

## Simulation of Trajectories and Comparison of Joint Variables for robotic Manipulator using Multibody Dynamics (MBD)

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### Abstract

This paper deals with simulation of different manipulator for a given value of joint variable and comparing workspace generated by both manipulators. In this work, the analytical model is validated and simulated using Multibody Dynamics (MBD) of Hyperworks. The objective of paper is to analyze singularities and variation of joint variable for both configurations while moving around a specified path. The link velocity is computed for both configuration and it is compared with analytical model. The trajectory used for manipulator is same and coding is done for forward and inverse kinematics using MATLAB 6.5.

**Keywords:** Joint Variable, MBD, Jacobian, Joint Velocity

### Introduction

A robot manipulator requires the knowledge of the end effectors' position and orientation for the instantaneous location of each joint as well as knowledge of the joint displacement for placing the end effector in a new location. Therefore, the direct and inverse kinematics is the fundamental problem of the most importance in the robot manipulator's position control. Certain applications like welding and assembly operation require that the end effector or tool should follow a given path. To achieve this, it is necessary to find corresponding motions of each joint, which will produce desired end effector motion.

The joint displacement that lead the end effector to a certain position and orientation  $T$  can be found by solving the kinematics model equations for unknown joint displacement and the location of the end effector is obtained by moving each joint by respective joint displacement.

For the manipulator, not only the final location of the end effector is important, but also the velocity with which the end effector would move to reach the final location is also an equally important concern. This requires the coordination of instantaneous end effector velocity and joint velocities.

The transformation from joint velocities to the end effector velocity is described by a matrix called Jacobian. The Jacobian matrix, which is dependant on manipulator configuration, is a linear mapping from velocities in joint space to velocities in Cartesian space. The inverse problem, where the joint velocities are to be determined for given end effector velocity is of practical importance and requires the inverse of the Jacobian. At certain location in joint space, the Jacobian matrix may lose rank and it may not be possible to find its inverse. These locations are referred as singular configuration.

The end effector of the manipulator is required to move in a particular path or trajectory to accomplish given task. The trajectory planning can be described as a time sequence of joint or end effector locations and derivatives of locations, which are generated, by interpolation or approximation. Also the position and orientation of the end effector can be defined in a Cartesian space. The inverse kinematics problem is set to find end effector velocity and subsequent solution of joint velocities and Jacobian.

### Mathematical Modeling

The articulated arm shown in Figure 1 consists of three links with rotary joints. The first link consists of rotation about vertical axis and other two links simulates the human arm with shoulder and elbow joint. The work volume of this configuration is spherically shaped.

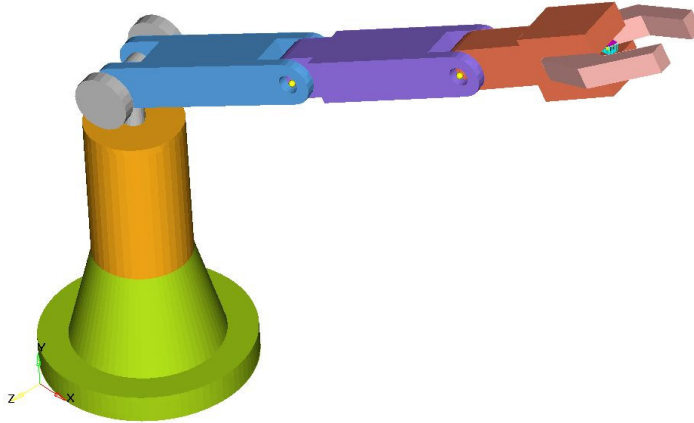


Figure 1: Articulated Arm Robot

The Table - 1 describes the D – H parameter for both manipulators.

**TABLE 1**  
D–H Parameter for RRR configuration

Joint	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	0	90°	0	$\theta_1$
2	L2	0	0	$\theta_2$
3	L3	0	0	$\theta_3$

### Forward Kinematics Equation

$${}^0T_1 = \begin{bmatrix} \cos \theta_1 & 0 & \sin \theta_1 & 0 \\ \sin \theta_1 & 0 & -\cos \theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 & 0 & L_2 \cos \theta_2 \\ \sin \theta_2 & \cos \theta_2 & 0 & L_2 \sin \theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_3 = \begin{bmatrix} \cos \theta_3 & -\sin \theta_3 & 0 & L_3 \cos \theta_3 \\ \sin \theta_3 & \cos \theta_3 & 0 & L_3 \sin \theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The overall transformation matrix for the end point of the arm is,

$${}^0T_3 = \begin{bmatrix} c_1c_{23} & -c_1s_{23} & s_1 & c_1(L_3c_{23} + L_2c_2) \\ s_1c_{23} & -s_1s_{23} & -c_1 & s_1(L_3c_{23} + L_2c_2) \\ s_{23} & c_{23} & 0 & L_3s_{23} + L_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where,  $c_1 = \cos \theta_1, s_1 = \sin \theta_1, c_2 = \cos \theta_2, s_2 = \sin \theta_2$

$$c_{23} = \cos(\theta_2 + \theta_3), s_{23} = \sin(\theta_2 + \theta_3)$$

### Velocity and Jacobian Calculation

The mapping of the linear angular velocity vector, from frame {1} to frame {2}, can have reference of two frames, the frame of description and the frame of differentiation. For the manipulator, the frame of differentiation is base frame, that is, frame {0}. The n – DOF manipulator is an open Kinematic chain with n links. The base, link 0, is the stationary reference and velocity of all other links is defined with respect to this.

The linear velocity of the  $i^{\text{th}}$  link is given by following equation:

$$v_i = \sum_{j=1}^i \frac{\partial({}^0T_i)}{\partial q_j} \dot{q}_j {}^iD_i$$

The angular velocity of  $i^{\text{th}}$  link is given by following equation:

$$\omega_i = \omega_{i-1} + {}^0R_{i-1z_{i-1}} \dot{q}_i$$

From the above equation, it is observed that the Cartesian velocities of the end effector are linearly related with the joint velocities. This relationship is shown as:

$$V_e(t) = J(q) \dot{q}$$

Where,  $V_e(t) = 6 \times 1$  Cartesian velocity vector =  $\begin{bmatrix} v \\ \omega \end{bmatrix}$

$$J(q) = 6 \times n \text{ Manipulator Jacobian or Jacobian Matrix} = \begin{bmatrix} j_{vi} \\ j_{\omega i} \end{bmatrix}$$

$$\dot{q} = n \times 1 \quad \text{vector of joint velocities}$$

### Results & Discussions from Mathematical model

The Figure 2 shows the trajectory generated by program. The Figure 3 and Figure 4 indicated velocity variation through given range of displacement.

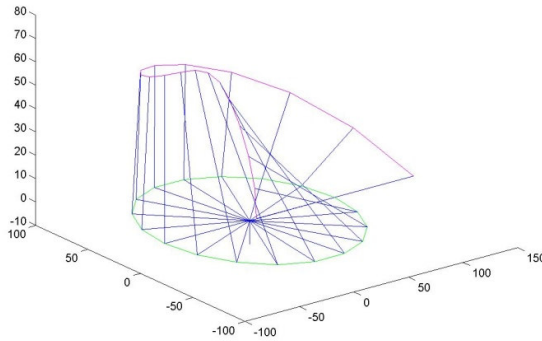


Figure 2: Displacement from mathematical model

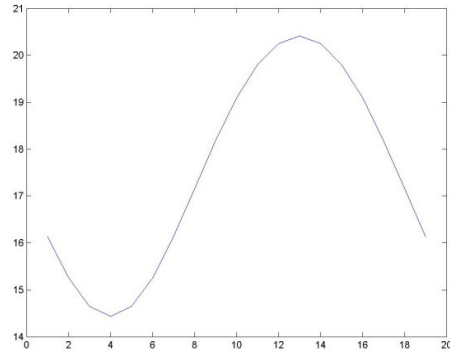


Figure 3 Velocity of Link6

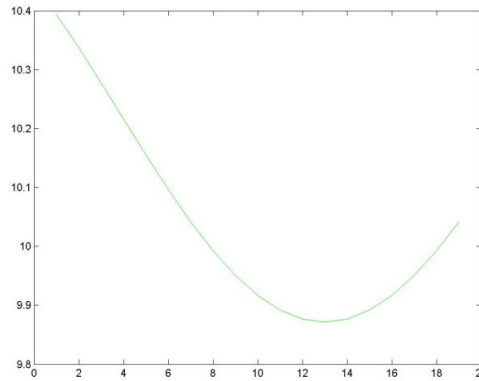


Figure 4 Velocity of End Effector

### Results & Discussion from simulation using MBD of Hyperworks

The virtual model is required to create to implement the concept. The mathematical model provides the basis for synthesizing the dimensions of the various bodies and parts to be assembled.

Looking into the type of robot and limiting displacement and velocities of various joints and links, the model can be created in any modeling software. In this work the model is created with the help of Pro/Engineer 5.0. The same is translated into \*.step files and imported into Multi Body Dynamics (MBD) environment of Hyperworks to carryout kinematic and dynamic analysis

The TABLE -2 gives information of Points and Bodies imported from the assembly.

TABLE - 2  
POINTS AND BODY INFORMATION

POINTS	LX	LY	LZ	Symmetry
<b>The Model</b>				
Global Origin	0.000000e+000	0.000000e+000	0.000000e+000	
BASE_3.PRT CG	0.000000e+000	6.181387e+000	0.000000e+000	
C1_2.PRT CG	0.000000e+000	4.500000e+001	-1.285693e-014	
R1_6.PRT CG	0.000000e+000	1.132219e+002	-3.535995e-014	
LINK2_2.PRT CG	3.301310e+001	1.280372e+002	-1.044825e+001	
LINK_6.PRT CG	9.694930e+001	1.541275e+002	-3.068328e+001	
E_F_3.PRT CG	1.613411e+002	1.824744e+002	-5.106250e+001	
PRT0001_2.PRT CG	1.836901e+002	1.938735e+002	-5.813569e+001	

Point_Link6_Rev	6.951310e+001	1.410372e+002	-1.280000e+001	
Point_EF3_Rev	1.340000e+002	1.684000e+002	-3.300000e+001	
R1_Point 9	5.000000e+000	1.140000e+002	1.700000e+001	
Point_link6_2	6.600000e+001	1.410000e+002	-2.400000e+001	
Point_EF3_Rev_2	1.310000e+002	1.684000e+002	-4.200000e+001	
Point_c1_R1_fixed	0.000000e+000	9.400000e+001	0.000000e+000	
<b>BODIES</b>	L.Mass	L.Ixx	L.Iyy	L.Izz
<b>The Model</b>				
Ground Body	0.000000e+000	0.000000e+000	0.000000e+000	0.000000e+000
BASE_3.PRT	4.371622e-001	1.928819e+002	3.050571e+002	1.928819e+002
C1_2.PRT	2.608259e-001	2.373244e+002	3.993896e+001	2.373244e+002
R1_6.PRT	4.294497e-002	1.006706e+001	9.186121e+000	2.259872e+000
LINK2_2.PRT	5.370621e-002	3.292466e+000	2.331516e+001	2.232774e+001
LINK_6.PRT	5.714977e-002	3.133241e+000	2.688925e+001	2.621251e+001
E_F_3.PRT	5.296037e-002	3.887337e+000	1.930244e+001	2.044936e+001
PRT0001_2.PRT	1.940694e-002	2.806132e+000	6.592891e+000	3.993766e+000

### Joint Information

TABLE – 3  
JOINT DEFINITION

Joint No	Joint Name	Joint Type	Body 1	Body 2
1	Rev_base_c2	Revolute Joint	Frame C2	Base frame
2	Fixed base	Fixed Joint	Base Frame	Ground Body
3	Fixed_R1_c1	Fixed Joint	Frame R1	Frame C2
4	R1_link2	Revolute Joint	Link2	Frame R1
5	Rev_link2_link6	Revolute Joint	Link2	Link6
6	Rev_link6_EF3	Revolute Joint	Link6	End Effectors
7	Fixed_Ef_part1	Fixed Joint	Link6	Part1

### Motion Information

TABLE – 4  
MOTION INFORMATION

Joint No	Joint Name	Motion Type	Property
1	Rev_base_c2	Displacement	STEP (TIME, 0, 0, 10,90D)
4	R1_link2	Displacement	STEP (TIME, 0, 0, 10,-90D)
5	Rev_link2_link6	Displacement	STEP (TIME, 0,-30, 10,60D)
6	Rev_link6_EF3	Displacement	STEP (TIME, 0,-30, 10,30D)

The following figure shows the results obtained from Multibody dynamics (MBD).

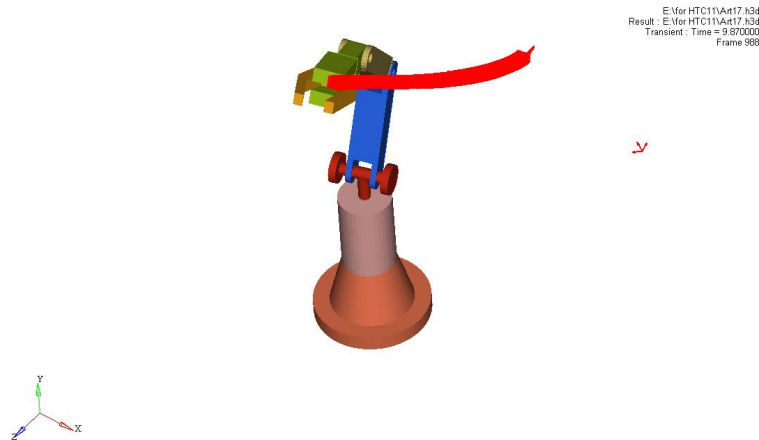


Figure 5: Tracing of End Effector

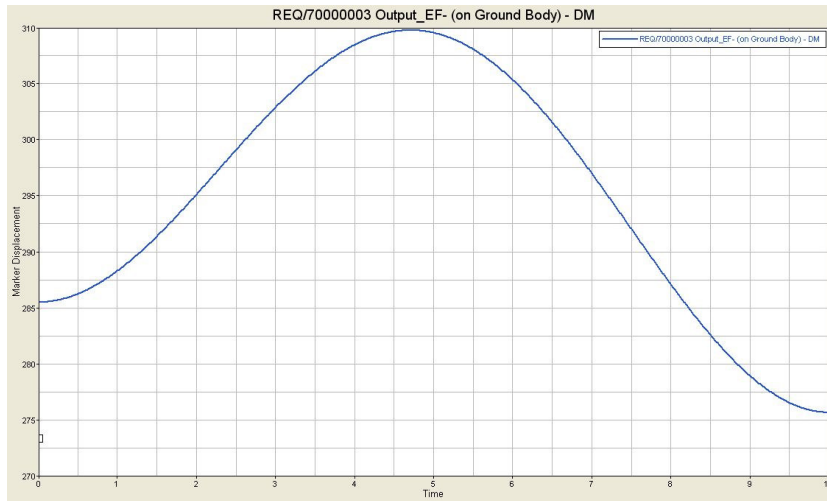


Figure 6: Displacement of End Effector (MBD)

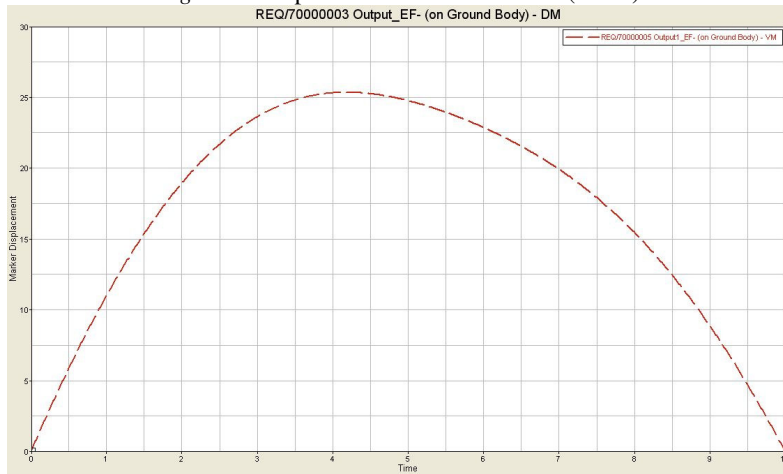


Figure 7: Velocity of End Effector (MBD)

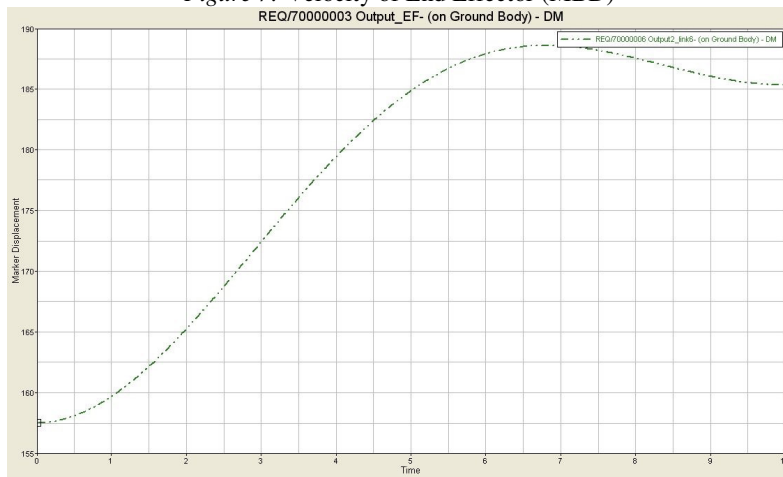


Figure 8: Displacement of Link6 (MBD)



Figure 9: Velocity of Link6

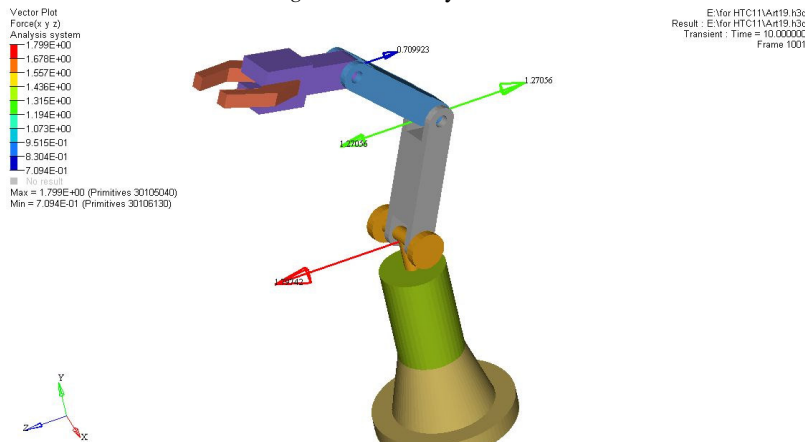


Figure 10: Forces on Joints

### Benefits Summary

Theoretical Model i.e. mathematical model is to be checked with some CAD modeling for feasibility. The kinematic and dynamic equations can provide strong basis for design of joints and numerical values can be obtained to select the actuators to drive the joints. However, it is necessary to prepare 3D CAD model and carryout Kinematic and Dynamic analysis to validate mathematical model.

The simulation of virtual ROBOT leads to actual feel of motion of all links and joints as per designed in the theoretical model. Also workspace of all the links can be plotted to analyze the maximum and minimum reach of various points on the links.

### Challenges

The methodology can be extended to analyze various manipulators for the same inputs and outputs. The selection of best suited manipulator can be made by such analysis. However, the mathematical modeling and programming is required facilitate solution for all type of configurations of robot.

### Conclusions

The software solution will enable the learner to develop the concept from basic mathematics to actual prototype development to achieve objectives.

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