Dynamic Analysis of Multiple Cracked Beam

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Dynamic Analysis of Multiple Cracked Beam.

Major Project

Submitted in partial fulfillment of the requirements

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Master of Technology in Mechanical Engineering (Design Engineering)

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May 2012

Declaration

This is to certify that

I). The thesis comprises my original work towards the degree of Master of Technology in Mechanical Engineering(Design Engineering) at Nirma University and has not been submitted elsewhere for Degree.

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This is to certify that the Major Project entitled"**Dynamic Analysis Of Multiple Cracked Beam.**" submitted by **Mr. Pratik P. Patel (10MMED13)** towards the partial fulfillment of the requirements for the degree of **Master of Technology** in Mechanical Engineering **(Design Engineering)** of Nirma University,Ahmedabad is the record of work carried out by him under my supervision and guidance. In my opinion,the submitted work has reached a level required for being accepted for examination.The results embodied in this major project,to the best of my knowledge,haven't been submitted to any other university or institution for award of any degree or diploma

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Abstract

Cracks in a vibrating structural component reduce safety and can cause catastrophic failure. The discontinuities like cracks and notches change the dynamic behavior of structure. The change in vibration parameters depends upon location and severity of the damage. So it is very important to study the effect of crack on vibration behavior of structure to achieve structural safety and performance. The detection of damage by using change in vibration parameters may prove a better way in structural health monitoring.

In present work, the vibration analysis is performed on isotropic beam with and without crack by Euler Bernoulli beam theory, finite element method, conventional finite element software(ANSYS) and by experimental method by using FFT analyzer. The results obtained by different methods shows close confirmation.

The various crack detection algorithms are studied and method to investigate multiple cracks location and its depth based on the natural frequencies is adopted. The analysis for crack detection is performed for beam for different crack location and crack depth. Natural frequencies of cracked beam are extracted from ANSYS. The method gives unique results for crack location and depth.

Key words: Cracked beam,mode shape,modal analysis,natural frequency

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Nomenclature

English symbol

a	Depth of crack (m)
h	beam thickness (m)
x	Distance of crack from left end (m)
D	Transverse dispacement (m)
F	Force (N)
A	Area (m^2)
I	Second area moment of inertia (m^4)
m_p	mass of beam (kg/m)
K	Rotational spring stiffness (Nm/rad)
L	Length (m))
U	Strain energy (J)
E	Young's Modulus (GPa)
M	Bending Moment (N/m^2)

Greek symbols

σ	Stress (Pa)
β	Crack location (non-dimensional, x/L)
ρ	Density(kg/m^3)
ϕ	Frequency Parameter

Subscripts

c	Crack
n_c	No crack
b	width of beam

Abbreviations

NDE	Non-destructive Evaluation
NDT	Non-destructive Technique
TMM	Transfer Matrix Method
EM	Energy Method

Chapter 1

Introduction

1.1 Preliminary remarks

The increasing demands of higher productivity and economical design lead to higher operating speeds of machinery and efficient use of materials through light weight structures. These trends make occurrence of resonant conditions more frequent during the operation of machinery and reduce the reliability of the system. It is required that structures must safely work during its service life. But, damages initiate a breakdown period on the structures. Cracks are among the most encountered damage types in the structures. Beam type structures are commonly used in steel construction and machinery industry. So the structural safety of beam is very important due to its practical importance. Cracks in a beam type structure may be hazardous due to static or dynamic loadings, so that crack detection plays an important role for structural health monitoring applications. Every structural component and structure has its certain vibration characteristics like natural frequencies, modal damping values, modal strain energy, maximum detection, etc. When any type of discontinuity or degenerative effect like crack is present in structural component, the vibration characteristics are changed due to change in stiffness at crack location. These changes are mode dependent. Hence it is possible to estimate the location and size of the crack by measuring the changes in vibration parameters.

Hence the knowledge of vibration characteristics of machine elements becomes essential to ensure adequate safety margins. Any variation of natural frequencies or other vibration characteristics will indicate either a failure or a need for maintenance of machine. The measurement of natural frequencies of vibration and the forces developed is necessary in the design and operation of active vibration isolation systems. The theoretically computed vibration characteristics may be different from the actual values due to the assumptions made in the analysis. Continuous systems are approximated as multi degree of freedom systems for simplicity. If the experimentally measured natural frequencies and mode shapes of a continuous system are comparable to the computed natural frequencies and mode shapes of the multi degree of freedom model, then the approximation will be valid one.

1.2 Motivation

The detection of crack by using vibration based techniques gives an effective global method of damage detection which is very helpful in structural health monitoring (SHM) of aging structures like bridges, aircraft, railway systems, wind power stations. It provides very convenient way for remote monitoring in off-shore structures. By controlling the damage state, more improvement in design becomes possible. The laminated high performance material used in aero engineering introduces a serious damage mechanism. So it requires a close monitoring of dynamic behavior during its operating condition.

In sudden overload, the aging occurs in the structures, subjected to continuous vibrations at moderately low amplitude and it creates crack due to fatigue loading. So SHM is very useful to make indication when damage occurs.

There are still many unsolved problems at every level. For example, at the lowest level of damage detection, the challenges are to increase the sensitivity, detect small amount of damage in early state. The other difficulty is to separate the effect of damage from the effects of change in environmental conditions.

1.3 Aims and scope

Keeping in view the above issues, investigation have been carried out with the following objectives.

- To study the dynamic behavior of beam with multiple cracks.
- To develop a scheme for modeling and analysis of transverse vibration of beam with multiple cracks
- To develop a method for the solution of inverse problem of determining crack position, size and orientation in beam.
- To carry out experiments to establish the accuracy of the theoretical prediction.

1.4 Methodology

In first stage, Euler's beam theory [17] is used to determined the mode shape behavior of uncracked beam at different natural frequency. In second stage, Natural Frequency Based Mode shape Method is used for find out the transverse cracks in beam at different location by Theocratical and Experimental Analysis.

1.5 Outline of Report

The report consists of Six chapter.The first chapter deals with Preliminary remarks and list of the objectives and methodology used for this work. and scope of Project. The second chapter reviews the relevant literature.The third chapter presents the modal analysis of the Uncracked beam.In fourth Chapter find the multiple cracks location in the beam by using Natural frequency based Mode Shape Method.The Fifth chapter Presents the Results obtained by Experimental for finding Crack location.The Summary and future work are presented in the Sixth chapter.

Chapter 2

Literature Review

2.1 Introduction

The structure suffering from damage changes its dynamic properties. The change in natural frequencies, modal damping values, modal strain energy, mode shapes can be easily identified. So many researchers have worked on detection of damage by using these modal parameters in different manners. The literature survey is carried out to introduce the present state of research work done on vibration based techniques for damage location. The subjects are classified in different sections with reference to methods used for dynamic analysis and crack detection techniques used in it.

2.2 Modeling of Crack

2.2.1 Dynamic behavior of cracked structure

The type of material, boundary conditions, dimensions of structure play important role to determine dynamic behavior of structure. The crack present in structure changes its dynamic behavior. The following features of crack greatly affects the dynamic behavior of structure:

- The position of crack

- The depth of crack
- The orientation of crack
- The number of cracks

2.2.2 Different types of cracks

The cracks can be classified in following way:

- Crack perpendicular to the beam axis is known as transverse crack.
- Crack parallel to the beam axis is known as longitudinal crack.
- Crack at an angle to the beam axis is known as slant crack.
- Cracks that open when the affected part of the material is subjected to tensile stresses and close when the stress is reversed are known as breathing cracks.
- Crack that always remains open is known as gaping crack or notch.
- Crack that is open on surface is known as surface crack.
- Crack that is not on surface is known as sub surface crack.

There is a wide variety in the way an open crack is modelled. They mostly differ in the way the local effects are accounted for. crack has modelled by appropriately reducing the section modulus of beam. According to the fracture mechanics principles, the crack occurring in a beam would reduce the local stiffness at the location of the crack. Dimarogonas and Papadopoulos [1] established a 5 x 5 flexibility matrix to model the vicinity of a crack; torsion was not included in the model later they extended the matrix formulation by adding torsion resulting in a full 6 x 6 flexibility matrix. In rotational spring approach, the reduced local stiffness at crack location is calculated using castigliano's second theorem as applicable to fracture mechanics formulation. The calculated local stiffness is then modeled by a rotational spring for the pure bending vibration of a cracked beam. To establish the vibration equation, the cracked beam was represented by two substructure connected by a rotating spring. A comprehensive review on vibration of a cracked structure is given by[2] by Dimarogonas

In the case of transverse vibration of slender beam, it is generally assumed that there is an extra angular rotation at the crack location proportional to the bending moment at the section. Hence a crack can be modelled as a massless rotational spring of infinitesimal length inserted at the location. Rizos and Asprgathos [3] have demonstrated the utility of the method for a cantilever beam of rectangular cross-section with a transverse edge crack. Spring stiffness calculated from crack size. Panigrahi and Ramakrushna [4] This paper contains an attempt to evaluate dynamic behaviors of beam structures with transverse crack subjected to external force. In this work theoretical expressions have developed for finding out the mode shapes and natural frequencies for beam with transverse crack using flexibility influence coefficients and local stiffness matrix. Crack depth and crack position are taken as main variable parameters. Suitable numerical models are considered, and the results are presented graphically. Further experimental and finite element analysis verifications are also done to prove the authenticity of the theory developed.

2.3 Crack Detection Methods

It is well known that when a crack develops in a component it leads to changes in its vibration parameters, e.g. a reduction in the stiffness and increase in the damping. These changes are mode dependent. Hence it may be possible to estimate the location and size of the crack by measuring the changes in vibration parameters. The vibration parameters could be structural parameters (i.e. mass, stiffness and flexibility) or modal parameters (i.e. natural frequencies, modal damping values and mode shapes). The vibration based methods of crack detection utilize one or more of these parameters as the basis for crack detection.

2.3.1 Crack detection based on Natural Frequency

Methods have been developed to detect the crack by measuring the changes in natural frequencies. These include the forward problem of determination of natural frequencies knowing the crack details and the inverse problem of determination of crack details from the natural frequencies.

Maiti and Nandwana [5] developed the method to detect the crack by using first three natural frequencies. The inclined edge crack is modeled by torsional spring approach. The frequency equation is derived by applying boundary conditions and equation is derived which relates change in stiffness due to crack location and natural frequencies. The crack location and depth is predicted by intersection of curves of stiffness vs crack location for first three modes.

Qian and Jiang [6] have used an element stiffness matrix of a beam with a crack is first derived from an integration of stress intensity factors, and then a finite element model of a cracked beam is established. This model is applied to a cantilever beam with an edge-crack, and the eigen frequencies are determined for different crack lengths and locations

Nandwana and Maiti [7] have also given same method for detection of location and size of a crack in a steeped cantilever beam.

Bamnios and Trochides [8] have given the analytical and experimental investigations provide a link between the change in natural frequencies of vibration and in mechanical impedance to the location and size of the crack for flexural vibrations. It is shown that the mechanical impedance of the beam changes substantially due to the presence of the crack and can be used as an additional defect information carrier. The results have been used to propose an improved method of non-destructive testing for simple beam structures.

2.3.2 Crack detection based on mode shape

A mode of vibration is characterized by a modal frequency and a mode shape, and are numbered according to the number of half waves in the vibration. For example, if a vibrating beam with both ends pinned displayed a mode shape of half of a sine wave (one peak on the vibrating beam) it would be vibrating in mode 1. If it had a full sine wave (one peak and one valley) it would be vibrating in mode 2. In the study of vibration, the mode shape describes the expected curvature of a surface vibrating at a particular mode. To determine the vibration of a system, the mode shape is multiplied by a function that varies with time, thus the mode shape always describes the curvature of vibration at all points in time, but the magnitude of the curvature will change. The mode Shape is dependent on the shape of the surface as well as the boundary conditions of that surface.

Mode of Fracture

Mode I: It is the opening mode. The dominant displacement is normal to the crack surface. Mode I is studied most with well developed experimental methods. It usually dominates in many engineering applications and is the most dangerous. **Mode II**: It is a sliding mode and the displacement is in the plane of the plate. The separation is antisymmetric through relative tangential displacement normal to the crack front. **Mode III**:This mode also causes sliding motion, but the displacement is parallel to the crack front, causing tearing.

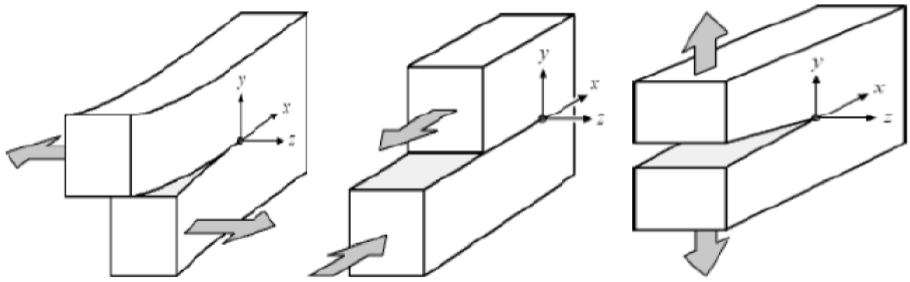

Figure 2.1: Mode of fracture

Rizos [3] have presented a method based on flexural vibration which requires measurement of amplitudes at any two location of the beam. The crack section is represented by a rotating spring. This method is useful for detection of both location and size and is demonstrated for cantilever beam with normal edge crack. Dado [9] stated crack detection algorithm for identification of transverse cracks in beams with different end conditions. The input data is first two natural frequencies of bending mode. The torsional spring approach is applied for Euler Bernoulli beam for modeling of crack. The prediction of crack location and depth is performed by preparing tables for different combinations of change in natural frequencies.

2.3.3 Detection of multiple cracks

Patil and Maiti [10] have proposed a method for detecting multiple cracks in a slender beam based on frequency measurement. In this method, the transverse vibration has been modeled through Transfer Matrix Method (TMM) and the crack is represented by a rotational spring. Beam is virtually divided into number of segments and each segments is considered to be associated with a damage parameter. This procedure gives a linear relationship explicitly between the changes in natural frequencies of the beam and damage parameters. Li [11] has also applied the Transfer Matrix method for multi-step beams with an arbitrary number of cracks and concentrated masses.
Patil and Maiti [12] have also developed a mehtod for prediction of location and size of multiple cracks based on measurement of natural frequencies has been verified experimentally for slender cantilever beams with two and three normal edge cracks. The analysis is based on energy method and representation of a crack by a rotational spring.A strategy to overcome failure in the prediction for cases with one of the cracks located near an anti-node has been suggested.
Mazanoglu and Yesilyurt [13] are presents the energy-based method for the vibration identification of non-uniform Euler-bernoulli beams having multiple open cracks.The distribution of the energy consumed is determined by taking into account not only the strain change at the cracked beam surface as in general but also the considerable effect of the stress field caused by the angular displacement of the beam due to bending.The RayleighRitz approximation method is used in the analysis.The method is adapted

to the cases of multiple cracks with an approach based on the definition of strain disturbance variation along the beam.
Khiem and Lien [14] have multi-crack detection for beam by natural frequencies has been formulated in the form of a non-linear optimization problem. The spring model of crack is applied to establish the frequency equation based on the dynamic stiffness of multiple cracked beam. The equation is the basic instrument in solving the multi-crack detection of beam. The set of crack parameters to be detected includes not only the crack position and depth, but also the quantity of possible cracks.
Caddemi and Calio [15] In this study,exact closed-form expressions for the vibration modes of the EulerBernoulli beam in the presence of multiple concentrated cracks are presented.The proposed expressions are provided explicitly as functions of four integration constants only,to be determined by the standard boundary conditions.The cracks, that are not subjected to the closing phenomenon,are modelled as a sequence of Diracs delta generalised functions in the flexural stiffness.The Eigen-mode governing equation is formulated over the entire domain of the beam without enforcement of any continuity conditions,which are already accounted for in the adopted flexural stiffness model.

2.4 Overview of literature review

Vibration based damage detection techniques has become the subject of interest for many researchers. A lot of work is done in this field by using natural frequencies and mode shapes as the damage indices. The significant work is carried out using wavelet analysis as the damage detection method. The dynamic analysis of beam like structures is carried out by many investigators to increase the structural safety of beam because beam is used as a structural element in most of the machine structures.

Chapter 3

Dynamic Analysis of Beam

3.1 Frequency equation and mode shape for beam

3.1.1 Lateral vibration of beam

Modal analysis of any structural element is required to find out fundamental natural frequency of system, which is lowest frequency from where structure starts vibrating. Modal analysis of beam is done in span wise configuration.

For deriving the equation of motion,consider the free body diagram of an element of a beam as shown in figure. M(x,t) is bending moment,V(x,t) is the shear force and F(x,t) is the external force per unit length of the beam.Since the inertia force acting on the beam is

$$\rho A(x)dx\frac{\partial^2 \omega(x,t)}{\partial x^2} \tag{3.1}$$

From the Euler Bernoulli beam theory, the relationship between bending moment and deflection can be expressed as

$$M(x,t) = EI(x)\frac{\partial^2 w}{\partial x^2}(x,t) \tag{3.2}$$

where E is the young modulus and I(x) is the moment of inertia.By inserting Equation (3.2) into (3.3),we obtain equation of motion for forced lateral vibration of non uniform beam

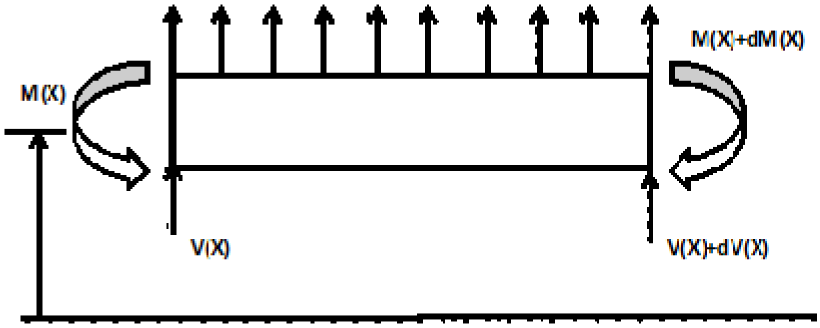

Figure 3.1: Beam element

$$\frac{\partial^2}{\partial x^2}[EI(x)\frac{\partial^2 w}{\partial x^2}(x,t)] + \rho A(x)\frac{\partial^2 w}{\partial x^2}(x,t) = F(x,t) \quad (3.3)$$

For a uniform beam Equation (3.4) becomes

$$EI\frac{\partial^4 w}{\partial x^4}(x,t) + \rho A\frac{\partial^2 w}{\partial t^2}(x,t) = F(x,t) \quad (3.4)$$

For free vibration we can write

$$c^2\frac{\partial^4 w}{\partial x^4}(x,t) + \frac{\partial^2 w}{\partial t^2} = 0 \quad (3.5)$$

By solving Equation (3.5) by method of separation of variable method,we get the solution,

$$c = \sqrt{\frac{EI}{\rho A}} \quad (3.6)$$

By solving Equation (3.6) by method of separation of variable method,we get the solution

$$w(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \tag{3.7}$$

Where c_1,c_2,c_3,c_4 in each case can be found from the boundary conditions. And the frequencies can be found from the equation:

$$\omega^2 = \beta^2 \frac{EI}{\rho A} \tag{3.8}$$

3.1.2 Frequency equation and mode shape function for different boundary conditions

The common boundary conditions are as follows:

1. simply supported end:

Deflection w=0 and bending moment

$$EI \frac{\partial^2 w}{\partial x^2} = 0 \tag{3.9}$$

2. Free end:

Bending moment

$$EI \frac{\partial^2 w}{\partial x^2} = 0 \tag{3.10}$$

Shear force

$$\frac{\partial}{\partial x}(\frac{\partial^2 w}{\partial x^2}) = 0 \tag{3.11}$$

3. Clamped end:

$$\frac{\partial w}{\partial x} = 0 \tag{3.12}$$

For simply supported beam:

The frequency equation is

$$W_n(x) = C_n \sin \beta_n x \tag{3.13}$$

The boundary conditions are satisfied by the values of = By solving above frequency equation, we can get the infinite number of nature frequencies and mode shapes.

Similarly for cantilever beam:

The frequency equation is

$$\cos \beta_n l \cosh \beta_n l = 0 \tag{3.14}$$

The mode shape function is

$$W_n(x) = C_n[\sin \beta_n x - \sinh \beta_n x - (\frac{\sin \beta_n l + \sinh \beta_n l}{\cos \beta_n l + \cosh \beta_n l})(\cos \beta_n x - \cosh \beta_n x)] \tag{3.15}$$

The frequency equation for this condition is satisfied for the values of = 1:87; 4:69; 7:85; 10:91 Similarly the frequency equations and mode shapes for fixed-fixed, fixed-pinned and pinned-free conditions can be derived by same method. By using this we can get infinite number of natural frequencies and mode shapes for any type of boundary conditions of beam. The derivation of frequency equation and mode shapes is explained in [5].

Table 3.1: Natural frequency of cantilever beam. (L=1m,b=0.050m,h=0.008m):

Frequency	ANALYTICAL	ANSYS	%, DIFFERENCE
$f_1 Hz$	13.628	13.621	0.051
$f_2 Hz$	85.40	85.35	0.070
$f_3 Hz$	237.91	238.95	0.4371

3.1.3 Comparison of Analytical and ANSYS results:

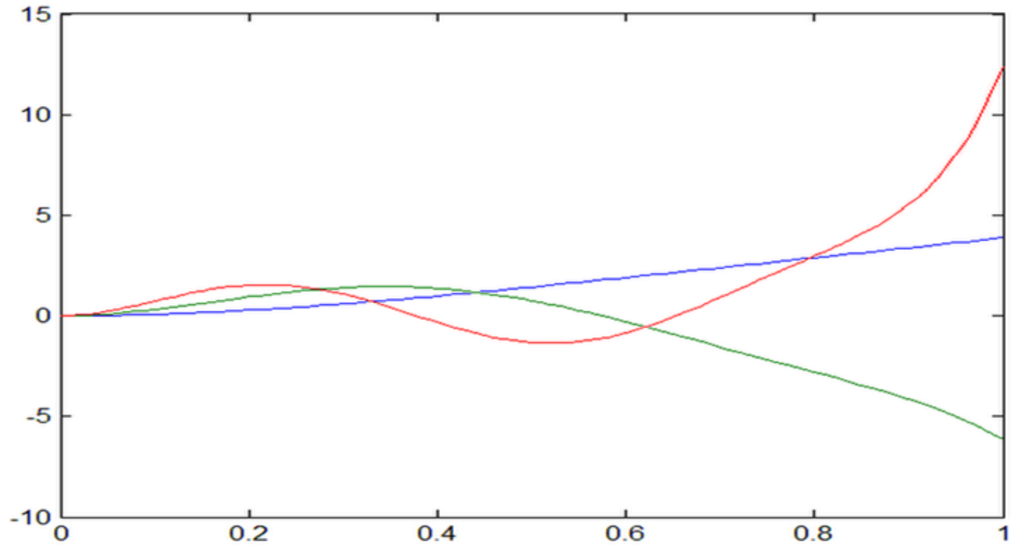

Figure 3.2: Mode shape for cantilever beam by analytical

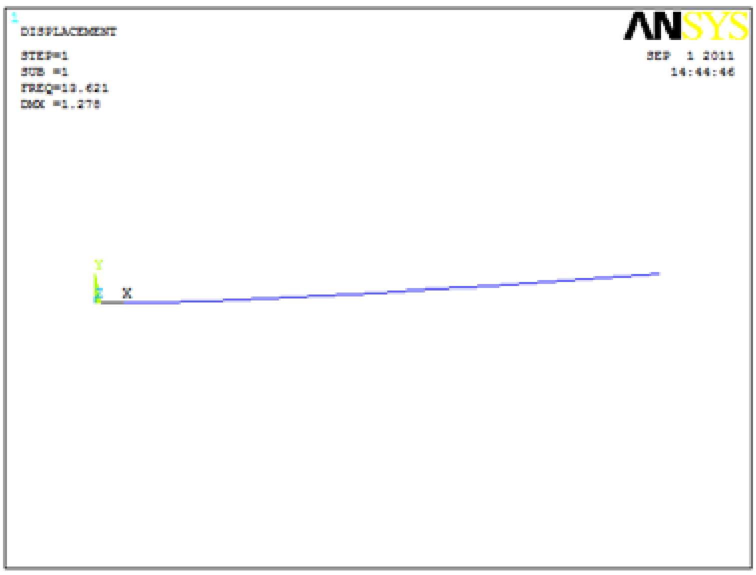

Figure 3.3: First Mode shape for cantilever beam by using ansys

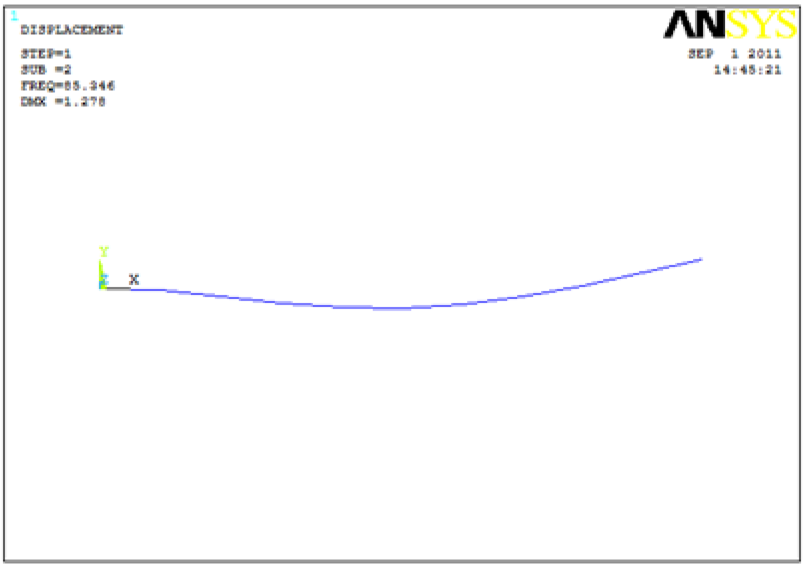

Figure 3.4: Second Mode shape for cantilever beam by using ansys

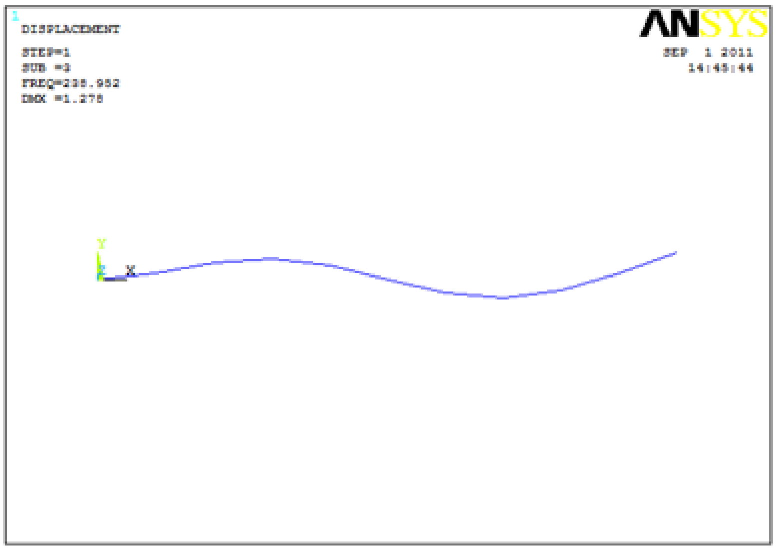

Figure 3.5: Third Mode shape for cantilever beam by using ansys

Chapter 4

Detection of cracks in Beam

4.1 Introduction

An uniform beam with n cracks located at ξ=x/L=$\beta_(j)$(j=1,2,3,...,n) is shown in Fig. 1.

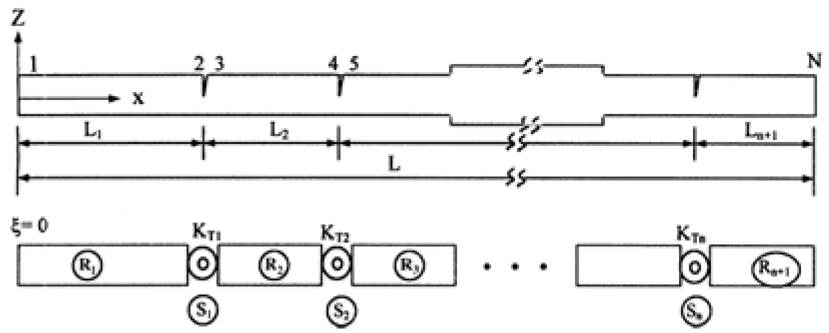

Figure 4.1: Beam with multiplecracks

For an EulerBernoulli beam the governing equation of motion is

$$\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 y(x,t)}{\partial x^2}\right]+\rho A\frac{\partial^2 y(x,t)}{\partial t^2}=0 \tag{4.1}$$

Through separation of variables y(x,t)=Z(x)cos(ω, t),the mode shape equation is obtained:

$$EI\frac{d^4 Z}{dx^4}-\rho A\omega_i^2 Z=0EI\frac{d^4 Z}{dx^4}-\rho A\omega_i^2 Z=0or\frac{d^4 Z}{dx^4}-\rho^4 Z=0 \tag{4.2}$$

where ρ is the mass density,A is the cross-sectional area,ω is natural frequency,E is the young modulus of elasticity,I is the moment of inertia and

$$p^4 = \frac{\rho A \omega^2}{EI} \tag{4.3}$$

The general solution of Eq. (4.2) can be written as follows:

$$Z(x) = C_1\left[\cos(px) + \cosh(px)\right] + C_2\left[\cos(px) - \cosh(px)\right]$$

$$+ C_3\left[\sin(px) + \sinh(px)\right] + C_4\left[\sin(px) - \sinh(px)\right] \tag{4.4}$$

Using this relation it is possible to relate displacement Z,slope θ=dZ/dx,bending moment M,shear force V at the two ends i and i-1 of an arbitrary segment:

$$\begin{bmatrix} Z \\ \theta \\ M \\ V \end{bmatrix}_i = \begin{bmatrix} A_i & B_i & \frac{C_i}{EI} & \frac{D_i}{EI} \\ p^4 D_i & A_i & \frac{B_i}{EI} & \frac{C_i}{EI} \\ EIp^4 C_i & EIp^4 D_i & A_i & B_i \\ EIp^4 B_i & EIp^4 C_i & p^4 D_i & A_i \end{bmatrix} \begin{bmatrix} Z \\ \theta \\ M \\ V \end{bmatrix}_{i-1} \tag{4.5}$$

$$A_i = \frac{\cos(pl_i) + \cosh(pl_i)}{2} A_i = \frac{\cos(pl_i) + \cosh(pl_i)}{2} \tag{4.6}$$

$$C_i = \frac{-\left[\cos(pl_i) + \cosh(pl_i)\right]}{2p^2}, D_i = \frac{-\left[\sin(pl_i) + \sinh(pl_i)\right]}{2p^3} \tag{4.7}$$

li=length of the segment

$$C_1 = \frac{Z_{i-1}}{2}, C_2 = -\frac{M_{i-1}}{2EIp^2}, C_3 = \frac{\theta_{i-1}}{2p}, C_4 = -\frac{V_{i-1}}{2EIp^3} \tag{4.8}$$

Shear force V acting on a section with its outer normal oriented in the positive x-direction is considered positive downward.
At a crack location, there is continuity in Z, M and V and a jump in θ. Utilizing these it is possible to relate the variables on the two sides of the crack:

$$\begin{bmatrix} Z \\ \theta \\ M \\ V \end{bmatrix}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1/\overline{K_i} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z \\ \theta \\ M \\ V \end{bmatrix}_{i-1} \tag{4.9}$$

where S_i is the transfer matrix and $\overline{K_i}$ is the rotational spring stiffness given by eq(1). For a step in a beam, the transfer matrix is just an identity matrix. This follows from the fact that there is continuity of deflection, slope, moment and shear force at the junction irrespective of the change in crosssections. For an intermediate support the beam has a discontinuity in shear force equal to the support reaction. This fact can be built into the corresponding transfer matrix.

4.1.1 Forward problem

For a given beam with geometric features such as a step, uniform segment, crack, etc., the overall transfer matrix $[\overline{H}]$, i.e.$[Z]_N$=$[\overline{H}][Z_1]$(Fig. 4.1), can be obtained from intermediate transfer matrices through simple multiplication

$$[\overline{H}] = [R_n]_{4\times4}\,[S_{n-1}]_{4\times4} \cdots [R_2]_{4\times4}\,[S_1]_{4\times4}\,[R_1]_{4\times4} \tag{4.10}$$

These matrices are obtainable from Eqs. (4.5) and (4.9), etc. By inserting the specified boundary conditions, it is possible to obtain a set of simultaneous equations [H][Z]= 0. Mostly two parameters out of the four are zero at the ends. This helps to obtain the characteristic matrix [H] as a matrix of size 2*2. This makes the proposed method very favourable. The characteristic equation of vibration is then given by

$$\det\left[H(\omega, \beta_1, \beta_2, \beta_3, ..., K_1, K_2, K_3, ...)\right]_{2\times 2} = 0 \tag{4.11}$$

where ω is natural frequency. For a cantilever beam with a single crack, the characteristic equation is as follows:

$$\begin{vmatrix} H_{11}^1 + \frac{H_{11}^2}{K} & H_{12}^1 + \frac{H_{12}^2}{K} \\ H_{21}^1 + \frac{H_{21}^2}{K} & H_{22}^1 + \frac{H_{22}^2}{K} \end{vmatrix} = 0 \tag{4.12}$$

where

$$H_{11}{}^1 = p^4 C_1 C_2 + p^4 B_1 D_2 + A_1 A_2 + p^4 D_1 B_2, \quad H_{11}^2 = EIp^4 A_1 D_2$$

$$H_{12}^1 = p^4 D_1 C_2 + p^4 C_1 D_2 + B_1 A_2 + A_1 B_2, H_{12}^2 = EIp^4 B_1 D_2 \tag{4.13}$$

$$H_{21}^1 = p^4 C_1 B_2 + p^4 B_1 C_2 + p^4 A_1 D_2 + p^4 D_1 A_2, H_{21}^2 = EIp^4 A_1 C_2 \tag{4.14}$$

$$H_{22}^1 = p^4 D_1 B_2 + p^4 C_1 C_2 + p^4 B_1 D_2 + A_1 A_2, H_{22}^2 = EIp^4 B_1 C_2 \tag{4.15}$$

Ai, Bi, Ci and Di (i = 1,2) are given by Eq. (4.5) except li is replaced by Li, where L_1= βL, L_2=(1-β)L and β is the nondimensional crack location. Explicitly Eq. (4.10) for cantilever and simply supported beams respectively are as follows:

$$4(1 + \cosh\lambda) + \frac{\lambda}{K}\sinh(\cos\lambda + \cos\lambda e) - \sin\lambda(\cosh\lambda + \cosh\lambda e) + 2\cosh(\lambda\beta)\sin(\lambda\beta)$$

$$-2\cos(\lambda\beta)\sinh(\lambda\beta) - 2\sinh(\lambda(1-\beta))\cosh(\lambda(1-\beta)) + 2\cos(\lambda(1-\beta))\sinh\lambda(1-\beta) = 0 \tag{4.16}$$

and

$$4\sin\lambda\sinh\lambda + \frac{\lambda}{K}\left\{\sinh\lambda(\cos\lambda - \cos\lambda e) - \sin\lambda(\cosh\lambda - \cosh\lambda e)\right\} = 0 \tag{4.17}$$

It is a bit difficult to derive Eq. (4.14) using this approach. Eqs. (4.16) and (4.17) can be re-written in a short form

$$\Delta_1 + \frac{\lambda}{K}\Delta_2 = 0 \tag{4.18}$$

Solving Eq. (4.12) or (4.18) it is possible to solve the forward problem and obtain the natural frequencies. In case there are n intermediate rigid supports, the size of the characteristic determinant (det[H]) increases by n. That is, the matrix size is then (n+2). The mode shape corresponding to a natural frequency can be obtained through Eq. (4.4).

4.1.2 Procedure for crack detection

For a single span beam with one crack Eq. (4.12) or (4.18) can serve as a basis for the detection of its location and size. But for the detection of multiple, say n, cracks the number of unknowns are 2n. It is then difficult to apply Eq. (4.18) directly. Here the approximate method of Hu and Liang [16] in conjunction with the TMM can be employed. This is explained in the following:
The Rayleigh quotient is given by

$$\omega^2 = \mu = \frac{\frac{1}{2}\int_0^L EI(x)\left(\frac{d^2Z}{dx^2}\right)^2 dx}{\frac{1}{2}\int_0^L \rho A Z^2 dx} = \frac{U}{V} \tag{4.19}$$

$$U = strainenergy = \frac{1}{2}\int_0^L EI\left(\frac{d^2Z}{dx^2}\right)^2 dx = \frac{1}{2}\int_0^L \psi dx, \psi = EI\left(\frac{d^2Z}{dx^2}\right)^2 \tag{4.20}$$

$$V = kineticenergy = \frac{1}{2}\int_0^L \rho A Z^2 dx \tag{4.21}$$

For small changes in Frequency

$$\frac{\Delta\mu}{\mu} = \frac{\Delta U}{U} - \frac{\Delta V}{V} \tag{4.22}$$

For a beam with a crack located normally to the beam axis and undergoing transverse vibration ΔV=0 Considering a crack to affect the mode shape locally and representing this effect by a damage parameter S,

$$\Delta U = \frac{1}{2}\int_0^L S\psi dx \tag{4.23}$$

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\int_0^L S\Psi dx}{\int_0^L \Psi dx} \tag{4.24}$$

If a beam is divided into m segments and the segment i is assumed to be associated with a damage parameter Si,

$$\frac{\Delta\omega}{\omega} = 2\sum_{i=1}^{m}\frac{1}{I_0}\int_{L_i}\psi dx S_i \tag{4.25}$$

for nth frequency

$$\frac{\Delta\omega_n}{\omega_n} = 2\sum_{i=1}^{m}\int_{L} g_n(\xi)\, L d\xi S_i \tag{4.26}$$

where

$$g_n(\xi) = \psi_n / I_{0n} \tag{4.27}$$

For a beam divided into m segments and q number of known frequencies,Eq. (4.26) gives rise to a set of simultaneous equations:

$$\left\{\frac{\Delta\omega_n}{\omega_n}\right\}_{q\times l} = 2\,[H]_{q\times m}\,\{S\}_{m\times l} \tag{4.28}$$

In the calculations some of the parameters Si are less than 0. Since a positive Si represents a reduction in section because of a crack, a negative Si indicates an increase of area of section. this is unrealistic. To handle the case Si ¡ 0, the corresponding segment is treated to be free of any damage and Si is set equal to zero. The calculations are repeated till a set of all positive damage parameters are obtained. Taking only one damage parameter Si of a segment i as nonzero at a time and keeping all others as zero, the change in a natural frequency Dxn due to one such damage in the segment is calculated. A variation of K with crack location b is obtained using Eq.

(4.18) and Dxn. This is repeated for three or more modes. Since,the stiffness of the spring is independent of the mode, the location where the three or more such curves intersect gives the crack location and the spring stiffness.

Result for simply supported beam

Table 4.1: Natural frequencies of simply supported beam with two cracks

case no.	crack location		Natural frequencies				
	$\beta 1$	$\beta 2$	$\omega 1$	$\omega 2$	$\omega 3$	$\omega 4$	$\omega 5$
1	uncracked beam		59.007	236.029	531.065	944.116	1475.18
2	0.25	0.45	58.625	235.142	528.096	942.515	1469.103

Table 4.2: Comparison of actual and predicted crack location for simply supported beam

case no	Actual data	Natural frequencies			Predicted data	
	Location β	$\omega 1$	$\omega 2$	$\omega 3$	Location β	%error
1	uncracked beam	59.007	236.029	531.065	-	-
2	0.25	58.915	235.314	530.233	0.25	0
3	0.45	58.719	235.884	529.051	0.443	-0.7

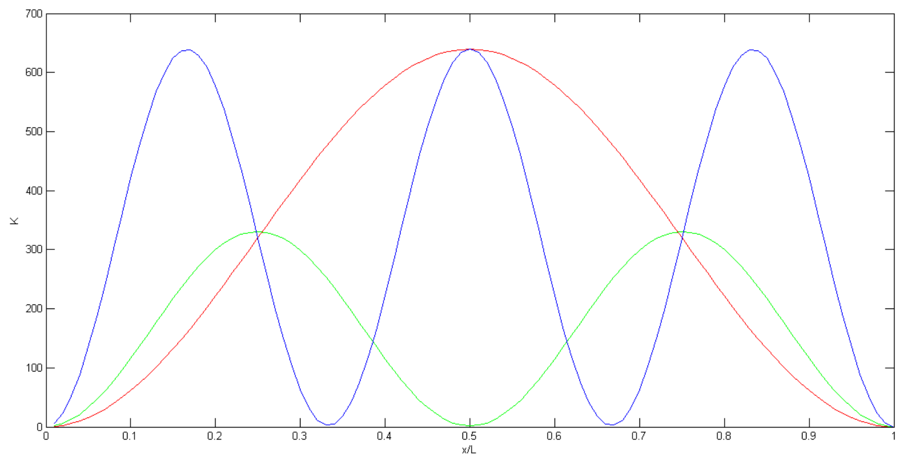

Figure 4.2: crack detection in simply supported beam

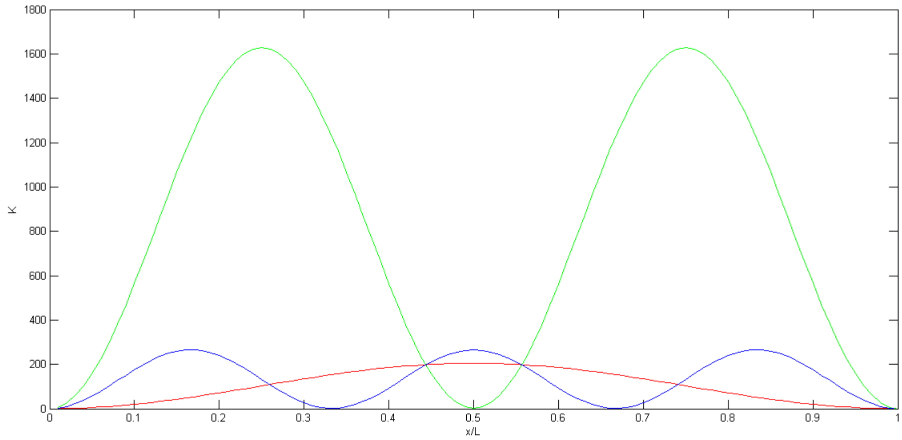

Figure 4.3: crack detection in simply supported beam

4.1.3 Result for cantilever beam

Table 4.3: Natural frequencies of cantilever beam with two cracks

case no.	crack location		Natural frequencies				
	$\beta 1$	$\beta 2$	$\omega 1$	$\omega 2$	$\omega 3$	$\omega 4$	$\omega 5$
1	uncracked beam		6.62	47.42	125.397	240.5539	392.892
2	0.1	0.4	6.243	46.32	123.69	237.834	386.782
3	0.1	0.5	6.18	45.938	122.3	235.487	384.782

Result for cantilever beam using ansys

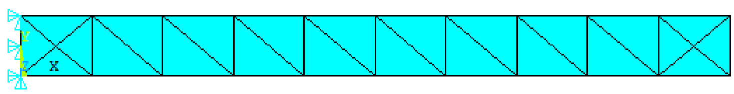

Figure 4.4: Natural frequency of uncracked beam

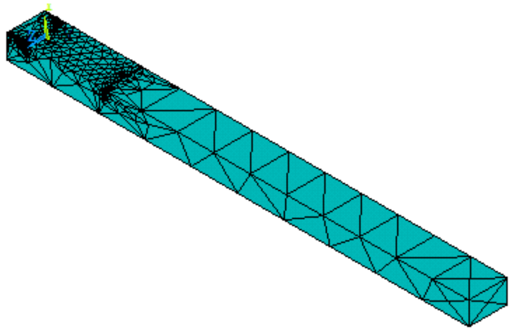

Figure 4.5: Natural frequency of cracked beam

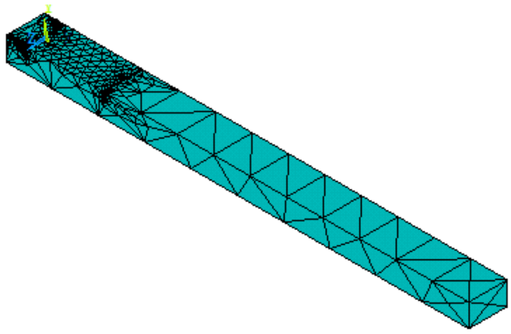

Figure 4.6: Natural frequency of cracked beam

4.1.4 Experimental study

The experimental modal analysis can be done by FFT analyzer. The FFT analyzer converts the time domain data to frequency domain data. The analysis of time domain data is very complex and it is not in appropriate form. From the frequency domain data, one can find resonance frequencies and the resonance conditions and amplitude can be determined easily in frequency domain data. In present work, the experimental analysis is done by using FFT analyzer.The experimental setup is as shown in the fig 5.1

Mild steel beams were considered for experimental. Geometric and material properties of these beams are given in Table 4.1. Beams are supported by cantilever support. Crack were made by wire-cut machining (EDM), using a wire of diameter of 0.25mm in size of 1mm.a/h=0.33,where h is beam thickness. Experimental set up of

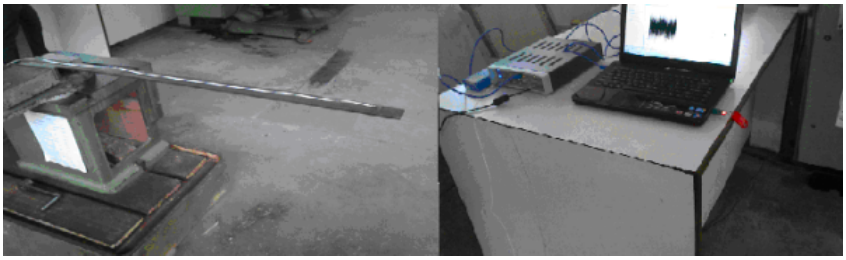

Figure 4.7: Experimental setup

Table 4.4: Geometric and material data for M.S.Beam.

Parameter	$L(m)$	$b_(m)$	$h_(m)$	$\rho_g(kg/m^3)$
Value	1	0.05	0.008	7860

cantilever supported beam is shown,One end of beam is fixed with the vice,FFT analyser is attached which is also shown in figure.Four Channel LSM Test.Xpress FFT Analyzer used for finding out the natural frequency of uncracked tube and cracked beam.To measure natural frequencies,the accelerometer is fix vertically at the end of the beam by applying a quick setting wax to the base of the accelerometer and pressing it for 5 to 10 second.From FFT analysis,the frequency corresponding to first few peaks are taken as natural frequencies.While testing a beam,it is tapped in the transverse direction and first three natural frequencies is measured.

4.1.5 Crack Identification Procedure

Using Equation 4.17,find the dimensionless local flexibility coefficient K for the first three natural frequency.After finding this K,use the Equation (4.19)- (4.20) and plot the Mode shape of crack beam.from this mode shape,at the crack location change in mode shape behavior Suddenly.so where the mode shape behavior change suddenly gives the crack location.

Table 4.5: Comparison of actual and predicted crack location for cantilever beam

case no	Actual data	Natural frequencies			Predicted data	
	Location β	$\omega 1$	$\omega 2$	$\omega 3$	Location β	%error
1	uncracked beam	6.538	47.2	124.6		
2	0.1	6.504	46.65	123.3	0.067	-3.3
	0.4	6.412	45.589	122.56	0.417	1.7
3	0.1	6.392	45.278	122.32	0.047	3.8
	0.5	6.21	44.573	120.923	0.469	-3.1

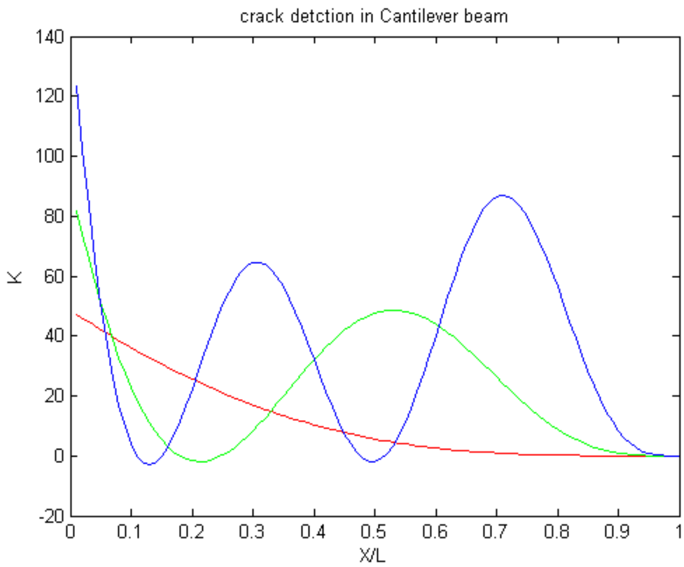

Figure 4.8: crack detection at $\beta = 0.1$

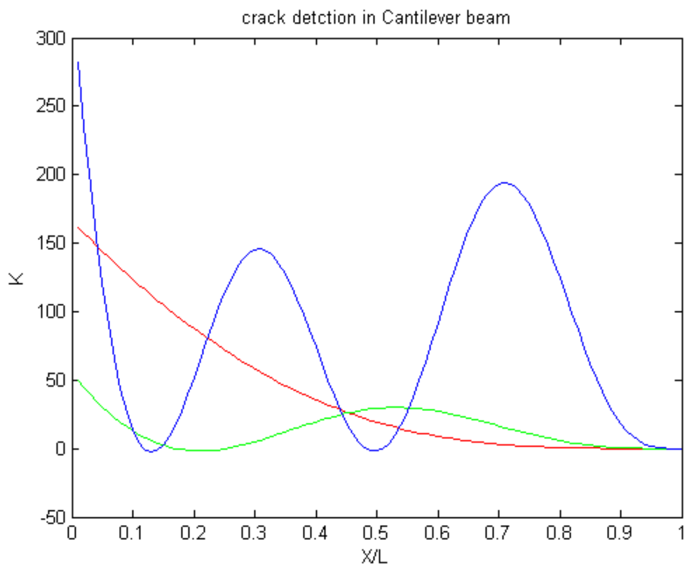

Figure 4.9: crack detection at $\beta = 0.4$

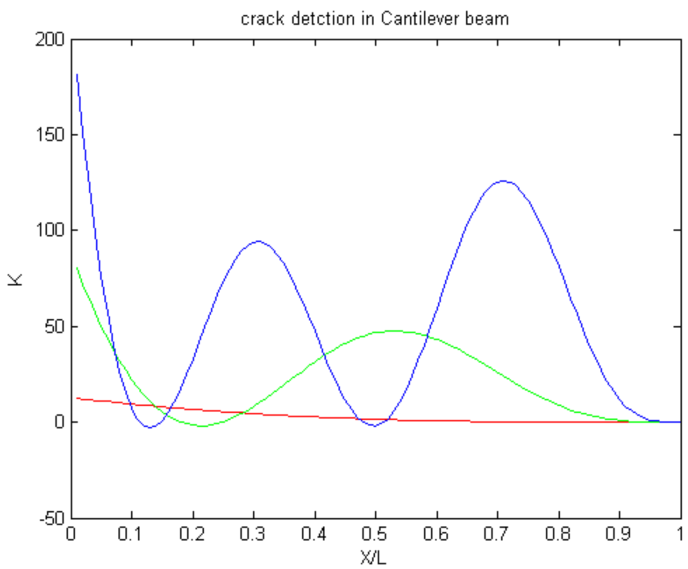

Figure 4.10: crack detection at $\beta = 0.1$

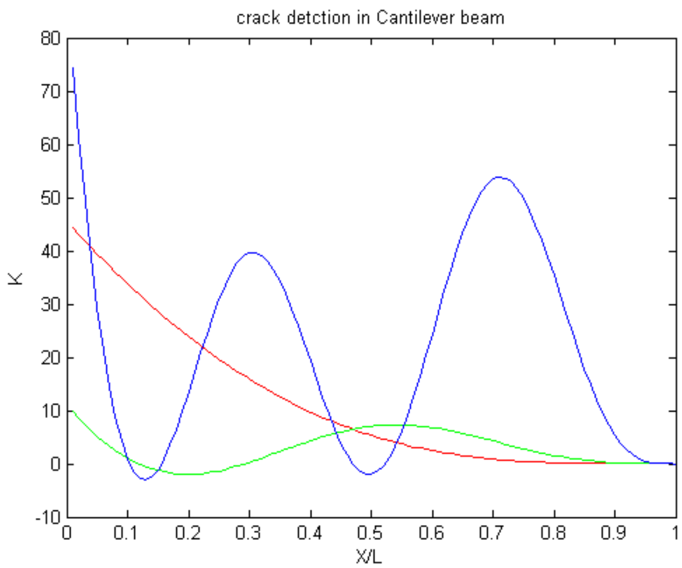

Figure 4.11: crack detection at $\beta = 0.5$

A technique, combining the TMM and representation of an open edge crack oriented normally to the axis by a rotational spring, has been proposed to solve the problem of detection of multiple cracks in beams with varying end conditions(fixed and simple supports).The accuracy of results in the inverse, or crack detection, problem is good.The maximum difference in the prediction of location is less than 10The method however suffers from one important limitation. That is, the maximum number of cracks that can be detected is less than equal to the number of segments into which the beam is virtually divided.

Chapter 5

Conclusion and Future Scope

5.1 Conclusion

Dynamic analysis has been done for Uncracked beam and its given very reasonable result between analytical and Ansys. The representation of a crack by a rotational spring in a straight beam for modeling its transverse free vibration for solving multiple cracks. The present effort to predict crack location by Natural Frequency Based Mode shape Method in the multiple cracked beam.

5.2 Future Scope

- This method is extended for detection of multiple cracks at different angle.
- The multiple cracked beam analysed under the external force.
- This method is extended for multiple crack in rotating shaft.

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