A General Solution for the Stresses around Internally Pressurized Circular hole in Symmetric Laminates

A. D S Sharma, B. Panchal Khushbu*, C. Patel Nirav
A. Professor, Mechanical Engg. Department, Institute of Technology, Nirma University,Ahmedabad
B. Assistant Professor, Shri Sad Vidya Mandal Institute of Technology, Bharuch.
C. Assistant Professor, Gandhinagar Institute of Technology, Gandhinagar

***Abstract*-- General solutions for determining the stress field around circular hole in infinite orthotropic plate subjected to internal pressure are obtained using Muskhelisvili's complex variable formulation. The generalized formulation thus obtained is coded and few numerical results are obtained using MATLAB 7.6. The effect of fibre orientation and material parameter on stress pattern around pressurized circular hole is studied. Plane stress models are prepared in ANSYS and results are compared with present method.**

***Index terms*—Fibre orientation, orthotropic plate, stress function, Pressurized Circular hole.**

I. INTRODUCTION

Holes and cut-outs are bound to be present in many engineering structures, which cause serious problems of stress concentrations. These hole/opening work as stress raisers and may lead to the failure of the structure/machine component. Many researchers have tried to find stress fields around holes and cut-outs in isotropic /anisotropic media.
Ukadgaonker and Rao[1] found stress patterns around different shaped holes using Muskhelisvili's [2] complex variable approach. J.Rezaeepazhand and M.Jafari[3] found the stress patterns around different central cut-outs in metallic plates using Lekhnitskii's[4] approach. Ukadgaonker and Rao[5] found resultant stresses on triangular hole boundary due to uniform pressure.

Using Muskhelisvili's [2] complex variable approach, the generalized solution for stress distribution around circular hole in symmetric laminate subjected to internal uniform pressure is obtained. The generalized solutions obtained are coded in MATLAB 7.6 and the effect of stacking sequence, fibre orientation and material property on stress pattern is studied for isotropic steel and for Eglass/epoxy [E_1=41GPa, E_2=10.4GPa, G_{12}=4.3GPa, □=0.28] with fibre orientation 0°, 0°/90°. The results are compared with the results obtained in ANSYS.

II. COMPLEX VARIABLE FORMULATION

A thin anisotropic plate is considered under generalized plane stress condition (Figure 1). The plate is assumed to be loaded in such a way that resultants lies in XOY plane. The stresses on top and bottom surface of plate as well as σ_z, τ_{xz} and τ_{yz} are zero everywhere within the plate.

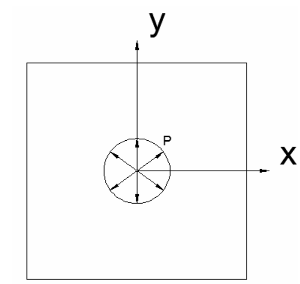


Fig. 1 Plate with circular hole

The stress components for plane stress conditions can be written in terms of Mushkhelishvili's complex function $\phi(z_1)$ and $\psi(z_2)$ as follows

$$\sigma_x = 2\,\mathrm{Re}\left[s_1{}^2\phi'(z_1) + s_2{}^2\psi'(z_2)\right]$$
$$\sigma_y = 2\,\mathrm{Re}\left[\phi'(z_1) + \psi'(z_2)\right]$$
$$\tau_{xy} = -2\,\mathrm{Re}\left[s_1\phi'(z_1) + s_2\psi'(z_2)\right] \qquad (1)$$

Where, s_1 and s_2 are complex constants of anisotropy, which are obtained from roots of characteristic equation arrived at from Hooke's law, Airy's stress function and strain-displacement compatibility condition.

The area external to a given circular hole, in Z-plane is mapped conformably to the area outside the unit circle in ζ plane using following mapping function.

$$z_j = \omega_j(\xi) = \frac{R}{2}\left[a_j\left(\frac{1}{\xi}\right) + b_j(\xi)\right]$$
$$a_j = (1 + is_j), \qquad b_j = (1 - is_j)\,; \quad \text{j=1,2.} \qquad (2)$$

III. GENERAL SOLUTION FOR UNIFORM PRESSURE AROUND THE HOLE

For the case of uniform pressure p around hole, the resultant stress X_n, Y_n on the hole boundary are given by

$$X_n = -p\cos(n,x) = -p\frac{dy}{ds},$$

$$Y_n = -p\cos(n,y) = -p\frac{dx}{ds} \qquad (3)$$

Where n is the positive direction of the outward normal to the hole boundary. The stress boundary conditions are given by

$$f_1^{\,0} = -\int_0^s Y_n ds + const. = -px,$$

$$f_2^{\,0} = -\int_0^s X_n ds + const. = -py, \qquad (4)$$

The stress functions are obtained using these boundary conditions $(f_1^{\,0}, f_2^{\,0})$ into Schwarz formula:

$$\psi(\xi) = \frac{i}{4\Pi(s_1 - s_2)}\int_\gamma \left[(s_1 f_1^{\,0} - f_2^{\,0})\left\{\frac{t+\xi}{t-\xi}\right\}\frac{dt}{t}\right]$$

$$\phi(\xi) = \frac{i}{4\Pi(s_1 - s_2)}\int_\gamma \left[(s_2 f_1^{\,0} - f_2^{\,0})\left\{\frac{t+\xi}{t-\xi}\right\}\frac{dt}{t}\right]$$

The stress functions and their derivatives obtained are given bellow,

$$\phi(\zeta) = \frac{-ipR}{2(s_1 - s_2)}\left[\frac{a_2}{\zeta}\right], \qquad \psi(\zeta) = \frac{-ipR}{2(s_1 - s_2)}\left[\frac{a_1}{\zeta}\right]$$

$$\phi'(\zeta) = \frac{-ipR}{2(s_1 - s_2)}\left[\frac{-a_2}{\zeta^2}\right], \psi'(\zeta) = \frac{-ipR}{2(s_1 - s_2)}\left[\frac{-a_1}{\zeta^2}\right] \qquad (5)$$

IV. RESULTS AND DISCUSSION

The stress functions obtained above are the generalized solutions. Using these functions stress field around circular hole under internal uniform pressure condition is presented. The basic procedure for the numerical solution is as follows: Take the value of elastic constants according to the material properties.

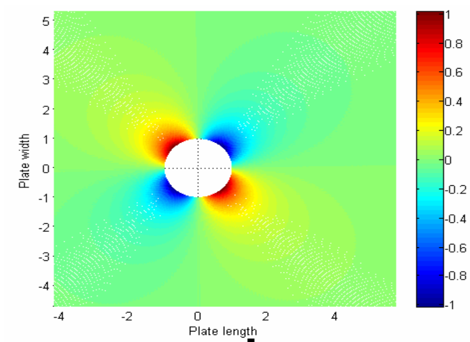
a. Calculate the compliance co-efficient, a_{ij} from generalized Hooke's Law.
b. Calculate the value of complex parameters of anisotropy s_1 and s_2 from the characteristic equation.
c. Calculate the constants: a_1,b_1,a_2,b_2 etc.
d. Evaluate the stress functions and their derivatives.
e. Evaluate stresses.

Figure [2] and [3] shows the stress distribution around the circular hole subjected to internal pressure in isotropic steel plate. Figure [3] shows the tangential and radial stress near the vicinity of hole. Radial stress varies from -1.00 to 0 and tangential stress varies from 0 to 1.00.

Figure [4], [5] and [6] shows the stress distribution around the circular hole subjected to internal pressure in glass/epoxy plate with 0° fibre orientation. The effect of anisotropy leads to increment in value of stresses. Stresses are comparatively higher in glass/epoxy than isotropic steel. The comparison of results are also seen in Figure [4], [5] and [6].

To understand the effect of fibre orientation and stacking sequence in the multilayered plate, the glass/epoxy plate with fibre orientation 0°/90° is taken. Figure [7], [8] and [9] shows the stress distribution around the circular hole subjected to internal pressure in anisotropic glass/epoxy plate with 0°/90° fibre orientation. The plane stress model has prepared using ANSYS and the results are obtained. In all cases the present method's results are in good agreement with the ANSYS's results.

The comparison of result's obtained for different material parameter from present method and ANSYS is tabulated in Table [I].



F
ig. 2(a) Present Method

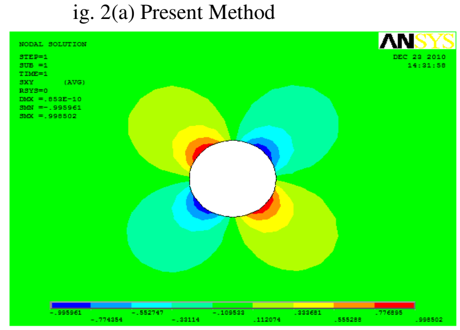


Fig. 2(b) ANSYS
Fig. 2 Shear stresses for Isotropic material

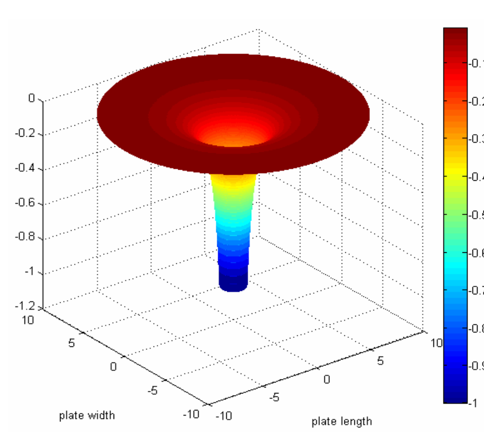


Fig. 3(a) Radial stress(Present Method)

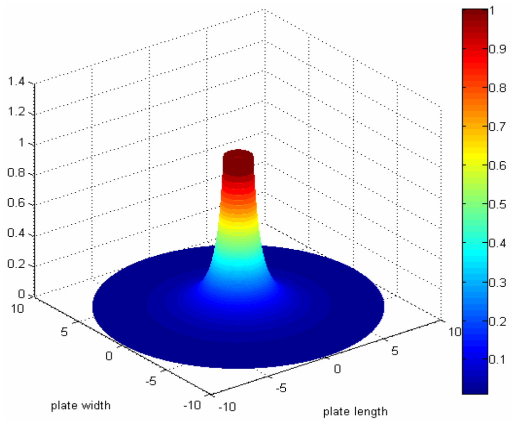


Fig. 3(b) Tangential stress (Present Method)

Fig. 3 Stresses in Radial and Tangential direction for Isotropic material

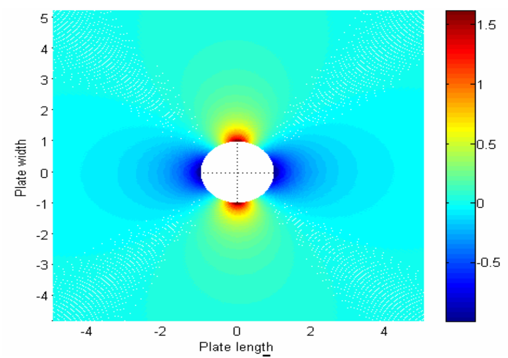


Fig. 4(a) Present Method

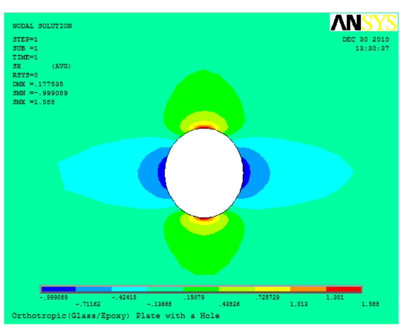


Fig. 4(b) ANSYS

Fig. 4 Stresses in X direction for Glass/Epoxy with 0° fiber orientation

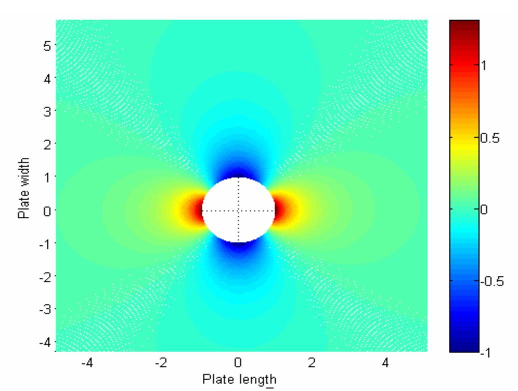


Fig. 5(a) Present Method

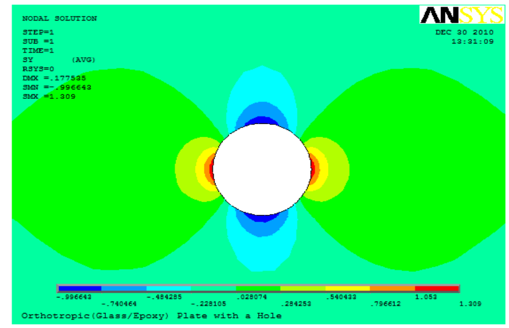


Fig. 5(b) ANSYS

Fig. 5 Stresses in Y direction for Glass/Epoxy with 0° fiber orientation

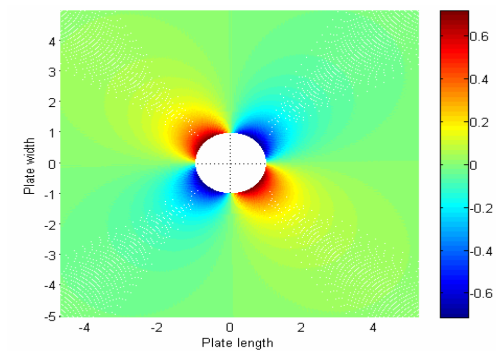


Fig. 6(a) Present Method

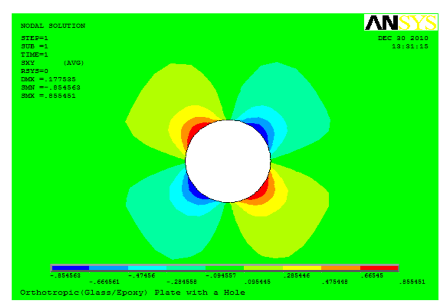


Fig. 6(b) ANSYS

Fig. 6 Shear Stresses for Glass/Epoxy with 0° fiber orientation

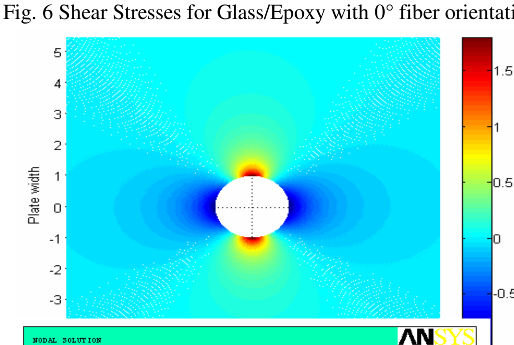


Fig. 7(a) Present Method

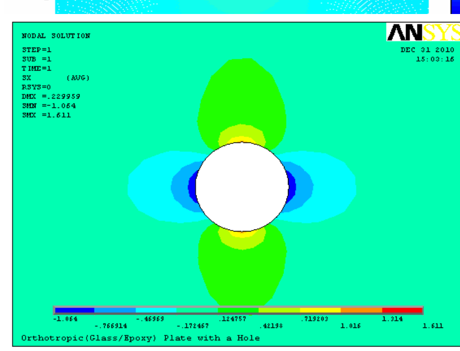


Fig. 7(b) ANSYS

Fig. 7 Stresses in X direction for Glass/Epoxy with 0°/90° fiber orientation

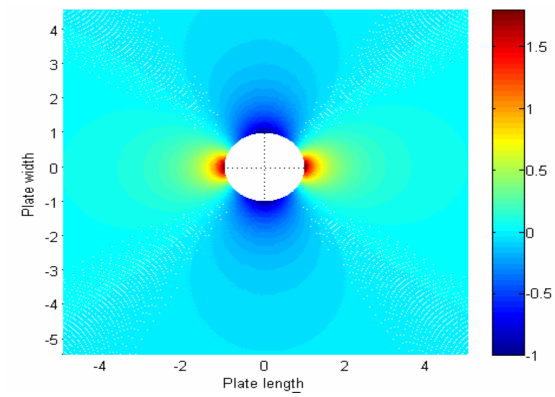


Fig. 8(a) Present Method

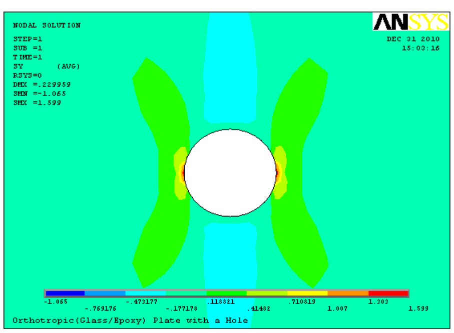


Fig. 8(b) ANSYS

Fig. 8 Stresses in Y direction for Glass/Epoxy with 0°/90° fiber orientation

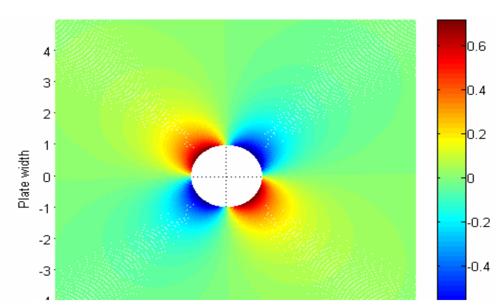


Fig. 9(a) Present Method

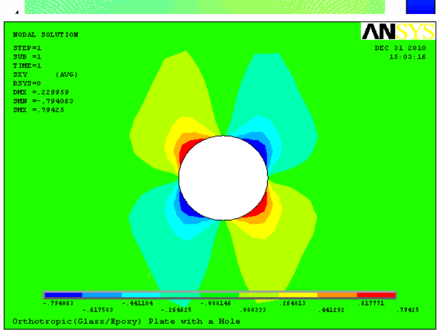


Fig. 9(b) ANSYS

Fig. 9 Shear stresses direction for Glass/Epoxy with 0°/90° fiber orientation

TABLE I.
COMPARISON OF RESULTS OF PRESENT METHOD WITH ANSYS RESULTS FOR CIRCULAR HOLE

Material	**Stress**	**Present Method**	**ANSYS**
Isotropic	Max σ_X	1.00	1.001
	Max σ_Y	1.00	1.001
	Max τ_{XY}	1.00	0.9955
Glass/Epoxy Fiber orientation =0°	Max σ_X	1.6521	1.555
	Max σ_Y	1.3085	1.209
	Max τ_{XY}	0.8537	0.8554
Glass/Epoxy Fiber orientation =0°/90°	Max σ_X	1.7916	1.611
	Max σ_Y	1.7916	1.599
	Max τ_{XY}	0.7164	0.7942

V. CONCLUSION

The generalized solution for infinite plate having circular hole subjected to internal pressure loading is obtained using Mushkhelisvili's complex variable method. The effect of various parameters on stress concentration around holes is studied. It is found that the fibre orientation, stacking sequence and material property has significant effect on stress distribution around hole. The formulation presented here, is a good tool for the designer to predict stress pattern around internally pressurized hole and to predict failure pattern of mechanical component and structures.

VI. REFERENCES

[1] V.G. Ukadgaonker, D.K.N. Rao, "A general solution for stresses around holes in symmetric laminates under in plane loading", *Journal of Composite Structures 49*, 2000, pp.339-354.

[2] N.I. Muskhelishvili, *Some basic problems of the mathematical theoryof elasticity*, P. Noordhooff Ltd., Groningen, The Netherlands, 1963.

[3] J. Rezaeepazhand, M. Jafari,," Stress concentration in metallic plates with special shaped cut-out", *Composite structure 52*, 2010,pp.96-102

[4] S.G. Lekhnitskii,, *Anisotropic plates*,. New York: Gordon and Breach, 1968.
[5] V.G. Ukadgaonker, D.K.N. Rao "Stress distribution around triangular holes in anisotropic plates", *Journal of Composite Structures 45*, 1999, pp.171-183.