Synthesis of Four Bar Mechanism for Function Generation with Optimum Transmission Angle

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09MME015

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Major Project

Submitted in partial fulfillment of the requirements

For the degree of

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Declaration

This is to certify that

i) The thesis comprises my original work towards the degree of Master of Technology in Mechanical Engineering (CAD/CAM) at Nirma University and has not been submitted elsewhere for a degree or diploma.

ii) Due acknowledgement has been made in the text to all other material used.

Tejal N. Patel

Certificate

This is to certify that the Major Project entitled " Synthesis of Four Bar Mechanism for Function Generation with Optimum Transmission Angle " submitted by Tejal N. Patel (09MME015), towards the partial fulfillment of the requirements for the degree of Master of Technology in Mechanical Engineering (CAD/CAM) of Nirma University of Science and Technology, Ahmedabad is the record of work carried out by her under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination. The results embodied in this major project, to the best of my knowledge, haven't been submitted to any other university or institution for award of any degree or diploma.

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Abstract

The important and complementary area called kinematic synthesis, where mechanisms are created to meet certain motion specifications, is touched upon only by a consideration of the simple aspect of planar linkage synthesis. Four-bar linkage has various applications like automobiles, endolite prosthetic knee, and mechanism for Steam Engines. Different methods are available for synthesis of the four-bar linkage and amongst these Freudenstein method has its useful characteristics in the synthesis problem. Kinematic synthesis by using complex Algebra is carried out, which gives results for synthesis of four bar mechanism for function generation. The formulation at three accuracy points has been taken. Chebychev Spacing formula are used for finding precision points from specified range of input motion.

General solutions for determining link lengths of four bar mechanism for function generation with optimum transmission angle is carried out. The method is useful to synthesis the four bar mechanism for function generation where function is polynomial equation of second order. Synthesis is also carried out by Dyad method. The transmission angle is important criteria for the design of mechanism by means of which the quality of motion transmission in a mechanism at its design stage can be judged. It helps to decide the ‘ Best ’ among a family of possible mechanisms for most effective force transmission. In precision point approach, the desired motion characteristics are achieved at precision points only and at all other points there is structural error. Simulation of mechanism in motion View (HYPER WORKS) software is carried out. The generalized formulation obtained using MATLAB 7.6.

Keywords: *Function Generation, Kinematic Dyad Synthesis, Optimum Transmission Angle, Structural Error.*

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Nomenclature

θ_2	Input Angle
γ	Transmission Angle when, $< 90°$
μ	Transmission Angle when, $> 90°$
β	Angle between coupler and driver
r_1, r_2, r_3, r_4	Link lengths of four bar mechanism
$f(x)$	Generator function
$F(x)$	Prescribed function
v_B, v_{BA}	Velocity of respective link
a_0, a_1, a_2	Constants of polynomial equation
k_1, k_2, k_3	Freudenstein's constant
n	Number of precision point
d	Dummy link
θ_d	Angle of Dummy link length
x_1, x_2, x_3	Precision points
y_1, y_2, y_3	Function generator points
x_0, x_f	Start and Final range of x
θ_s, θ_f	Start and final range of θ
ϕ_s, ϕ_f	Start and final range of ϕ
s, l	Shortest and Longest link lengths
p, q	Remaining two link lengths
T_2, T_4	Torque of input and output link
F	Force exerted by coupler
yc	Calculated y values
$D_1, D_2, D_3, ...D_n$	Design parameters
${\theta_2}^{1j}$	Input angle in j^{th} position
${\theta_3}^{1j}$	Coupler angle in j^{th} position
${\theta_4}^{1j}$	Output angle in j^{th} position

Chapter 1

Introduction

1.1 Preliminary Remarks

The four-bar linkage has a long history in both the theoretical kinematics literature as well as a variety of applications. The engineering applications range from James Watt's mechanism for steam engines to modern applications,such as, the frames in mountain bicycles and artificial knees for prosthetics. The four-bar linkage is the simplest possible closed-loop mechanism, and has numerous uses in industry also for simple devices found in automobiles, toys, etc. Mechanisms involving a finite number of links possess an inherent error and it is the task of the designer to reduce this error to a sufficiently low value. Freudenstein method formulation is available for synthesis of different mechanisms at three accuracy points[12].
In the initial stage of kinematic design of any machinery, whether it is a geometrical or analytical the knowledge of transmission angle is necessary. Because of the wide spread use of the four bar linkage the criteria related to quality of such linkage is most important. It is compared by some of the ratios, angles and other parameter of mechanism that tells whether a mechanism is good one or poor one. Many such parameters have been defined like, velocity ratio, torque ratio, transmission angle etc. They refers to as indices of merit.

1.2 Mechanism

A mechanism is a group of links interacting with each other through joints to complete required motion or force transmission. Those mechanism only having lower pairs, such a mechanism is called linkage. Mechanism is obtain if one of the links is fixed to the ground. To emphasis their similarities and difference Mechanism may be categorizes in several different ways. It divides mechanism in to, Planner Mechanism and Spherical Mechanism. Planner mechanism is one in which all particles describe plane curves in space and all these curves lie in parallel planes that is, the loci of all points are plane curves parallel to a single common plane. This characteristic makes it possible to represent the locus of any chosen point of a planner mechanism in its true size and shape on a single drawing. The motion transformation of any such mechanism is called coplanar mechanism. The plane four bar linkage, the cam and follower and slider crank mechanism are familiar example of planar mechanism. The majority of mechanism in use today are planar linkage. They include only revolute and prismatic pairs. Spherical Mechanism is one in which each link has a point that remains stationary as the linkage moves.

Mechanism may be classify as Snap Action Mechanism, Clamping, Indexing Mechanism, Rocking Mechanism. In each case the output mechanism swings through an angle less than 360°. As example four bar linkage, Cam Follower, Slider Offset Mechanism, Slider Crank Mechanism, Scotch Yock Mechanism etc.

1.3 Synthesis of mechanism

1.3.1 Definition

Design is most probably termed as Synthesis. Synthesis is the process of prescribing the sizes, shapes, material compositions and arrangements of the parts so that the resulting machine will perform the prescribed task. Synthesis of mechanism is done by graphical methods or by analytical means.They are classified as following ways.

1.3.2 Types of Synthesis

(1) Type Synthesis

Type synthesis refers to the kind of mechanism selected. It may be linkage, a gear system, belts and pulley or cam system.

(2) Number Synthesis

Number synthesis deals with the number of links and joints or parts that are required to obtain certain mobility.

(3) Dimensional Synthesis

Dimensional synthesis determines the dimensions of the individual link.

1.3.3 Classification of types of Synthesis

(1) Function Generation

A frequent requirement in design is that of causing an output member to rotate, oscillate or reciprocate according to the specified function of time or input motion, this is called the function generation. To generate the function y = f (x) where x would represent the motion of the input crank link and linkage would design so that the motion of output rocker would approximate the function y.

(2) Path Generation

When a point on a coupler is to be guided along a prescribed path is said to be a path generation problem.

(3) Motion Generation

A mechanism is designed to guide a rigid body in a prescribed path is known as motion generation path.

1.4 Transmission Angle

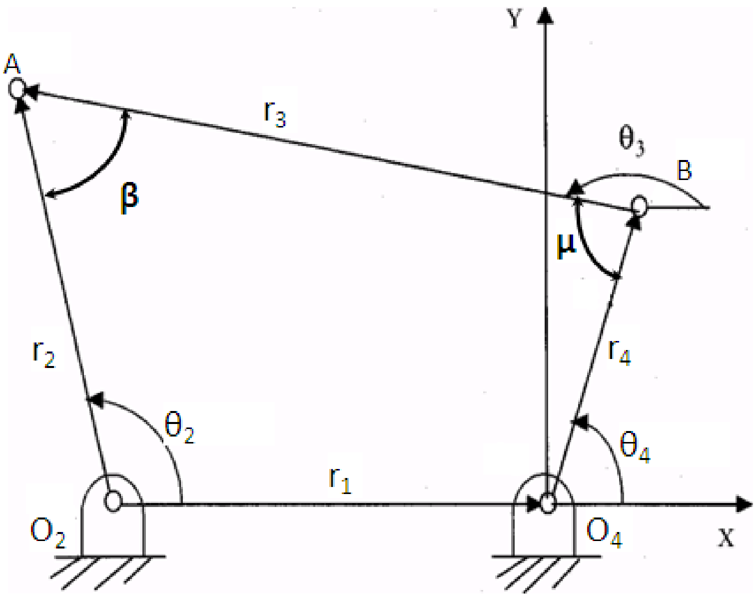


Figure 1.1: Transmission Angle in Four bar Mechanism

Figure (1.1) shows Transmission Angle in four bar mechanism. In figure, O_2O_4AB is four bar mechanism. Where O_2O_4 is the fix link. O_2A indicates the input crank. BO_4 is output follower and AB is coupler because it couples the motion of crank and follower. The link lengths are r_1, r_2 ,r_3, and r_4. And θ_1, θ_2, θ_3, θ_4 are angles of respective links measured, anticlockwise from horizontal x Axis. As first link is fix to the ground so, the length of this link is assume as unity and respective angle of motion is consider as 0°. The angle between coupler and follower is known as Transmission Angle. It is denoted by μ. Another important angle is angle between crank and coupler. It is denoted by β. The value of them is found by equation (1.1) and (1.2).

$$\mu = \cos^{-1}\left(\frac{{r_3}^2 + {r_4}^2 - {r_1}^2 - {r_2}^2 + 2r_1r_2\cos\theta_2}{2r_3r_4}\right) \tag{1.1}$$

$$\beta = cos^{-1}\left(\frac{{r_3}^2 - {r_4}^2 - {r_1}^2 + {r_2}^2 + 2r_1r_2cos\theta_4}{2r_3r_4}\right) \tag{1.2}$$

Transmission Angle is the absolute value of acute angle of the pair of angles at a intersection of the two linkages and varies continuously from some minimum to some maximum value as the linkage goes through its range of motion. Transmission Angle is a smaller angle between the direction of velocity difference vector of driving link and the direction of absolute velocity vector of output link. It varies throughout the range of operation and is most favorable when it is 90°. The recommended transmission angle is 90° ± 50°[13]. In mechanism having a reversal of motion, transmission angle must be investigated for both directions of motion transmission. Transmission of motion is impossible when transmission angle is 0° or 180°. If transmission angle is zero, no torque can be realized on output link, i.e. dead center position. A large transmission angle does not necessarily guarantee the low fluctuation of torque. Very small or very large transmission angle results in large error of motion, high sensitivity to manufacturing error, noisy and unacceptable mechanism. It is not the absolute value of transmission angle but its deviation from 90° that is significant.

1.4.1 Maximum and Minimum Transmission Angles

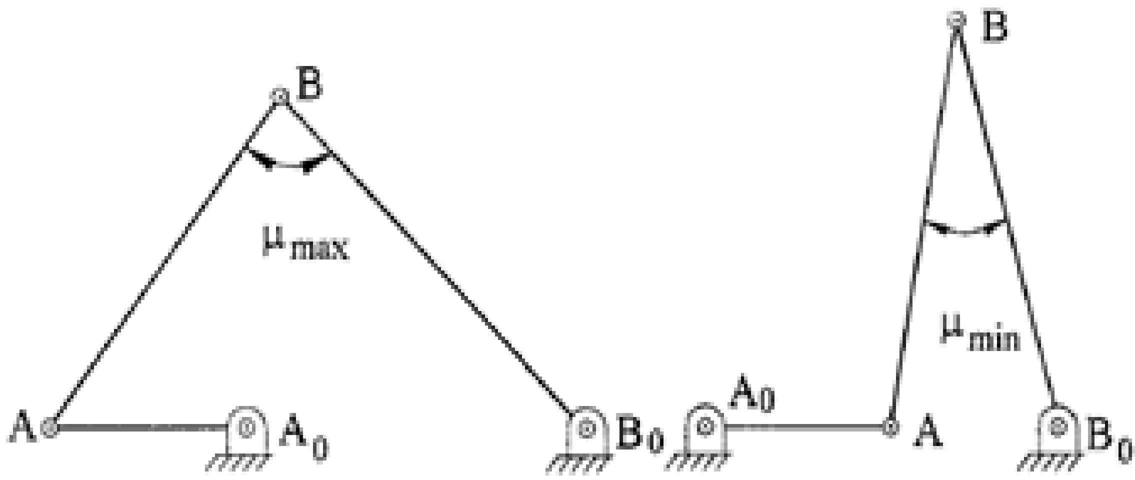


Figure 1.2: Min-Max value of Transmission Angle

The transmission angles at the extreme positions of a double rocker linkage will also be the minimum and maximum values of transmission angle for the entire motion of mechanism. In case of crank-rocker as shown in figure (1.3), the transmission angle will be minimum when input crank angle is 0° and maximum when input crank angle is 180°. These occur twice in each revolution of the driving crank and not occur at the extreme positions of the linkage. The deviation of transmission angle from 90° is the measure of reduction in effectiveness of force transmission. So the aim in linkage design is to proportionate the links so that these deviations are as small as possible, especially in the presence of appreciable joint friction. If the range of operation is sufficiently small, it seems as if can be obtained a linkage with optimum variation of transmission angle if it is set equal to 90° in the designed position.

Among the family of possible four-bar linkages, there is one linkage that has a minimum transmission angle, which is greater then the minimum transmission angles of all the others. This is called optimum transmission angle and this particular linkage has the best dimensions for most effective force transmission. If the designer tries to optimize the linkage with respect to its force transmission characteristics simultaneously with optimum transmission angle synthesis, it increases the difficulty of problem extensively. Therefore combined force transmission and synthesis studies have been restricted to relatively simple linkages. Figure (1.4) shows the min max values of transmission angle for crank rocker mechanism.
The minimum and maximum value of transmission angle of four-bar mechanism can be find out by equation (1.3).

$$cos\mu_{min,max} = \left(\frac{{r_3}^2 + {r_4}^2 - {r_1}^2 - {r_2}^2 + 2r_1r_2}{2r_3r_4} \pm \frac{r_1r_2}{r_3r_4} \right) \tag{1.3}$$

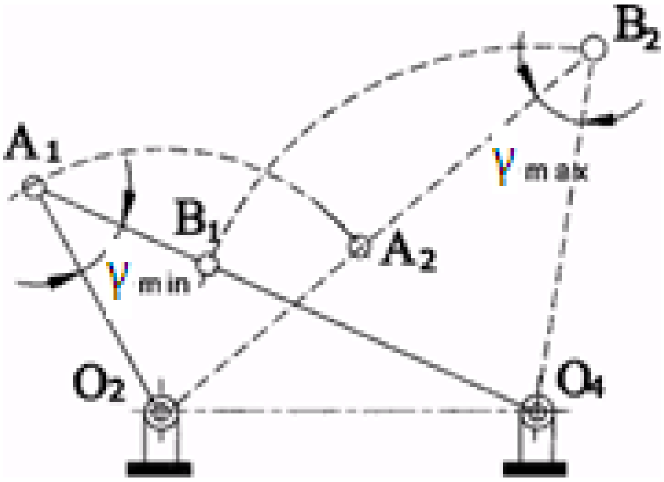


Figure 1.3: Double-Rocker Mechanism

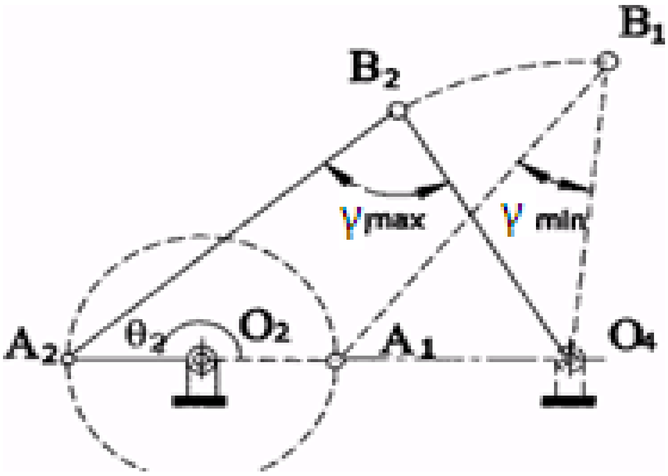


Figure 1.4: Crank-Rocker Mechanism

1.4.2 Constant Transmission Angle

In some of the applications like direct contact mechanism (Cam and Follower Mechanism) and Trammel Mechanism, the transmission angle is kept constant. In such cases, the transmission angle depends on the shape of the curved surface of the output link. If the curve is of the form of a logarithmic spiral having a constant rise, then the it remains constant for some part of motion.

1.4.3 Importance of Transmission Angle

The mechanism designed with maximum transmission angle criterion will have minimum force acting along the coupler and on the bearings resulting in small friction torque at the shafts. Though a good transmission angle is not a cure-all for every design problem, however, for many mechanical applications it can guarantee for the performance of linkage at higher speed without unfavorable vibrations. When transmission Angle is right angle, most effective force transmission takes place and the accuracy of output motion is less sensitive to manufacturing tolerances of link lengths and clearance between joints and change of dimensions due to thermal expansion. Mechanisms having transmission angle too much deviated from $90°$, exhibit poor operational characteristics like noise and jerk at high speeds. If it is $0°$, self-locking takes place. Thus the transmission angle of a mechanism provides a very good indication of the quality of motion, the accuracy of its performance, expected noise output and its costs in general. In other words, it is a simple and useful coefficient of performance for mechanisms for non-uniform transmission of motion. It serves as a basis for comparing mechanisms.

1.5 Mechanical Advantage

Mechanical Advantage of the linkage is the ratio of output torque exerted by the driven link to the necessary input torque required at the driver. Figure (1.5) indicates the Mechanical Advantage for four bar mechanism.
Mechanical Advantage $\propto \sin\gamma$, and $\propto 1/\sin\beta$, As γ and β changes, Mechanical Advantage changing continuously as the linkage moves.
When $\sin\beta = 0$, The Mechanical Advantage $= \infty$. Thus at such a position only a small input torque is necessary to overcome a large output torque load. This is the case when the driver is directly inline with the coupler, is said to be **Toggle position** as shown in figure (1.6). This principle is utilize in **Rocker Crusher Mechanism.** As Transmission Angle become small Mechanical Advantage decreases, and even small

amount of friction will causes the mechanism to lock or jam. It diminishes when Transmission Angle is much less than a right angle. To avoid this, a Common rule of thumb is that a four bar linkage should not be used in the region where the Transmission Angle is less than 45° or 50 ° [9]. The best 4-bar linkage based on the quality of its force transmission, will have Transmission Angle that deviates from 90° by the smallest amount.

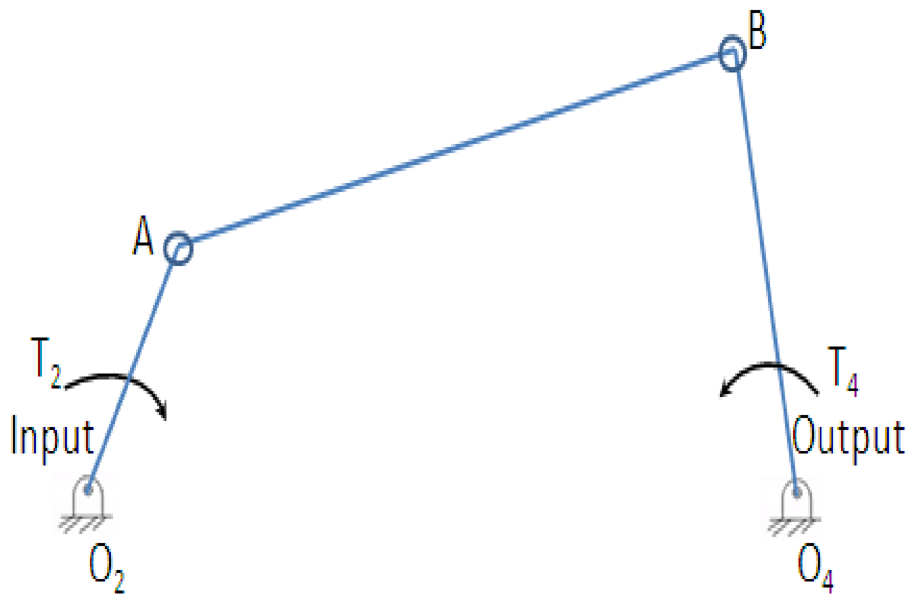


Figure 1.5: Mechanical Advantage

1.6 Basic theorems and formulae

(1) Chebychev Spacing

In function generation problem the output is related to the input through a function y = f (x) and it is required to obtain the dimension of a linkage to satisfy this relationship. Generally, A linkage synthesis problem does not have exact solution over

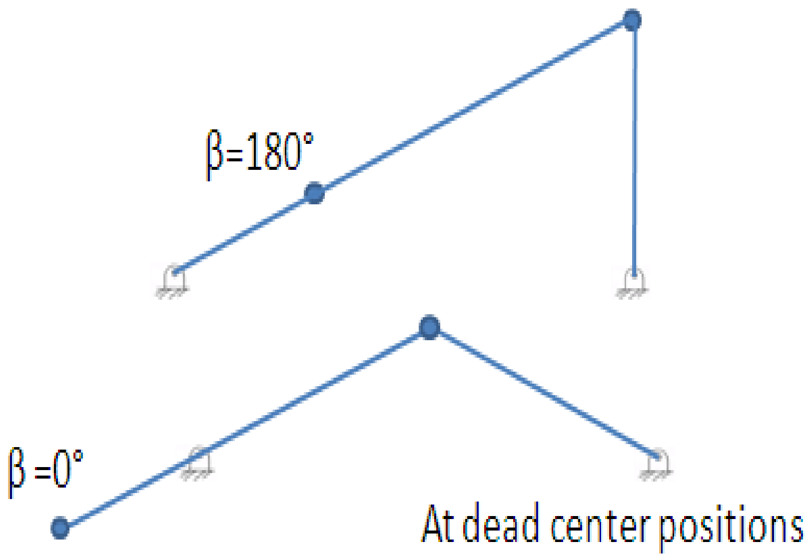


Figure 1.6: Toggle Position

it's entire range of travel. However it is usually possible to design a linkage which exactly satisfies the desired function at a few chosen positions known as precision points. For n accuracy positions in the range $X_0 < X < X_n+_1$

$$X_i = \frac{X_{n+1}+X_0}{2} - \frac{X_{n+1}-X_0}{2} \frac{\cos(2i-1)\pi}{2n}$$

where i = 1 to n.

(2) Spacing of Accuracy Points When a mechanism is designed to generate a given function or trace a given curve, it is not possible in general to obtain a mathematically exact solution. But that the mechanism fits the function or curve at only a finite number of points, the accuracy points. The number of these accuracy points is equal to the number of fixed parameters that may be used in the synthesis, is known as Precision Points. The problem considered here is that of spacing the accuracy points within the interval of function generation to minimize the errors between accuracy points. Consider the function f (x) to be approximated in a given interval of variation of x by means of a mechanism which generates the function F(x; D_1,D_2,...,D_n) where

D_l ,...,D_n are the values of n design parameters in the linkage. The difference between these two functions is the structural error. [f(x) - F(x, D_1, D_2 ,..., D_n)]. The structural error can be find out by plotting against x.

(3) Structural Error

It is assumed that the design deviates very slightly from the desired function between the precision and that the deviation is within acceptable limit. The difference between the function prescribed and the function produced by the design linkage is known as the Structural Error. For most of the cases this error may be 3 to 4%[12].

(4) The Angular velocity ratio theorem

The angular velocity ratio of any two bodies in planer motion with respect to a third body is inversely proportional to the segments in to which the common instant center cuts the line of centers.

(5) The Grashof's Condition

It is very simple relationship which predicts the behavior of a four bar linkage, whether it is rotating or oscillating. $s+l \leq p+q$ [9][12], where "s" and "l" is smallest and largest link length and "p" and "q" are remaining two link lengths.

1.7 Motivation

Extensive studies have been made on synthesis of four bar mechanism for Function Generation, Motion Generation, and Path Generation globally and find applications. In the recent past, around three decades, attempts have been carried out for function generation. Very few studies are based upon the structural error directly because the expression for error is, in general, more difficult to obtain then the expression for the overall error. Need for generalized solution is the need of the hour for function generation where function is polynomial equation of second order considering optimum transmission angle by using Freudenstein's equation as well as Dyad synthesis.

1.8 Aim and Scope of the Work

The aim of the present work is to obtain general solutions for function generation with optimum transmission angle with function using polynomial equation of second order. And also find the structure error. It is expected that these general solutions will be useful to study as well as design the four bar mechanism.

1.9 Description of the Problem

Given multiple sets of precision points, the objective is to synthesis a single mechanism that can trace each set of precision points. This work provides synthesis procedure making use of Chebychev Spacing. By using this technique to synthesis four bar mechanism for function generation with optimum transmission angle is carried out. To developing a mathematical expression leading to development of a code. Present work focuses on to obtain a generalized code for getting the values of link lengths of four bar mechanism for function generation, where function is polynomial equation of second order. And find structural Error for generated function. Comparison between these two method is also carried out.

1.10 Methodology

Initially using function generation method taking function y = f (x), By taking equation $y = a_0 + a_1x + a_2x^2$ where, a_0, a_1, a_2, are constants of polynomial equation. Using chebychev spacing and Freudenstein's equation MAT LAB generalized code is formulated. Also function generation by use of Complex Algebra can be done. The comparison between Freudenstein's synthesis and Dyad synthesis is carried out. Finding the structural error for generated function. The results obtained thereof will be compared with software simulation in Motion View (HYPER WORKS).

Chapter 2

Literature Review

2.1 Introduction

The pioneer of modern kinematics in the United States is generally considered to be Freudenstein. His paper, probably marks the beginning of the shift in emphasis from graphical to analytical methods. The expression '**Approximate Synthesis**', was widely used during this era to denote precision-position synthesis to approximate a given function.

Because of the limitation of the maximum precision points, We can not synthesis 'exactly' our necessity or desired linkage mechanism. To overcome this imperfection also synthesis procedure making use of Cheybychev Spacing. This technique has been developed for function generation considering Transmission Angle.

Very few studies are based upon the structural error directly because the expression for error is, in general, more difficult to obtain then the expression for the overall error.

2.2 Function generation problems

2.2.1 Synthesis by Freudenstein's Equation

The criteria for the design of mechanism are low fluctuation of input torque, compact in size and links proportion, good in force transmission, low periodic bearing loads, less vibrations, less wear, optimum transmission angle and higher harmonics. The transmission angle is an important criterion for the design of mechanism as was pointed out by Bali[13].
For many mechanical applications it can guarantee for the performance of linkage at higher speed without unfavorable vibrations. The solution space for the feasible mechanisms is also reduced. Therefore, an attempt is made to bring the entire information about transmission angle in spatial 4-bar, planar 4-, 5-, 6- and 7-link mechanisms under one umbrella.

Dr. V. B. Math, Sharangouda , S.G.Sarganachari [1] used cheybychev spacing method for synthesis of four bar mechanism. This technique has been developed for synthesis of four link mechanism for function generation considering transmission angle.

Eres Soylemez [15] solved Transmission Angle problem for slidercrank mechanisms is the determination of the dimensions of planar slidercrank mechanisms with optimum transmission angle for given values of the slider stroke and corresponding crank ratio. The solution is obtained as the root of a cubic equation within a defined range, which can be easily implemented on computer.

J. Jesus Ceravantes Sanchez, Emillio. J. Gonzalez Galvan,[8] Provides careful definition of the design coefficient may improve the kinematic synthesis of spherical 4R linkages intended for function generation for three or four precision points. The design process based on a simple system of linear equations whose solution is obtained in closed form. Several applications, examples are presented to prove the feasibility and the validity of the proposed method.

2.2.2 Synthesis by Dyad Equations

Hong Zhou [6] synthesis adjustable function generation linkages using the optimal pivot adjustment. Adjustable four-bar linkages generates flexible output motions using the same set of hardware.
Mc Govern and Sandor [5] utilized complex number method to synthesis adjustable linkages for function and path generations.

Norsinnira Zainul Azlan and Yamoura Hiroshi [11] synthesis under actuated Anthropomorphic finger mechanism for grasping and pinching with optimized parameter. Under actuated finger mechanism is beneficial in the anthropomorphic applications in which it reduced the finger size, weight and power consumption due to less number of actuators compared to its number of degree of freedom.

E. Ngale Haulin, A. A. Lakis, R Vinet [3] optimize synthesis planar four link mechanism used in a hand prothesis. The optimal synthesis of a planar four link mechanism is carried out with reference to n positions of the output and coupling bars. The optimal mechanism obtained is with a minimum acceptable angle of transmission. This crossed four link mechanism is used in the development of a hand prosthesis.

Srinivas S. Bali and Satish Chand, [14] synthesis four-bar mechanism with variable topology for motion between extreme positions. An analytical method of synthesis of a planar five-bar mechanism with variable topology separated position is suggested. The method is useful to reduce the solution space. The motion between two extreme positions of a function generating mechanism is considered.

Hai-Jun Su [16] synthesis Bistable Compliant four-bar mechanism using Polynomial Homotopy. They formulate and solve the synthesis equation for a compliant four-bar linkage with three specified equilibrium configuration in the plane.

2.3 Structural Error

Dr.V. P. Singh, B. S. Thakur, S. Sharma,[2] formulate the structural error problem. Three accuracy points has been extended to make it applicable to four accuracy points for the synthesis problem. Structural error varies in an unpredictable way, so optimization approach by least square technique is used for minimizing the structure error and the results obtained are compared with that of Galerkins Method.

W. SUN Communicated by E. J. Haug [18] determines the initial dimensions of the links of a path-generating mechanism by the closure error function of Freudenstein's equation for a four-bar linkage. The sum of the absolute values of the errors of the output angles is then taken as the cost function for optimization. Successive one-dimensional optimization in the coordinate directions is then used for the design of four-bar function generators.

Todor Stollov Todorov [17] describes a new dimensional synthesis method. The position function of the four-bar mechanism is presented by the Freudenstein's equation and it is minimized by the Chebyshev's best approximation theory. The target function is used as an exactly satisfied equation and Freudenstein's equation is considered as a Chebyshev's polynomial. In some cases the method provides possibilities to find simple solutions of the synthesis tasks.

2.4 Optimization of Transmission Angle

Ming Lun, Yonghou leu [10] development of an analytical method for the design of function generation mechanism with simultaneous optimum transmission behavior and optimum structural error.

Ibrahim Uzmay [7] optimize the Transmission Angle for slider crank mechanism with joint clearance. Joints with clearance at contact points as known crank-pin center and piston-pin center are considered, and their effects on the kinematic characteristics and transmission quality of the model mechanism are investigated.

Dr. V. B. Math, Sharangouda, S. G. Sarganachari[1] done the Synthesis of Slider Crank Mechanism with Optimum Transmission Angle. And transmission angle is find out as root of cubic equation.

H. Zhou, Edmund H.M. Cheung [6] introduce the concept of orientation structural error of the fixed link and presents a new optimal synthesis method of crank-rocker linkages for path generation. The orientation structural error of the fixed link effectively reflects the overall difference between the desired and generated path.

F. Freudenstein, E. J. F. Primrose [4] describes classical Transmission Angle problem. The classical Transmission Angle problem for the dimensions of a plane crank and rocker linkage with optimum Transmission Angel variation for given value of a rocker swing angle and corresponding crank rotation.
The Transmission Angle optimization is formulated as a constrained minimization problem using Langrangian Multipliers and in the case of a plane four-bar linkage a closed-form solution is obtained.

Chapter 3

Mathematical Formulation

3.1 Introduction

Very first formulating the equation for finding Transmission Angle, Mechanical Advantage, and Angle β. They can be utilized for plotting the various graphs from generalized code in MAT LAB. By use of Chebychev Spacing technique and Freudenstein's synthesis, equation for finding link lengths can be formulated.

Formulation for getting structural error can be done for generated function. Kinematic Synthesis by complex algebra (Dyad method) is carried out for function generation.

A. Loop Closer Equation for Four-Bar Mechanism

Figure (3.1)[12] shows a four-bar linkage mechanism with various link lengths as r_1, r_2, r_3, and r_4. All the angles are measured counter clockwise from the x axis, which is along the fixed vector. Considering the closed loop along O_2ABO_4.

If Angle of O_2, and O_4 is zero then,

$$\vec{R_A}=\vec{R_{AO_2}}+\vec{R_{O_2}}$$
$$\vec{R_{O_4}}=\vec{R_{O_4O_2}}+\vec{R_{O_2}}$$
$$\vec{R_B}=\vec{R_{BO_4}}+\vec{R_{O_4}}$$
$$\vec{R_A}+\vec{R_{BA}}=\vec{R_{BO_4}}+\vec{R_{O_4}}$$

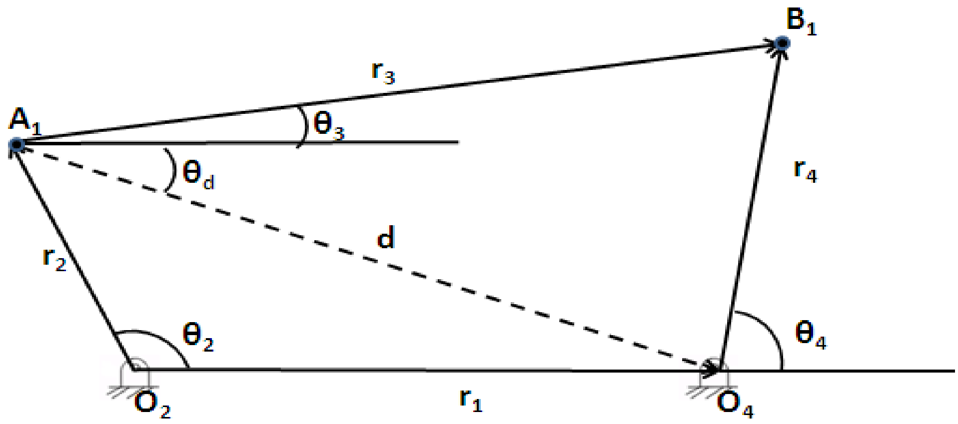


Figure 3.1: Four-Bar Mechanism

$$\vec{R_B}=\vec{R_{BA}}+\vec{R_A}$$
$$\vec{R_B}=\vec{R_{BO_4}}+\vec{R_{O_4}}$$

$$\vec{R_{BO_4}}+\vec{R_{O_4}}=\vec{R_{BA}}+\vec{R_A} \tag{3.1}$$

The equation (3.1) known as loop closer equation.

From these equations,

$r_4 + r_1 = r_3 + r_2$

$r_1+r_4=r_2+r_3$

Using complex exponential notation,

$r_1e^{i\theta_1}+r_4e^{i\theta_4}=r_2e^{i\theta_2}+r_3e^{i\theta_3}$

θ_2 varies between 0° to 360°, and

θ_3 and θ_4 are variables.

$r_1+r_4e^{i\theta_4}=r_2e^{i\theta_2}+r_3e^{i\theta_3}$

These equation is known as loop closer equation in complex notation form.

B. Equation for finding length of Dummy link

For finding the equation for Dummy link length,considering the loop at left side of four bar linkage, it can be write,

$r_2+d=r_1$.

so, $d=r_1-r_2$,

Again using complex exponential notation,

$de^{i\theta_d}=r_1e^{i\theta_2}-r_2e^{i\theta_2}$

$de^{i\theta_d}=r_1-r_2e^{i\theta_2}$

Multiply by complex conjugates,

$d(e^{i\theta_d})(de^{-i\theta_d})=(r_1-r_2e^{i\theta_2})(r_1-r_2e^{-i\theta_2})$

$$d^2 = {r_1}^2 + {r_2}^2 - 2r_1r_2\text{Cos}\theta_2 \tag{3.2}$$

Thus, equation for getting value of dummy link can be achieved.

C. Equation for Output Angle

Considering right side of the four bar mechanism,

$d+r_4=r_3$

Now, $r_3=d+r_4$

$r_3e^{i\theta_3}=de^{i\theta_d}+r_4e^{i\theta_4}$

There are two unknowns, θ_3 and θ_4

$(r_3e^{i\theta_3})(r_3e^{i\theta_3})=(de^{i\theta_d}+ r_4e^{i\theta_4})(de^{-i\theta_d}+r_4e^{-i\theta_4})$

${r_3}^2=d^2+{r_4}^2+dr_4(e^{-i\theta_4}e^{i\theta d}+e^{-i\theta_d}e^{i\theta_4})$

Replaced by sin and cos,

${r_3}^2=d^2+{r_4}^2+dr_4(\cos\theta_4\text{-}i\sin\theta_4)(\cos\theta_d+i\sin\theta_d)+(\cos\theta_d\text{-}i\sin\theta_d)(\cos\theta_4+ i\sin\theta_4)$

${r_3}^2=d^2+{r_4}^2+dr_4(2\cos\theta_d\cos\theta_4+ 2\sin\theta_d\sin\theta_4)$

${r_3}^2=d^2+{r_4}^2+2dr_4\cos(\theta_d\text{-}\theta_4)$

so, $\theta_d\text{-}\theta_4=\cos^{-1}({r_3}^2\text{-} d^2\text{-} {r_4}^2)/(2dr_4)$

$$\theta_4 = \theta_\text{d} - \cos^{-1}\left(\frac{{r_3}^2-\text{d}^2-{r_4}^2}{2r_4\text{d}}\right) \tag{3.3}$$

From equation (3.3) value of follower angle can be find out.

D. Equation for angle of Dummy link length

From , $r_2+d=r_1$

$r_2=r_1-d$

$(r_2e^{i\theta_2})= r_1e^{i\theta_1}- de^{i\theta_d}$

Multiply by complex conjugates,

$(r_2e^{i\theta_2})(r_2e^{-i\theta_2})=(r_1- de^{i\theta_d})(r_1-de^{-i\theta_d})$

${r_2}^2={r_1}^2+d^2- 2r_1d\cos\theta_d$

$\cos\theta_d={r_1}^2+d^2-{r_2}^2)/2r_1d$

$$\theta_d=\cos^{-1}\left(\frac{{r_1}^2+d^2-{r_2}^2}{2r_1d}\right) \tag{3.4}$$

From equation (3.4) dummy link length angle can be find out.

E. Equation for finding θ_3

Now $r_3=d+r_4$

From this equation for θ_3 can be written as,

$$\theta_3 = \cos^{-1}\left(\frac{d\cos\theta_d+r_4\cos\theta_4}{r_3}\right) \tag{3.5}$$

From equation (1.5) angle of coupler link can be find out.

F. Equation for finding Transmission Angle[12]

By use of cos rule,

${r_3}^2+{r_4}^2-2\ r_3\ r_4\cos\gamma=d^2$

$\cos\gamma={r_3}^2+{r_4}^2-d^2/2r_3r_4$

$$\gamma=\cos^{-1}\left(\frac{{r_3}^2-d^2+{r_4}^2}{2r_3r_4}\right) \tag{3.6}$$

From equation (1.6) Transmission Angle can be find out. Same way equation for β can be find out.

3.2 Function Generation formulation

3.2.1 Synthesis by Freudenstein's Equation

For function generation $y = a_0 + a_1x + a_2\, x^2$, where a_0 , a_1 , a_2, are constants of polynomial equation.
Let, subscripts s, f, and i indicate start, final, and intermediate values in the range.
than x_s<x< x_f, and $x_1 \neq x_s$

Form Chebychev Spacing, three precision point approach all the values of x can be obtained. After finding x, from function generator all y value can be obtained.
x_1 gives, $y_1 = a_0 + a_1x_1 + a_2{x_1}^2$
x_2 gives, $y_2 = a_0 + a_1x_2 + a_2{x_2}^2$
x_3 gives, $y_3 = a_0 + a_1x_3 + a_2{x_3}^2$
If θ is input angle and ϕ is output angle and
Range of them is from 0° to 360°

A. Equation for finding values of θ and ϕ
Range of $\theta = \theta_f$ - θ_s and
Range of $\phi = \phi_f$ - ϕ_s
It can be assumed the linear relationship between x and input angle.
Figure (3.2) indicates a linear relationship between x and θ.
From figure, following equation can be written.

$$\frac{x_i - x_s}{x_f - x_s} = \frac{\theta_i - \theta_s}{\theta_f - \theta_s} \tag{3.7}$$

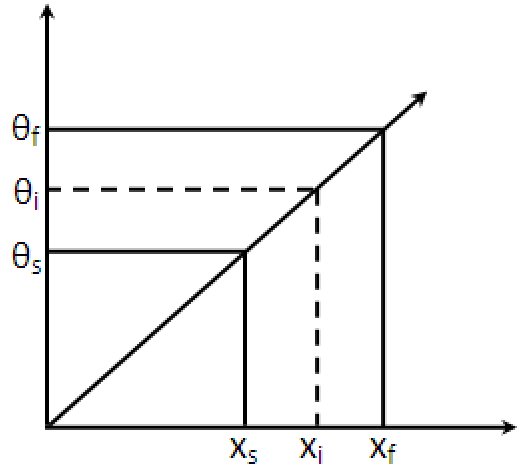


Figure 3.2: Linear relationship between x and θ

$$\theta_i = \theta_s + \frac{\theta_f - \theta_s}{x_f - x_s}\ (x_i - \mathrm{x}_s) \tag{3.8}$$

$$\theta_i = \theta_s + \frac{\Delta\theta}{\Delta \mathrm{x}}\ (x_i - \mathrm{x}_s) \tag{3.9}$$

Same way, linear relationship between values of y and ϕ can be established .

$$\frac{y_i - \mathrm{y}_s}{y_f - y_s} = \frac{\phi_i - \phi_s}{\phi_f - \phi_s} \tag{3.10}$$

$$\phi_i = \phi_s + \frac{\Delta\phi}{\Delta \mathrm{y}}\ (y_i - \mathrm{y}_s) \tag{3.11}$$

From equation (1.9) and (1.11) intermediate values of input and output angles can be find out.

Now, by considering links to be vectors,

Displacement along X axis can be written as,

$r_2\cos\theta + r_3\cos\beta = r_4 + r_3 \cos\phi$

And Along Y axis,

$r_2\sin\theta + r_3\sin\beta = r_4\sin\phi$

By squaring and adding these two equations and comparing with freudenstein's equation solution can be achieved.

Freudenstein's equation can also be written as

$k_1\cos\phi + k_2\cos\theta + k_3 = \cos(\theta - \phi)$

Where, $k_1 = r_1/r_2$,

$k_2 = -r_1/r_4$,

$k_3 = (r_2^2 - r_3^2 + r_4^2 + r_1^2)/(2\times r_2 \times r_4)$,

Either value of r_1 or r_4 can be assumed to be unity to get the proportionate value of other parameters.

For three precision points three equations can be find out.

$k_1\cos\phi_1 + k_2\cos\theta_1 + k_3 = \cos(\theta_1 - \phi_1)$

$k_1\cos\phi_2 + k_2\cos\theta_2 + k_3 = \cos(\theta_2 - \phi_2)$

$k_1\cos\phi_3 + k_2\cos\theta_3 + k_3 = \cos(\theta_3 - \phi_3)$

Where, k_1, k_2, k_3 are Freudenstein constants.

k_1, k_2, k_3 can be evaluated by **Gaussian elimination** method or by **cramer's rule.**

$$\Delta = \begin{vmatrix} \cos\theta_1 & \cos\phi_1 & 1 \\ \cos\theta_2 & \cos\phi_2 & 1 \\ \cos\theta_3 & \cos\phi_3 & 1 \end{vmatrix} \tag{3.12}$$

$$\Delta_1 = \begin{vmatrix} \cos(\theta_1 - \phi_1) & \cos\theta_1 & 1 \\ \cos(\theta_2 - \phi_2) & \cos\theta_2 & 1 \\ \cos(\theta_3 - \phi_3) & \cos\theta_3 & 1 \end{vmatrix} \tag{3.13}$$

$$\Delta_2 = \begin{vmatrix} \cos\phi_1 & \cos(\theta_1 - \phi_1) & 1 \\ \cos\phi_2 & \cos(\theta_2 - \phi_2) & 1 \\ \cos\phi_3 & \cos(\theta_3 - \phi_3) & 1 \end{vmatrix} \tag{3.14}$$

$$\Delta_3 = \begin{vmatrix} \cos\phi_1 & \cos\theta_1 & \cos(\theta_1-\phi_1) \\ \cos\phi_2 & \cos\theta_2 & \cos(\theta_2-\phi_2) \\ \cos\phi_3 & \cos\theta_3 & \cos(\theta_3-\phi_3) \end{vmatrix} \tag{3.15}$$

$k_1 = \Delta_1/\Delta$, $k_2 = \Delta_2/\Delta$, $k_3 = \Delta_3/\Delta$,

By inputting the value of r_1

$r_2 = r_1/k_1$

$r_4 = -r_2/k_2$

$r_3 = (r_2^2+r_4^2+r_1^2) / (2*k_3*r_2*r_4)$

From these equations link lengths of four bar mechanism can be synthesized .

3.2.2 Synthesis by Dyad Method

By using complex number synthesis of four-bar mechanism for function generation is carried out. It is a very powerful approach to the synthesis of planar linkages, to takes advantage of the concept of precision points. Because the links may not change lengths during the motion, the magnitudes of these complex vectors do not change from one position to next, but angles vary. By writing equations at several precision points set of simultaneous equations can be obtained that may be solved for the unknown magnitudes and angles. A four-bar linkage like the great majority of other planar linkages can be thought of as a combination of vector pairs called 'Dyads'.

Let us consider two positions of a vector triad consisting of three vectors r_2, r_3, r_4. Considering, $r_1 = 1$ and $n = 3$ as number of precision points. Figure (3.3) three precision position for four bar mechanism.

By using Loop Closer equation,

$$r_2 e^{i\theta_2{}^{ij}} + r_3 e^{i\theta_3{}^{ij}} = r_1 + r_4 e^{i\theta_4{}^{ij}} \tag{3.16}$$

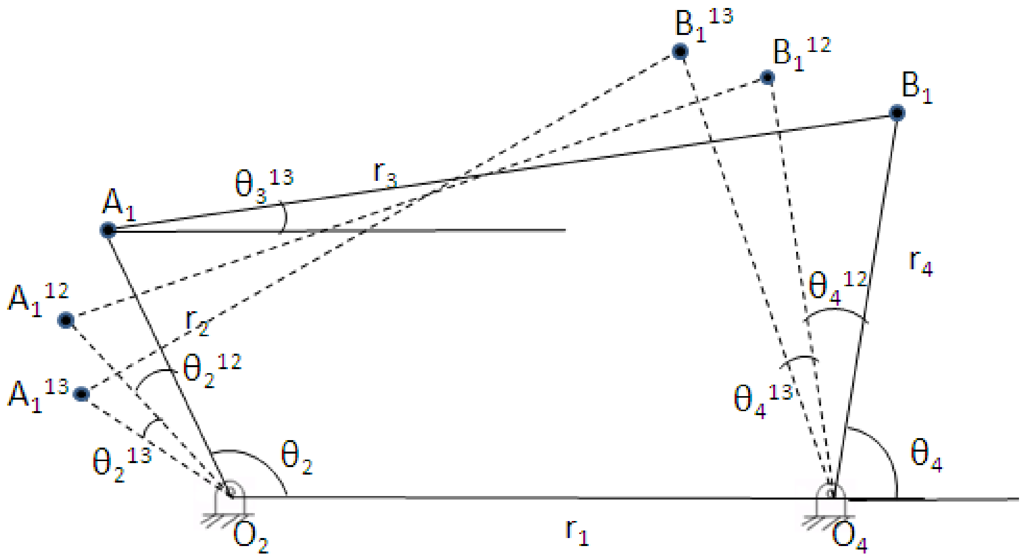


Figure 3.3: Three precision positions of Four-Bar Mechanism

In first position,

Assuming $\theta_2 = 0°$

$$r_2 e^{i\theta_2} + r_3 e^{i\theta_3} = r_1 + r_4 e^{i\theta_4} \tag{3.17}$$

For j = 2,

$$r_2 \mathrm{e}^{i{\theta_2}^{12}} + r_3 \mathrm{e}^{i{\theta_3}^{12}} = r_1 + r_4 \mathrm{e}^{i{\theta_4}^{12}} \tag{3.18}$$

For j = 3,

$$r_2 + r_3 \mathrm{e}^{i{\theta_3}^{13}} = r_1 + r_4 \mathrm{e}^{i{\theta_4}^{13}} \tag{3.19}$$

Equation (1.18) - (1.19) is known as Dyad equations for three precision points.

Considering range of input and output angle is 90°.

Say for example, Start range of input angle $\theta_s = 30°$, and

Final range of input angle $\theta_f = 120°$.

If Start range of output angle is 50° and final range of output angle $\phi_s = 140°$.
Start range of independent variable x, as $x_s = 1$ and final range of x, $x_f = 3$.
If number of precision points = 3, then
By use of chebychev formula, $[x_1\ x_2\ x_3] = [1.13,\ 2.00,\ 2.80]$
From function generation, $[y_1\ y_2\ y_3] = [4.5,\ 9.0,\ 14.9]$.
From the values of intermediate x, values of intermediate input crank angle will be obtained. The values are like
$[\ \theta_1\ \theta_2\ \theta_3\ ] = [0.6288,\ 1.3090,\ 1.9892]$.
From the values of dependent variable y, values of intermediate output rocker angle will be obtained. The values are like,
$[\phi_1\ \phi_2\ \phi_3\ ] = [\ 0.9452,\ 1.5272,\ 2.3055]$.

Transforming dyad equations in to complex rectangular form. if real and imaginary components are separated, two algebraic equations are obtained for each position.
For first position,

$$r_2cos\theta_2{}^{12} + r_3cos\theta_3{}^{12} = r_1cos\theta_1{}^{12} + r_4cos\theta_4{}^{12} \tag{3.20}$$

$$r_2sin\theta_2{}^{12} + r_3sin\theta_3{}^{12} = r_1sin\theta_1{}^{12} + r_4sin\theta_4{}^{12} \tag{3.21}$$

As, $sin\theta_1 = 0$, $cos\theta_1 = -1$
By putting these values in (1.20) and (1.21),

$$r_2cos\theta_2{}^{12} + r_3cos\theta_3{}^{12} - r_4cos\theta_4{}^{12} - 1 = 0 \tag{3.22}$$

$$r_2sin\theta_2{}^{12} + r_3sin\theta_3{}^{12} - r_4sin\theta_4{}^{12} = 0 \tag{3.23}$$

In order to eliminate the coupler angle θ_3 from the equations, moving all the terms except those involving r_3 to the right hand side and squaring both the sides,

$$r_3{}^2cos^2\theta_3{}^{12} = (r_1 - r_2cos\theta_2{}^{12} - r_4cos\theta_4{}^{12})^2 \tag{3.24}$$

$$r_3{}^2 sin^2\theta_3{}^{12} = (-r_2 sin\theta_2 - r_4 sin\theta_4{}^{12})^2 \tag{3.25}$$

Adding these two equations and expanding the right hand side of equation for three precision points, three equation can be obtained. Finally, three equations and three unknown can be evaluated by MAT LAB software by using "solve function". And that gives the values of synthesized link lengths.

3.2.3 Advantages of the Dyad Kinematic synthesis

The Dyad technique by Sandor and Erdman permits handling any types of four bar-linkage mechanism and is extensively used for the synthesis of single degree of freedom mechanism.
Also it is to be noted that unlike the graphical methods, it is not limited by drawing accuracy.

3.3 Structural Error

First of all, Take the rounding values of function generated link lengths. From values of x, values of input angle θ can be find out. By using Freudenstein's equation, output angles can be found out by " fzero function " in MAT LAB 7.6. From these values of output angle ϕ, all the values of y can be obtained. These values are known as 'calculated y' and denoted by 'y_c'. After plotting n v/s y and n v/s y_c, the plot for structural error can be obtained.

3.4 Optimization of Transmission Angle

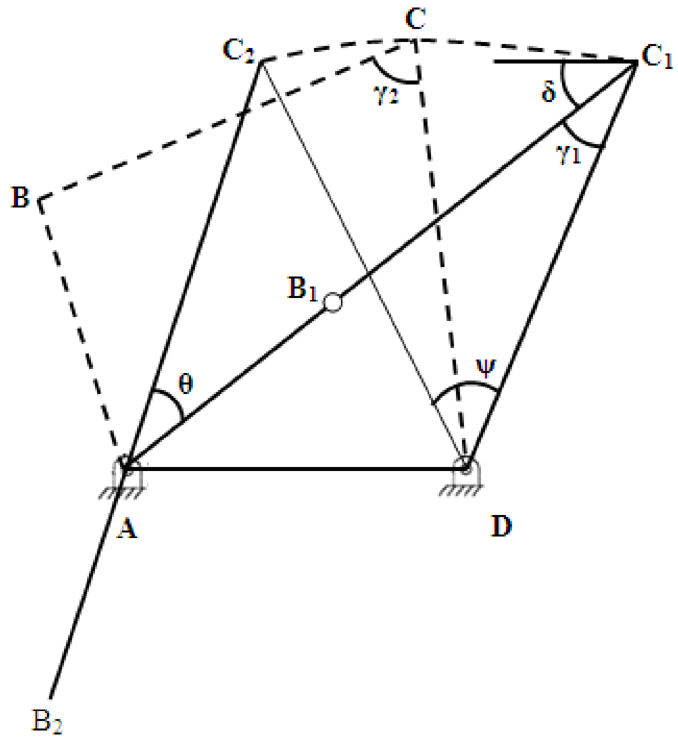


Figure 3.4: Dead center position of four bar mechanism

The two positions considered are the extreme positions. The maximum transmission angle is to be 180°. Hence it is assumed that the mechanism operates between two extreme positions where the maximum and minimum values of transmission angle (40° and 180°) occur. For the prescribed input(θ_1) and output (ψ_1) motions, the links r_2, r_3, r_4 can be determined by determining the angular motions of the coupler link between two extreme positions of it. Figure (3.4) shows the dead center position of four bar mechanism.

When the transmission angle is 0° or 180°, that means two much deviates from the ideal one (90°) the mechanism exhibits poor transmission characteristics. If the transmission angle reaches 180°, the mechanism will be self locked and further motion is seized. It is desirable for the some of the applications like Toggling mechanism, circuit breakers etc.

Figure (3.3) shows the Four-Bar Mechanism and dead center position. Where ABCD is four bar mechanism.

AB_1C_1D is folded position of given four bar mechanism.

AB_2C_2D is extended position.

Input angle $\theta = \angle C_1AC_2$.

The rocker DC oscillates between two extreme positions DC_1 and DC_2 with corresponding rocker swing angle ψ and crank positions AB_1 and AB_2.

Considering $\tau = (\theta + \psi)/2$

$AB_1 = r_2$, $B_1C_1 = r_3$, $C_1D = r_4$, $AD = r_1$, $\delta = \angle C_2C_1A$, $\phi =$ Crank Rotating Angle.

So, r_2, r_3, r_4, r_1 are link lengths.

If r_3 is taken as unity then equation of Transmission Angle is as follow.

$$cos\gamma = \frac{{r_3}^2 + 1 - {r_2}^2 - {r_1}^2}{2r_3} + \frac{r_2 r_1}{r_3}\cos\phi \tag{3.26}$$

Where ϕ is rotating Crank Angle.

Transmission Angle μ is an Acute Angle expressed as $\mu = \gamma$ if $\gamma < 90°$ or $\mu = 180°$ - γ if $\gamma \geq 90°$.

Hence, $cos\mu = \mid cos\gamma \mid$.

In $\triangle AC_1C_2$, by using the sine law, the equations for link length can be obtained.

$$r_2 = \frac{\sin\left(\frac{\psi}{2}\right)\cos\left(\delta + \frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)} \tag{3.27}$$

$$r_3 = \frac{\sin\left(\frac{\psi}{2}\right)\sin\left(\delta + \frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)} \tag{3.28}$$

Using sin law in the $\triangle AC_1D$ gives,

$$r_4 = \frac{2}{\sin\theta}\sqrt{\cos^2\left(\frac{\theta}{2}\right)\sin^2\left(\delta + \frac{\theta}{2}\right) - \cos^2\tau\left(\cos^2\left(\frac{\theta}{2}\right) - \cos^2\left(\delta + \frac{\theta}{2}\right)\right)} \tag{3.29}$$

Substituting these three equations in to equation (3.26),

$$\cos\gamma = \pm\mathrm{N}+\mathrm{M}\cos\phi \tag{3.30}$$

Where,

$$\mathrm{N} = \frac{\cos\tau\left(\cos^2\left(\frac{\theta}{2}\right)-\cos^2\left(\delta+\frac{\theta}{2}\right)\ \right)}{\cos^2\left(\frac{\theta}{2}\right)\ \sin\left(\delta+\frac{\theta}{2}\right)} \tag{3.31}$$

And,

$$\mathrm{M}=\frac{\cos\left(\delta+\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)}\sqrt{1-\frac{\cos^2\tau\left(\cos^2\left(\frac{\theta}{2}\right)\ -\cos^2\left(\delta+\frac{\theta}{2}\right)\ \right)}{\cos^2\left(\frac{\theta}{2}\right)\ \sin^2\left(\delta+\frac{\theta}{2}\right)}} \tag{3.32}$$

The value of $cos\gamma$ is maximum when ϕ=0°, and minimum when ϕ=180°. So,

$$\cos\mu_{\mathrm{min}} = \mathrm{N}+\mathrm{M} \tag{3.33}$$

With given ψ and θ, μ_{min}=0. When δ is minimum or maximum. Thus μ_{min} must have a maximum value $(\mu_{min})_{max}$.

In this case, δ labeled with δ^*.

Rearranging equation (3.32) yields,

$$\mathrm{M}=\sqrt{\left(\frac{1}{n}-1\right)(n-\mathrm{N}^2)} \tag{3.34}$$

Where,

$$\mathrm{n} = 1-\frac{\cos^2\left(\delta+\frac{\theta}{2}\right)}{\cos^2\left(\frac{\theta}{2}\right)} \tag{3.35}$$

The equation (3.33) can be expressed as

$$\cos\mu_{\mathrm{min}} = \mathrm{N}+\sqrt{\left(\frac{1}{n}-1\right)(n-\mathrm{N}^2)} \tag{3.36}$$

Differentiating equation (3.36) with respect to δ yields,

$$\frac{\mathrm{d}(\cos\mu_{\mathrm{min}})}{d\delta}=\left(1-\sqrt{\frac{\frac{1}{n}-1}{\frac{n}{\mathrm{N}^2}-1}}\right)\left(\frac{\mathrm{dN}}{d\delta}-\frac{\mathrm{N}}{2n}\frac{\mathrm{dn}}{d\delta}\left(\sqrt{\frac{\frac{n}{\mathrm{N}^2}-1}{\frac{1}{n}-1}}+1\right)\right) \tag{3.37}$$

Differentiating equations (3.31) and (3.35) with respect to δ yields,

$$\frac{\sin^2\left(\frac{\theta}{2}\right)}{\sin^2\left(\delta+\frac{\theta}{2}\right)}-\sqrt{\frac{\frac{n}{\mathrm{N}^2}-1}{\frac{1}{n}-1}}=0 \tag{3.38}$$

Substituting equations (3.31) and (3.35) in to equation (3.38) and rearranging it yields the uniform relational expression of ψ, θ, and δ^*,

$$\left(\frac{\cos^2\left(\frac{\theta}{2}\right)}{\cos^2\tau}-1\right)\sin^6\left(\delta^*+\frac{\theta}{2}\right)+\sin^2\left(\frac{\theta}{2}\right)\sin^4\left(\delta^*+\frac{\theta}{2}\right)-\sin^4\left(\frac{\theta}{2}\right)\cos^2\left(\delta^*+\frac{\theta}{2}\right)=0 \tag{3.39}$$

Substituting the ψ and θ in to equation (3.39) yields δ^*, So that $(\mu_{min})_{max}$ can be obtained by using equation (3.36).

3.4.1 Example

Four-bar mechanism with θ=33° and ψ=49°.

Substituting ψ and ϕ in to equation (3.39) yield δ^*=7° and from equation (3.36) $(\mu_{min})_{max}$=16°.

Optimized link lengths are, [r_1, r_2, r_3, r_4] = [1, 1.09, 0.60, 0.45]

Chapter 4

Results And Discussion

The generalized solutions for getting equations of input angle, transmission angle and mechanical advantage is carried out. Generalized code for getting link lengths for function generation is obtained. And finding out the structural error, These all are coded using MAT LAB 7.6. Optimization of transmission angle is also carried out.

4.1 Transmission Angle

Methodology for finding all the dimension of mechanism are as follows.

a. Choose the values of link lengths r_1, r_2, r_3, r_4.

b. Choose the range of Input angle θ_2.

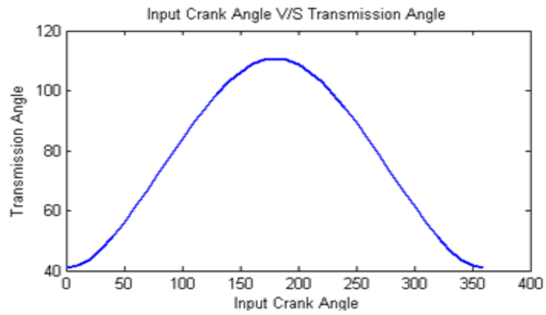
c. Calculate the values of d, θ_d, θ_4, γ, β, and Mechanical advantage.

d. Analyze the relationship between θ_2 and γ.

e. Analyze the relationship between θ_2 and Mechanical Advantage.

f. Analyze the relationship between γ and Mechanical Advantage.

Example: Four bar mechanism link lengths are $r_1 = 1000$, $r_2 = 400$, $r_3 = 900$, $r_4 = 800$ and Input crank angle varies from $0°$ to $360°$.



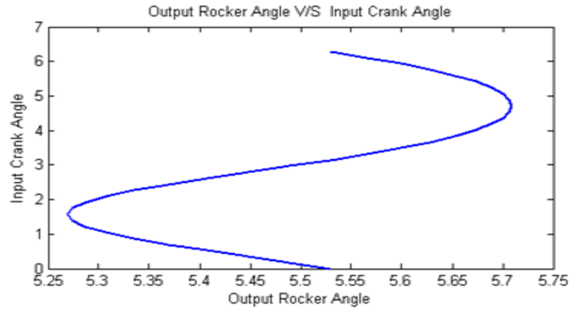
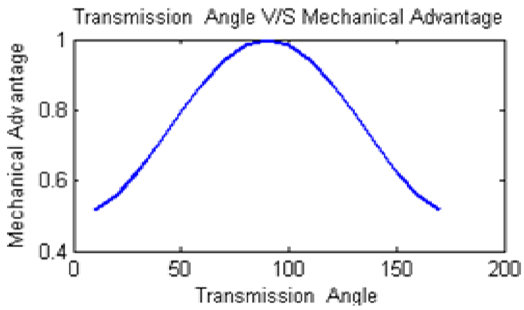


Figure 4.1: (A)Input Angle v/s μ, and(B)Output Angle v/s Input Angle



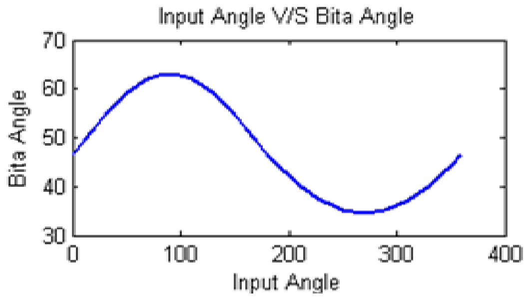


Figure 4.2: (A) μ v/s Mechanical Advantage and(B) Input v/s β angle

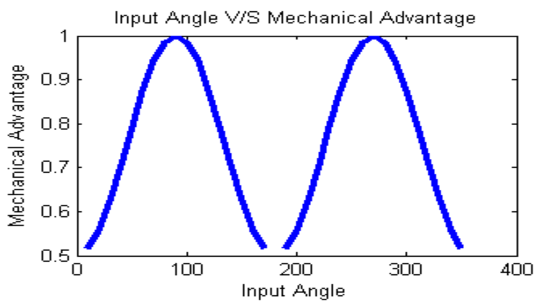


Figure 4.3: Plot for Input v/s Mechanical Advantage

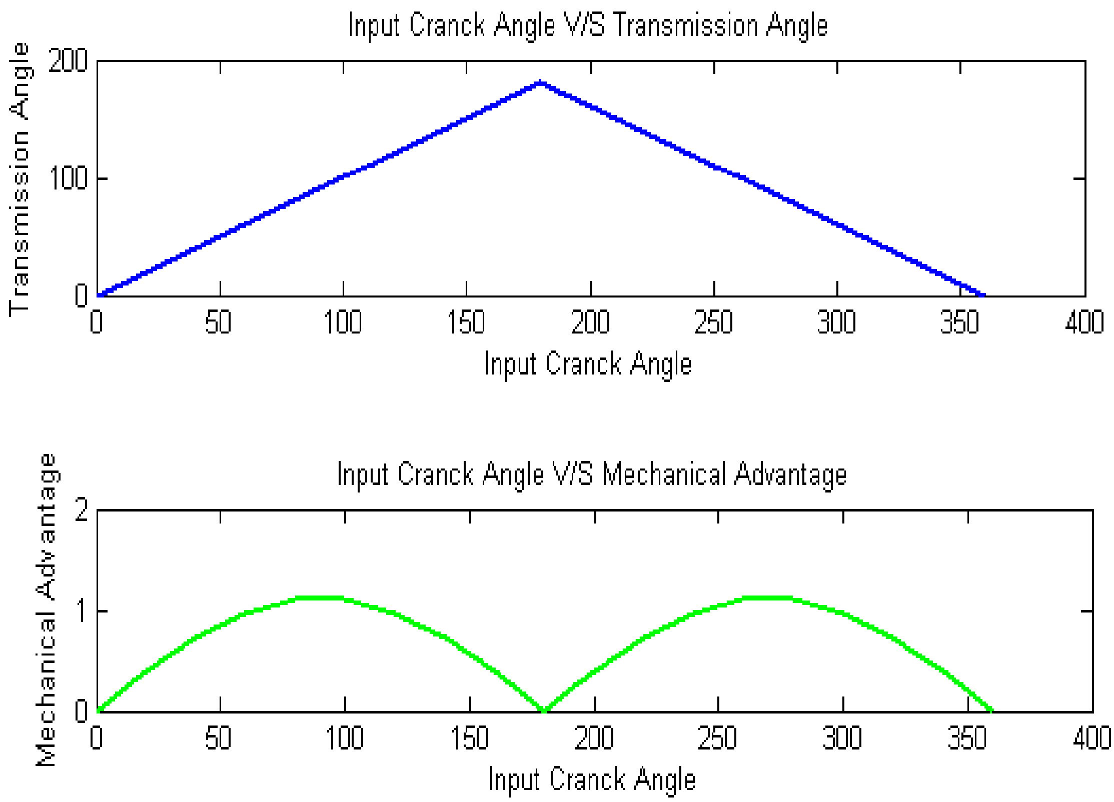


Figure 4.4: Plot for Same link lengths four bar mechanism

From figure (4.1A), Transmission angle is minimum when Input angle is 0° and maximum 100° when Input angle is 180°. Figure (4.1B) shows the characteristic curve of Output and Input angle. Figure (4.2A) shows plot between Transmission angle and the Mechanical Advantage. When Transmission Angle is 90° at that time the value of Mechanical Advantage is maximum. Figure (4.2B) shows relation between input angle and β. Figure (4.3) shows plot for input angle v/s mechanical Advantage. From the graph, Mechanical Advantage becomes unity at input angle 90°. Figure (4.4) shows the plot for Input Angle v/s Transmission Angle and input angle and mechanical advantage for same link length. From the first graph Transmission Angle is 0° at 0° Input Angle and maximum value 180° at 180° Input Angle, it is reducing simultaneously as crank complete its rotation of motion. Mechanical Advantage is 0 at 0° input Angle. And that of maximum at 90° and 270° Input Angle.

4.2 Function Generation

4.2.1 Synthesis by Freudenstein's Equation:

a. Choose the value x_0, x_f, n, a_0, a_1 and a_2.

b. Calculate the values of x_1, x_2, and x_3 from Chebychev Spacing.

c. Calculate the values of y_1, y_2, and y_3 from function generator y = f (x).

d. Choose the value of θ_s, θ_f, ϕ_s and ϕ_f.

e. Calculate the values of intermediate values of θ and ϕ.

f. Calculate the values k_1, k_2, and k_3 from Freudenstein's equation.

g. Choose the value of fixed link length r_1.

h. Obtain the link lengths r_2, r_3, and r_4 from Freudenstenstein's constants.

i. Analyze the relationship between θ_2 and γ.

Example

For function $y = 1 + 2x + x^2$, start rage of $x = 1$ and final range of $x = 3$, number of precision point $n = 3$. Values of constants of polynomial equation $a_0 = 1$, $a_1 = 2$ and $a_2 = 1$, θ_s= 30°, θ_f= 120°, ϕ_s=50° and ϕ_f=140°, Take fixed link length r_1=1.
[r_1, r_2, r_3, r_4] = [1, 1.02, 0.77, 0.98]

Table I: Result Table for function generated Links by Freudenstein Equation

Sr. no.	x	y	θ	ϕ	k
1	1.1340	4.5538	36.028°	50°	0.9804
2	2	9	75°	88.5648°	−1.0101
3	2.866	14.9462	113.9711°	140°	1.1943

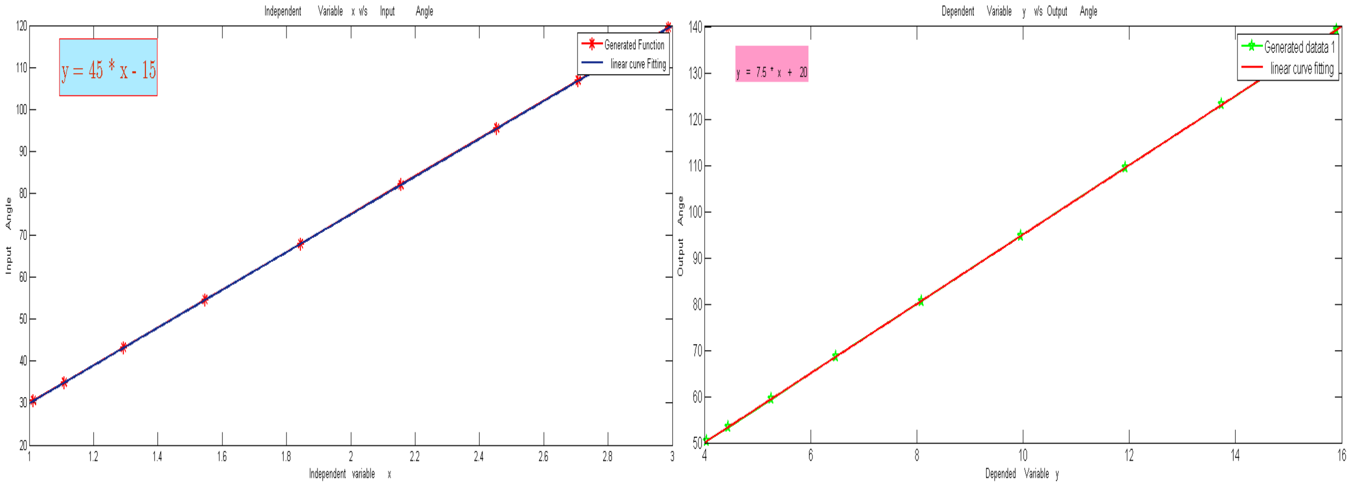


Figure 4.5: Linear Relationship between(A)x v/s Input Angle and (B) y v/s Output Angle

Figure (4.5A) shows the linear relationship between x and Input Angle. The curve is fitted with linear equation of motion. The line of equation is found as $y = 45x$ - 15. Same way linear relationship between y and Output Angle is shown in figure (4.5B). That fitted with linear equation which gives line of equation as $y = 7.5x + 20$.

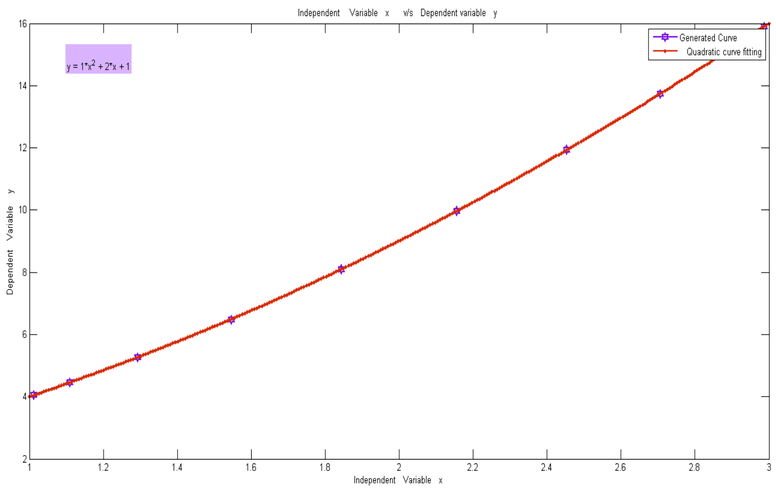


Figure 4.6: Parabolic relationship between x and y

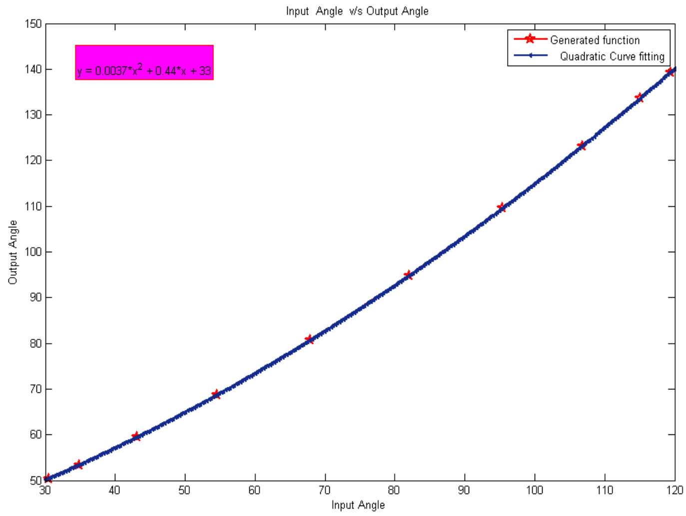


Figure 4.7: Parabolic relationship between Input Angle and output Angle

Figure (4.6) shows the parabolic relationship between independent variable x and dependant variable y. The another figure shows the parabolic relationship between Input Angle and Output Angle. If curve fitting for parabolic curve is carried out then the blue colored curve is generated curve. This curve fitted by quadric equation, as shown in figure as red curve.Which are almost intersect at all the points and gives the equation of parabola as y = $x^2 + 2x + 1$. Same way if Input Angle against Output Angle is plotted as shown in figure (4.7). That also gives parabolic relationship between them. The curve which is highlighted by red portion is shows the curve for generated function and that is fitted with quadric equation which is blue in color. Which is almost intersect the generated function. Equation of curve is given by $y = 0.0037x^2 + 0.44x + 33$.

4.2.2 Synthesis by Dyad Equations

a. Calculate the values of x_1, x_2, and x_3 from Chebychev Spacing.

b. Calculate the values of y_1, y_2, and y_3 from function generator y = f(x).

c. Choose the value of θ_s, θ_f, ϕ_s and ϕ_f.

d. Calculate the values of intermediate value of θ and ϕ.

e. Derive equations for Dyad using complex algebra.

f. Choose the value of fixed link length r_1.

g. Solve the equations in MATLAB and find the unknowns from Dyad equations.

h. Obtain the link length r_2, r_3, and r_4.

i. Analyze the relationship between θ_2 and γ.

j. Compare results with results of Freudenstein's method.

Example A mechanism will be synthesized to generate the function $y = 1+2x+x^2$. Start rage of $x = 1$ and final range of $x = 3$, Number of precision points $n = 3$. Values of constants of polynomial equation $a_0 = 1$, $a_1 = 2$ and $a_2 = 1$, θ_s= 30°, θ_f= 120°, ϕ_s = 50° and ϕ_f = 140°, Take fixed link length r_1=1. For input range is $\Delta\theta = 90°$ and the output range is $\Delta\phi = 90°$.
Link lengths are, [r_1, r_2, r_3, r_4] = [1, 0.99, 0.78, 0.96].

4.3 Structural Error

a. Choose the value x_0, x_f, n, a_0, a_1 and a_2.

b. Calculate the values of x_1, x_2, and x_3 from Chebychev Spacing.

c. Calculate the values of y_1, y_2, and y_3 from function generator y = f (x).

d. Choose the value of θ_s, θ_f, ϕ_s and ϕ_f.

e. Calculate intermediate value of θ and ϕ.

f. Choose the rounding values of previous link lengths.

g. Calculate the values k_1, k_2, and k_3 from input values of link lengths.

h. Calculate the intermediate values of ϕ from values of k and θ.

i. Calculate the intermediate values y_c from ϕ.

j. Analyze the relationship between n and y, x and y_c.

Example

For function $y = 1 + 2x + x^2$, start rage of $x = 1$ and final range of $x = 3$, Number of precision point $n = 10$. Values of $a_0 = 1$, $a_1 = 2$, $a_2 = 1$, $\theta_s = 30°$, $\theta_f = 120°$. link lengths $r_1 = 100$, $r_2 = 102$, $r_3 = 77$, $r_4 = 98$, Find the structural error.

Table II: Result Table for Structural Error for Freudenstein Synthesis

Sr. no.	x	θ	ϕ	y	y_c	%Error
1	1.0123	30.5540°	51.6242°	4.0494	4.3347	−7.045
2	1.1090	34.9047°	53.6616°	4.4479	4.5763	−2.868
3	1.2929	43.1802°	59.0596°	5.2574	5.2486	0.167
4	1.5460	54.5704°	68.2879°	6.4822	6.4248	0.885
5	1.8436	67.9604°	80.5744°	8.0859	8.0039	1.0140
6	2.1564	82.0396°	94.5822°	9.9631	9.8038	1.5989
7	2.4540	95.4296°	108.8890°	11.9301	11.6261	2.548
8	2.7071	106.8198°	122.2016°	13.7426	13.2823	3.349
9	2.8910	115.0953°	133.3626°	15.1399	14.5925	3.615
10	2.9877	119.4460°	140.7588°	15.9017	15.3529	3.454

table (2) shows result table for structural error for freudenstein's method.

Table III: Result Table for Structural Error for Dyad synthesis

Sr. no.	x	θ	ϕ	y	y_c	%Error
1	1.0123	30.5540°	51.6242°	4.0494	4.2166	−4.1290
2	1.1090	34.9047°	53.6616°	4.4479	4.4882	−0.9.60
3	1.2929	43.1802°	59.0596°	5.2574	5.2079	0.9415
4	1.5460	54.5704°	68.2879°	6.4822	6.4384	0.6788
5	1.8436	67.9604°	80.5744°	8.0859	8.0766	0.1150
6	2.1564	82.0396°	94.5822°	9.9631	9.9443	0.1887
7	2.4540	95.4296°	108.8890°	11.9301	11.8519	0.6555
8	2.7071	106.8198°	122.2016°	13.7426	13.6269	0.8419
9	2.8910	115.0953°	133.3626°	15.1399	15.1150	0.1645
10	2.9877	119.4460°	140.7588°	15.9017	16.1012	1.2546

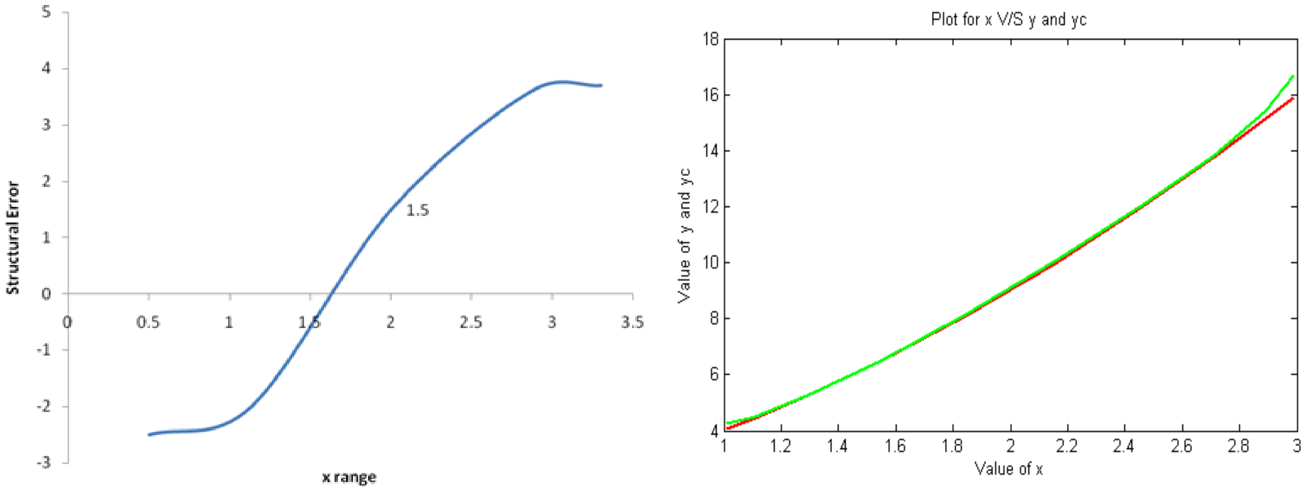


Figure 4.8: Plot for(A)x range v/s Structural Error and (B)x v/s y and y_c

Table (3) shows the result table for structural error for dyad synthesized link lengths. Figure (4.8A) shows the plot for x range v/s structural error.

Figure (4.8B) shows the plot for x v/s y values and y_c values. The red color curve is plot for x and y values which is prescribed function. The green colored curve plot for x and y_c values which is generated one. So it can be concluded from this plot that, the desired output can not be achieved due to the structural error. It can be concluded that error is more at end points.

4.4 Comparison Between Freudenstein's and Dyad Synthesis

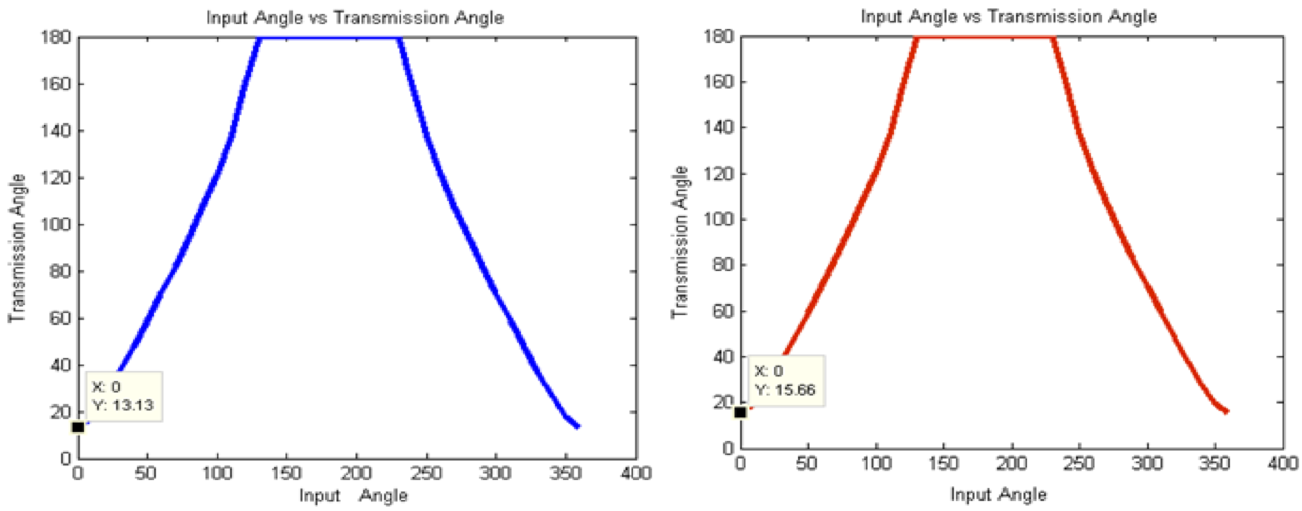


Figure 4.9: plot for Input Angle v/s Transmission Angle for (A)Freudenstein's and (B)Dyad Synthesis

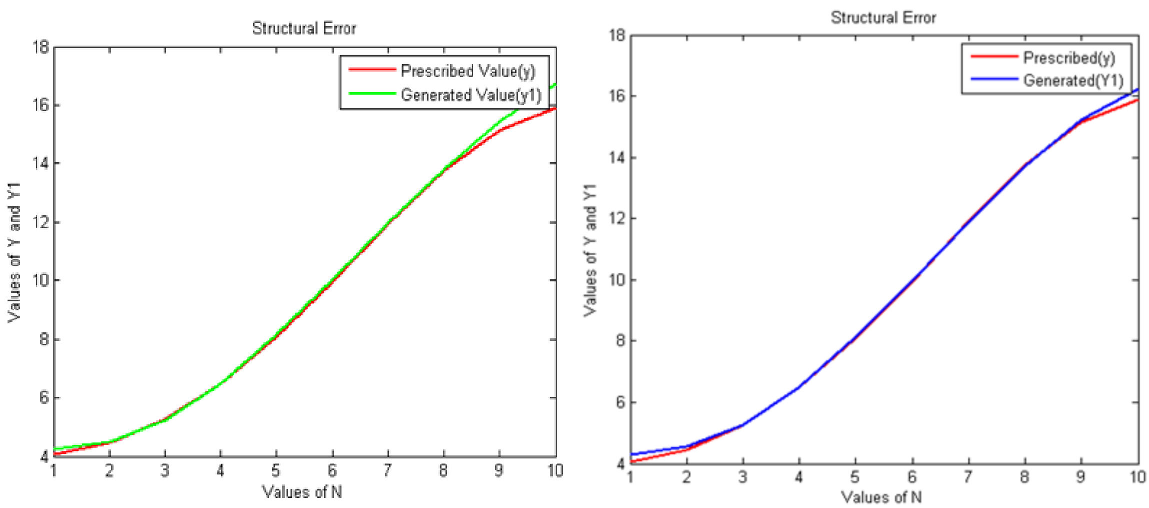


Figure 4.10: Structural Error for (A)Freudenstein's and(B)Dyad Synthesis

Figure (4.9) shows the comparison between Freudenstein's and Dyads methods for Input Angle v/s Transmission Angle. The Fredenstein method gives minimum value of Transmission Angle is 13.13 at 0° Input Angle.
In Dyad method of synthesis the minimum value of Transmission Angle is 16.66 at 0° Input Angle.

If it is plotted n values against y and n values against y_c gives structural error. Figure (4.10) shows comparison for structural error for both the method. In Freudenstein's method the maximum structural error is 3.45% and in Dyad synthesis it is 1.25%.

Table (4) shows the comparison between synthesized link lengths.

Table IV: Result Table for comparison of Function generated link lengths by Freudenstein's and Dyads synthesis

Sr. no.	Link Length	Freudenstein Synthesis	Dyad Synthesis	%Differences
1	r_1	1.00	1.00	0.0%
2	r_2	1.02	0.99	-0.0303%
3	r_3	0.77	0.78	0.0128%
4	r_4	0.98	0.96	-0.020%

4.5 Software Simulation

After synthesizing the link lengths of planar four bar linkages, simulation of that with software is carried out. For that preparing the model in dimensions whichever is getting by Freudenstein's equation.
The curve of input angle verses output angle achieved in the software is shown in figure (4.11). It is plotted after importing the plot file in to the Microsoft Excel file. The equation of parabolic curve is achieved.

Figure (4.12) shows the Motion View [HYPER WORKS] model for synthesized link lengths for function generation. For preparing the model in Motion View software, the following steps can be carried out.

a. Locate the end points and mid points of four bar mechanism in new file by use of point tool.
   Add points p_0 to p_6.

b. Create bodies and name them as crank body, coupler body, follower body, by use of body too.
   Ground body already created by default in the software.
   Taking CM Co-ordinate system for body. Assume the properties of all the body.
   Take mass of body as 1000 unit. Take all inertia properties as 1.

c. Taking cylinder as graphic connectivity with two points and related body as radius 5.

d. Create joints 0,1,2,3 as revolute joint with two body and corresponding one point.Take rotational vector as global z axis.

e. Give motion 0 to connectivity at ON JOINT, as joint 0, by taking property as a velocity option.

f. Create output 0 with entity ON JOINT join 0 with property displacement. Take reference marker as global marker by default. Again create output 1 as property displacement to the entity ON JOINT on joint 3 with by default global reference marker.

g. Then go for checking of the model and run after saving the model.

h. Animate the model with transition option.

i. Create the plot related to animation. Take marker displacement on crank body to YAW position on x axis and required output as OUTPUT 1 on follower body at YAW position on y axis.

j. Compare this plot with curve which found in MATLAB code.

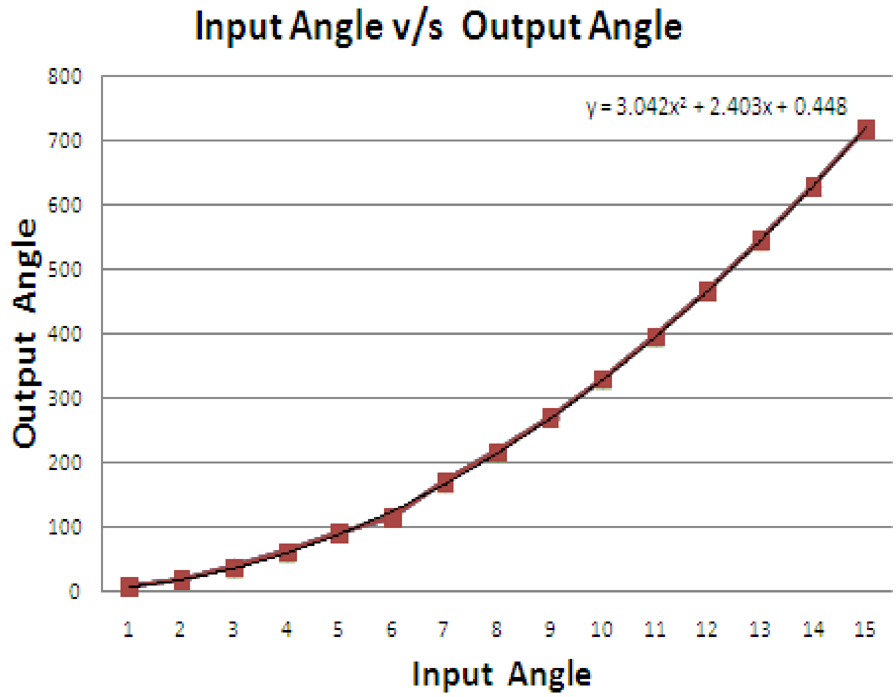


Figure 4.11: Parabolic Curve

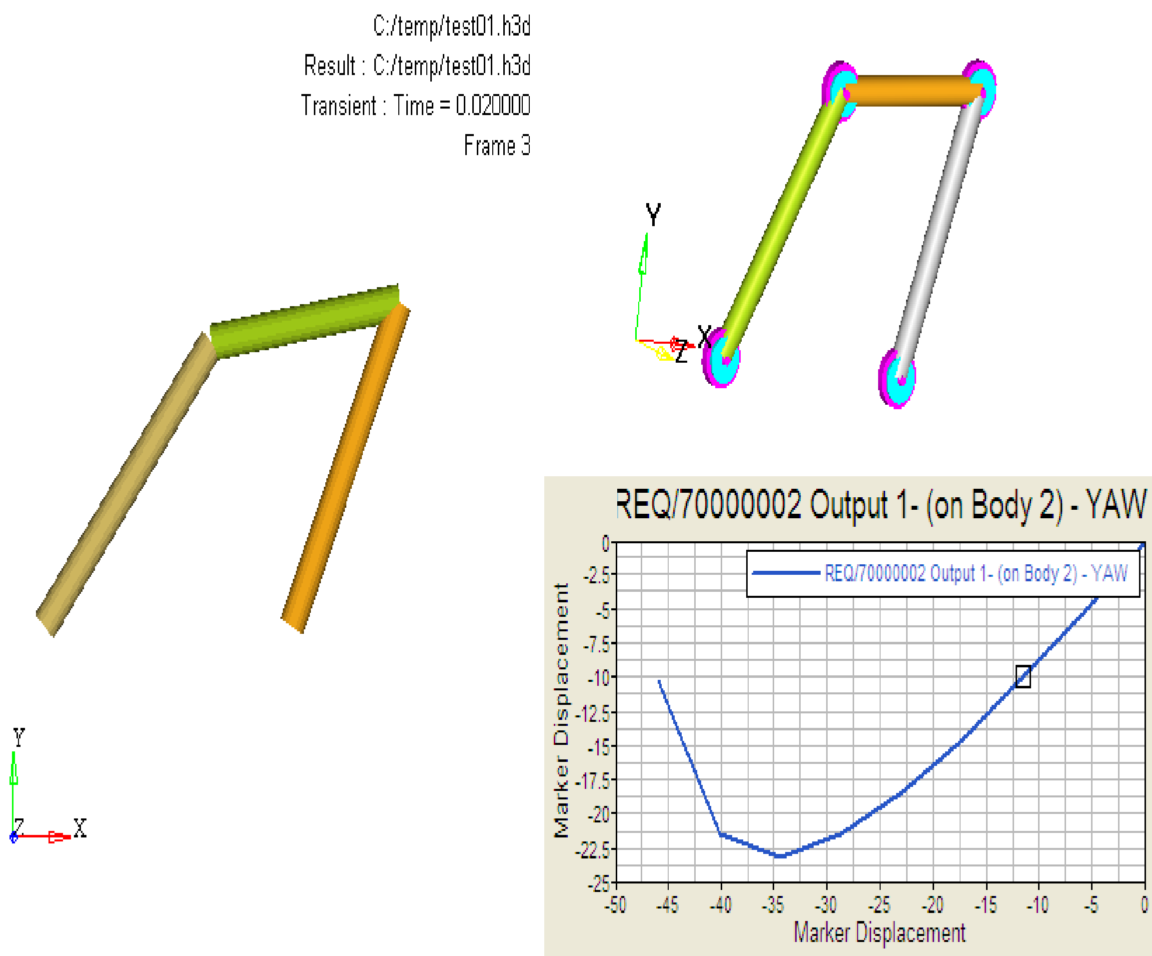


Figure 4.12: Software Simulation

By software simulation, the parabolic curve is achieved which is same as found in MAT LAB generated curve as shown in figure (1.12) The characteristic is same for both the graphs. So it can be concluded that the same output is achieved in simulation software as well as in analytical synthesis for synthesis of four-bar mechanism for function generation, where function is polynomial equation of second order.

Chapter 5

Conclusion and Future Scope

5.1 Conclusion

The general solution presented is very much useful to study of Synthesis of four bar mechanism for function generation with using Chebychev Spacing and Freudenstein's equation, where function is polynomial equation of second order. Analytical method has been developed for the approximate generation of functions, using a four-bar linkage having three-accuracy points. Numerical results demonstrate that Freudenstein Method in function generation is an effective and convenient method and it possesses possibilities to find the best solution of the task. Although the degree of this effectiveness, to a large extent, depends on the required function.

Generally smooth structural error distributions remaining within tolerable bounds are obtained. Free parameters,such as, length and rotation of input link and sub domain choice, which are to be found in formulation, will provide the designer with some means for selecting the most suitable one of the resulting designs.

The general solution presented is very much useful to study of Synthesis of Four-Bar Mechanism for function generation with using Chebychev Spacing for three accuracy points, where function is polynomial equation of second order. Function gives parabolic curve as output. Analytical Synthesis using Complex Algebra for Synthesis of four bar mechanism for function generation is carried out.

The synthesis of Four-Bar Mechanism for function generation with optimum transmission is carried out with help of MATLAB 7.6. Generated function satisfies the Grashof's condition, and gives fully rotating mechanism. Also find out the structural error for generated function.

5.2 Future Scope

Freudenstein Method and Kinematic Synthesis using Complex Algebra can be easily extended to cover other areas of applications. These include the kinematic design of planar, spherical and spatial mechanisms with revolving, sliding and rolling pairs, whose displacement functions exhibit suitable patterns according to several criteria.

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