SIMPLIFIED METHOD FOR ANALYSIS AND DESIGN OF HELICAL STAIRS

BY

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SIMPLIFIED METHOD FOR ANALYSIS AND DESIGN OF HELICAL STAIRS

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Master of Technology in Civil Engineering

(Computer Aided Structural Analysis & Design)

By

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Declaration

This is to certify that

a. The thesis comprises of my original work towards the Degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) at Nirma University and has not been submitted elsewhere for a degree.

b. Due acknowledgement has been made in the text to all other material used.

Narendra K. Patel
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Certificate

This is to certify that the Major Project entitled "Simplified Method for Analysis and Design of Helical Stairs" submitted by Narendra K. Patel (09MCL009), towards the partial fulfillment of the requirements for the degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) of Nirma University, Ahmedabad is the record of work carried out by him under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination. The results embodied in this major project, to the best of my knowledge, haven't been submitted to any other university or institution for award of any degree or diploma.

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Abstract

The design of reinforced and prestressed concrete helicoidal stairs has recently very famous among the architect. Bergman suggests an approximate method of analysis, which reduces the analysis to that of a horizontal bow girder. More accurate procedures based on an analysis of the longitudinal elastic axis of the girder as a three dimensional indeterminate structure have been presented by Fuchsteinner, Gedizli, Cohen,and Holmes.

Analysis of helicoidal stairs require long and complex equations. The aim of Present study is to develop a simplified method for analysis of helicoidal stairs. Normally, helicoidal stair is an indeterminate to the six degree. But in this report a simplification such as symmetrical loading condition is made in such a way that it reduces to two degree. Also it is further assumed that torsional modulus is taken as one half of the elastic modulus by neglecting poisson's ratio of the material and Ix/Iy is neglected due to stair slab width is very wide as compare to its thickness. Based on above assumption a stair with equal under symmetrical loading, displacement are calculated from virtual integral with application of principle of least work and two redundant are determined by solving the elastic equation and then moment and shear force are calculated.

Analysis and design of helical stair is carried out using Simplified method and Scordelis method. The comparison of Simplified method and Scordelis method is also presented. Moment and force are consider as unknown redundants. For different values of angle the table is provided to get values of unknown redundants. A spread sheet programme is developed to calculate design moment and forces. Design example is presented and structural detailing is provided.

Nomenclature

b .... Width of Stair

D .... Thickness of Waist Slab

f_{ck} .... Characteristic cube compressive strength of concrete

f_y .... Characteristic strength of steel

M_r .... Bending Moment

M_s .... Lateral Moment

M_t .... Torsional Moment

R .... Centreline Radius

X_x .... A horizontal force in the direction of the X-axis

X_r .... A moment acting about the X-axis

α .... Angle of Slope of stair (Vertical angle)

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 General

Staircases are used for the purpose of giving access to different floor of a structure. Stairs must be provided in almost all buildings,either low-rise or high-rise, even if adequate numbers of elevators are provided. Stairs consist of rises, runs (or treads), and landings. The total steps and landings are called a staircase.

1.2 History

The stairs are one of the oldest buildings in architectural history, they have always played a central role in the history of humanity, although it is difficult to tell exactly in which year they were born, it is believed his appearance was by the year 6000 before Christ. The stairs seems to change shape with the change of architectural eras, reflecting the trends used in different ages and revealing the talent of those who designed them.

The first stair in the history was wood trunk stair (Figure 1.1), these kind of stairs were used to acquire strategic positions for survival. In a basic sense, the first use which was given to the stairs was to overcome the difficulties presented by the terrain,

Figure 1.1: Wood Trunk Stair

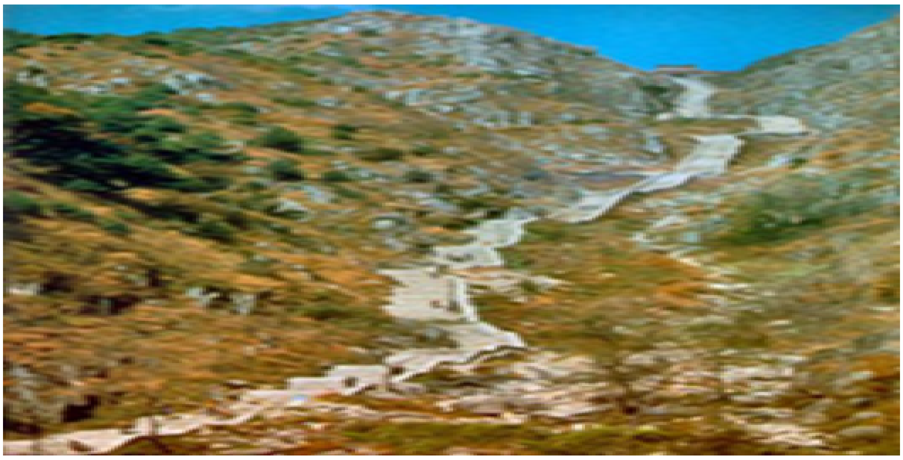


Figure 1.2: Stairs In Mountain

such as valleys or mountains (Figure 1.2), the goal was to be able to pass these difficulties as soon as possible, move up often meant moving to a place of greater security, then this could have meant at that time the difference between life and death, it was very important to move quickly, hence the importance of the stairs.

In the history of the stairs they first emerged as a solution to a problem, although, years later it was found in China the first granite staircase leading to the sacred mountain in Tai Shan, this indicates that one of the utilities that was given to the stairs in his story was for religious purposes. Confucius in one of his stories said to have gone up this ladder to the top in the year 55 BC. The ladder was used in a metaphoric way reach the divine height and establish a connection between earth and sky. Other examples of stairs built for religious purposes are: the biblical Jacob's ladder, the tower of Babel, which was a helical tower, the pyramids of Egypt that had stairs, the celestial ladder of Shantung in China, the stairs in India, a peculiarity of the stairs in India is that they had also scientific utility. All these stairs have something in common, they symbolize the rise of the light, the sun, and a way in to the gods path. Later in the history of stairs, spiral stairs wede used in clastles for military reasons, the proliferation of spiral stairs in castles was not casual, they allow a strategic position to the soldier who defended the castle, these spiral staircases and railings were built in order to make the solder placed in top an advantage, this soldier would have his right hand full of space to move his sword, while the woldier placed on the bottom would constantly hit the wall whily fighting, because he would have blocked part of the range of motion of his right, besides, his head would be easy to reach for his opponent, the lack of handrails was not casual, the aim was to push the opponent over the edge of the stair.

1.3 Stairs In Present

The end of the nineteenth century is regarded by many as the golden era of construction of stairs, Peter Nicholson developed a mathematical system for stairs and railings approaching the art of the stairs to the workers of wood and metal. By the end of 1980 Eva Jiricna in London started designing stairs in glass and stainless steel which gave the stairs a sleek and futuristic look.
Today it is increasingly common to exit the conventional design of iron and wood and move on to different materials such as stainless steel, glass and titanium.

1.4 Type Of Stairs

There are different types of stairs, which depend mainly on the type and function of the building and on the architectural requirements. The most common types are as follows.

1.4.1 Single-flight stairs

The structural behaviour of a flight of stairs is similar to that of a one-way slab supporting at both ends. The thickness of the slab is referred as the waist (slab) in Figure 1.3. When the flight of stairs contains landing, it may be more economical to provide beams at B and C between landing (Figure 1.4).

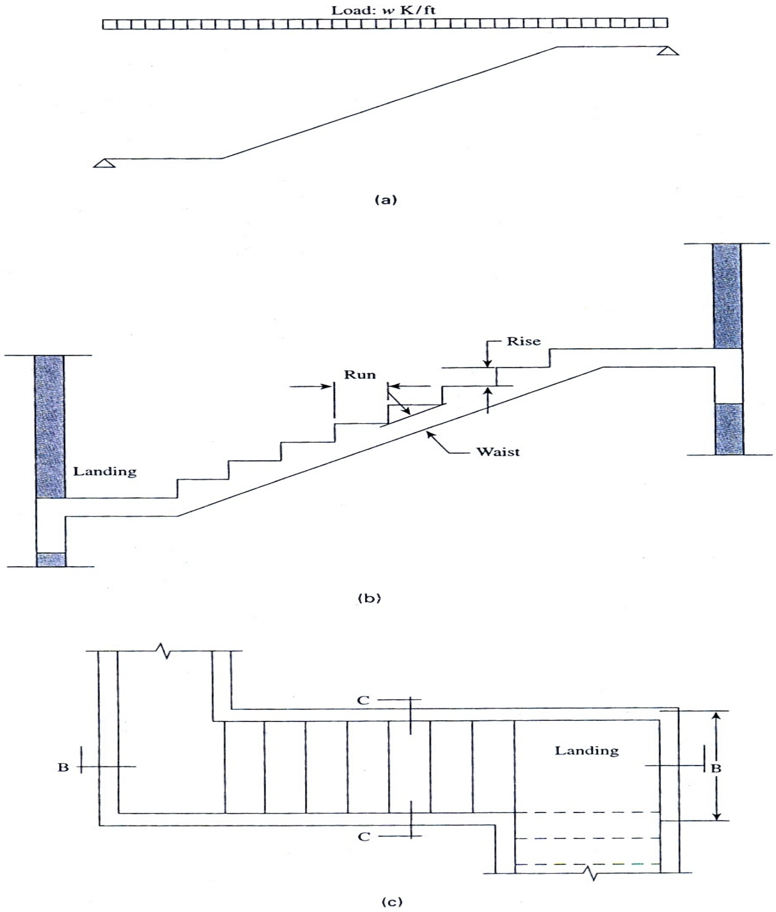


Figure 1.3: a)Loads b) Section at B-B c) Plan

1.4.2 Double-flight stairs

It is more convenient in most buildings to build the staircase in double flight between floors.The types commonly used are quarter-turn(Figure 1.5(a)) and closed or open well stairs as shown in Figure 1.5(b) and Figure 1.6.

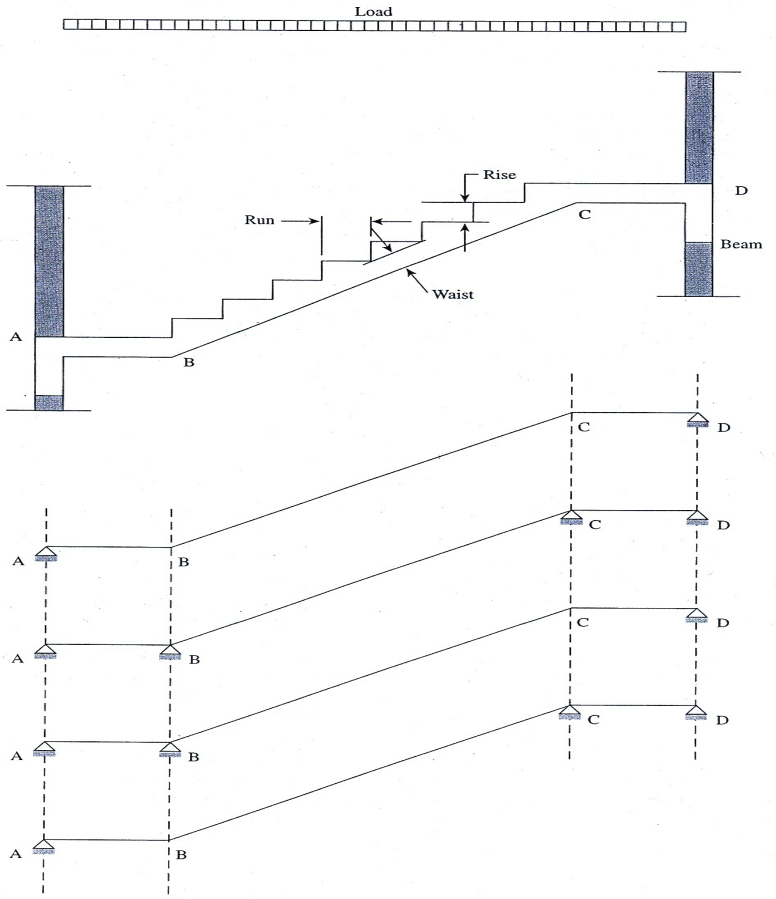
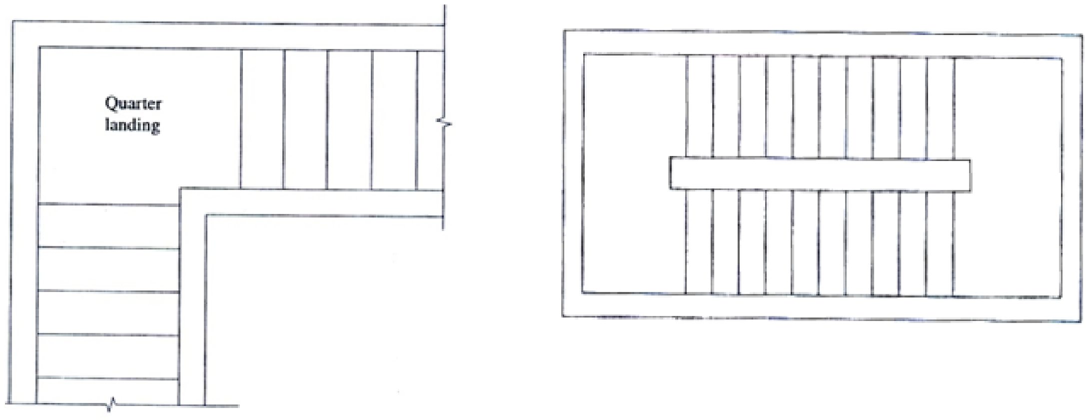


Figure 1.4: Supporting system of single flight

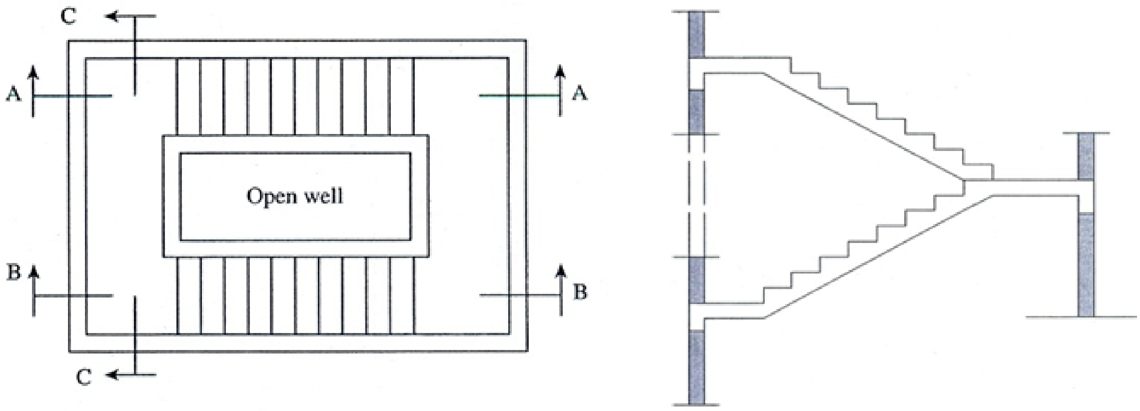
1.4.3 Three or more flights of stairs

In some cases, Where the overall dimensions of the staircase are limited, three or four flight may be adopted (Figure 1.7)



a) b)

Figure 1.5: a) Quarter turn stair b) Closed Well Stair



a) b)

Figure 1.6: (a)Open Well Stair (b) Section at B-B

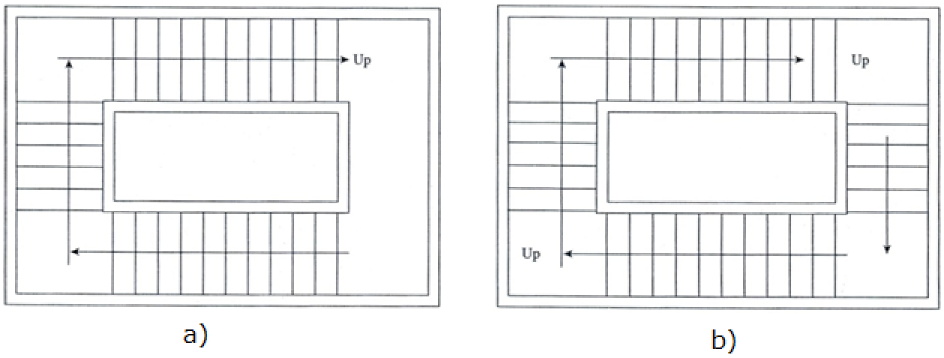


Figure 1.7: a) Three flight b) Four flight

1.4.4 Cantilever Stairs

Cantilever Stairs are used mostly in fire-escape stairs, and they are supported by concrete wall or beams. The stair step may be of the full-flight type, Projecting from one side of the wall, the half-flight type, projecting from both sides of the supporting wall, or of the semi spiral type as shown in Figure 1.8. In this type of stairs, each step acts as a cantilever, and the main reinforcement is placed in the tension side of the run and the bars are anchored within the concrete wall. Shrinkage and temperature reinforcement is provided in the transverse direction.

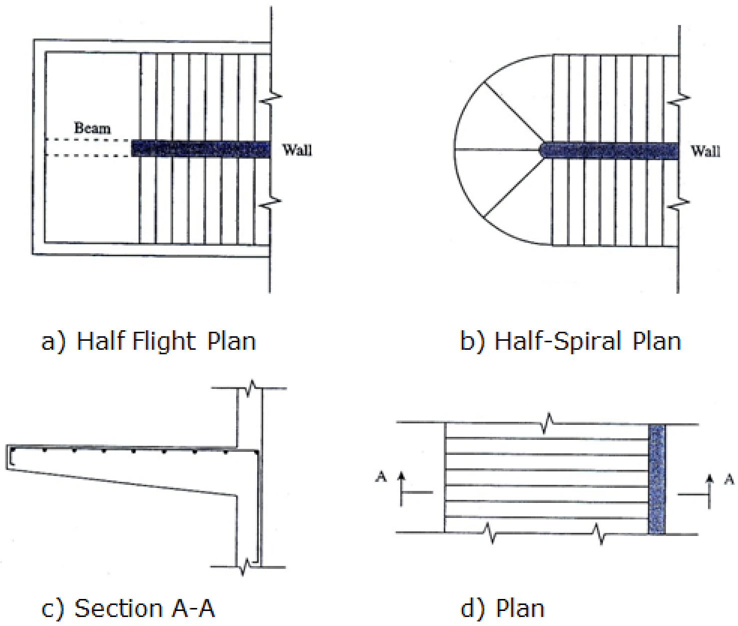


Figure 1.8: Steps projecting from one or two sides of the supporting wall

1.4.5 Precast flight of stairs

The speed of construction in some project requires the use of precast flight of stairs (Figure 1.9). The flight may be cast separately and then fixed to cast-in-place landing. In other cases, the flight, including the landing, are cast and then placed in position on their supporting walls or beams. They are designed as simply supported one-way slab with the main reinforcement at the bottom of the stair waist.

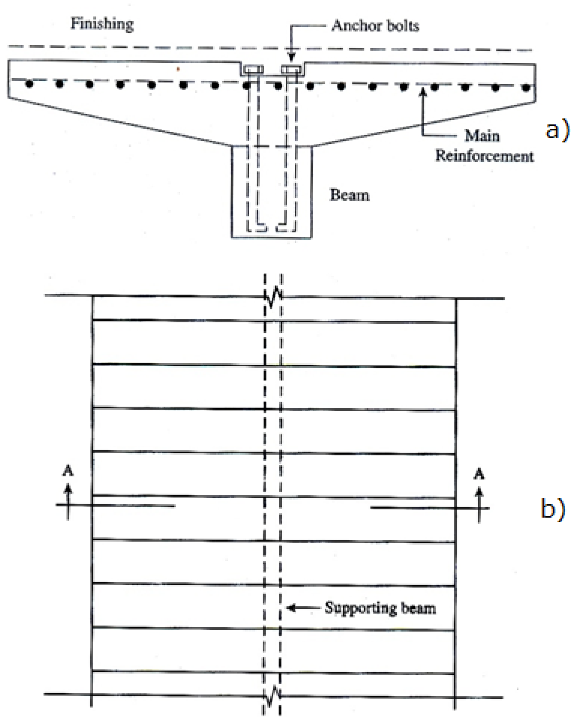


Figure 1.9: Precast cantilever stair supported by central beam (a)section A-A (b) Plan

1.4.6 Free-standing staircase

In this type of stairs, the landing projects in the air without any support at its end (Figure 1.10). The stairs behave in a springboard manner, causing torsional stresses in the slab.

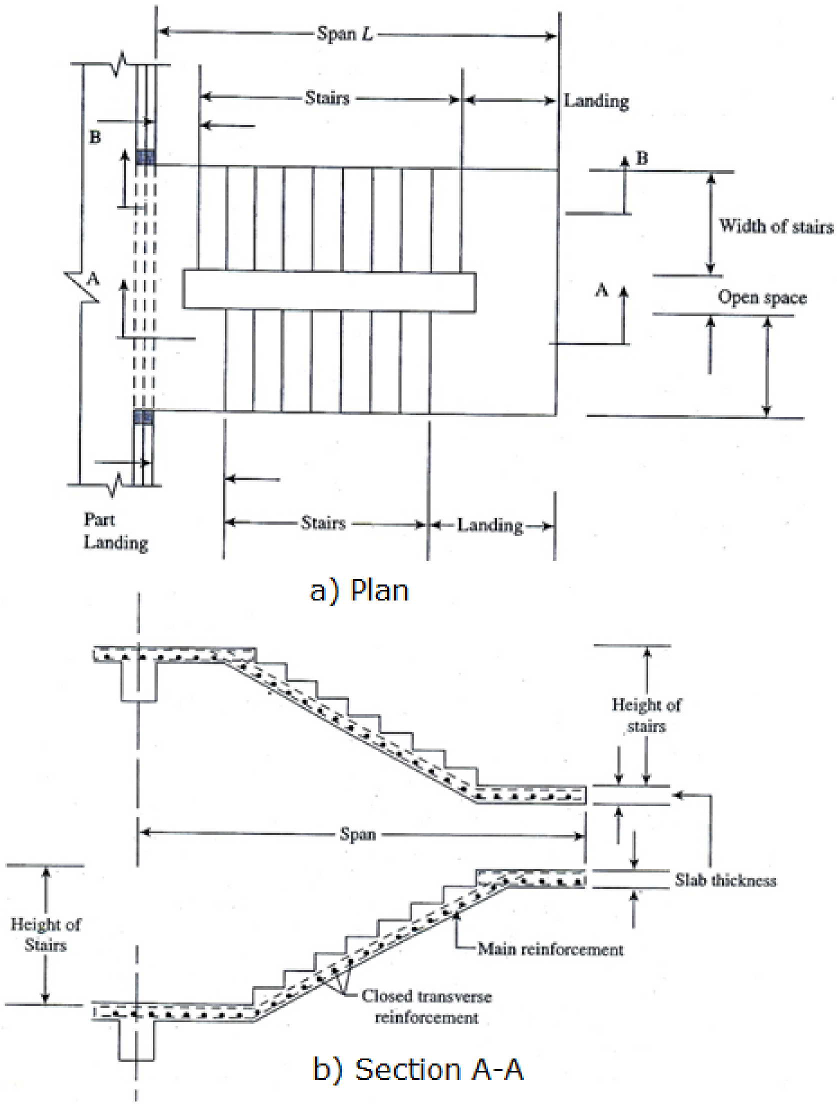


Figure 1.10: (a) Plan of free standing staircase (b) Section of free standing staircase

1.4.7 Run-riser stairs[15]

Run-riser stairs are stepped underside stairs that consist of a number of runs and risers rigidly connected without the provision of the normal waist slab (Figure 1.11).This type of stairs has an elegant appearance and is sometimes favored by architects.The structural analysis of run-riser stairs can be simplified by assuming that the effect of axial forces is negligible and that the load on each run is concentrated at the end of the run (Fig. 1.12(a)). For the analysis of a simply supported flight of stairs. consider a simple flight of two runs, ABC. subjected to a concentrated load P at B' (Figure 1.12(a)). Because joints B and B' are rigid, the moment at joint B is equal to the moment at B', or

$$M_B = M'_B = \frac{PS}{2} \tag{1.1}$$

where s is the width of the run.The moment in rise BB' is constant and is equal to PS/2 when the rise is absent the stair ABC act as a simply supported beam and the maximum bending moment occurs at mid-span with value

$$M_B = \frac{PL}{4} = \frac{PS}{2} \tag{1.2}$$

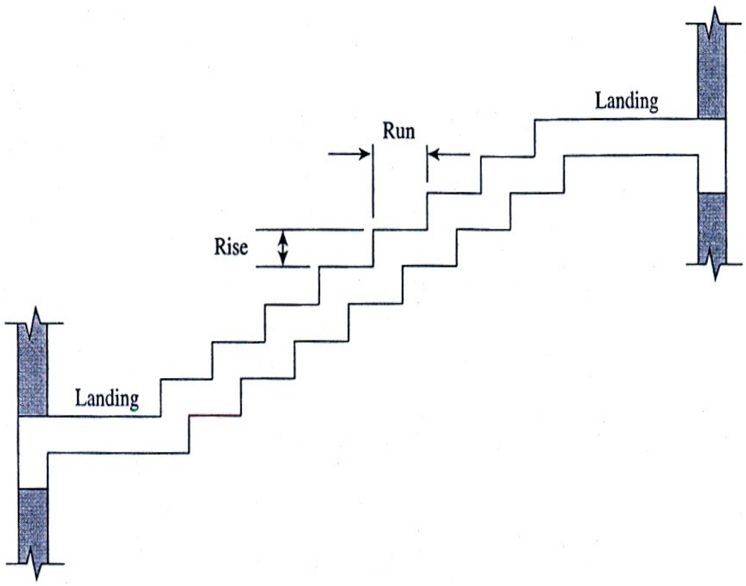


Figure 1.11: Cross Section of Run riser staircase

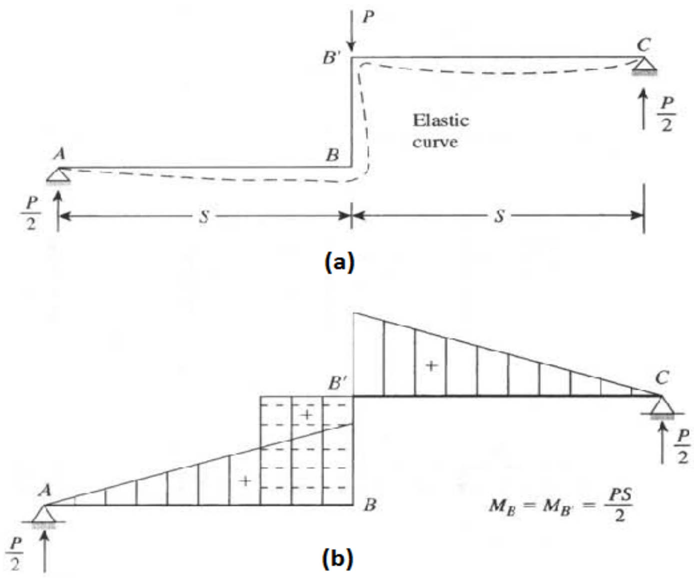


Figure 1.12: (a) Elastic Curve (b) Bending Moment Diagram

1.4.8 Helical stairs

A helical staircase is a three-dimensional structure, which usually has a circular shape in plan.

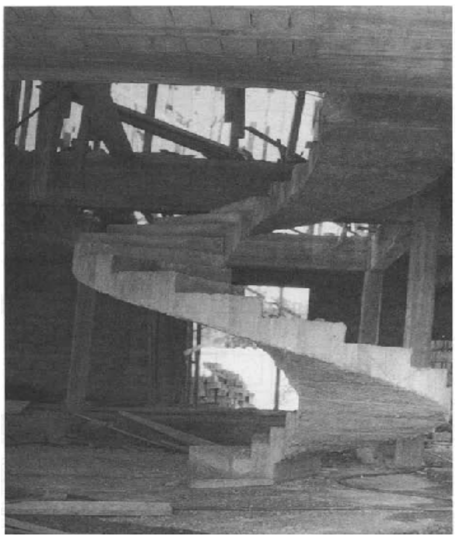


Figure 1.13: Helical Stair

It is distinctive type of stairs used mainly in prestigious buildings, posh bungalows,entrance hall,theater foyer,and special low-rise office buildings (Figure 1.13).The forces acting at any section may consist of vertical moment, lateral moment, torsional moment, axial force, shearing force across the waist of the stairs, and radial horizontal shearing force.

1.5 Objective of Work

The main objective of the present study are :

- To study the Scordelis Method for the analysis of Helical Stair.
- Simplify the Scordelis Method.
- To elaborate the Simplified Method for the analysis of Helical Stair
- Compare Scordelis and Simplified Method.
- To develop the table based on simplified method to find redundant directly for different horizontal angle.
- To provide design and detail of Helical Stair.

1.6 Scope of Work

In order to achieve above objectives, scope of major project work is as follows:

- Study the behaviour of helical stair.
- Study of scordelis method.
- To simplify the scordelis method.
- Analysis example of helical stair based on scordelis and simplified method.
- To compare the vertical moment, lateral moment and torsional moment by simplified and scordelis method.
- Design and Detail of helical stair.

1.7 Organization of Major Project

The organisation of chapters in this Project is as follows.
Chapter 2 covers the literature review from research paper and books. It gives overall idea about method of analysis.
Chapter 3 includes analysis of helical stairs based on simplified method and scordelis method and analysis example based on this method.Parametric study incorporates the influence of horizontal angle and centreline radius on vertical,lateral and torsional moment.
Chapter 4 presents the design of helical stairs based on analysis by simplified and scordelis method. It also includes detail drawing.
Chapter 5 summarizes the work done in the major project. It consists summary of work done, various conclusions obtained from the study and future scope of work.

Chapter 2

Literature Survey

2.1 General

Recently,Helical staircase have been constructed,supported only at the top and bottom.Although they are circular in plan projection,in elevation their description is helicoidal.Various analyses are available to solve such a complicated problem.From each analysis,torsional moment,bending moment,shear forces and axial thrusts are resulted.The geometry of each helical staircase affects the application of load and hence the results.This subject has been throughly reviewed in depth by various researchers.

2.2 Literature Review

Various papers have been referred for Analysis of Helical Stairs.Some of the important papers and books are summarized below.

A.C.Scordelis [**1**] analyze helicoidal girder for a uniform vertical load of 1 lb per lineal ft of horizontal projection of the girder longitudinal axis. The girder, which is fixed at the ends, is statically indeterminate to the sixth degree. By selecting the redundants at mid span, and using principles of symmetry, all but two of the redundants become equal to zero. And gives general equations for the determination of the redundant at mid span of a uniformly loaded helicoidal girder fixed at the ends and tabulate results for the mid span redundant of 510 different girders having rectangular cross sections, the variables being horizontal angle,angle of slope, and width-depth ratio of cross section.

Morgan [**14**]developed one of the first method for helical stair. It is based on the strain energy method.It is based on freely supported flight. The loading is assumed to be symmetrical and structure is divided at centre. Also derived various expression for helical staircase flight with far end fixed using strain energy method. various moments including torsional moments are computed. The analysis also gives shears and axial thrusts.

Cusen [**8**] analysed helical stair with fixed ends indeterminate to sixth degree. By considering symmetrical loading and cut at the midspan two redundant forces become equal to zero. To redundants force are horizontal force in radial direction and bending moment acting in tangential plane. The general equations of vertical moment , lateral moment ,torsional moment , thrust shearing force and radial horizontal shearing force are given. By using this equation Charts are developed for the Design of Helical Stairs with Fixed Supports

Himat T. Solanki [**3**] analyze helicoidal girders by Castigliano's Theorem and provides the equations of Moment and Forces.

Cohen [**6**] analyzed helical staircase as a determinate and indeterminate conditions. The distributed load can be non uniform with a non uniform bending moment per unit length of the curve. General equations of equilibrium are related to three loaded axes. An element of an arc is consider for twisted curve. For statically indeterminate staircase the equations of equilibrium are not sufficient and in addition, equations of deformation and angular rotation at any point is considered. The determinate beam staircase involves cantilevers with supported beams and beam with three supports.

Scordelis [**16**] Presented Study of helicoidal girder,fixed at the ends,subtending a horizontal angle of 180^o,and having a slope of 30^o.Four Girders,each of having a different uniform rectangular cross section through out its length are investigated.The determination of influence lines for end reaction due to vertical load is illustrated using analytical and experimental methods.The effect of width to depth ratio of cross section on end reaction and internal forces is discussed.

Cusens and Trirojna [17] Presented Method of Analysis for 80^o helicoidal Staircase fixed at both ends investigate the behaviour of a half scale model under uniform load.The total angle in plan is 80° and width of the staircase is 2 m and thickness is 0.15 m. Test are described on half scale models under uniformly distributed loading. Construction and testing of two models. first model is loaded with sand bags each weighing 40 kg. First crack at midspan and the top and bottom ends of the staircase occurred simultaneously at a load of 2040 kg. Second model loaded with pig iron bars weight approximately 25 kg each. First Crack observed at a load of 1490 kg.

Chatterjee [4] has discussed the analysis of helical girder as a curve beam and as a space structure.Based on this method a design example is given.

Designers Handbook by Reynolds and Steedman [5] provides the charts to find out moment and forces in helical stairs.

Chapter 3

Helical Stairs

3.1 Introduction

A helical staircase is a three-dimensional structure, which usually has a circular shape in plan (Figure 3.1). It is a distinctive type of stairs used mainly in entrance halls, theater foyers, and special low-rise office buildings. The cost of a helical stair is much higher than that of a normal staircase. The stairs may be supported at some edges within adjacent walls or may be designed as a free-standing helical staircase, which is most popular. The structural analysis of helical staircases is complicated and was discussed by Morgan[14] based on strain energy method, Scordelis [1] based on virtual work method and Cusens[8] by strain energy method. Under load, the flight slab will be subjected to torsional stresses throughout. The upper landing will be subjected to tensile stresses, whereas compressive stresses occur at the bottom of the flight. The forces acting at any section may consist of vertical moment, lateral moment, torsional moment, axial force, shearing force across the waist of the stairs, and radial horizontal shearing force. The main longitudinal reinforcement consists of helical bars placed in the concrete waist of the stairs and runs from the top landing to the bottom support. The transverse reinforcement must be in a closed stirrup form to resist torsional stresses or in a U-shape lapped at about the mid width of the stairs.

Figure 3.1: Helical stair

3.2 Analysis

3.2.1 Scordelis Method

The geometry of a helicoidal girder may be defined, as shown in Fig 3.2, in terms of its centerline radius R, horizontal angle 2Φ ,angle of slope α, width b,and depth h. The helicoid may be either left-hand or right-hand as shown in Fig. 3.2.

The helicoidal girder will be analyzed for a uniform vertical load of 1 lb/ft of horizontal projection of the girder longitudinal axis. The girder, which is fixed at

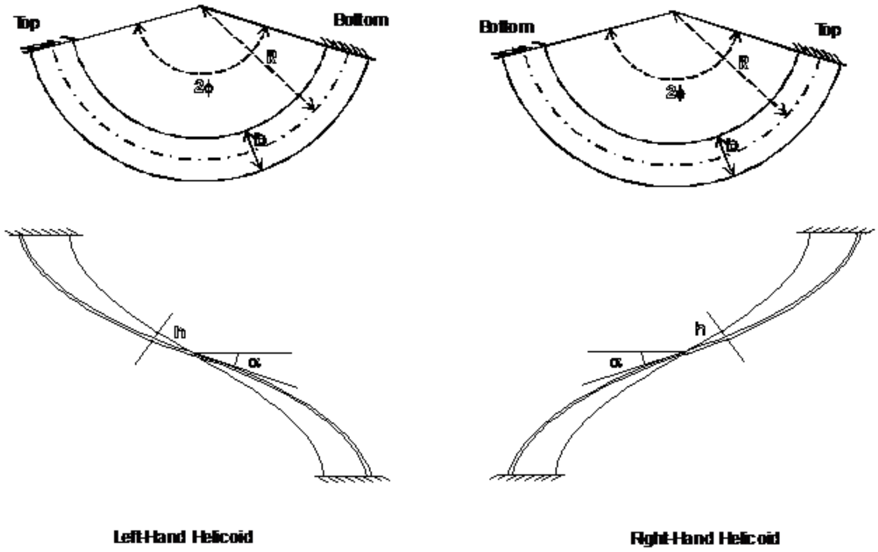


Figure 3.2: Geometry of Helicoidal Girders[1]

the ends, is statically indeterminate to the sixth degree. By selecting the redundants at midspan, and using principles of symmetry, all but two of the redundants become equal to zero. This greatly simplifies the problem. The two redundants at the midspan cut are X_x, a horizontal force in the direction of the X-axis; and X_r, a moment acting about the X-axis. These are shown with their positive directions in Fig. 3.3. A moment vector is shown with a double arrowhead. The vector indicates the axis about which the moment acts and the right hand rule should be used to determine the direction of the moment. Using the principle of superposition the displacements in the direction of the redundants may be written as follows:

$$\delta_{xw} + X_x\delta_{xx} + X_r\delta_{xr} = 0 \tag{3.1}$$

$$\delta_{rw} + X_x\delta_{rx} + X_r\delta_{rr} = 0 \tag{3.2}$$

In the above equations the δ term indicate relative displacements of the two ends of the girder at the midspan cut :

δ_{xw}= Relative linear displacement in the direction of the x-axis due to uniform load of 1 lb/ft of horizontal projection with the redundant equal to zero;

δ_{rw}= Relative angular displacement about the x-axis due to uniform load of 1 lb

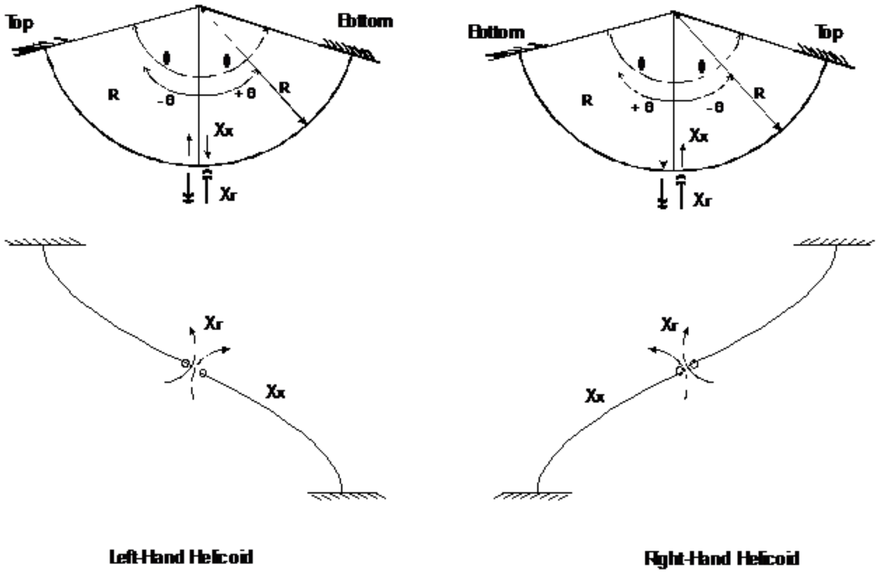


Figure 3.3: Redundant Positive Direction[1]

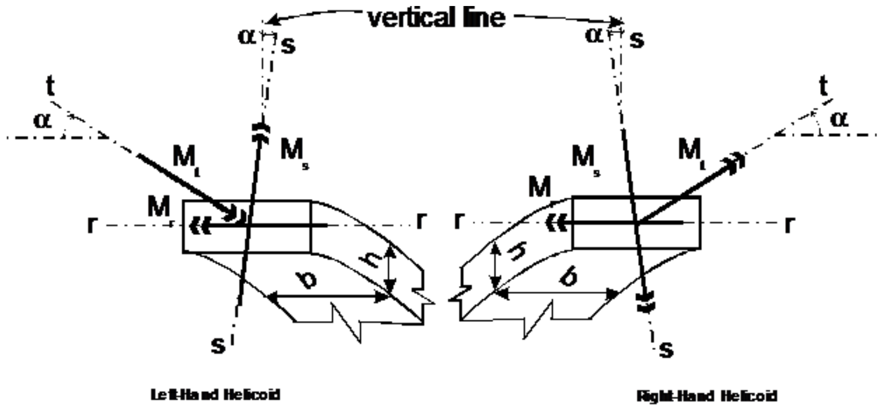


Figure 3.4: Bending and Torsional Moments Positive Direction[1]

per lineal foot of horizontal projection with the redundant equal to zero;

δ_{xx}= Relative linear displacement in the direction of the x-axis due to X_x= 1;

δ_{rx}= Relative angular displacement in the direction of the x-axis due to $X_x = 1$

δ_{xr}= Relative linear displacement in the direction of the x-axis due to X_r= 1;

δ_{rr}= Relative angular displacement in the direction of the x-axis due to $X_r = 1$;

By Maxwell law of reciprocal,displacement $\delta_{xr} = \delta_{rx}$.To solve equations 3.1 and 3.2 for the redundants, five displacements must be determined δ_{xw}, δ_{rw} ,δ_{xx} ,$\delta_{xr} = \delta_{rx}$ and δ_{rr} Any method may be used to determine the displacements, one of the most direct being the method of virtual work. For example

$$\delta_{xw} = \int_{-\Phi}^{\Phi} \frac{m_{rx} m_{rw}}{EI_r} Rd\Phi + \int_{-\Phi}^{\Phi} \frac{m_{sx} m_{sw}}{EI_x} Rd\Phi d\Phi + \int_{0}^{-\Phi} \frac{m_{tx} m_{tw}}{GJ_t} Rd\Phi \quad (3.3)$$

m_{rw}, m_{sw} and m_{tw} represent bending and torsional moments in the girder due to the uniform load of 1 kN/m of horizontal projection with the redundants equal to zero. Referring to Fig. 3.3 and Fig. 3.4, at any location θ from midspan these may be expressed by statics as follows:

$$m_{rw} = -R^2(1 - \cos\theta) \quad (3.4)$$

$$m_{sw} = -R^2(\theta - \sin\theta)\sin\alpha \quad (3.5)$$

$$m_{tw} = -R^2(\theta - \sin\theta)\cos\alpha \quad (3.6)$$

m_{rx} ; m_{sx} and m_{tx} represent bending moments and torsional moment in the stair due to $X_x = 1$

$$m_{rx} = -R(\theta\sin\theta)\tan\alpha \quad (3.7)$$

$$m_{sx} = R(\sin\theta)\cos\alpha + R(\theta\cos\theta)\sin\alpha\tan\alpha \quad (3.8)$$

$$m_{tx} = -R(\sin\theta)\sin\alpha + R(\theta\cos\theta)\sin\alpha \quad (3.9)$$

m_{rr}, m_{sr} and m_{tr} represent bending and torsional moments in the stair due to $X_r = 1$

$$m_{rr} = \cos\theta \quad (3.10)$$

$$m_{sr} = \sin\theta\sin\alpha \quad (3.11)$$

$$m_{tr} = -\sin\theta\cos\alpha \quad (3.12)$$

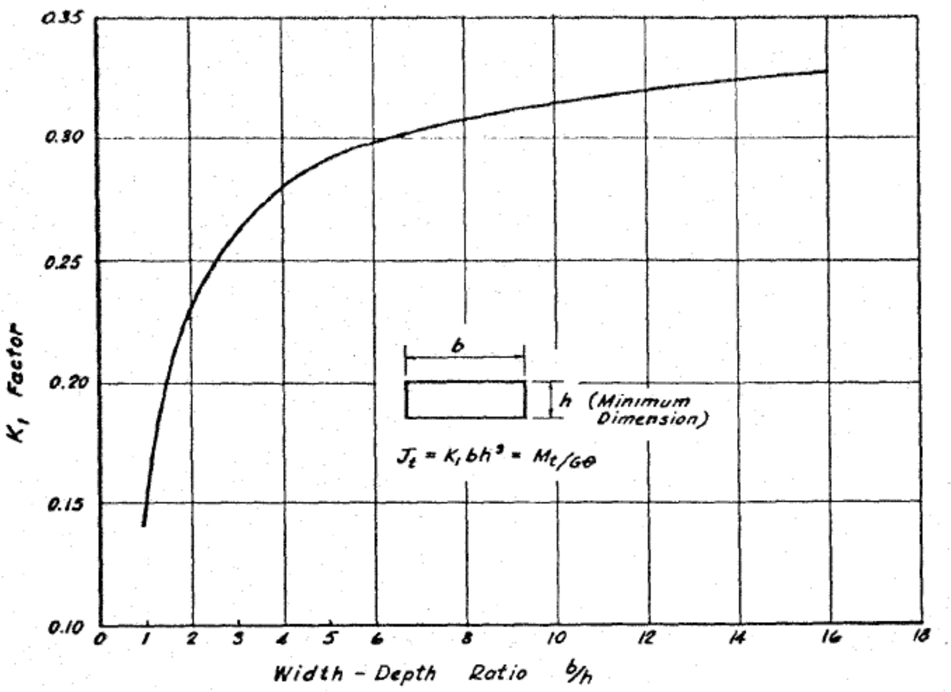


Figure 3.5: Torsion Constant for Rectangular Sections[18]

EI_r and EI_s represent the bending stiffness about the r and s-axes, respectively, and GJ_t represents the torsional stiffness about the t-axis. For rectangular cross sections $EI_r$and EI_scan be easily calculated. G can be taken equal to 3/7 E for concrete and J can be calculated by means of the expression $J_t = k_1bh^3$ in which k_1 is a constant depending on the width-depth ratio of the cross section.

General formulas for each of the five displacements needed in the solution of Equations 3.1 and 3.2 have been derived in terms of the horizontal angle ϕ ,angle of slope α, centerline radius R and the bending and torsional stiffness.

$$\begin{aligned}\delta_{xw} &= \frac{R^4 \tan\alpha \sec\alpha}{EI_r}[\sin\Phi - \Phi\cos\Phi - \frac{1}{8}(\sin 2\Phi - 2\Phi\cos 2\Phi)] \\ &+\frac{R^4 \sec\alpha \sin\alpha}{EI_s}[(\Phi/2 - \sin\Phi + \Phi\cos\Phi - \frac{\sin 2\Phi}{4})\cos\alpha \\ &-(2\Phi\cos\Phi + (\Phi^2 - 2)\sin\Phi - \frac{1}{8}(\sin 2\Phi - 2\Phi\cos 2\Phi))\tan\alpha\sin\alpha] \\ &+\frac{R^4 \sin\alpha}{GJ_t}\end{aligned}$$

$$[(3-\Phi^2)\sin\Phi - 3\Phi\cos\Phi - \frac{\Phi}{2} + \frac{3}{8}\sin 2\Phi - \frac{\Phi\cos 2\Phi}{4}] \tag{3.13}$$

$$\begin{aligned}\delta_{rw} &= \frac{R^3\sec\alpha}{EI_r}[\Phi/2 - \sin\Phi + \frac{\sin 2\Phi}{4}] \\ &+\frac{R^3\sin^2\alpha\sec\alpha}{EI_s}[\Phi/2 - \sin\Phi - \frac{\sin 2\Phi}{4} + \Phi\cos\Phi] \\ &+\frac{R^3\cos\alpha}{GJ_t}\,[\frac{\Phi}{2} - \sin\Phi - \frac{\sin 2\Phi}{4} + \Phi\cos\Phi]\end{aligned} \tag{3.14}$$

$$\begin{aligned}\delta_{xx} &= \frac{R^3\sec\alpha\tan^2\alpha}{EI_r}(\frac{\Phi^3}{6} - (\frac{\Phi^2}{4} - \frac{1}{8}) \\ &\sin 2\Phi - \frac{\Phi\cos 2\Phi}{4}) + \frac{R^3\sec\alpha}{EI_s}([\frac{\Phi}{2} - \\ &\frac{\sin 2\Phi}{4}]\cos^2\alpha + \frac{1}{4}[\sin 2\Phi - 2\Phi\cos 2\Phi]\sin^2\alpha \\ &[\frac{\Phi^3}{6} - (\frac{\Phi^2}{4} - \frac{1}{8})\sin 2\Phi + \frac{\Phi\cos 2\Phi}{4}] \\ &\tan^2\alpha\sin^2\alpha) + \frac{R^3\sec\alpha\sin^2\alpha}{GJ_t} \\ &([\frac{\Phi}{2} - \frac{sin 2\Phi}{4}] - \frac{1}{4}[\sin 2\Phi - 2\Phi\cos 2\Phi] + \\ &[\frac{\Phi^3}{6} - (\frac{\Phi^2}{4} - \frac{1}{8})\sin 2\Phi - \frac{\Phi\cos 2\Phi}{4}])\end{aligned} \tag{3.15}$$

$$\begin{aligned}\delta_{rr} &= \frac{R\sec\alpha}{EI_r}[\frac{\Phi}{2} + \frac{\sin 2\Phi}{4}] + \frac{R\sec\alpha\sin^2\alpha}{EI_s} \\ &[\frac{\Phi}{2} - \frac{sin 2\Phi}{4}] + \frac{R\cos\alpha}{GJ_t}[\frac{\Phi}{2} - \frac{sin 2\Phi}{4}]\end{aligned} \tag{3.16}$$

$$\begin{aligned}\delta_{rx} &= \delta_{xr} = -\frac{R^2\sec\alpha\tan\alpha}{EI_r}[\frac{1}{8}(\sin 2\Phi - 2\Phi\cos 2\Phi)] \\ &+\frac{R^2\sin\alpha\sec\alpha}{EI_s}[(\frac{\Phi}{2} - \frac{\sin 2\Phi}{4})\cos\alpha\end{aligned}$$

$$+\frac{1}{8}(\sin 2\Phi - 2\Phi\cos 2\Phi)\tan\alpha\sin\alpha] + \frac{R^2\sin\alpha}{GJ_t[-(\frac{\Phi}{2} - \frac{sin2\Phi}{4}) + \frac{1}{8}(\sin 2\Phi - 2\Phi\cos 2\Phi)]} \quad (3.17)$$

Uniform vertical load (impose load), torque loading and internal forces -deal load produce an eccentricity with respect to the center line of the stair (The stair having high width- center line radius ratio $\frac{b}{R}$. This may be expressed as follows

$$e = \frac{b^2}{12R} \quad (3.18)$$

m_{rt},m_{st} and m_{tt} represent bending and torsional moment in the girder due to uniform torque loading of 1 lbft

$$m_{rt} = -R(1 - \cos\theta) \quad (3.19)$$

$$m_{st} = R\sin\theta\cos\alpha \quad (3.20)$$

$$m_{tt} = R\sin\theta\cos\alpha \quad (3.21)$$

To facilitate the contribution of torque loading, Table B of Scordelis paper[2] gives results for X_r/R and X_x for a wide range of variables. Using these redundants and from Equations 3.4 to 3.12 and Equations 3.18 to 3.21, the final bending and torsional moment at any section of the girder may be found as,

$$M_r = m_{rw} + em_{rt} + X_x m_{rx} + X_r m_{rr} \quad (3.22)$$

$$M_s = m_{sw} + em_{st} + X_x m_{sx} + X_r m_{sr} \quad (3.23)$$

$$M_t = m_{tw} + em_{tt} + X_x m_{tx} + X_r m_{tr} \quad (3.24)$$

3.2.2 Simplified Method

Structurally helicoidal are mostly used in the design of staircases. The helicoidal stair analysis considering it as an indeterminate to the sixth degree space structure has been investigated (Bangash 1999; Born 1962; Fuchssteiner 2000; Koseoglu 1980) . If it is assumed to be fixed at its ends and assuming symmetrical loading and by cutting at mid span, only two redundants remain to be solved, the other being equal to zero (Chatterjee 1988; Cusen 1966; Santathadaporn 1966; Scordelis 1960a; Scordelis 1960b).

The analysis of a stair with both ends fixed as shown in Fig. 3.2 is indeterminate to the six degree. If the loading is symmetrical the problem reduces to only two redundants and other redundants becomes equal to zero. To solve these, the stair is cut at the center and the displacements due to external load and two unknown redundants are calculated from the virtual integral, whose general expression is as follows:

$$EI_x\delta_{1,2} = \int M_{x1}M_{x2}ds + \int M_{y1}M_{y2}(\frac{I_x}{I_y})ds + \int M_{t1}M_{t2}(\frac{EI_x}{GJ})ds \qquad (3.25)$$

The direction of redundants in positive direction are shown in Fig. 3.3 and Fig. 3.4, it shown the positive direction of bending and torsional moments. Using the principle of least work (superposition) for finding out the displacements in the direction of the redundants, the elastic equation may be written as follows:

$$\delta_{xw} = X_x\delta_{xx} + X_r\delta_{xr} \qquad (3.26)$$

$$\delta_{rw} = X_x\delta_{rx} + X_r\delta_{rr} \qquad (3.27)$$

in Which,

δ_{xw}= Relative linear displacement in the direction of the x-axis due to uniform load of 1 lb per ft of horizontal projection with the redundant equal to zero;

δ_{rw}= Relative angular displacement about the x-axis due to uniform load of 1 lb per ft of horizontal projection with the redundant equal to zero;

δ_{xx}= Relative linear displacement in the direction of the x-axis due to X_x= 1;

δ_{rx}= Relative angular displacement in the direction of the x-axis due to $X_x = 1$

δ_{xr}= Relative linear displacement in the direction of the x-axis due to X_r= 1;

δ_{rr}= Relative angular displacement in the direction of the x-axis due to $X_r = 1$;

By Maxwell law of reciprocal,displacement $\delta_{xr} = \delta_{rx}$. From Fig. 3.3 and Fig. 3.4, for any location of θ from mid span, the expression may be written as follows (Scordelis 1960a; Scordelis 1960b):

$$m_{rw} = -R^2(1 - \cos\theta) \tag{3.28}$$

$$m_{sw} = -R^2(\theta - \sin\theta)\sin\alpha \tag{3.29}$$

$$m_{tw} = -R^2(\theta - \sin\theta)\cos\alpha \tag{3.30}$$

in which,m_{rw} ,m_{sw} , and m_{tw} represent bending moments and torsional moment in the stair due to the uniform load of 1 lb per foot of horizontal projection with redundants equal to zero.

$$m_{rx} = -R(\theta\sin\theta)\tan\alpha \tag{3.31}$$

$$m_{sx} = R(\sin\theta)\cos\alpha + R(\theta\cos\theta)\sin\alpha\tan\alpha \tag{3.32}$$

$$m_{tx} = -R(\sin\theta)\sin\alpha + R(\theta\cos\theta)\sin\alpha \tag{3.33}$$

in which, m_{rx} ; m_{sx} and m_{tx} represent bending moments and torsional moment in the stair due to $X_x = 1$

$$m_{rr} = \cos\theta \tag{3.34}$$

$$m_{sr} = \sin\theta\sin\alpha \tag{3.35}$$

$$m_{tr} = -\sin\theta\cos\alpha \tag{3.36}$$

m_{rr}, m_{sr} and m_{tr} represent bending and torsional moments in the stair due to $X_r =$ 1.

Uniform vertical load (impose load), torque loading and internal forces -deal load produce an eccentricity with respect to the center line of the stair (The stair having high width- center line radius ratio $\frac{b}{R}$. This may be expressed as follows (Scordelis 1960a; Scordelis 1960b):

$$e = \frac{b^2}{12R} \tag{3.37}$$

Therefore:

$$m_{rt} = -R(1 - \cos\theta) \tag{3.38}$$

$$m_{st} = R\sin\theta\sin\alpha \tag{3.39}$$

$$m_{tt} = R\sin\theta\cos\alpha \tag{3.40}$$

m_{rt}, m_{st}, and m_{tt} represent bending and torsional moments in the stair due to an uniform vertical load (torque load) of 1 lbft with redundants equal to zero. From Equation 3.25,3.28 to Equation 3.36 and Figs 3.3 and 3.4, the following equation can be expressed:

$$EI_r\delta_{xw} = \int_0^{\phi} \frac{m_{rx}m_{rw}}{\cos\alpha} Rd\phi + \int_0^{\phi} \frac{m_{sx}m_{sw}}{\cos\alpha}\frac{I_r}{I_s} Rd\phi + \int_0^{-\phi} \frac{m_{tx}m_{tw}}{\cos\alpha}\frac{EI_r}{GJ} Rd\phi \tag{3.41}$$

The torsional rigidity may be taken as:

$$GJ = \frac{2EI_rI_s}{I_r + I_s} \tag{3.42}$$

Then,

$$\frac{EI_r}{GJ} = \frac{1}{2}(\frac{I_r}{I_s} + 1) \tag{3.43}$$

The stair slab width is very wide as compare to its thickness i.e. I_s much greater than I_r and hence the ratio of $\frac{I_r}{I_s}$ could be neglected. Then Eq. can be reduced to:

$$EI_r\delta_{xw} = \int_0^{\phi} \frac{m_{rx}m_{rw}}{\cos\alpha} Rd\phi + \int_0^{-\phi} \frac{m_{tx}m_{tw}}{2\cos\alpha} Rd\phi \tag{3.44}$$

Now,

$$\delta_{xw} = \int_0^{\phi} \frac{[-R(\theta \sin\theta)\tan\alpha][-R^2(1-\cos\theta)]}{\cos\alpha} R d\theta +$$
$$\int_0^{-\phi} \frac{[-R(\theta\sin\theta)\sin\alpha + R(\theta\cos\theta)\sin\alpha][-R^2(\theta-\sin\theta)\cos\alpha]}{2\cos\alpha} R d\theta \qquad (3.45)$$

Eq. 3.45 simplifies to:

$$\delta_{xw} = R^4 \tan\alpha \sec\alpha[(\sin\phi - \phi\cos\phi) - (\frac{1}{8})$$
$$(\sin 2\phi - 2\phi\cos 2\phi)] + (\frac{R^4 \sin\alpha}{2})[3(\sin\phi - \phi\cos\phi)$$
$$-\phi^2\sin\phi - (\frac{\phi}{2}) + (\frac{3}{8})\sin 2\phi - (\frac{1}{4})\phi\cos 2\phi] \qquad (3.46)$$

and the same way the other displacements δ_{rw},δ_{xr},δ_{rr}, and $\delta_{xr} = \delta_{rx}$ can be determined from Equations 3.28 to 3.36, the integral simplifies to:

$$\delta_{rw} = R^3 \sec\alpha[\frac{\phi}{2} + \frac{\sin 2\phi}{4} - \sin\phi] + \frac{R^3\cos\alpha}{2}$$
$$[\frac{\phi}{2} - \frac{\sin 2\phi}{4} - (\sin\phi - \phi\cos\phi)] \qquad (3.47)$$

$$\delta_{xx} = R^3 \sec\alpha \tan^2\alpha[\frac{\phi^2}{2}(\frac{\phi}{3} - \frac{\sin 2\phi}{2} + \frac{1}{8}$$
$$(\sin 2\phi - 2\phi\cos 2\phi)] + \frac{R^3 \sec\alpha \sin^2\alpha}{2}[(\frac{\phi}{2} - \frac{\sin 2\phi}{4}) - \frac{1}{4}(\sin 2\phi$$
$$-2\phi\cos 2\phi)] + [\frac{\phi^2}{2}(\frac{\phi}{3} + \frac{\sin 2\phi}{2}) - \frac{1}{8}(\sin 2\phi - 2\phi\cos 2\phi)] \qquad (3.48)$$

$$\delta_{rr} = R\sec\alpha[\frac{\phi}{2} + \frac{\sin 2\phi}{4} + \frac{R\cos\alpha}{2}[\frac{\phi}{2} - \frac{\sin 2\phi}{4}] \qquad (3.49)$$

$\delta_{rx} = \delta_{xr} = -R^2 \sec\alpha\tan\alpha[\frac{1}{8}(\sin 2\phi - 2\phi\cos 2\phi)]+$

$$\frac{R^2\sin\alpha}{2}[-(\frac{\phi}{2} - \frac{\sin 2\phi}{4}) + \frac{1}{8}(\sin 2\phi - 2\phi\cos 2\phi)] \qquad (3.50)$$

The redundants can be expressed as:

$$X_X = -\frac{\delta_{xw}\delta_{rx} - \delta_{rw}\delta_{xx}}{\delta_{xr}\delta_{rx} - \delta_{rr}\delta_{xx}} \tag{3.51}$$

$$X_r = -\frac{\delta_{rw}\delta_{xr} - \delta_{xw}\delta_{rr}}{\delta_{xr}\delta_{rx} - \delta_{rr}\delta_{xx}} \tag{3.52}$$

The final bending and torsional moments at any section of the stair may be found by the following equations.

$$M_r = m_{rw} + em_{rt} + X_x m_{rx} + X_r m_{rr} \tag{3.53}$$

$$M_s = m_{sw} + em_{st} + X_x m_{sx} + X_r m_{sr} \tag{3.54}$$

$$M_t = m_{tw} + em_{tt} + X_x m_{tx} + X_r m_{tr} \tag{3.55}$$

The thrust,shearing forces across the bottom (waist slab) of stair and radial horizontal shearing forces due to the uniform load of 1 lb/ft of the horizontal projection can be expressed as:

Thrust:

$$P_n = -X_r \sin\theta\cos\alpha - R\theta\sin\alpha \tag{3.56}$$

Shearing Force:

$$S_n = R\theta\cos\alpha - X_r\sin\theta\sin\alpha \tag{3.57}$$

Radial Horizontal Shearing Force

$$S_h = -X_r\cos\theta \tag{3.58}$$

3.3 Illustrative Example

3.3.1 Configuration of Helical Stair

Data:

(1) Angle of Slope of stair $\alpha = 25^o$

(2) Horizontal Angle $2\phi = 240^o$

(3) Width of Stair b = 1.2 m = 3.937 ft

(4) Centerline Radius R = 1.5 m = 4.92 ft

(5) Thickness of Waist Slab, D = h = 0.175 m = 0.574 ft

(6) Riser = 0.15 m = 0.492 ft

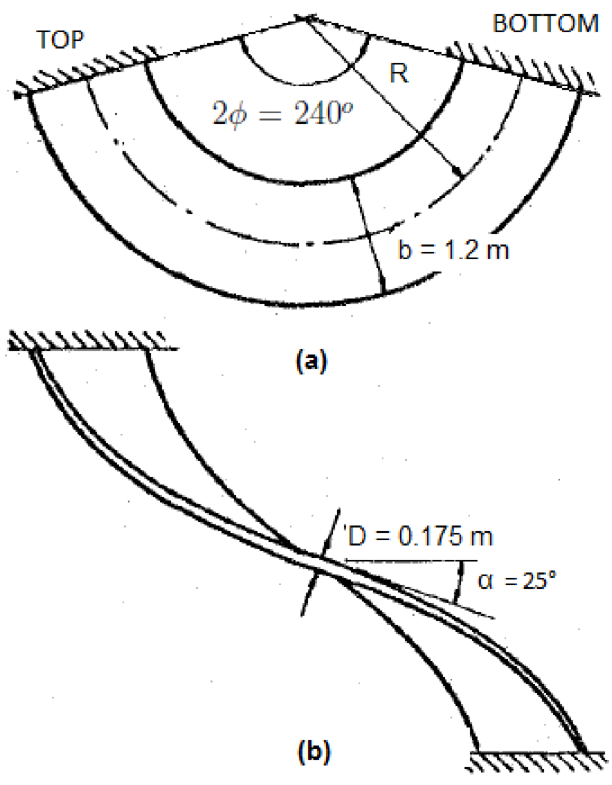


Figure 3.6: Geometry of Helicoidal Stair:(a) Plan (b) Elevation

3.3.2 Load Calculation

$$\text{Arc length in plan} = 2\pi R \times \frac{2\phi}{360} = 2\pi \times 1.5 \times \frac{240}{360} = 6.28m$$
$$Now, \tan\alpha = \frac{H}{\text{arc length in plan}} \therefore H = \tan 25 \times 6.28 = 2.92m$$

Self weight of waist slab = 0.175 × 25 = 4.38 kN/m^2

Self weight of Step = 1/2 × 0.15 × 25 = 1.875 kN/m^2

Floor finish = 0.04 × 25 = 1 kN/m^2

Plaster = 0.25 kN/m^2

Railing = 0.25 kN/m^2

Total = 7.755 kN/m^2

DL = (7.755/cos25) = 8.56 kN/m^2

LL = 3 kN/m^2

Total load = 11.56 kN/m^2 of plan area.

Total load, W = 11.56 × 1.2 =13.87 kN/m = 950.35 lb/ft

3.3.3 Analysis using Scordelis Method

Step 1. Obtain the redundants X_r and X_x

From the Scordelis Table B [2],

For α = 25° and b/h = 6.86

X_r/R = -0.646776 and X_x = -1.04577

Thus, X_r = -3.18214 and X_x = -1.04577

Step 2. Calculation of eccentricity.

From Equation 3.18

Eccentricity, e = 0.26253 ft

Step 3. Calculation of vertical, lateral and torsional moment.

From Equation 3.22,

$$M_r = m_{rw} + em_{rt} + X_x m_{rx} + X_r m_{rr}$$

$$M_r = [-R^2(1 - \cos\theta) + (0.26253)(-R(1 - \cos\theta)) + (-1.04577)(-R)(\theta \sin\theta) \tan\alpha$$
$$+(-3.18214) \cos\theta] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$

So, by putting the value of R, θ and α,

$$M_r = [-4.92^2(1 - \cos 2.094) + (0.26253) - R(1 - \cos 2.094) + (-1.04577)$$
$$(-4.92)(120 \sin 2.094) \tan 0.436 + (-3.18214) \cos 2.094] \times 950.35$$

M_r = -30700.38 lbft = -41.63 kNm

Now, From Equation 3.23,

$$M_s = m_{sw} + em_{st} + X_x m_{sx} + X_r m_{sr}$$

$$M_s = [-R^2(\theta - \sin\theta) \sin\alpha + (0.26253)R \sin\theta \cos\alpha + (-1.04577)R(\sin\theta) \cos\alpha$$
$$+R(\theta \cos\theta) \sin\alpha \tan\alpha + (-3.18214) \sin\theta \sin\alpha] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$

So, by putting the value of R, θ and α,

$$M_s = [-4.92^2(2.094 - \sin 2.094)\sin 0.436 + (0.26253)4.92\sin 2.094\cos 0.436$$
$$+(-1.04577)4.92(\sin 2.094)\cos 0.436 + 4.92(2.094cos2.094)\sin 0.436\tan 0.436$$
$$+(-3.18214)\sin 2.094\sin 0.436] \times 950.35$$

$$M_s = \text{-15428.72 lbft} = \text{-20.92 kNm}$$

Now, From Equation 3.23,

$$M_t = m_{tw} + em_{tt} + X_x m_{tx} + X_r m_{tr}$$

$$M_t = [-R^2(\theta - \sin\theta)\cos\alpha + (0.26253)R\sin\theta\cos\alpha +$$
$$(-1.04577) - R(\sin\theta)\sin\alpha + R(\theta\cos\theta)\sin\alpha + (-3.18214) - \sin\theta\cos\alpha] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$
So, by putting the value of R, θ and α,

$$\begin{aligned} M_t = {} & [-4.92^2(2.094 - \sin 2.094)\cos 0.436 + \\ & (0.26253)4.92\sin 2.094\cos 0.436 + (-1.04577) - 4.92(\sin 2.094)\sin 0.436 + \\ & 4.92(2.094\cos 2.094)\sin 0.436 + (-3.18214) - \sin 2.094\cos 0.436] \times 950.35 \end{aligned}$$

$$M_t = \text{-23067.01 lbft} = \text{-31.28 kNm}$$

Similarly, Vertical Moment M_r, Lateral Moment M_s and Torsional Moment M_t for different value of θ are calculated and given in following table 3.1.

Table 3.1: Moments using Scordelis method

θ (°)	120	96	72	48	24	0	-24	-48	-72	-96	-120
M_r(kN/m)	-41.63	-30.71	-20.28	-11.69	-6.06	-4.10	-6.06	-11.69	-20.28	-30.71	-41.63
M_s(kN/m)	-20.92	-15.75	-11.23	-7.21	-3.52	0.00	3.52	7.21	11.23	15.75	20.92
M_t(kN/m)	-31.28	-18.17	-9.16	-3.80	-1.17	0.00	1.17	3.80	9.16	18.17	31.28

3.3.4 Analysis using Simplified Method

Step 1. Obtain the redundants X_r and X_x

Putting Value $\phi = 2.094$ radian and $\alpha = 0.436$ radian in Equation 3.46 to 3.50,

$$\begin{aligned}\delta_{xw} &= R^4 \tan(0.436)\sec(0.436)[(\sin(2.094) - \\ &(2.094)\cos(2.094)) - (\frac{1}{8})(\sin 2(2.094) - 2(2.094)\cos 2(2.094))] + \\ &(\frac{(R)^4 \sin(0.436)}{2})[3(\sin(2.094) - (2.094)\cos(2.094)) - \\ &(2.094)^2 \sin(2.094) - (\frac{2.094}{2}) + (\frac{3}{8})\sin 2(2.094) - (\frac{1}{4})(2.094)\cos 2(2.094)]\end{aligned}$$

δ_{xw}= 1.08 R^4

$$\begin{aligned}\delta_{rw} &= (R)^3 \sec(0.436)[\frac{2.094}{2} + \frac{\sin 2(2.094)}{4} - \sin(2.094)] + \\ &\frac{(R)^3 \cos(0.436)}{2}[\frac{2.094}{2} - \frac{\sin 2(2.094)}{4} - (\sin(2.094) - (2.094)\cos(2.094))]\end{aligned}$$

δ_{rw}=-0.33 R^3

$$\begin{aligned}\delta_{xx} &= R^3 \sec(0.436)\tan^2 0.436[\frac{2.094^2}{2}(\frac{2.094}{3} - \\ &\frac{\sin 2(2.094)}{2} + \frac{1}{8}(\sin 2(2.094) - 2\phi\cos 2(2.094))] + \\ &\frac{R^3 \sec(0.436)\sin^2(0.436)}{2} \\ &[[(\frac{2.094}{2} - \frac{\sin 2(2.094)}{4}) - \frac{1}{4}(\sin 2(2.094) - 2(2.094)\cos 2(2.094))] +\end{aligned}$$

$$[\frac{(2.094)^2}{2}(\frac{2.094}{3}+\frac{\sin 2(2.094)}{2})-\frac{1}{8}(\sin 2(2.094)-2(2.094)\cos 2(2.094))]]$$

δ_{xx}=0.77 R^3

$$\begin{aligned}\delta_{rr} &= R\sec(0.436)[\frac{2.094}{2}+\frac{\sin 2(2.094)}{4}+ \\ &\frac{R\cos(0.436)}{2}[\frac{2.094}{2}-\frac{\sin 2(2.094)}{4}] \\ &\delta_{rr}=1.49R\end{aligned}$$

$$\begin{aligned}\delta_{rx} &= \delta_{xr}=-(R)^2\sec(0.436)\tan(0.436) \\ &[\frac{1}{8}(\sin 2(2.094)-2(2.094)\cos 2(2.094))]+\frac{R^2\sin(0.436)}{2} \\ &[-(\frac{2.094}{2}-\frac{\sin 2(2.094)}{4})+\frac{1}{8}(\sin 2(2.094)-2(2.094)\cos 2(2.094))] \\ &\delta_{rx}=\delta_{xr}=-0.31R^2\end{aligned}$$

The redundants can be expressed as:

$$\begin{aligned}X_X &= -\frac{\delta_{xw}\delta_{rx}-\delta_{rw}\delta_{xx}}{\delta_{xr}\delta_{rx}-\delta_{rr}\delta_{xx}} \\ &=-0.08R^2 \\ &=-1.93\end{aligned} \tag{3.59}$$

$$\begin{aligned}X_r &= -\frac{\delta_{rw}\delta_{xr}-\delta_{xw}\delta_{rr}}{\delta_{xr}\delta_{rx}-\delta_{rr}\delta_{xx}} \\ &=-1.44\end{aligned} \tag{3.60}$$

Step 2. Calculation of eccentricity.

From Equation 3.37

Eccentricity, e = 0.26253 ft

Step 3. Calculation of vertical, lateral and torsional moment.

From Equation 3.53,

$$M_r = m_{rw} + em_{rt} + X_x m_{rx} + X_r m_{rr}$$

$$M_r = [(-R)^2(1-\cos\theta) + (0.26253)(-R)(1-\cos\theta) + (-1.93)(-R)(\theta\sin\theta)\tan\alpha$$
$$+(-1.44)\cos\theta] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$

So, by putting the value of R, θ and α,

$$M_r = [(-4.92)^2(1-\cos 2.094) + (0.26253)(-R)(1-\cos 2.094) + (-1.93)$$
$$(-4.92)(120\sin 2.094)\tan 0.436 + (-1.44)\cos 2.094] \times 950.35$$

$$M_r = \text{-25406.17 lbft} = \text{-34.45 kNm}$$

Now, From Equation 3.54,

$$M_s = m_{sw} + em_{st} + X_x m_{sx} + X_r m_{sr}$$

$$M_s = [(-R)^2(\theta-\sin\theta)\sin\alpha + (0.26253)R\sin\theta\cos\alpha + (-1.93)R(\sin\theta)\cos\alpha$$
$$+R(\theta\cos\theta)\sin\alpha\tan\alpha + (-1.44)\sin\theta\sin\alpha] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$

So, by putting the value of R, θ and α,

$$M_s = [(-4.92)^2(2.094 - \sin 2.094)\sin 0.436 + (0.26253)4.92 \sin 2.094 \cos 0.436$$
$$+(-1.93)4.92(\sin 2.094)\cos 0.436 + 4.92(2.094 cos 2.094)\sin 0.436 \tan 0.436$$
$$+(-1.44)\sin 2.094 \sin 0.436] \times 950.35$$

$$M_s = \text{-19138.82 lbft} = \text{-25.95 kNm}$$

Now, From Equation 3.55,

$$M_t = m_{tw} + em_{tt} + X_x m_{tx} + X_r m_{tr}$$

$$M_t = [(-R)^2(\theta - \sin\theta)\cos\alpha + (0.26253)R\sin\theta\cos\alpha +$$
$$(-1.93) - R(\sin\theta)\sin\alpha + R(\theta\cos\theta)\sin\alpha + (-1.44) - \sin\theta\cos\alpha] \times W$$

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$
So, by putting the value of R, θ and α,

$$\begin{aligned} M_t \;=\; & [(-4.92)^2(2.094 - \sin 2.094)\cos 0.436 + \\ & (0.26253)4.92 \sin 2.094 \cos 0.436 + (-1.93) - 4.92(\sin 2.094)\sin 0.436 + \\ & 4.92(2.094\cos 2.094)\sin 0.436 + (-1.44) - \sin 2.094 \cos 0.436] \times 950.35 \end{aligned}$$

$$M_t = \text{-22682.97 lbft} = \text{-30.76 kNm}$$

Step 3. Calculation of Thrust , Shearing force and Radial horizontal shear force.

At support, $\theta = 120 \times \pi/180 = 2.094$ and $\alpha = 25 \times \pi/180 = 0.436$

So, by putting the value of R, θ and α,

$$\begin{aligned} P_n &= -X_r \sin\theta \cos\alpha - R\theta \sin\alpha \\ &= (-1.44)\sin 2.094 \cos 0.436 - (4.92)2.094 \sin(0.436) \\ &= 1141.86lb = 5.08kN \end{aligned} \quad (3.61)$$

Shearing Force:

$$\begin{aligned} S_n &= R\theta \cos\alpha - X_r \sin\theta \sin\alpha \\ &= (4.92)2.094 \cos 0.436 - (-1.44)\sin 2.094 \sin 0.436 \\ &= 9949.69lb = 44.26kN \end{aligned} \quad (3.62)$$

Radial Horizontal Shearing Force

$$\begin{aligned} S_h &= -X_r \cos\theta \\ &= (-1.44)\cos 2.094 \\ &= -3363.86lb = -14.96kN \end{aligned} \quad (3.63)$$

Similarly, Vertical Moment M_r, Lateral Moment M_s, Torsional Moment M_t,Thrust P_n , Shearing force S_n and S_h for different value of θ are calculated and given in following table :

Table 3.2: Moments and Forces using Simplified Method

Angle	120	96	72	48	24	0	-24	-48	-72	-96	-120
Mr(kN/m)	-34.45	-25.90	-18.75	-13.45	-10.21	-9.12	-10.21	-13.45	-18.75	-25.90	-34.45
Ms(kN/m)	-25.95	-22.64	-18.43	-13.11	-6.84	0.00	6.84	13.11	18.43	22.64	25.95
Mt(kN/m)	-30.76	-19.97	-12.18	-6.76	-2.97	0.00	2.97	6.76	12.18	19.97	30.76
Pn(kN)	5.08	12.25	14.75	12.79	7.35	0.00	-7.35	-12.79	-14.75	-12.25	-5.08
Sn(kN)	44.26	37.07	28.93	19.89	10.14	0.00	-10.14	-19.89	-28.93	-37.07	-44.26
Sh(kN)	-14.96	-3.13	9.25	20.02	27.34	29.92	27.34	20.02	9.25	-3.13	-14.96

3.3.5 Comparison of Scordelis and Simplified Method

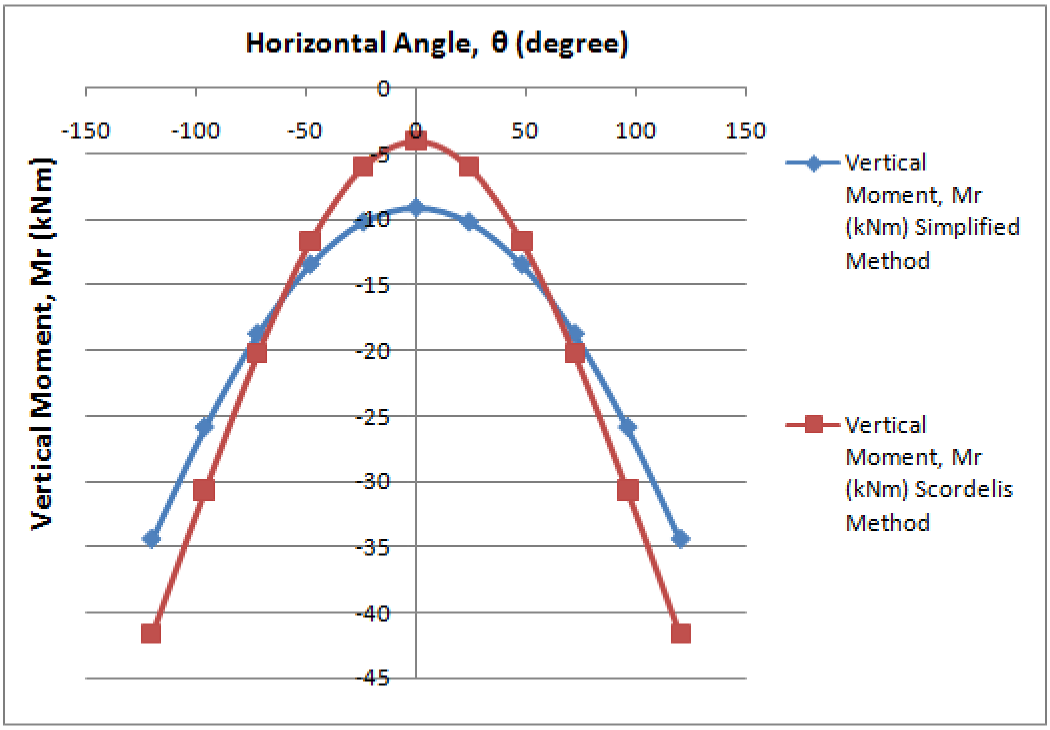


Figure 3.7: Comparison of Vertical Moments, M_r (kNm)

Figure 3.7 shows the comparison of the vertical moments at horizontal angle θ to -θ by simplified method and scordelis method. The results are quite comparable with each other. The vertical moment obtained at support by scordelis method is 1.2 times higher than computed by simplified method. While at center, scordelis method gives lesser moment compared to simplified method.

Figure 3.8 shows the comparison of the lateral moments at horizontal angle θ to -θ by simplified method and scordelis method. Here also, the results are quite comparable with each other. But, the lateral moment obtained at support by simplified method is 1.24 times higher than computed by scordelis method.

Figure 3.9 shows the comparison of the torsional moments at horizontal angle θ to -θ by simplified method and scordelis method. The results shows that torsional moment computed by simplified method and scordelis method are well fitted to each other.

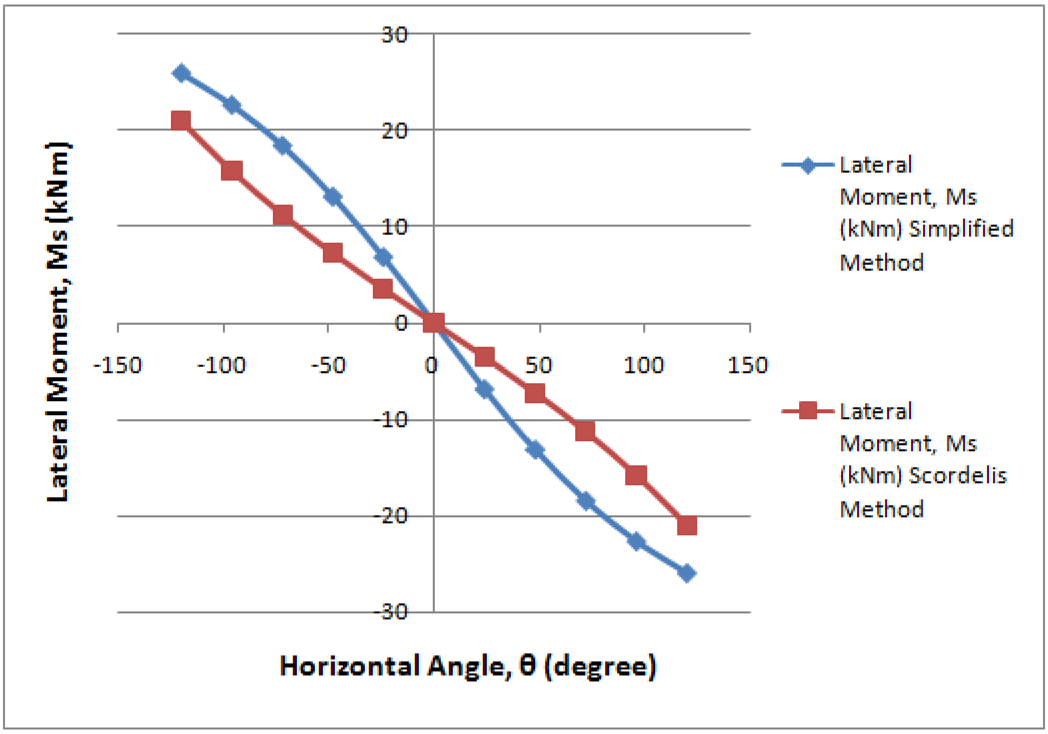


Figure 3.8: Comparison of Lateral Moments, M_s (kNm)

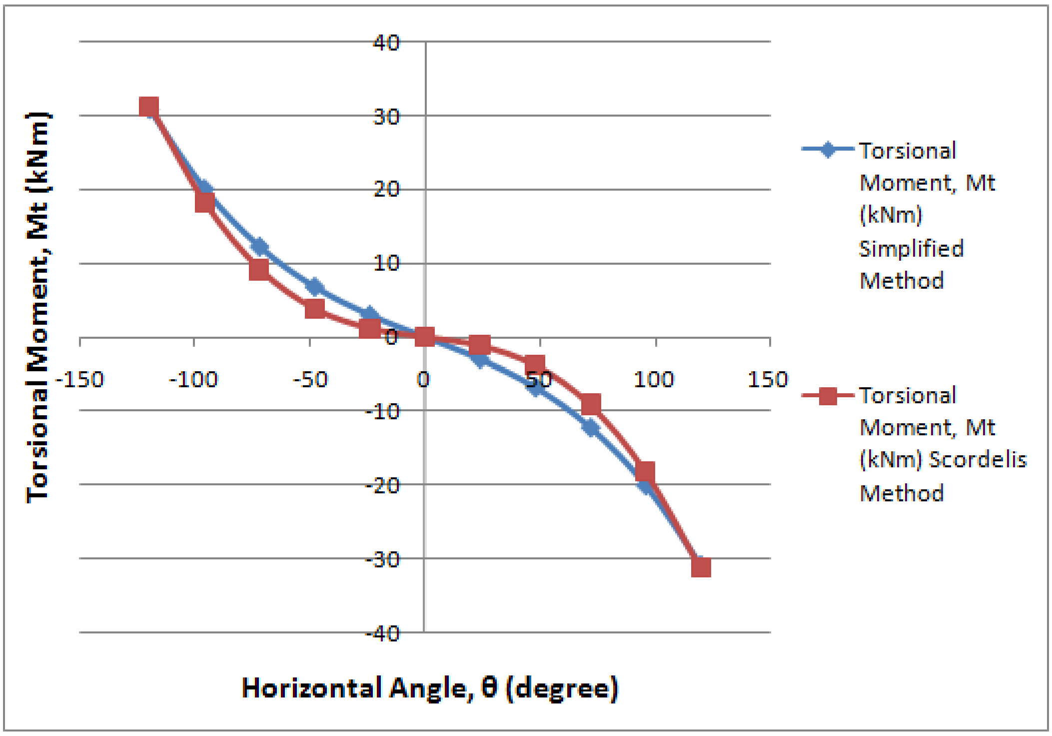


Figure 3.9: Comparison of Torsional Moments, M_t (kNm)

3.3.6 Parametric Study

Table 3.3: Values of Redundants for simplified method

ϕ	$\alpha = 10°$		$\alpha = 20°$		$\alpha = 30°$		$\alpha = 40°$	
(degree)	X_x/R^2	X_r/R	X_x/R^2	X_r/R	X_x/R^2	X_r/R	X_x/R^2	X_r/R
45	-0.001376	-2.96	-0.001376	-1.44	-0.001377	-0.90	-0.001378	-0.62
60	-0.004399	-3.06	-0.004402	-1.48	-0.004407	-0.94	-0.004412	-0.64
75	-0.010933	-3.20	-0.010942	-1.55	-0.010957	-0.98	-0.010976	-0.67
90	-0.023231	-3.36	-0.023260	-1.63	-0.023304	-1.03	-0.023359	-0.71
105	-0.044422	-3.57	-0.044504	-1.73	-0.044632	-1.09	-0.044793	-0.75
120	-0.078770	-3.80	-0.078998	-1.84	-0.079358	-1.16	-0.079822	-0.80
135	-0.131944	-4.07	-0.132535	-1.97	-0.133492	-1.25	-0.134770	-0.86
150	-0.211086	-4.36	-0.212493	-2.11	-0.214822	-1.33	-0.218040	-0.92
165	-0.324197	-4.64	-0.327201	-2.25	-0.332273	-1.42	-0.339493	-0.97
180	-0.477932	-4.85	-0.483558	-2.35	-0.493195	-1.47	-0.507207	-1.01
195	-0.672889	-4.92	-0.681817	-2.37	-0.697218	-1.48	-0.719819	-1.00
210	-0.897502	-4.76	-0.908901	-2.27	-0.928511	-1.40	-0.957114	-0.93
225	-1.126351	-4.32	-1.137054	-2.04	-1.155226	-1.23	-1.181143	-0.79
240	-1.329565	-3.63	-1.335098	-1.69	-1.344250	-1.00	-1.356772	-0.62
255	-1.488323	-2.82	-1.485340	-1.29	-1.480256	-0.74	-1.473002	-0.44
270	-1.601080	-2.00	-1.588754	-0.90	-1.568508	-0.50	-1.541125	-0.28
285	-1.676372	-1.25	-1.655900	-0.55	-1.622619	-0.29	-1.578267	-0.15
300	-1.722689	-0.59	-1.696174	-0.25	-1.653156	-0.11	-1.595962	-0.04
315	-1.743384	-0.01	-1.713077	0.02	-1.663724	0.04	-1.597707	0.05
330	-1.736406	0.50	-1.704364	0.26	-1.651799	0.18	-1.580684	0.13
345	-1.696282	0.95	-1.664235	0.46	-1.611177	0.29	-1.538402	0.20
360	-1.617070	1.34	-1.586377	0.64	-1.535112	0.39	-1.463867	0.26

Table 4.1 shows the value of redundants X_x/R^2 and X_r/R for different horizontal angle(ϕ) and Vertical angle (α). The table 4.1 is developed using Equations 3.51 and 3.52. These values are useful in calculating vertical, lateral, torsional moments and also to calculate thrust, shearing force and radial horizontal shearing force.

Chapter 4

Design and Detailing of Helical Stairs

4.1 Introduction

In this chapter design of helical stair has been carried out by limit state method using the analysis results of simplified method and scordelis method as obtained in chapter 3. The detail design procedure has been illustrated in following sections. It also cover the detailing of the helical stair.

4.2 Design of Helical stairs

(1)Angle of Slope of stair $\alpha = 25°$

(2)Horizontal Angle $2\phi = 240°$

(3)Width of Stair b = 1.2 m

(4)Centerline Radius R = 1.5 m

(5)Thickness of Waist Slab D = 0.175 m

(6)Riser = 0.15 m

(7)$f_{ck} = 25\ N/mm^2$

(8)f_y = 415 N/mm^2

$$\text{Arc length in plan} = 2\pi R \times \frac{2\phi}{360} = 2\pi \times 1.5 \times \frac{240}{360} = 6.28m$$
$$Now, \tan\alpha = \frac{H}{\text{arc length in plan}} \therefore H = \tan 25 \times 6.28 = 2.92m$$

Self wt.=0.175× 25 = 4.38 kN/m^2

Steps wt. = 1/2 × 0.15 × 25) = 1.875 kN/m^2

Floor finish = 0.04× 25 = 1 kN/m^2

Plaster = 0.25 kN/m^2

Railing = 0.25 kN/m^2

Total = 7.755 kN/m^2

DL = (7.755/cos25) = 8.56 kN/m^2

LL = 3 kN/m^2

Total load = 11.56 kN/m^2 of plan area.

Total laod, W = 11.56 × 1.2 = 13.87 kN/m

4.2.1 Design using the Results of Simplified Method

Design section (1.2m × 0.175m).

At support

Vertical moment, M_r = -34.45 kNm

Horizontal moment, M_s = -25.95 kNm

Torsional moment, M_t = -30.76 kNm

Shear Force, V = 44.26 kNm

At mid-span

Vertical moment, M_r = -9.12 kNm

Step 1. Determination of design moments, shear and torsion at supports.

Design Bending Moment,

M_u = 34.45 × 1.5 = 51.68 kNm

Design Torsional Moment,

T_u = 30.76 × 1.5 = 46.14 kNm

Design Shear Force,

V_u = 44.26 × 1.5 = 66.39 kNm

Step 2. Determination of equivalent moments and longitudinal reinforcement at supports.

The longitudinal reinforcement shall be designed to resist an equivalent bending moment, M_{el}, given by

M_{el} = M_u + M_t (As specified in IS 456 : 2000)

$$M_{el} = M_u + T_u \frac{(1 + D/b)}{1.7}$$

$$M_{el} = 51.68 + 46.14 \frac{(1 + 0.175/1.2)}{1.7}$$

$$M_{el} = 51.68 + 31.10 = 82.78 \text{ kNm}$$

Here,

b = 1200 mm

D = 175 mm

d' = 30 mm

d = 145 mm

Now, Area of steel

$$A_{st} = \frac{0.5 \times f_{ck}}{f_y} \left[1 - \sqrt{1 - \frac{4.6 \times M_u}{f_{ck} \times b \times d^2}} \right] \times b \times d$$

$$A_{st} = \frac{0.5 \times 25}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 82.78 \times 10^6}{25 \times 1200 \times 145^2}} \right] \times 1200 \times 145$$

$$A_{st} = 1942 mm^2$$

Thus, provide 10 Nos. of 16 mm Dia. at top of the support. (For 1/4 th. arc length on either supports)

$$A_{st(prov.)} = 2010mm^2$$

Step 3. Determination of equivalent shear at supports.

Equivalent shear, V_e, shall be calculated from the formula,

$$V_e = V_u + 1.6\frac{T_u}{b} \qquad \text{(As specified in IS 456 : 2000)}$$

$$V_e = 66.39 + 1.6\frac{46.14}{1.2} = 127.91kN$$

Step 4. Determination of shear stress and check with the maximum allowable shear stress value at supports.

$$\tau_{ve} = \frac{V_e}{bd} \text{ (As specified in IS 456:2000)}$$

$$\tau_{ve} = \frac{127.91 \times 10^3}{1200 \times 145} = 0.7351N/mm^2$$

Now, for M25 concrete grade, as per Table 20 (Specified in IS 456 : 2000)

Maximum allowable shear stress, τ_{cmax} = 3.1 N/mm^2

Thus, $\tau_{ve} < \tau_{cmax}$, Hence, o.k.

Step 5. Calculation of transverse reinforcement for shear and torsion at supports.

$$A_{sv} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)} \qquad (4.1)$$

where,

s_v = spacing of the stirrup reinforcement

b_1 = centre-to-centre distance between corner bars in the direction of the width

b1 = 1200 - 30 - 30 = 1140 mm

d_1 = centre-to-centre distance between comer bars = 175 - 30 - 30 = 115 mm

$$\frac{A_{sv}}{s_v} = \frac{46.14 \times 10^6}{1140 \times 115 \times (0.87 \times 415)} + \frac{66.39 \times 10^3}{2.5 \times 115 \times (0.87 \times 415)}$$

$$\frac{A_{sv}}{s_v} = 1.614mm$$

but the total transverse reinforcement shall not be less than following given formula.

$$A_{sv} \not< \frac{(\tau_{ve} - \tau_c)b.s_v}{0.87f_y} \tag{4.2}$$

where,

τ_c = shear strength of the concrete as per Table 19 of IS 456 : 2000,

Here,

$$P_t = 100 \times \frac{A_{st(prov.)}}{bd} = 100 \times \frac{2010}{1200 \times 145} = 1.156\%$$

For M25 grade of concrete and P_t = 1.156%,

τ_c = 0.67 N/mm^2

$$\therefore \frac{A_{sv}}{s_v} \not< \frac{(0.7351 - 0.67) \times 1200}{0.87 \times 415}$$

$$\therefore \frac{A_{sv}}{s_v} \not< 0.216mm$$

Provide maximum of Equations 4.1 and 4.2

Thus, (A_{sv}/s_v) = 1.614

Provide 4 - legged 8 mm dia. closed hoops stirrups.

$$s_v = \frac{4 \times (\pi/4) \times 8^2}{1.614} = 125mm$$

Now, as specified in IS 456 : 2000, the spacing of the stirrups shall not exceed the least of x_1, $(x_1 + y_1)/4$ and 300 mm

where,

x_1 = the shorter dimension of the stirrup = 175 - 20 - 20 + 8 + 8 = 151 mm

y_1 = the longer dimension of the stirrup = 1200 - 20 - 20 + 8 + 8 = 1176 mm

Thus, the spacing of the stirrups should not exceed 150 mm.

The required spacing of the stirrups = 125 mm, Hence ok.

Provide 4 - legged 8 mm dia. 125 mm c/c stirrups at supports. (For 1/4 th. arc length on either supports)

Step 6. Determination of design lateral moment and reinforcement at supports.

Design Lateral Moment,

M_{us} = 25.95 × 1.5 = 38.93 kNm

Here, b = 175 mm and d = 1200 -30 = 1170 mm

Now, Area of steel,

$$A_{st} = \frac{0.5 \times 25}{415} \left[1 - \sqrt{1 - \frac{4.6 \times 38.93 \times 10^6}{25 \times 175 \times 1170^2}} \right] \times 175 \times 1170$$

$$A_{st} = 92.9mm^2$$

Minimum A_{st} = $(0.85bd/f_y)$ (As specified in IS 456 : 2002)

Thus, $A_{st(min.)}$ = (0.85 × 175 × 1170)/415 = 420 mm^2

Provide 2 Nos. 16 mm dia. bars at each face.

Step 7. Determination of design moment and reinforcement at mid-span.

As, the mid-span moment is also negative, provide reinforcement on top at mid-span.

Design Bending Moment,

M_u = 9.12 × 1.5 = 13.68 kNm

Now, Area of steel

$$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 13.68 \times 10^6}{25 \times 1200 \times 145^2}}\right] \times 1200 \times 145$$

$$A_{st} = 268mm^2$$

$A_{st(min.)}$ = (0.85 × 175 × 1170)/415 = 420 mm^2

Provide 5 Nos. 12 mm dia. bars on top of mid-span.

Provide min. shear reinforcement as 4- legged 8 mm dia. 150 mm c/c at mid-span.

Step 8. Determination of bottom reinforcement.

As there is not any moment at bottom, the whole area of bottom is in compression.

So, provide minimum longitudinal reinforcement as 5 Nos. 12 mm dia. bars through out the section

The following page shows the detail drawing of helical stair.

4.2.2 Design using the Results of Scordelis Method

Design section (1.2m × 0.175m).

At support

Vertical moment, M_r = -41.63 kNm

Horizontal moment, M_s = -20.92 kNm

Torsional moment, M_t = -31.28 kNm

Shear Force, V = 44.26 kNm

At mid-span

Vertical moment, M_r = -4.10 kNm

Step 1. Determination of design moments, shear and torsion at supports.

Design Bending Moment,

$M_u = 41.63 \times 1.5 = 62.45$ kNm

Design Torsional Moment,

$T_u = 31.28 \times 1.5 = 46.92$ kNm

Design Shear Force,

$V_u = 44.26 \times 1.5 = 66.39$ kNm

Step 2. Determination of equivalent moments and longitudinal reinforcement at supports.

The longitudinal reinforcement shall be designed to resist an equivalent bending moment, M_{el}, given by

$M_{el} = M_u + M_t$ (As specified in IS 456 : 2000)

$$M_{el} = M_u + T_u \frac{(1 + D/b)}{1.7}$$

$$M_{el} = 62.45 + 46.92 \frac{(1 + 0.175/1.2)}{1.7}$$

$$M_{el} = 62.45 + 31.63 = 94.08 \text{ kNm}$$

Here,

b = 1200 mm

D = 175 mm

d' = 30 mm

d = 145 mm

Now, Area of steel

$$A_{st} = \frac{0.5 \times f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 \times M_u}{f_{ck} \times b \times d^2}}\right] \times b \times d$$

$$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 94.08 \times 10^6}{25 \times 1200 \times 145^2}}\right] \times 1200 \times 145$$

$$A_{st} = 2305mm^2$$

Thus, provide 12 Nos. of 16 mm Dia. at top of the support. (For 1/4 th. arc length on either supports)

$$A_{st(prov.)} = 2412mm^2$$

Step 3. Determination of equivalent shear at supports.

Equivalent shear, V_e, shall be calculated from the formula,

$$V_e = V_u + 1.6\frac{T_u}{b} \qquad \text{(As specified in IS 456 : 2000)}$$

$$V_e = 66.39 + 1.6\frac{46.92}{1.2} = 128.95kN$$

Step 4. Determination of shear stress and check with the maximum allowable shear stress value at supports.

$$\tau_{ve} = \frac{V_e}{bd} \text{ (As specified in IS 456:2000)}$$

$$\tau_{ve} = \frac{128.95 \times 10^3}{1200 \times 145} = 0.7411N/mm^2$$

Now, for M25 concrete grade, as per Table 20 (Specified in IS 456 : 2000)

Maximum allowable shear stress, τ_{cmax} = 3.1 N/mm^2

Thus, $\tau_{ve} < \tau_{cmax}$, Hence, o.k.

Step 5. Calculation of transverse reinforcement for shear and torsion at supports.

$$A_{sv} = \frac{T_u s_v}{b_1 d_1 (0.87 f_y)} + \frac{V_u s_v}{2.5 d_1 (0.87 f_y)} \tag{4.3}$$

where,

s_v = spacing of the stirrup reinforcement

b_1 = centre-to-centre distance between corner bars in the direction of the width

= 1200 - 30 - 30 = 1140 mm

d_1 = centre-to-centre distance between comer bars = 175 - 30 - 30 = 115 mm

$$\frac{A_{sv}}{s_v} = \frac{46.92 \times 10^6}{1140 \times 115 \times (0.87 \times 415)} + \frac{66.39 \times 10^3}{2.5 \times 115 \times (0.87 \times 415)}$$

$$\frac{A_{sv}}{s_v} = 1.631mm$$

but the total transverse reinforcement shall not be less than following given formula.

$$A_{sv} \not< \frac{(\tau_{ve} - \tau_c) b.s_v}{0.87 f_y} \tag{4.4}$$

where,

τ_c = shear strength of the concrete as per Table 19 of IS 456 : 2000,

Here,

$$P_t = 100 \times \frac{A_{st(prov.)}}{bd} = 100 \times \frac{2412}{1200 \times 145} = 1.386\%$$

For M25 grade of concrete and $P_t = 1.386\%$,

$\tau_c = 0.725$ N/mm^2

$$\therefore \frac{A_{sv}}{s_v} \nless \frac{(0.7411 - 0.725) \times 1200}{0.87 \times 415}$$

$$\therefore \frac{A_{sv}}{s_v} \nless 0.0535mm$$

Provide maximum of Equations 4.1 and 4.2

Thus, $(A_{sv}/s_v) = 1.631$

Provide 4 - legged 8 mm dia. closed hoops stirrups.

$$s_v = \frac{4 \times (\pi/4) \times 8^2}{1.631} = 123mm$$

Now, as specified in IS 456 : 2000, the spacing of the stirrups shall not exceed the least of x_1, $(x_1 + y_1)/4$ and 300 mm

where,

x_1 = the shorter dimension of the stirrup = 175 - 20 - 20 + 8 + 8 = 151 mm

y_1 = the longer dimension of the stirrup = 1200 - 20 - 20 + 8 + 8 = 1176 mm

Thus, the spacing of the stirrups should not exceed 150 mm.

The required spacing of the stirrups = 125 mm, Hence ok.

Provide 4 - legged 8 mm dia. 120 mm c/c stirrups at supports. (For 1/4 th. arc length on either supports)

Step 6. Determination of design lateral moment and reinforcement at supports.

Design Lateral Moment,

$M_{us} = 20.92 \times 1.5 = 31.38$ kNm

Here, b = 175 mm and d = 1200 -30 = 1170 mm

Now, Area of steel,

$$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 31.38 \times 10^6}{25 \times 175 \times 1170^2}}\right] \times 175 \times 1170$$

$$A_{st} = 74.78mm^2$$

Minimum A_{st} = (0.85bd/f_y) (As specified in IS 456 : 2002)

Thus, $A_{st(min.)}$ = (0.85 × 175 × 1170)/415 = 420 mm^2

Provide 2 Nos. 16 mm dia. bars at each face.

Step 7. Determination of design moment and reinforcement at mid-span.

As, the mid-span moment is also negative, provide reinforcement on top at mid-span.

Design Bending Moment,

M_u = 4.10 × 1.5 = 6.15 kNm

Now, Area of steel

$$A_{st} = \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 6.15 \times 10^6}{25 \times 1200 \times 145^2}}\right] \times 1200 \times 145$$

$$A_{st} = 118.88mm^2$$

$A_{st(min.)}$ = (0.85 × 175 × 1170)/415 = 420 mm^2

Provide 5 Nos. 12 mm dia. bars on top of mid-span.

Provide min. shear reinforcement as 4- legged 8 mm dia. 150 mm c/c at mid-span.

Step 8. Determination of bottom reinforcement.

As there is not any moment at bottom, the whole area of bottom is in compression.

So, provide minimum longitudinal reinforcement as 5 Nos. 12 mm dia. bars through out the section

4.2.3 Comparison of Simplified and Scordelis Method

Table 4.1: Result Comparison of Simplified and Scordelis Method

Parameter	Simplified Method	Scordelis Method
Longitudinal Reinforcement Required at Support, A_{st} (mm^2)	1942	2305
Shear Stress, τ_{ve} (N/mm^2)	0.7351	0.7411
Transverse Reinforcement Required at Support, A_{sv}/s_v (mm)	1.614	1.631
Lateral Reinforcement Required at Support, A_{st} (mm^2)	92.9	74.78
Longitudinal Reinforcement Required at Mid-span, A_{st} (mm^2)	268	118.88

It is evident from Table 4.1 that longitudinal reinforcement required at supports from simplified method is 18.7% less than longitudinal reinforcement required from scordelis method. The shear stress and the required transverse reinforcement at support caclulated by both the method are very similar. In simplified method lateral reinforcement required at support is 24% higher as compared to scordelis method. Longitudinalreinforcement required at mid-span by simplified mathod is quite higher than computed by scordelis method, but, both are less than minimum required reinforcement as specified by codal limit.

Chapter 5

Summary and Conclusions

5.1 Summary

Recently, helical stairs have been constructed, supported only at the top and bottom. Although, they are circular in plan projections, in elevation their description is helicoidal. Various analysis are available to solve such a complicated problems and required long and complex equations. From each analysis, torsional moments, bending moments, shear forces and axial thrusts are resulted. The geometry of each helical stair affects the application of load and hence the result. The subject has been throughly reviewed in depth by various researchers.

The main objective of the study is to explore the simplified method for the analysis of helical stairs. In simplified method, displacement are calculated from virtual integral with application of the principle of least work and redundants are determined by solving the elastic equations and moments and forces are calculated.

In this report, simplified method has been developed. The analysis and design of helical stairs have been carried out by simplified method and as well as scordelis method and the results are compared in terms of vertical, horizontal and torsional moments.

The parametric study is also carried out to evaluate the redundants X_x/R^2 and X_r/R for different horizontal angle (ϕ) and vertical angle(α).

5.2 Conclusions

Based on work carried out following conclusions are made.

- The simplified method can be applied to calculate bending moments, torsional moment and the forces for the helical stair design. This method is limited to helical stair with equal angle and symmetrical loading case.
- The I_r/I_s ratio can be neglected without significant errors in calculating displacements, forces and moments.
- From the results of simplified and scordelis method, the vertical moment obtained at supports by scordelis method is increased by 20.84% as compared to simplified method.
- While the lateral moment is increased by 24% in simplified method.
- The torsional moment computed by simplified method and scordelis method are well fitted to each other.

5.3 Future scope of work

- To develop Simplified method for helical stair using six degree of freedom.
- FEM analysis of helical stair can be carried out.
- Testing can be carried out using prototype helical stair and observed the actual behaviour.

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