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Stress analysis of infinite composite plate with circular hole

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Abstract-
Complex variable method is used to obtain solution for determining s tress concentration around elliptical cutout in infinite composite plate subjected to in plane tension/shear at infinity. For obtaining stress functions, Schwarz formula is evaluated for given boundary conditions. The generalized formulation is coded using MATLAB 7.0 and stress field around the elliptical opening is obtained. Results obtained are compared with existing literature.

Index Terms: **stress function, biaxial loading, infinite plate**

I. INTRODUCTION

Laminated composite materials are widely used in the aerospace and transportation industries.
Holes and cut-outs are bound to be present in many engineering structures, which cause serious problems of stress concentrations due to the geometry discontinuity. These problems gain more importance particularly in structures made of composite materials as the materials are brittle in nature and possess anisotropy. Therefore more attention has been paid by many researchers to the stress concentration problems.
Solutions for this problem have been given by Lekhnitskii [1] using the series method, Savin [2] by conformal mapping and Cauchy integrals and Green [3] by complex variables in curvilinear coordinates. The paper aims to provide a generalised solution for stress concentration around elliptical holes in infinite composite plate subjected to various kinds of loading conditions. The results obtained thereof are accurate and less time consuming as compared to computational techniques.

II. BOUNDARY CONDITIONS AT INFINITY

The boundary conditions [4] for different in-plane loading conditions are as follows:

$$\sigma_{x'}^{\infty} = \lambda\sigma \,;\; \sigma_{y'}^{\infty} = \sigma \,;\; \tau_{x'y'}^{\infty} = 0 \text{ At } z \rightarrow \infty \qquad (1)$$

Where, $\sigma_{x'}^{\infty}$, $\sigma_{y'}^{\infty}$, stresses applied about x', y' axes at infinity. λ=0 and λ=1 explains uniaxial and biaxial loading conditions, respectively. By applying stress invariance into above boundary conditions, boundary conditions about XOY can be written explicitly as:

$$\sigma_x = \frac{P}{2}[(\lambda+1)+(\lambda-1)\cos 2\alpha]$$
$$\sigma_y = \frac{P}{2}[(\lambda+1)-(\lambda-1)\cos 2\alpha]$$
$$\tau_{xy} = \frac{P}{2}[(\lambda-1)\sin 2\alpha] \qquad (2)$$

III COMPLEX VARIABLE FORMULATION

The stress functions for single hole problem, can be obtained using complex variable approach [5] and can be written as follows,

$$\sigma_x = 2\,Re\,[s_1^2\,\phi'_0(z_1) + s_2^2\,\psi'_0(z_2)]$$
$$\sigma_y = 2\,Re\,[\phi'_0(z_1) + \psi'_0(z_2)]$$
$$\tau_{xy} = 2\,Re\left[s_1\,\phi'_0(z_1) + s_2\,\psi'_0(z_2)\right] \qquad (3)$$

Where $\phi(z), \psi(z)$ are complex potentials of the complex variable $z = x + iy$

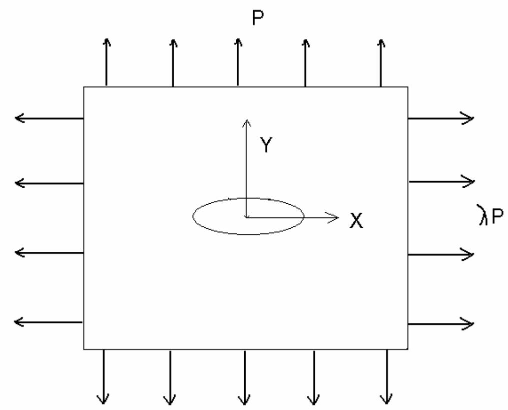

Fig 1 Arbitrary biaxial loading

In order to find stress distribution around a hole in z-plane, it is mapped to a region outside unit circle in ζ-plane, which has origin at ζ=0. The mapping function is given as follows.

$$z = \omega(\xi) = \frac{R}{2}\left(\xi + \frac{m}{\xi}\right)$$

The stress functions in ζ-plane are given by

$$\phi(z_1) = \phi_1(z_1) + \phi_0(z_1)$$
$$\psi(z_2) = \psi_1(z_2) + \psi_0(z_2) \qquad (5)$$

In absence of hole stress functions $\phi_1(z_1)$ and $\psi_1(z_2)$ can be written as

$$\phi_1(z_1) = B\,z_1$$
$$\psi_1(z_2) = (B' + iC')\,z_2 \qquad (6)$$

Where the B, C, B', C' are loading constant and are given by,

$$B = \frac{\sigma_x + \sigma_y}{4};\ B' = \sigma_x - \sigma_y;\ C = 0;\ C' = \tau_{xy}$$

The boundary condition on the fictitious hole are given by,

$$f_1 = 2\,Re\,[\phi_1(z_1) + \psi_1(z_2)]$$
$$f_2 = 2\,Re\,[s_1\,\phi_1(z_1) + s_2\,\psi_1(z_2)] \qquad (7)$$

Now considering the plate with hole and negative boundary conditions without external loading, the stress boundary conditions on the hole are given by,

$$f_1^0 = -f_1 \qquad f_2^0 = -f_2 \qquad (8)$$

By using Schwarz formula and given boundary conditions stress function $\phi_0(\xi)$ and $\psi_0(\xi)$ can be obtained.

$$\phi_0(\xi) = \frac{a_3}{\xi} + \frac{b_3\,m}{\xi}$$
$$\psi_0(\xi) = \frac{a_4}{\xi} + \frac{b_4\,m}{\xi} \qquad (9)$$

Where a_3, a_4, b_3, b_4 are complex constants

Now considering the stress functions $\phi(z_1), \psi(z_2)$ for the given plate by superposition of stress functions from Eq. (6) and (9) as per Eq. (5) and introducing them into Eq. (3) the stress around the hole are obtained as follows,

$$\sigma_x = \sigma_x^{\infty} + 2\,Re\,[s_1^2\,\phi'_0(z_1) + s_2^2\,\psi'_0(z_2)]$$
$$\sigma_y = \sigma_y^{\infty} + 2\,Re\,[\phi'_0(z_1) + \psi'_0(z_2)]$$
$$\tau_{xy} = \tau_{xy}^{\infty} - 2\,Re\,[s_1\,\phi'_0(z_1) + s_2\,\psi'_0(z_2)] \qquad (10)$$

Where,

$$\phi'_0(z_1) = \frac{\phi'_0(\xi)}{\omega'_1(\xi)}, \qquad \psi'_0(z_2) = \frac{\psi'_0(\xi)}{\omega'_2(\xi)}$$
$$\omega'_1(\xi) = \frac{dz_1}{d\xi}, \qquad \omega'_2(\xi) = \frac{dz_2}{d\xi} \qquad (11)$$

Applying the transformation, the stresses in Cartesian coordinate are transformed into orthogonal coordinate system (ρ,θ) using the following relations;

$$\sigma_\theta + \sigma_\rho = \sigma_y + \sigma_x$$
$$\sigma_\theta - \sigma_\rho + 2i\tau_{\theta\rho} = (\sigma_y - \sigma_x + 2i\tau_{xy})e^{2i\alpha} \qquad (12)$$

IV RESULTS AND DISCUSSION

The stress functions obtained above are the generalized solutions. Using these functions stress field around hole under different loading conditions can be obtained. The results are discussed for graphite epoxy $(\pm45)_{4s}$, graphite epoxy $(0\text{-}90)_{4s}$ and plywood.

The following cases have been considered.

1-Plate subjected to uniaxial tension at infinite distance.

2-Plate subjected to biaxial tension at infinite distance.

4.1-Plate subjected to uniaxial tension at infinite distance

The infinite plate with single hole is subjected to uniaxial tension (λ=0).The tangential stresses and shear stress are calculated using MATLAB 7.0.

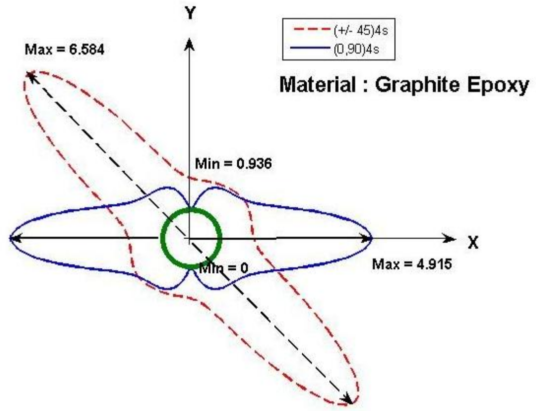

Fig 2 Uniaxial Loading - Tangential Stress

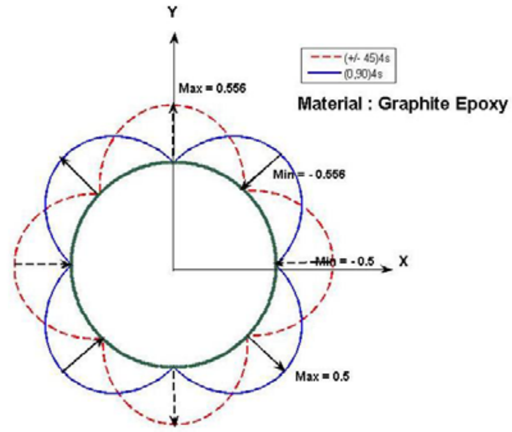

Fig 3 Uniaxial Loading - Shear Stress

4.2-Plate subjected to biaxial tension at infinite distance

The infinite plate with single hole is subjected to biaxial tension (λ=1).The tangential stresses and shear stress are calculated using MATLAB 7.0.

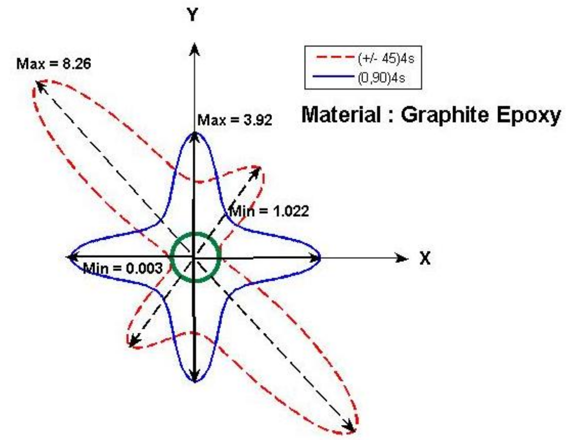

Fig 4 Biaxial Loading - Tangential Stress

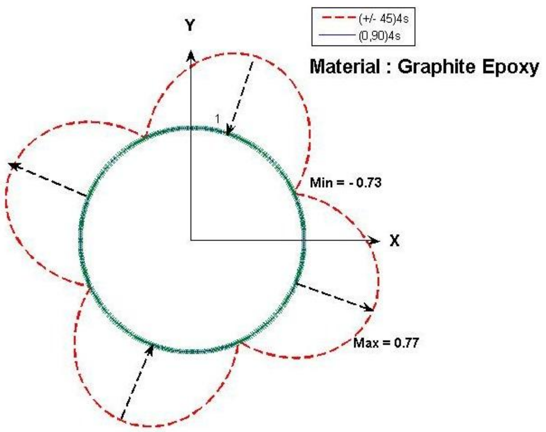

Fig 5 Biaxial Loading -Shear Stress

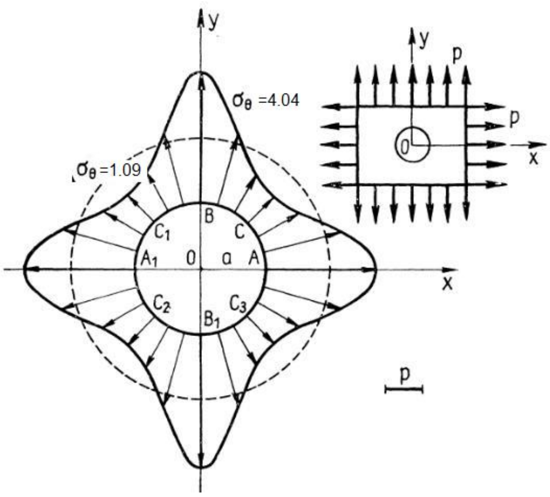

Fig 6 Stress pattern in plywood by S.G. Lekhnitskii[1]

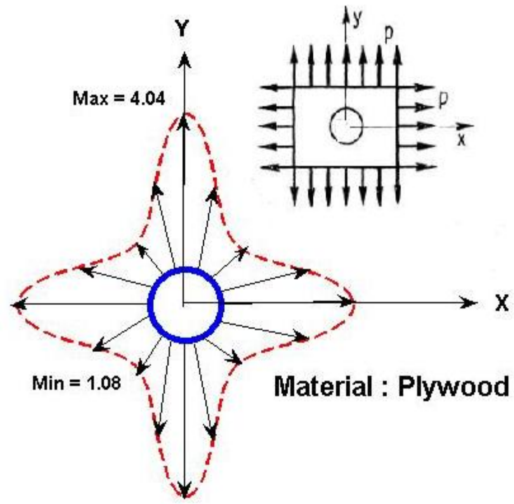

Fig 7 Polar plot of stress distribution (Present method)

V CONCLUSIONS

The general solution presented in this paper is very much useful to study stress field around circular opening under different type of in-plane loading. A detailed parametric study can be easily made using the present solution by merely introducing the mapping function, the orientation angle and the biaxial loading factor. The computer implementation is easier and faster than finite element method and the results are quite comparable. With slight modification in mapping function same formulation can be used to find stress fields around various hole shapes.

VI REFERENCES

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