Seismic response reduction of building using semi-actively controlled magnetorheological (MR) damper

S.P. Purohit* and **N.K. Chandiramani**†
*Research Scholar, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400 076, email: p7purohit@civil.iitb.ac.in
†Associate Professor, Department of Civil Engineering, Indian Institute of Technology Bombay, Powai, Mumbai – 400 076, email: naresh@civil.iitb.ac.in

The magnetorhelogical (MR) damper is a semi-active device in which the yield stress of the fluid is controlled via the applied input voltage. They produce sizeable damping force with small amount of input voltage and power requirement. MR damper exhibits non-linear relation between the damping force and input voltage/states. For the present study, a modified Bouc-Wen model developed by Spencer et al. [1] is considered which is given by following set of equations to determine damper force.

$$f = c_1 \dot{y} + k_1 (x - x_0) \quad \text{(1)}$$

$$\dot{y} = \frac{1}{(c_0 + c_1)} \{\alpha z + c_0 \dot{x} + k_0 (x - y)\} \quad \text{(2)}$$

$$\dot{z} = -\gamma |\dot{x} - \dot{y}| z |z|^{n-1} - \beta(\dot{x} - \dot{y}) |z|^n + A(\dot{x} - \dot{y}) \quad \text{(3)}$$

$$\alpha = \alpha_a + \alpha_b u,\ c_0 = \alpha_{0a} + \alpha_{0b} u,\ c_1 = c_{1a} + c_{1b} u \quad \text{(4)}$$

$$\dot{u} = -\eta (u - \mathrm{v}) \quad \text{(5)}$$

Here $x, \dot{x}$ and f are the damper displacement, velocity and force, respectively and z is the evolutionary variable. Other model parameters and their numerical values are as defined by Spencer et al. [1]. The upper limit of evolutionary variable z is given as,

$$z_u = \left(\frac{A}{\gamma + \beta} \right)^{\frac{1}{n}} \quad \text{(6)}$$

Neglecting the stiffness terms of Equations (1)-(2), being small compared to damping terms, and using the steady state solution of Equation (5) i.e., $u = \mathrm{v}$, along with Equation (6), the upper bound of MR damper force f is approximated as [2],

$$f \approx \frac{(c_{1a} + c_{1b} \mathrm{v})}{\left[(c_{0a} + c_{1a}) + (c_{0b} + c_{1b} \mathrm{v}) \right]} \left[(\alpha_a + \alpha_b \mathrm{v}) z_u + (c_{0a} + c_{0b} \mathrm{v}) \dot{x} \right] \quad \text{(7)}$$

Replacing z_u by z in Equation (7), an inverse relationship between damper force and applied voltage is established as follows,

$$\begin{aligned} &\{c_{1_b} \alpha_b z + c_{1_b} c_{0_b} \dot{x}\} \mathrm{v}^2 + \{(c_{1a} \alpha_b + c_{1b} \alpha_a) z + (c_{1a} c_{0b} + c_{1b} c_{0a}) \dot{x} - (c_{0b} + c_{1b}) f\} \mathrm{v} \\ &+ \{\mathrm{c}_{1\mathrm{a}} \alpha_a z + c_{1a} c_{0a} \dot{x} - (c_{0a} + c_{1a}) f\} = 0 \end{aligned} \quad \text{(8)}$$

Two new voltage laws, Inverse Quadratic Voltage Law (IQVL) and Inverse On Off Voltage Law (IOOVL) are developed based on Equation (8) by substituting the desired damping force f_d for f to determine required voltage. A linear optimal control strategy based on measured output feedback termed Optimal Static Output Feedback (OSOF) is used to obtain desired damping force f_d. A three story building with MR damper attached between ground floor and first floor (refer Figure 1) is considered to assess the effectiveness of OSOF control strategy with IQVL, IOOVL and an existing Clipped Voltage Law (CVL). The building is subjected to N-S component of the 1940 El Centro ground motion measured at Imperial Valley. Since building model is a scaled model, the time scale of ground motion data is scaled down by five times the recorded rate.

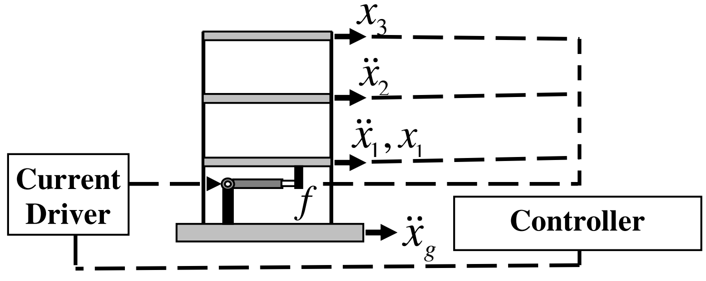

Figure 1. Three story building with MR damper

The equation of motion for the three story building is given by,

$$\boldsymbol{M}_s\,\ddot{\boldsymbol{x}}+\boldsymbol{C}_s\,\dot{\boldsymbol{x}}+\boldsymbol{K}_s\,\boldsymbol{x}=\boldsymbol{G}\,f-\boldsymbol{M}_s\boldsymbol{L}\ddot{x}_g \quad (9)$$

The state space equation representing building dynamics (Equation (9)) and the output equation are,

$$\dot{\boldsymbol{q}}=\boldsymbol{A}\,\boldsymbol{q}+\boldsymbol{B}\,f+\boldsymbol{E}\,\ddot{x}_g \quad (10)$$

$$\boldsymbol{y}=\boldsymbol{C}\,\boldsymbol{q}+\boldsymbol{D}\,f \quad (11)$$

The matrix co-efficients of Equation (9) – (10) and their numerical values are as defined by Dyke et al. [3]. The control input (i.e., damper force) using OSOF control strategy is given by [4],

$$\boldsymbol{f}_d=-\boldsymbol{K}\,\boldsymbol{y} \quad (12)$$

where, $\boldsymbol{K}$ is the constant feedback gain matrix to be determined. The quadratic Performance Index (PI) defined as, $J=\frac{1}{2}\left[\int_0^\infty\left(\boldsymbol{q}^T\boldsymbol{Q}\boldsymbol{q}+f_d^T\,R\,f_d\right)\mathrm{d}t\right]$ is used. Here $\boldsymbol{Q}=\boldsymbol{C}^T\hat{\boldsymbol{Q}}\boldsymbol{C}$ is the positive semi-definite state weighting matrix with $\hat{\boldsymbol{Q}}_{33}=1$ and R is the positive definite control input weighting matrix. Following Lewis and Syrmos [4], design equations for OSOF control to determine $\boldsymbol{K}$, which minimizes an optimal cost (i.e., PI) $J=0.5\,tr\,(\boldsymbol{PX})$ is given as,

$$\boldsymbol{A}_C^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A}_C+\boldsymbol{C}^T\boldsymbol{K}^T R\boldsymbol{K}\boldsymbol{C}+\boldsymbol{Q}=\boldsymbol{0} \quad (13)$$

$$\boldsymbol{A}_C\boldsymbol{S}+\boldsymbol{S}\boldsymbol{A}_C^T+\boldsymbol{X}=\boldsymbol{0} \quad (14)$$

$$R^{-1}\boldsymbol{B}^T\boldsymbol{P}\boldsymbol{S}\boldsymbol{C}^T(\boldsymbol{C}\boldsymbol{S}\boldsymbol{C}^T)^{-1}=\boldsymbol{K} \quad (15)$$

Here, $\boldsymbol{X}\equiv\boldsymbol{q}(0)\boldsymbol{q}^T(0)=\boldsymbol{I}$. Coupled nonlinear matrix Equations (13)-(15) are solved by Moerder–Calise iterative algorithm.

To compare the performance of OSOF control strategy, an existing control like Passive On and LQG with COC voltage law are considered and implemented. The building problem is solved using MATLAB and peak response quantities like interstory drift, displacement, and acceleration are obtained as shown in Table 1.

Table 1. Peak response quantities of building subjected to El Centro Ground Motion

Control Strategy	Displacement (cm)	Interstory Drift (cm)	Acceleration (cm/sec²)	MR Damper Force (N)	Performance Index
Uncontrolled	0.547	0.547	873.69	-	-
	0.835	0.318	1069.4		
	0.971	0.202	1408		
Passive On	0.079	0.079	273.96	964.69	-
	0.1952	0.157	495.96		
	0.3044	0.11	767.15		
LQG – COC (R = 10^{-17})	0.1204	0.1204	757.4	969.72	7.007
	0.1876	0.098	733.08		
	0.2177	0.106	735.37		
OSOF – COC (R = 10^{-17})	0.1203	0.1203	711.19	809.21	5.2378
	0.1739	0.100	383.19		
	0.2392	0.0796	553.86		
OSOF – IQVL (R = 10^{-06})	0.1274	0.1274	717.68	768.5	5.589
	0.1883	0.115	468.98		
	0.2532	0.0806	561.17		
OSOF – IOOVL (R = 10^{-08})	0.1149	0.1149	783.01	905.79	5.5149
	0.1578	0.1033	438.1		
	0.2277	0.0797	554.58		

It is concluded from the present study that OSOF control works well with MR damper. A reduction in maximum peak interstory drift and PI is obtained when using OSOF control as compared to passive-on/LQG – CVL control. The peak values of accelerations are also reduced via OSOF control, except when considering first storey accelerations using passive-on control.

References

1. Spencer BF Jr., Dyke SJ, Sain MK, Carlson JD, 1996, Phenomenological model of a magnetorhelogical damper, *ASCE Journal of Engineering Mechanics*, 123(3), 230-238.
2. Change Chih-chen, Zhou Li, 2002, Neural network emulation of inverse dynamics for a magnetorhelogical damper, *ASCE Journal of Engineering Mechanics*, 128(2), 231-239.
3. Dyke SJ, Spencer BF Jr., Sain MK, Carlson JD, 1996, Seismic response reduction using mangetorhelogical dampers, *Proceedings of IFAC World Congress*, San Franscisco, California, June 30 – July 5.
4. Lewis FL, Syrmos VL, 1995, *Optimal Control,* John Wiley & Sons, Inc., New York, 359-370.