m
Sag – 3 m
Location - Ahmedabad
C/C Spacing between columns - 3m

LOAD INTENSITY

Dead Load

Self Weight of Cable (Considering 32 mm dia.	Open Spiral	0.05	kN/m
Strand)		0.05	KIN/III
Roofing Material (PTFE coated Fabric)		13.5	N/m ²
Live Load		0.75	kN/ m ²
Wind Load (Refer Appendix A)			
Static wind Load	Pressure	Suction	
	0.82	-1.93	kN/m
Dynamic wind Load	Pressure	Suction	
	0.87	-2.05	kN/m
Pre-Tension provided	220.00		kN
Base shear due to wind -			
X-Direction			
Shear due to wind pressure on walls		1012.33	kN
Shear due to wind pressure on roof		327.66	kN
Total Base Shear V _{bx}		1340	kN
Base shear due to Seismic Forces – 149 kN (Re	efer Appendix	A)	

Governing is Wind loading condition.

6.



LOAD CASE	T _{max} (kN) - At Support	T (kN) - At center	Vertical Reaction
DEAD LOAD	229.69	229.43	55.06
DEAD LOAD + LIVE LOAD	470.73	463.80	111.31
D. L. + L.L + WIND LOAD (STATIC - Pressure)	558.54	549.19	131.81
D. L. + L.L + WIND LOAD (STATIC - Suction)	264.11	262.89	63.09
D. L. + L.L + WIND LOAD (Dynamic - Pressure)	563.98	554.48	133.07
D. L. + L.L + WIND LOAD (Dynamic - Suction)	251.31	250.45	60.11
D. L. + WIND LOAD (STATIC - Pressure)	317.51	314.81	75.56
D. L. + WIND LOAD (STATIC - Suction)	23.08	28.52	6.84
D. L. + WIND LOAD (Dynamic - Pressure)	322.95	320.10	76.82
D. L. + WIND LOAD (Dynamic - Suction)	10.28	16.07	3.86

Table 6.1 Force in Single Cable for different load combinations

Design of Cable supporting Roof

Design Tension =1.5 x 563.98 = 846 kN (Governing load case - DL+LL+WL_P) Area of Cable Required - $\frac{846}{1800}$ = 470mm² Provide Open Spiral Strand - 32 mm diameter Change in Length of Cable - 0.00041m Change in Sag of Cable (as per linear) – 0.00149 m (As per nonlinear – 0.00 m) Permissible Deflection – $\frac{L}{325} = 0.15m$ Hence, safe in deflection **Design of Anchor Cable** Inclination of Cable to horizontal 6.85 Degree Inclination of Anchor Cable 60 Degree Cable is passed over pulleys Vertical Force on top of Pier 833.53 kΝ Horizontal Force on top of Pier 416.94 kΝ 5420.27 Moment at Base kN.m Cable is passed over smooth rollers Vertical Force on top of Pier 833.33 kΝ Tension in Anchor Cable 1679.85 kΝ

Design Tension in Anchor Cable	2519.78	kN
Area of Anchor Cable Required	1400	mm ²

Provide 52 mm diameter open spiral strand for Anchor cable inclined at 30 degree to the horizontal, having an area of 1657 mm² and minimum breaking load capacity of 2570 kN.

Frequency and time period for first three modes of vibration

	Frequency	Time Period
f ₁₁	0.20	5.10
f ₁₂	0.39	2.55
f ₁₃	0.59	1.70

Ratio of frequencies f12/f11 = 0.4 / 0.2 = 2

Safe in flutter considering first two modes of frequency

Increase the stiffness by stiffer roofing material for more appropriate results and safety in flutter under worst wind load case.

6.1.2 Parametric Study of Single Cable

A study of maximum, minimum tension in cable and displacement is undertaken with change in sag, span and column spacing based on approximate method. Comparison of governing load cases for Static and dynamic wind are also undertaken. For few of the cases – nonlinear, linear and approximate methods are compared.

Length – 51 m, Width - 40 m and 50 m Height – 13 m Sag /Span ratio = 0.06, 0.08, 0.1 and 0.12 Location - Ahmedabad C/C Spacing between columns – 1.5m and 3m

Study of results for different cases- parameters indicates similar variation. Thus, only graphs of one case are indicated in the report. For single cable results of span length -50 m for 3 m sag having cable spacing of 3 m are presented.





Fig.6.2 Effect of change in Sag on Maximum and Minimum Tension of 40 m span

6.1.2.2 Variation of Maximum and Minimum Tension with Sag for Single Cable - 50 M Span (Pretension = 220 kN)



Fig.6.3 Effect of change in Sag on Maximum and Minimum Tension of 50 m span

6.1.2.3 Percentage Variation of Maximum and Minimum Tensile Force with increase in 10 m span



Fig.6.4 Effect of change in Span on Maximum Tension (percentage increase)



Fig.6.5 Effect of change in Span on Minimum Tension (percentage decrease)





Fig.6.6 Effect of change in cable spacing on Maximum Tensile force



Fig.6.7 Effect of change in cable spacing on Minimum Tensile force



6.1.2.5 Comparison of force due to Static and Dynamic Wind Force

Sag / Span

Fig.6.8 Effect on Maximum Tension due to Static and Dynamic wind force (percentage increase)



Fig.6.9 Effect on Minimum Tension due to Static and Dynamic wind force

6.1.2.6 Comparison of Static Linear, Nonlinear and Approximate Forces

Load Case	Approximate	Linear	Non-Linear
Pre-Tension	220.00	219.22	219.80
DL	229.43	229.06	229.06
DL+LL	463.80	455.20	460.53
DL+LL+WL_SP	549.19	541.20	543.3
DL+LL+WL_SS	262.89	253.82	266.73
DL+LL+WL_DP	554.48	546.45	548.35
DL+LL+WL_DS	250.45	240.18	253.61
DL+WL_SP	314.81	305.22	313.4
DL+WL_SS	28.52	17.83	31.66
DL+WL_DP	320.10	310.45	318.55
DL+WL_DS	16.07	4.20	18.3

Table 6.2 Static Approximate, Linear and Nonlinear forces of cable (50 m with 3 m sag)

Table 6.3 Percentage variation of Static Linear/Approximate and Nonlinear forces of cable

Load Case	W.R.T Approximate	W.R.T Linear
Pre-Tension	-0.09	0.26
DL	-0.16	0.17
DL+LL	-0.71	1.17
DL+LL+WL_SP	-1.07	0.39
DL+LL+WL_SS	1.46	5.09
DL+LL+WL_DP	-1.11	0.35
DL+LL+WL_DS	1.26	5.59
DL+WL_SP	-0.45	2.68
DL+WL_SS	11.01	77.57
DL+WL_DP	-0.49	2.61
DL+WL_DS	13.88	335.71

Negative sign indicates the decrease in nonlinear forces with respect to approximate case, whereas the positive results with linear case indicates higher forces in nonlinear analysis.



Load Case

Fig.6.10 Variation in maximum tension - Approximate, Linear and Non linear

6.1.2.7 Displacement comparison – Linear, Nonlinear and Approximate



Load Case

Fig.6.11 Displacements at Center



6.1.2.8 Effect of anchor cable angle with horizontal

Angle with Horizontal

Fig.6.12 Cable tension / Reaction with change in Anchor angle

	Frequency	Time Period	Frequency	Time Period
	Approximate Analysis		SAP Analysis	
f ₁₁	0.20	5.10	0.15	6.67
f ₁₂	0.39	2.55	0.32	3.13
f ₁₃	0.59	1.70	0.60	1.67

Table 6.4 Frequency and Time period as per Approximate and Non-linear Analysis

6.1.3 Observations for single cable roof based on parametric study

- Maximum Tension in cable occurs under Dead load + Live Load + Wind load pressure combination and minimum tension is for Dead load + Wind loadsuction case.
- With increase in sag the maximum tension in cable decreases whereas the minimum tension increases.
- With change in cable spacing from 1.5 to 3 m the increase in force is 27% to 39% for both static and dynamic wind cases for 40 m span, whereas it is 30% to 43% for 50 m span. Decrease in minimum tension for 40 m span is 21% to 48% for static wind and 23% to 53% for dynamic wind, whereas for

50 m span it is 33% to 82% for static wind and 35% to 91% for dynamic wind. Lesser percentage increase for higher value of sag/span is observed.

- For same sag/span ratio and different span length the variation in maximum tension in cable with change in cable spacing from 1.5 to 3 m is observed to vary by 27% to 43% for static and dynamic wind. The percentage variation decreases with increase in Sag/span ratio.
- Increase in 10 m span of cable results into increase in maximum tension by 7% to 10% for 0.06, 6% to 9% for 0.08, 5.65% to 8.4% for 0.1 and 5.2% to 7.9% for 0.12 sag/span ratio.
- Increase in 10 m span of cable results into decrease in minimum tension by 20% to 86% for 0.06, 14% to 44 % for 0.08, 10% to 27 % for 0.1 and 9% to 23 % for 0.12 sag/span ratio.
- Difference in maximum tension for static and dynamic wind is not very high i.e. 1 to 2% higher forces in dynamic case. The decrease in minimum tension i.e. residual force in cable shows large variation for cable spaced at 3 m as compared to 1.5 m spacing and decreases with increase in sag/span ratio.
- Approximate method of analysis gives higher value of force as compared to nonlinear analysis, whereas the linear analysis results in large decrease in value. This indicates the proficiency of approximate analysis and the effect of large displacements.
- Approximate method gives lesser value of displacement and linear results into higher value of displacement as compared to nonlinear analysis.
- Increase in anchor cable angle with the horizontal results into large increase in force in cables.
- Frequency calculated using approximate method can be used as preliminary data for checking flutter.

6.2 CABLE TRUSS – DOUBLE CABLE SYSTEM

Various forms of cable truss can be formed combining hanging cable to resist the downward load and the hogging cable to resist the upward load. The diagonal or vertical bracings/ struts resist non-uniform load without large deflections. For understanding the behaviour and load transfer system parametric study of a simply supported cable truss is considered.

This analysis is based on the assumption that the loads are applied as uniformly distributed; along practically the system is subjected to various point loads. Only wind calculations are carried out and are considered critical.

6.2.1 Analysis and design based on Approximate Method

Length – 200 m			
Width - 100 m			
Height – 21 m (13 m – Column + 8 m Rise	of Cable truss)	
Sag of top and bottom cable –8 m			
Location - Ahmedabad			
C/C Spacing between columns -1.5 m			
C/C Distance between supporting struts of	truss – 6 m		
Load Intensity			
Self Weight of Bottom Cable		0.107	kN/m
Self Weight of Top Cable		0.107	kN/m
Roofing Material (PTFE Coated Fabric)		13.5	N/m ²
Weight of Strut		0.31	kN/m
Live Load		0.75	kN/ m ²
Load Calculations			
Roofing Dead Load	D.L	0.02025	kN/m
Weight of Top Cables		0.107	kN/m
Weight of Bottom Cables	qw	0.107	kN/m
Weight of spreaders		0.46	kN/m
Live Load	L.L	1.125	kN/m
Wind Load			
Static Wind load- X		-2.28	kN/m
Dynamic Wind Load - X		-2.38	kN/m
Gust Factor obtained is – 2.04			
Apply Pretension (initially) to top cables	T_{pre}	120	kN
---	-------------------	--------------	---------------
Maximum Tension Required in Bottom cables	T_{max}	230	kN
For double curved cable roof structure use of	external	pressure coe	efficients of
curved roof as per Table- 15 (IS: 875-1987)	is best	suitable, as	covering of
roofing material is supported on top cable.			
Length of Top and Bottom Cable	$L_{ct} = L_{cb}$	101.71	m
Frequency of top cable	\mathbf{f}_{u}	5.2	n
Frequency of bottom cable	f_{b}	6.4	n

Hence, Safe against resonance under self weight and pretension

LOAD CASE	Tu	Tb	ΔL	Δf	fu1	fb1
DEAD LOAD (qd)	116.0	230.4	0.15	0.36	5.2	6.4
DEAD LOAD + LIVE LOAD (p)	36.0	335.0	0.05	-0.25	2.9	7.8
DL + LL + WL (Static)	198.3	122.7	0.25	-0.01	6.8	4.7
DL + LL + WL (Dynamic)	204.9	114.0	0.26	-0.01	6.9	4.5
DL + WL (Static)	278.2	18.1	0.36	-0.08	8.0	1.8
DL + WL (Dynamic)	284.9	9.4	0.37	0.10	8.1	1.3
			L	0.21		

Table 6.5 Force in Cable Truss for different load combinations

Permissible Deflection

 $\frac{L}{325} = 0.31$ m

Hence, safe in deflection for dead + live load loading condition along with no resonance condition.

Design of top cable (Governing load case is DL + WL (Dynamic))					
Design Axial Tension in cable (1.5x284.9)	428	kN			
Area of cable required	238	mm ²			

Provide 28 mm diameter Open- Spiral Strand-Galvanized with Minimum Breaking load of 745 kN and area of 480 mm².

Design of bottom cable	(Governing Load case is DL +LL)
------------------------	---------------------------------

Design Axial Tension in cable (1.5 x 335)	503	kN
Area of cable required	280	mm ²

Provide 32 mm diameter Open- Spiral Strand-Galvanized with Minimum Breaking load of 970 kN and area of 628 mm². Larger size of diameter is required to have some amount of residual tension in top and bottom cable, under every load condition, as well as to control deflection.

Provide 200 mm NB Heavy Pipe section as struts, to keep the cables in position – to resist a compressive load of 12 kN.

Design of anchor cable		
Inclination of Cable to horizontal	4.60	Degree
Inclination of Anchor Cable	60	Degree
Cable is passed over pulleys		
Vertical Force on top of Pier	505.16	kN
Horizontal Force on top of Pier	265.22	kN
Cable is passed over smooth rollers		
Vertical Force on top of Pier	505.16	kN
Tension in Anchor Cable	1064.3	kN



Fig.6.13 Cable truss – Elevation

Fig.6.13 is not the scale and only for understanding and reference.



6.2.2 Parametric Study of Double Cable truss

A study of maximum and residual tension in top and bottom cables is undertaken with change in sag, span and column/cable spacing based on approximate method.

Comparison of governing load cases for Static and dynamic wind are also undertaken. Comparison of nonlinear, linear and approximate method for 100 m cable span with 8 m sag is presented for change in forces and displacement.

Frequency and time period as per approximate method and nonlinear analysis (based on SAP) is presented.

The parameters selected for study are as follows Length – 100 m Width - 80 m, 100 m and 120 m Height – 13 m (supporting column) Sag /Span ratio = 0.06, 0.08, 0.1 and 0.12 Location - Ahmedabad C/C Spacing between columns – 1.2 m, 1.5m and 1.8m Number of struts – 17

Study of results for different cases- parameters indicates similar variation. Thus, only graphs of one case are indicated in the report. For cable truss results of 100 m span and 8 m sag having distance between two trusses of 3 m are presented.

Observations based on all the graphs are described in next section.



6.2.2.1 Effect of change in Initial Pretension Of Top Cable

Pretension Force in Top Cable (kN)





Pretension Force in Top Cable (kN)

Fig.6.16 Effect of Pretension on Bottom cable tensile force

6.2.2.2 Effect of Change in Cable Span and sag- for 1.5 m column spacing



Fig.6.17 Effect of Change in Cable span on initial pre-tension and residual tension



Fig.6.18 Effect of Change in Cable span on top and bottom cable – tension force

6.2.2.3 Effect of Change in Column Spacing

The proportionality effect is observed for value of initial pretension and final tension on top cable as well as bottom cable.

For study of this parameter, residual tension in top cable is fixed to 9.4 KN, as well as other parameters are kept as constant. The area of top cable is 480 mm^2 and that of the bottom cable is kept as 628 mm^2 .

Column	Pre-Tension	T _u	T _b	Residual T_{b}
spacing				
1.2	117.8	285.6	314.1	52.7
1.5	117.4	284.9	335	36
1.8	117	284.2	355.9	19.3

Table 6.6 Force in Cable Truss with different column spacing

6.2.2.4 Variation of Force- Static and Dynamic

Тор	Bottom	Тор	Bottom	Тор	Bottom	Тор	Bottom
Cable	Cable	Cable	Cable	Cable	Cable	Cable	Cable
D.L + L	.L.+WL-S	D.L + L.	L.+WL-D	D.L -	⊦WL-S	D.L +	- WL-D
80.90	5.30	87.60	-3.40	160.80	-99.30	167.50	-108.00
100.90	25.30	107.60	16.60	180.80	-79.30	187.50	-88.00
120.90	45.30	127.60	36.60	200.80	-59.30	207.50	-68.00
140.90	65.30	147.60	56.60	220.80	-39.30	227.50	-48.00
160.90	85.30	167.60	76.60	240.80	-19.30	247.50	-28.00
180.90	105.30	187.60	96.60	260.80	0.70	267.50	-8.00
200.90	125.30	207.60	116.60	280.80	20.70	287.50	12.00
220.90	145.30	227.60	136.60	300.80	40.70	307.50	32.00

Table 6.7 Force in top and bottom cables with change in pre-Tension from 0 to 140 kN

As design of cable roofs is mainly governed by wind, the effect of increase in pretension is presented only for load cases that include wind. Pretension is increased from 0 to 140 kN with an increment of 20 kN at each step. For dead and live load combination also the increase is found to be linear.



Pre-Tension in Top Cable(kN)

Fig.6.19 Percentage Variation in Cable Truss Tension - Static and Dynamic Wind Force

6.2.2.5 Variation of Force- Nonlinear, Linear and Approximate Analysis



Fig.6.20 Nonlinear, Linear and Approximate Analysis- Top and Bottom Cable forces



6.2.2.6 Displacements- Nonlinear, Linear and Approximate Analysis

0.5

Load Case

Fig.6.21 Nonlinear, Linear and Approximate Analysis- Displacement comparison

	Approxim	ate Method	SAP		
	Frequency	Time Period	Frequency	Time Period	
f11	5.98	0.17	6.32	0.16	
f12	9.16	0.11	8.4	0.12	
f13	11.96	0.08	13.2	0.08	

Table 6.8 Cable Truss - Frequency and Time period

6.2.3 Observations for double cable roof truss based on parametric study

- Increase in cable tension for all load combinations is same as increase in value of pretension. To prevent slacking of cable Dead load + Dynamic wind load is the governing load case for sagging cable and Dead load + Live load for hogging cable of the truss.
- So For same value of residual tension in bottom cable (kept as 9.4 kN), the pretension and residual tension are observed to increase in top cable for all spans of truss. With increase in sag/span ratio these values decreases.
- Increase in 20 m span of cable truss increases tension in hogging cable of the truss by 15% to 25% and increase in tension of sagging cable by 20% to 27%.

- Increase in column/cable truss spacing by 0.3 m increases tension in sagging cable by 6-7% whereas
- Hogging cable tension increases from 3 to 6% with decrease in pretension from 140 to 0 kN and for sagging cable decreases from 6 to 165% for Dead load+ Live load + wind load case. This is not similar for dead load + wind load case. The increase in top cable tension is 2 to 4% and bottom cable shows increase from 6 to 35% upto pretension 100 kN, and then after it decreases from 1200 to 21%.
- Approximate method of analysis gives higher values of tension in cables whereas the linear analysis gives lower values as compared to nonlinear analysis.
- Preliminary values of displacement and frequency can be based on approximate method as less variation is observed as compared to nonlinear analysis. The displacement plot indicates higher displacement for linear analysis and reduction in value for nonlinear case.

6.3 CABLE NET

Various shapes – Synclastic and Anticlastic, conical and many others can be formed as discussed in previous chapter. A saddle shape roof is considered here by combining hanging cable to resist the downward load and the hogging cable to resist the upward load. It is important to understand that such shapes can only be formed only if the edges are curved in elevation. Rigid edges of steel beams are considered. Flexible cables if used as edge cables can also be used.

For understanding the behaviour and load transfer system and parametric study a simply supported cable truss is considered. This analysis is based on the discrete method of analysis- which is an exact method. Only wind calculations are carried out and are considered critical as per literature review. A factor of 1.5 is applied to the coefficients of flat roof, due to the limitation of available data for present code of practice.

6.3.1 Analysis and design based on Exact Analysis (SAP 2000)

Length – 14 m Width – 14 m Height - 6 m (5 m - Column + 1 m Rise of hogging beam)Sag and Rise of edge beams/cables - 1 m Location - Ahmedabad C/C Spacing between columns - 14 m Spacing of cables in both directions – 3.5 m Load Intensity **Dead Load** Self Weight of Cable (Macalloy-Galvanized Full 0.107 kN/m Locked Coil Strand - 44 m dia) Roofing Material (PTFE-Fabric) 0.0135 kN/m^2 Live Load 0.07 $\gamma = h / I$ [Table 2 - IS: 875 (Part 2) - 1987] θ 6.95 Degree kN/m^2 0.75 Wind Load - Considering 1.5 as Windward Leeward Factor Static wind Load -1.31 -0.66 kN/m^2 kN/m^2 -1.36 Dynamic wind load -0.68 Gust Factor obtained is 2.30 Load kN/ m² Loads for Cable Spacing - 3.5 m Joint load CASE I - Dead Load 0.91 0.01 CASE II - Live Load 0.75 9.19 CASE IIIa - W.L (Static Suction - Windward Face) -1.31 -16.10

CASE IIIb - W.L (Static Suction - Leeward Face)	-0.66	-8.05
CASE IVa - W.L (Dynamic Suction - Windward Face)	-1.36	-16.64
CASE IVb - W.L (Dynamic Suction - Leeward Face)	-0.68	-8.32

Load Cases

CASE I - Dead Load + Live Load CASE II- Dead Load + Live Load + W.L (Static) CASE III- Dead Load + Live Load + W.L (Dynamic) CASE IV- Dead Load +W.L (Static) CASE V- Dead Load + W.L (Dynamic)

Element	PRETENSION = 70 kN (Z-direction)								
No	Pre- Tension	DL	CASE I	CASE II	CASE III	CASE IV	CASE V		
	Sagging Cables								
1 and 4	70.053	73.24	108.18	42.63	40.45	8.326	6.17		
2 and 3	68.672	71.67	106.01	41.81	39.67	8.2	6.08		
5 and 8	70.053	73.3	108.76	43.44	41.26	8.05	5.88		
6 and 7	68.672	71.87	106.58	42.62	40.48	7.93	5.8		
9 and 12	70.053	73.3	108.18	96.6	96.2	61.82	61.43		
10 and 11	68.672	71.8	106.01	94.64	94.26	60.58	60.19		
			Hoggir	ng Cables	•	•			
13 and 16	70.053	66.87	32.48	79.5	81.1	114.56	116.2		
14 and 15	68.672	65.56	31.88	77.9	79.46	112.23	113.82		
17 and 20	70.053	66.78	31.117	79.64	81.26	114.98	116.6		
18 and 19	68.672	65.47	30.516	78.04	79.63	112.65	114.23		
21 and 24	70.053	66.87	32.48	79.48	81.08	114.56	116.18		
22 and 23	68.672	65.55	31.88	77.9	79.46	112.23	113.82		

Table 6.9 Force in Cable elements of cable net for different load combinations - Wind in Z

Table 6.10 Force in Cable elements of cable net for different load combinations- Wind in X

		PI	RETENSIO	DN = 70 kľ	N (X-direct	ion)	
Element No	Pre- Tension	DL	CASE I	CASE II	CASE III	CASE IV	CASE V
			Saggin	g Cables			
1 and 4	70.053	73.23	108.18	61.11	59.55	26.78	25.24
2 and 3	68.672	71.8	106.01	59.94	58.41	26.307	24.81
5 and 8	70.053	73.32	108.76	60.78	59.18	25.14	23.53
6 and 7	68.672	71.87	106.58	59.2	58.05	24.68	23.1
9 and 12	70.053	73.23	108.18	61.11	59.55	26.78	25.24
10 and 11	68.672	71.8	106.01	59.94	58.41	26.307	24.81
			Hoggin	g Cables			
13 and 16	70.053	66.78	32.48	97.78	100	132.82	135.05
14 and 15	68.672	68.67	31.88	95.83	98	130.12	132.3
17 and 20	70.053	66.78	31.12	97.12	99.34	132.85	135.08
18 and 19	68.672	68.67	30.52	95.16	97.38	130.16	132.34
21 and 24	70.053	66.87	32.48	43.24	43.6	77.67	78.13
22 and 23	68.672	65.56	31.88	42.24	42.77	76.23	76.57

Design of sagging cables (Governing load Case-DL+L	L)	
Maximum Value of Cable Tension	108.76	kN
Design Value of Cable tension	163.14	kN
Area of cable required	90.63	mm ²
Select Cable- Spiral strand	16	mm
Area of cable provided	157	mm ²
Minimum Breaking strength of cable provided	240	kN
Safe cable		
Design of hogging cables (Governing load case-DL+N	WL(Dynamic-X)	1
Maximum Value of Cable Tension	135.08	kN
Design Value of Cable tension	202.62	kN
Area of cable required	112.57	mm ²
Select Cable- Spiral strand	16	mm
Area of cable provided	157	mm²
Minimum Breaking strength of cable provided	240	kN
Safe cable		

6.3.1.2 Design of beam

6.3.1.1 Design of cables

As fixed conditions result into lesser value of design moments and from the geometry selected it is easier to connect with moment connection, it is preferred to consider fix end conditions for beam-column conditions. With biaxial design of beam the results for selected section and its stresses are:

Table 6.11 De	esign forces	in	beams
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Beam Forces		Mx (kN m)	Vy (kN)	My (kN m)	Vx (kN)	Axial Force (kN)
Fix	Sagging Beam	50	1	-536	-115	-180
support	Hogging Beam	-40	-3	-551	162	-96

Beam properties: 4ISA 200x200x25

A - 4.00 x 10^4 mm² I_x - I_y - 9.04 x 10^8 mm⁴ r_x - r_y - 160 mm

 $Z_x - Z_y - 4.52 \times 10^6 \text{ mm}^3$



Fig. 6.22 Beam section - 4ISA200x200x25

Sagging beam

Bending Stress ratio = 0.83 < 1.33 (33% higher for wind load) Shear Stress ratio - (Major axis) = 0.001Shear Stress ratio - (Minor axis) = 0.058Hogging beam Bending Stress ratio = 0.821 < 1.33 (33% higher for wind load) Shear Stress ratio - (Major axis) = 0.001Shear Stress ratio - (Minor axis) = 0.082 s

6.3.1.3 Design of column

As the structure is symmetrical in plan, change in wind direction will result in change in forces in column in the same manner. So, maximum design values are considered. Although 4 angle sections with no spacing in between are sufficient for design, due to required connection at its end the dimensions are increased in both the directions. This enables easier connection of beam to column.

Table 6.12 Design force in column

Column Forces	My (kNm)	$\lambda \lambda \mu (zN)$	My (kN m)	Vx	Axial Force
		VY (KN)	My (KN111)	(kN)	(kN)
	53	14	146	46	16

Column Properties: 4ISA 200x200x25 (Battened)

A - 3.75 x 10⁴ mm²

 $I_x = I_y - 3.37 \times 10^9 \text{ mm}^4$

 $r_x = r_{y-300} \text{ mm}$ and $Z_x = Z_y - 9.62 \times 10^6 \text{ mm}^3$

Bending Stress ratio = 0.174 < 1.33 (33% higher for wind load)



Fig.6.23 Battened Column Section - 4ISA200x200x25

Size of batten plates: 550 x 400 x 10 thick. Use 5-12 Φ HSFG bolts connecting batten plate and column.

6.3.1.4 Design of beam splice

As the span is larger than the available lengths of sections, splices will be required for beam and are preferred at 5.25 m distance from column centre.

Beam Splice	Mx (kN m)	Vy (kN)	My (kN m)	Vx (kN)
Sagging Beam	10	4	160	57
Hogging Beam	63	6	123	55

Table 6.13 Design force for beam splice



Fig.6.24 Beam Splice Details

6.3.1.5 Design of cable to beam connection

The axial force of cables is transferred to the beams with the standardized connections. Swaged socket connections along with thickness required for gusset plate are available from Macalloy Company standards are used here.

As the thickness of connecting member required is 25 mm, angles of 10 mm thickness are used with a 5 mm plate at centre.

Two angle sections are preferred to satisfy the required thickness of gusset plate. The angles are welded to the beam along all its sides.



Fig.6.25 Cable-Beam Connections -Plan and Elevation

6.3.1.6 Design of cable to cable connection – clamp design

The function of clamps is similar to that of the struts in cable trusses. The forces from supported cable to supporting cable are transferred from the clamps.

High strength friction grip bolts of Grade 8.9 are preferred as horizontal movement of net also takes place along with vertical deflection.

25 mm diameter High strength friction grip bolts are thus provided with 25 mm thick plate to connect the cables at their intersection. This will enable the transfer of forces from one cable to another, neglecting the possibility of any cable becoming slack.



Fig.6.26 Cable-Cable Connections

6.3.1.7 Design of roof covering – PTFE coated Fabric

As width of available fabric selected is 4 m, the cables are spaced at 3.5 m. It is considered that the manufactured material is safe in both wrap and weft

direction for the loading considered. The seaming to cables is usually as per manufacturer's instruction and not scope of designer.

6.3.1.8 Design of beam-column connection

The above specified values of maximum moment and shear in both the beams for design of beam are at the beam-column connection. Thus, the moment resistant connections are designed for both of these. As the value of moment is very high, welding as well as bolted connections are used.



Fig.6.27 Beam Column Connections

6.3.1.9 Design of column base plate and anchor bolts

Due to large moments the anchor bolts are provided to resist the uplifting.

6.3.1.10 Design of foundation

As moment is higher as compared to axial compressive forces, the foundation is subjected to uplifting and can also cause failure. It is thus preferred to design the strap beam for resisting this moment and safely distributing it into both the columns





6.3.1.11 Serviceability criteria

(a) Deflection of net

Although this is not specified in any code, we consider the same as of the steel structures as L / 325. Permissible deflection – 43 mm Maximum Vertical deflection is 0.00051 mm Hence, it is within permissible limit.

(b) Deflection of beam

Permissible – L / 325 = 43 mm Sagging beam – 21.87mm Hogging beam – 16.33 mm Hence, safe in deflection

(c) Failure due to Fluttering

For the first three modes of vibrations the frequency and time period of the net is

Mode	Time Period	Frequency
1	3.53	0.282
2	1.98	0.50
3	1.39	0.717

Ratio of frequency's $f_{n1}/f_{n2} = \frac{0.50}{0.28}$

= 1.80 < 2

Hence, the structure might get into failure due to fluttering. This is the aerodynamic process and only under dynamic wind forces the failure of roof can take place. The remedies to overcome problems due to fluttering and deflection are as specified below:

6.3.1.12 Remedies to overcome failure due to deflection and flutter

1. Decrease in column spacing to reduce deflection and load reactions on beam.

2. Increase in diameter of cables or increase stiffness of roofing material, to overcome flutter.



- 113 -



- 114 -





- 115 -





- 116 -

6.3.2 Parametric Study of Cable Net

A study of maximum, minimum tension in sagging/hogging cables and displacement is undertaken with change in sag, span and column spacing based on exact method. Comparison of governing load cases for Static and dynamic wind are also undertaken. For few of the cases – nonlinear, linear and approximate methods are compared.

Length – 14m, 21m and 28 m, Width -14m, 21m and 28 m Height – 5 m (Column support)

Sag /Span ratio = 0.07, 0.10, and 0.14

Location - Ahmedabad

C/C Spacing between cables –3.5 m

Study of results for different cases- parameters indicates similar variation. Thus, only graphs of one case are indicated in the report. For cable net results of span length -14m for 1 m sag having cable spacing of 3.5 m are presented.

Wind effects in both X and Z direction is different on the sagging and hogging cables. Thus, a comparative plot is created and study for windward and leeward forces is presented.



6.3.2.1 Effect of wind in Z-direction on all cables

Fig.6.34 Effect of wind in Z-direction – 14 m cable net (1 m sag and 70 kN pretension)



6.3.2.2 Effect of wind in X-direction on all cables

Fig.6.35 Effect of wind in X-direction – 14 m cable net (1 m sag and 70 kN pretension)

6.3.2.3 Comparative plot of wind in X-direction and Z-direction



Fig.6.36 Sagging cable- comparative plot for wind in X and Z direction



Load Case

Fig.6.37 Hogging cable- comparative plot for wind in X and Z direction





Fig.6.38 Effect of change in sag – 14 m cable net (70 kN pretension)

6.3.2.5 Effect of Change in Pre-Tension



Fig.6.39 Effect of change in pretension on sagging cables - 14 m cable net (1 m sag)



Fig.6.40 Effect of change in pretension on hogging cables 14 m cable net (1 m sag)



6.3.2.6 Non-Linear Analysis – Static V/S Dynamic

Fig.6.41 Percentage Decrease in sagging cable force for static to dynamic force



Fig.6.42 Percentage Increase in hogging cable force for static to dynamic force

6.3.2.7 Effect of change in Span

Centre three cables of 21 x 21 m and 28 x 28 m net are considered for comparison of forces to the cable net of 14 x 14 m, with same cable spacing for all load cases.

		Pre-		DL+LL	DL+LL	DL+	DL+
Cable	DL	Tension	DL+LL	+WL(S)	+WL(D)	WL(S)	WL(D)
C1	73.24	70.053	108.18	42.63	40.45	8.326	6.17
C2	73.3	70.053	108.76	43.44	41.26	8.05	5.88
C3	73.27	70.053	108.18	96.6	96.2	61.82	61.43
C4	66.87	70.053	32.48	79.5	81.1	114.56	116.2
C5	66.78	70.053	31.12	79.64	81.26	114.98	116.6
C6	66.87	70.053	32.48	79.48	81.08	114.56	116.18

Table 6.14 14 x 14 m Cable net with Sag/Span – 0.07

Table 6.15	5 21 x 21	m Cable net	with Sag/Span	- 0.07
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		Pre-		DL+LL	DL+LL	DL+	DL+
Cable	DL	Tension	DL+LL	+WL(S)	+WL(D)	WL(S)	WL(D)
C2	110.06	105.14	163.38	60.43	57.87	6.56	3.98
C3	110.31	105.14	163.38	66	63.64	13.44	11.06
C4	110.07	105.14	162.38	139.95	139.32	85.16	84.52
C7	101	105.14	47.62	117.5	119.24	170.27	171.98
C8	100.62	105.14	46.9	117.14	118.87	169.9	171.62
C9	101	105.14	47.62	117.51	119.24	170.27	171.98

Table 6.16 28 x 28 m Cable net with Sag/Span – 0.07

		Pre-		DL+LL	DL+LL	DL	DL+
Cable	DL	Tension	DL+LL	+WL(S)	+WL(D)	+WL(S)	WL(D)
C 3	142.84	136.26	208.6	71.23	69.34	4.27	2.38
C4	142.76	136.18	208.6	83.93	82.27	19.2	17.55
C 5	142.84	136.26	208.6	169.04	168.44	99.71	99.08
C10	129.48	136.7	62.56	155.53	156.8	220.83	222.06
C11	129.56	136.15	62.42	155.56	156.82	220.98	222.23
C12	129.48	136.7	62.56	155.53	156.8	220.83	222.06

		Pre-		DL+LL	DL+LL	DL+	DL+
Cable	DL	Tension	DL+LL	+WL(S)	+WL(D)	WL(S)	WL(D)
C2	1.50	1.50	1.51	1.42	1.43	0.79	0.65
C3	1.50	1.50	1.50	1.52	1.54	1.67	1.88
C4	1.50	1.50	1.50	1.45	1.45	1.38	1.38
C7	1.51	1.50	1.47	1.48	1.47	1.49	1.48
C8	1.51	1.50	1.51	1.47	1.46	1.48	1.47
C9	1.51	1.50	1.47	1.48	1.47	1.49	1.48

Table 6.17 Percentage increase in forces from 14 to 21 m

Table 6.18 Percentage increase in forces from 14 to 21 m

		Pre-		DL+LL	DL+LL	DL	DL
Cable	DL	Tension	DL+LL	+WL(S)	+WL(D)	+WL(S)	+WL(D)
C 3	1.95	1.95	1.93	1.67	1.71	0.51	0.39
C4	1.95	1.94	1.92	1.93	1.99	2.39	2.98
C 5	1.95	1.95	1.93	1.75	1.75	1.61	1.61
C10	1.94	1.95	1.93	1.96	1.93	1.93	1.91
C11	1.94	1.94	2.01	1.95	1.93	1.92	1.91
C12	1.94	1.95	1.93	1.96	1.93	1.93	1.91

6.3.2.8 Nonlinear V/S Linear and Approximate analysis



Fig.6.43 Linear, Non-Linear and Approximate analysis _ Cable net 14 m with 1 m sag

	Approximate Analysis		Linear Analysis	
	Hogging	Sagging	Hogging	Sagging
DL	-0.25	0.38	104.86	95.56
DL+LL	-14.84	5.07	3.33	-0.47
DL+LL+WIND_S	-1.36	-39.38	0.20	2.60
DL+LL+WIND_D	-2.07	-41.47	0.18	2.94
DL+WIND_S	-13.39	-21.33	-0.15	26.38
DL+WIND_D	-13.67	-28.21	-0.12	37.12

Table 6.19 Percentage variation of approximate and linear analysis w.r.t. nonlinear

6.3.2.9 Deflection of cables – Nonlinear and Approximate analysis



Fig.6.44 Linear, Non-Linear and Approximate analysis -Deflection_ Cable net 14 m with 1 m sag

6.3.2.10 Frequency and Time Period – Nonlinear and Approximate analysis

Table 6.20 Frequency and Tim	e period as per Approximate an	nd Non-linear Analysis (Cable Net)
------------------------------	--------------------------------	------------------------------------

	Frequency	Time Period	Frequency	Time Period
	Approximate Analysis		SAP Analysis	
f ₁₁	0.13	7.69	0.28	3.53
f ₁₂	0.13	7.69	0.50	1.98
f ₁₃	0.26	3.85	0.72	1.39

6.3.3 Observations for Saddle shape cable net

- Scables subjected to tension force at some inclination, results into horizontal and vertical force component on the supporting members. This requires biaxial design of the supporting structural member. Force in horizontal direction being higher results into higher width of member as compared to required depth.
- For steel structure, design of connections is critical and requires higher size of column for provision of connections.
- High moment at the base of the column occurs as compared to axial force. To prevent footing failure either the tension piles, ground anchors or strap beam is required.
- Fluttering effect checked at first two modes of vibration results into safe structure, whereas at higher modes this results in to failure due to flutter.
- As wind on leeward and windward direction is different the cable net is subjected to unsymmetrical loading. Although the design forces are higher for wind in X-direction the residual forces in cable with wind in Z-direction requires analysis for both the cases.
- Increase in sag increases force in hogging cables and the forces decrease in sagging cables.
- Linear increase in tension of cable is observed for increase in pretension for all load combinations.
- 5% to 25 % decrease in tension is observed from static to dynamic wind for sagging cables and 0.6% to 2% increase for hogging cable.
- Increasing span to 1.5 times and 2 times result into increase in force of cables to approximately similar values. This is not the case for dead load + wind load (Static and dynamic) for cables on leeward side.
- Approximate method gives less tension force in sagging cable for dead load and live load as compared to nonlinear and higher tension for all other load cases and cables.

- So Linear analysis results into lesser tension in cables.
- Displacement value for approximate and nonlinear case varies largely, whereas for linear case the variation is very less as compared to nonlinear analysis.
- Approximate method of analysis being based on certain assumption does not result into similar frequency as compared to exact results of software.

7.1 SUMMARY

Steel space frames and mainly tensile structures may be used efficiently to form roofs, walls and floors for projects such as shopping arcades, but their real supremacy is in providing roof cover for sports stadia, exhibition halls, aircraft hangars and similar major structures.

An extensive literature survey, concerned with both practical and theoretical aspects of cable roofs, is presented. Some aspects included are: structural systems, cable types and their properties, connection and anchorages, roof loads approximate and exact methods of analysis and design of saddle roof.

As the initial shape of a cable roof depends on the internal force distribution, it cannot be described by simple geometrical models. Special iterative methods have to be utilized in order to find the pretension configuration of the roof.

Although cable mechanics of single cable is easy to analyze, the behaviour of cable truss and net are critical to understand. Iterations can only result into most economical system for span-sag ratio and connections.

Nonlinear behaviour of cable roofs make the task more difficult to analyze and design manually. Use of matrix methods and iterative procedure is the only solution to converge to optimum results.

7.
7.2 CONCLUSIONS

Increasing use of cables for large span structures is not only because of the aesthetic appeal but also due to the advantage of high strength to weight ratio. The lightness of cable gives an expanded impression of space and its characteristics curvilinear form provides a fresh alternative from the regular orthogonal shape buildings.

Based on the observations for selected parameters during this work, the conclusions drawn are:

- All cable systems are effective for wide span. Each system has its own distinct characteristics which makes it attractive for certain conditions and thereby more suitable for particular architectural applications.
- Simply suspended cables provide economical solution only if the deflection is not stringent.
- Cable beams are simple and attractive which are usually employed for buildings orthogonal in plan.
- ∞ Cable nets although can cost high provide excellent anticlastic shapes.
- Pretension is must for any cable roof, as wind force leads to slacking of cables.
- Approximate method of analysis gives higher values of tension in cables whereas the linear analysis gives lower values as compared to nonlinear analysis.
- Preliminary values of displacement and frequency can be based on approximate method as less variation in forces is observed as compared to nonlinear analysis results.
- Displacement with linear analysis is higher as compared to nonlinear analysis for single cable, cable truss as well as cable net.
- As linear increase in forces of cables is observed with increase in pre-tension,
 for any cable roofs a preliminary calculation can be carried out with any value

of pretension and the final value can be easily calculated observing the required increase or decrease in tension so as to resist the slacking of cables.

- Solution Wind is the critical design factor that governs the behaviour of cable roofs.
- See Very less variation in static and dynamic wind force is observed which is based on certain assumptions in present code of practice. This calls for wind tunnel tests and preparation of codes for such structures, with provisions for wind coefficient. More detail description for flutter for higher modes of frequency is also required.

7.3 FUTURE SCOPE OF WORK

Based on the present study of cable roofs the work can be extended further both analytically and experimentally.

7.3.1 Analytical

- Software preparation for nonlinear analysis of cable structures
- Solution of exact methods of nonlinear analysis
- Parametric study for Hyper Paraboloid roof
- Analysis and Design of Hyper Paraboloid roof
- Analysis and Design of Saddle shape cable net for larger span
- Analysis and Design of Tensegrity structures
- Analysis and design of cable net with flexible boundary conditions

7.3.2 Experimental

- So Wind tunnel test Preparation of Wind coefficient for different cable systems
- Cable nets effect of pretension, study of deflection for symmetrical and unsymmetrical loading. (Comparison between experimental and theoretical results)

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APPENDIX-A

A.1 GENERAL

In this section Wind load and Earthquake load calculation for single cable is presented. Different loads are considered as per Indian standard as follows:

a) Wind load as per IS: 875-(Part-III)-1987.

b) Earthquake load as per IS: 1893-(Part-I)-2002.

There are no external pressure coefficients are defined in IS: 875-(part-III)-1987 for singly curved roof. So use the external pressure coefficients of free standing double sloped roof as per Table-8 for walls as per Table 4 are taken in to consideration. For double cable truss and cable net the calculations are similar and tables used for wind are specified as in chapter 6.

Building Data						
Length of structure	L	51		m		
Width of structure		В	50		m	
Height of structure		h	13		m	
C/C Spacing of Columns		c/c	3		m	
Wind Data						
Location		Ahmedabad		Δnn	endix-A	
Basic Wind speed	V _b	39	m/s			
Type of Structure		S1		Tab	le 1	
Probable mean life of str.		50	years			
Category of Structure		1		Clau	ise	
		-		5.3.	2.1	
Class of Structure		Δ		Clau	ise	
				5.3.	5.3.2.2	
Terrain Category		TC2				
k ₁	k_1	0.92		Tab	le 1	
Terrain, height, Str. Size Factor	k ₂					
Upto Height	10	0.98		Tab	le 33	
	13	1.02				
Slope of Ground	Θ	3	degree			

A.2 SINGLE CABLE ROOF - STATIC WIND ANALYSIS

k	k ₃	1		
	Height	10	13	
Vz	$V_{b} k_{1} k_{2} k_{3}$	35.16	36.60	
p _z (N / m ²)	0.6 V _z ²	741.84	803.63	
Permeability	High			
	20	%		
Shape coefficient	C _f	1.2	Fig. 4	
WALL - Wind force	C _f (C _{pe} -C _p i) A _e p _d			
	h/W	0.26		
	L/W	1.02		

External Pressure coefficient	Surface (Table 4)					
Wind Angle	А	В	С	D	Local	
0	0.7	-0.2	-0.5	-0.5	-0.8	
90	-0.5	-0.5	0.7	-0.2	-0.8	
Internal Pressure Coefficient	Positive Coefficient 0					
		Negative	-0.7			

Wind Angle	Cni	(Cpe-Cpi)				
	Срі	А	В	С	D	Local
0	0.7	0	-0.9	-1.2	-1.2	-1.5
	-0.7	1.4	0.5	0.2	0.2	-0.1
90	0.7	-1.2	-1.2	0	-0.9	-0.1
	-0.7	0.2	0.2	1.4	0.5	-0.1

ROOF FORCES - Pressure coefficient for Free standing double sloped roof

Roof Angle		degree
Solidity Ratio	1	
Positive Coefficient	0.34	
Negative Coefficient	-0.8	
Design Force	0.82	kN/m
	-1.93	kN/m

Wind Angle	Cni		FOF	RCE IN kN/	′m	
	Cp:	А	В	С	D	Local
0	0.7	0.00	-2.17	-2.89	-2.89	-3.62
	-0.7	3.38	1.21	0.48	0.48	-0.24
90	0.7	-2.89	-2.89	0.00	-2.17	-0.24
	-0.7	0.48	0.48	3.38	1.21	-0.24

A.3 SINGLE CABLE ROOF - DYNAMIC WIND ANALYSIS

Location		Ahmedabad			
Basic Wind speed	V _b	39	m/s		
				Table 1: Class of	
Type of Structure		S1		Structure	
Probable mean life of str.		50	years		
Category of Structure		1		Clause 5.3.2.1	
Class of Structure		A		Clause 5.3.2.2	
Terrain Category		TC2			
k ₁	k ₁	0.92		Table 1	
Terrain, height, Str. Size					
Factor	k ₂				
Upto Height	10	0.67		Table 3	
	13	0.72			
Slope of Ground	Θ	3	Degree		
k	k ₃	1		Clause 5.3.3.	
	Height	10	13		
Vz	$V_{b} k_{1} k_{2} k_{3}$	24.04	25.83	Clause 5.3	
p _z (N / m ²)	$0.6 V_z^2$	346.74	400.42	Clause 5.4	
Permeability	High				
	20	%		Clause 6.2.3.	
Shape coefficient	C _f	1.2		Fig. 4	
Wind Force	$C_f (C_{pe}-C_p i) A_e p_d$			Clause 6.3	
WALL	h/W	0.26			
	L/W	1.02			
External Pressure		1	1		
coefficient	Surface (Table 4)				

Wind Angle	А	В	С	D	Local
0	0.7	-0.2	-0.5	-0.5	-0.8
90	-0.5	-0.5	0.7	-0.2	-0.8
Internal Pressure	Positi	ve Coefficie	0.7	Clause	
Coefficient	Negat	ive Coefficie	-0.7	6.2.3.	

Wind Angle	Срі					
Wind Angle		А	В	С	D	Local
0	0.7	0	-0.9	-1.2	-1.2	-1.5
	-0.7	1.4	0.5	0.2	0.2	-0.1
90	0.7	-1.2	-1.2	0	-0.9	-0.1
	-0.7	0.2	0.2	1.4	0.5	-0.1

Wind blowing parallel to Shorter Direction of building						
Along Wind Load	C _f (C _{pe} -C _{pi}) A _e	Clause 8.3				
Gust Factor G Calculation						
Peak Factor x Roughness factor = gf r	1.35		Figure 8			
L(h)	1000					
Су	10		Figure 8			
Cz	12					
Cz h / L(h)	0.156					
Breadth of Structure Normal to wind	50					
$\lambda = Cy b / Cz h$	3.21	m				
Background Factor B	0.7					
Time Period	0.17		Figure 9			
Frequency f ₀	6.04	sec				
Fo	36.50	Hz				
Size Reduction Factor S	0.003					
fo L(h) / Vh	233.95		Figure 10			
Available Energy E	0.015					
Damping Coefficient	0.02		Figure 11			
Ø for bolted steel structure	0		(Table 34)			
Gust Factor G	2.13					

ROOF FORCES							
Roof Angle							degree
Solidity Ratio				1			
Positive Coefficient	t		0	.34			
Negative Coefficier	nt		-	0.8			
Design Force			0.87		kN/m		
		-2.05				kN/m	
Wind Angle			FO	RCE IN I	<n m<="" td=""><td>ו</td><td></td></n>	ו	
Wind Angle	Срі	Α	В	C		D	Local
0	0.7	0.00	-2.30	-3.07	-3	8.07	-3.84
	-0.7	3.58	58 1.28 0.51 0.		.51	-0.26	
90	0.7	-3.07	-3.07	0.00	-2	2.30	-0.26
	-0.7	0.51	0.51	3.58	1	.28	-0.26

Base shear due to wind

X-Direction	
Shear due to wind pressure on walls	1012.33
Shear due to wind pressure on roof	327.66
Total Base Shear V _{bx}	1339.99

A.4 SINGLE CABLE ROOF – SEISMIC ANALYSIS

Sr.							
No.	Component	No.	L	В	н	Weight	Weight
			m	m	m	kN/m	kN
Α	Columns						
1	Considering ISMB 600	34	13	1	1	1.23	543.66
В	Walls / Cladding						
1	X-Direction						
	Asbestos Sheeting	2	51	1	13	0.159	210.83
	Side Rails - Considering						
	ISA 125 x 75 x 6	16	51	1	1	0.09	73.44
2	Z-Direction						
	Asbestos Sheeting	2	50	1	13	0.159	206.70
	Side Rails - Considering						
	ISA 125 x 75 x 6	16	50	1	1	0.09	72.00

С	Cable Assembly						
	X-Direction - cable +						
1	roofing dead load	17	0.27	1	1	1	4.59
F	Live Load - over roof	0.25	50	51	1	0.75	478.13
	TOTAL SEISMIC WEIGHT					1589.35	

Description	Value	Units	Reference Code
Building Parameters			
L (X-direction)	51	m	
B (Z-direction)	50	m	
H (Y-direction)	13	m	
Total Seismic Weight	1589.35	kN	
Location	Ahmedabad		
Zone (Z)	III		Annex E
Zone (Z)	0.16		Table 2
Importance Factor (I)	1.5		Table 6
Building Frame Systems	OMRF		
Response Reduction Factor (R)	3		Table 7
Type of Structure	Steel		
Brick Infill Category	No		
Time period Tx (sec)	0.58	sec	0.085 h ^{0.75} Clause
Time period Tz (sec)	0.58	sec	7.6.1
Soil Type	Medium Soil		
Sa/g (X-Direction)	2.34		Clause 6 4 5
Sa/g (Z-Direction)	2.34		
Ah (X-Direction)	0.093		$A_h = (ZIS_a / 2Rg)$
Ah (Z-Direction)	0.093		Clause 6.4.2.
Base Shear (Vb-x)	149	kN	
Base Shear (Vb-z)	149	kN	

APPENDIX-B

// PROGRAM FOR SADDLE SHAPE CABLE NET WITH RIGID BOUNDARY // /*

Terminology

```
NCx - number of cables in x direction
```

NCy - number of cables in y direction

Xa, Xb, Xc and Xd - X co-ordinate of the supporting nodes

```
Ya, Yb, Yc and Yd - Y co-ordinate of the supporting nodes
```

```
Za, Zb, Zc and Zd - Z co-ordinate of the supporting nodes
```

fx - Sag of Cable in X-Direction

```
fy - Rise of Cable in Y-Direction
```

```
a - Constant
```

```
b - Constant
```

```
Lx - Length of cable in X-Direction
```

```
Ly - Length of cable in Y-Direction
```

```
Lex - Increment in node value in X-Direction
```

```
Ley - Increment in node value in Y-Direction
```

```
nnodex - Number of nodes in X-direction for single cable
```

```
nnodey - Number of nodes in Y-Direction for single cable
```

```
*/
```

```
#include<stdio.h>
```

```
#include<conio.h>
```

```
#include<math.h>
```

```
void main()
```

{

```
float i,j,NCx,NCy,Xa,Ya,Za,Xb,Yb,Zb,Xc,Yc,Zc,Xd,Yd,Zd,nnodex,nnodey,Node;
float fx,fy,Lx,Ly,Lex,Ley,a,b,Lx1,Ly1,X,Y,Z;
char ch1[30],ch2[30];
clrscr();
FILE*f1,*f2;
printf("Enter name of input file:");
gets(ch1);
```

```
f1=fopen(ch1,"r");
```

```
printf("Enter name of output file:");
gets(ch2);
f2=fopen(ch2,"w");
fscanf(f1,"%f %f %f",&Xa,&Ya,&Za);
fscanf(f1,"%f %f %f",&Xb,&Yb,&Zb);
fscanf(f1,"%f %f %f",&Xc,&Yc,&Zc);
fscanf(f1,"%f %f %f",&Xd,&Yd,&Zd);
fscanf(f1,"%f",&fx);
fscanf(f1,"%f",&fy);
fscanf(f1,"%f",&NCx);
fscanf(f1,"%f",&NCy);
Lx = (Xb - Xa);
Lx1 = (Xc - Xd);
Ly = (Yd - Ya);
Ly1 = (Yc - Yb);
a = (4*fx)/(Lx*Lx);
b = (4*fy)/(Ly*Ly);
Lex = Lx/(NCx+1);
Ley = Ly/(NCy+1);
nnodex = NCx+2;
nnodey = NCy+2;
fprintf(f2,"Coordinates of the four edges are:");
fprintf(f2,"\n%f \t%f \t%f",Xa,Ya,Za);
fprintf(f2,"\n%f \t%f",Xb,Yb,Zb);
fprintf(f2,"\n%f \t%f",Xc,Yc,Zc);
fprintf(f2,"\n%f \t%f",Xd,Yd,Zd);
fprintf(f2,"\nThe Rise of Cable in X-direction is:\t %f",fx);
fprintf(f2,"\nThe Sag of Cable in Y-direction is:\t %f",fy);
fprintf(f2,"\nThe value of constant a is:\t %f",a);
fprintf(f2,"\nThe value of constant b is:\t %f",b);
fprintf(f2,"\nNumber of Nodes in X-direction is: \t %f",nnodex);
fprintf(f2,"\nNumber of Nodes in Y-direction is: \t %f",nnodey);
fprintf(f2,"\nThe nodes of the cable net are:");
X =0;
```

Y =0;

```
Z =0;
Node = 0;
fprintf(f2,"\nNode No. Co-ordinate X \tCo-ordinate Y \tCo-ordinate Z");
for(i=1;i<=nnodey;i++)
{
  for(j=1;j<=nnodex;j++)
  {
    Node = Node + 1;
    X = Xa + Lex*(j-1);
    Y = Ya + Ley*(i-1);
    Z = (a*X*X)-(b*Y*Y);
    fprintf(f2,"\n%f \t%f \t%f",Node,X,Y,Z);
    }
  }
  getch();
```

```
}
```

INPUT FILE

3

OUTPUT FILE

Coordinates of the four edges are:			
-7	7	0	
7	7	0	
7	-7	0	
-7	-7	0	
The Rise of Cable in X-direction is:	1		
The Sag of Cable in Y-direction is:	1		

The value of constant a is:	0.020		
The value of constant b is:	0.020		
Number of Nodes in X-direction is:	5		
Number of Nodes in Y-direction is:	5		
The nodes of the cable net are:			
Nada Na	Co-ordinate	Co-ordinate	Co-ordinate
Node No.	Х	Y	Z
1	-7	7	0
2	-3.5	7	-0.75
3	0	7	-1
4	3.5	7	-0.75
5	7	7	0
6	-7	3.5	0.75
7	-3.5	3.5	0
8	0	3.5	-0.25
9	3.5	3.5	0
10	7	3.5	0.75
11	-7	0	1
12	-3.5	0	0.25
13	0	0	0
14	3.5	0	0.25
15	7	0	1
16	-7	-3.5	0.75
17	-3.5	-3.5	0
18	0	-3.5	-0.25
19	3.5	-3.5	0
20	7	-3.5	0.75
21	-7	-7	0
22	-3.5	-7	-0.75
23	0	-7	-1
24	3.5	-7	-0.75
25	7	-7	0

APPENDIX-C

USEFUL WEBSITES

- http://www.ingentaconnect.com
- http://books.google.com
- http://www.macalloy.com
- http://www.asfi.net
- http://www.intents.be/default2.asp
- http://www.corusconstruction.com
- http://www.ifai.com
- http://www.nycroads.com/crossings/williamsburg
- http://en.wikipedia.org/wiki/Cable-stayed_bridge
- http://www.nicee.org
- http://www.sciencedirect.com
- http://www.csiberkeley.com
- http://www.lightweightstructures.com
- http://www.tensiledesigns.com
- http://www.asce.org
- http://www.tensilestructures.com
- http://www.geigerengineers.com
- http://www.columbia.edu
- http://www.fabricarchitecture.com
- http://architecturalfabrics.com
- http://www.ufsinc.com
- http://www.tensinet.com
- http://www.tecnun.es/labcad/membranes/index_main.htm
- http://www.forten32.com/
- http://www.technet-gmbh.com/index.htm
- http://www.tensys.com/

APPENDIX-D

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