MOMENT RESULTANT FOR SYMMETRIC ANISOTROPIC PLATE WITH SUPER ELLIPTICAL SHAPE HOLE

J. Dave[1] and D. Sharma[2]

[1,2] Mechanical Engineering Department, Institute of Technology, Nirma University, Ahmedabad

In present work, distribution of moment resultant around hole of super elliptical shape in symmetric laminated plate is obtained by Muskhelishvili's complex variable approach. The anisotropic plate with super elliptical shaped hole is subjected to moment at infinity is considered and analytical solutions are obtained. In this paper, effect of various parameters like material, hole geometry, loading angle, staking sequencing etc. on moment distribution is studied for symmetrical laminates. The material considered for study are Glass/Epoxy, Graphite/Epoxy, Boron/Epoxy etc. and cross ply, angle ply, 16 ply laminates are considered for obtaining results.

Keywords: *Moment resultant, anisotropic material, complex variable, symmetrical laminates*

1 Introduction

The solution of stress analysis problems is fascinating to researchers for many years. The solution of the problem is become difficult when it contains any discontinuity. The discontinuity may be due to environmental effect like corrosion, wear or damage. Many times, it is due to design needs to provide some access or opening into components in terms of holes of different shape. Such holes or discontinuity leads to stress concentration when the component is under different loading conditions. Today, the use of composite material have been increased to a great extent in the application of aerospace, aircraft, marine, sports, automobile etc. hence, it is necessary to increase research in the area of stress analysis with anisotropy consideration. The designers can be equipped with handy solution tool which gives better results with minimum time.

Savin [1] and Lekhnitskii [2] have provided formulation for bending based on thin plate theory for anisotropic media. They presented solution for circular and elliptical hole in various anisotropic materials. For isotropic material, Goodier [3] gave the solution for bending and twisting based on thin plate theory. Prasad and Shuart [4] provided closed-form solution for the moment distribution of an infinite anisotropic plate using fundamental equation given by Lekhnitskii [2]. Ukadgaonkar and Rao [5] and Ukadgaonkar and Khakhandaki [6] has extended Muskhelishvili's [7] complex variable approach and provided superposition method to solve problem of stress analysis with different hole shape using conformal mapping and Gao's [8] arbitrary biaxial loading condition. A general solution is given by Ukadgaonker and Rao [9] for bending of symmetric laminates with holes based on the formulations of Savin[1] and Lekhnitskii[2] that considers any shape of hole in symmetric laminates subjected to remotely applied bending or twisting moments. The moment distribution around circular, elliptical and triangular hole is studied by various parameter like stacking sequence, material and geometry by Sharma and Patel [10] using Muskhelishvili's [7] stress function.

This paper focuses on the solution for moment resultant for anisotropic infinite plate with super elliptical hole. Super ellipse represents family of curve by single equation. By changing various constant in the equation many shapes can be obtained. Here, Muskhelishvili's stress functions are extended to obtained general solution for super elliptical hole in symmetrical laminates.

2 Analytical Formulation

Considering displacement u and v in x and y direction respectively and corresponding strains $\varepsilon_x, \varepsilon_y$ and γ_{xy} can be presented as a function of deflection $w(x, y)$ of mid plane in z direction by following equations.

$$u = -zw,_x, v = -zw,_y$$
$$\varepsilon_x = -zw,_{xx}\ \varepsilon_y = -zw,_{yy}\ \gamma_{xy} = -2zw,_{xy} \quad (1)$$

[1] Research Scholar, jatin.dave@nirmauni.ac.in
[2] Professor, dss_iit@yahoo.com

Taking strain – stress relation ship and substituting values of strain from Eq. (1) in it and considering h as a height of laminate from mid plane, we obtain following equation for stresses.

$$\sigma_x = -z\frac{h^3}{12}(D_{11}w,_{xx} + D_{12}w,_{yy} + 2D_{16}w,_{xy}),$$
$$\sigma_x = -z\frac{h^3}{12}(D_{12}w,_{xx} + D_{22}w,_{yy} + 2D_{26}w,_{xy}), \qquad (2)$$
$$\tau_{xy} = -z\frac{h^3}{12}(D_{16}w,_{xx} + D_{26}w,_{yy} + 2D_{66}w,_{xy})$$

The flexural stiffness matrix $D_{ij,}(i,j=1,2,6)$ can be obtained by conventional relations. The moments can be expresses as follows.

$$M_x = -\frac{h^3}{12}(D_{11}w,_{xx} + D_{12}w,_{yy} + 2D_{16}w,_{xy}),$$
$$M_y = -\frac{h^3}{12}(D_{12}w,_{xx} + D_{22}w,_{yy} + 2D_{26}w,_{xy}), \qquad (3)$$
$$M_{xy} = -\frac{h^3}{12}(D_{16}w,_{xx} + D_{26}w,_{yy} + 2D_{66}w,_{xy})$$

Introducing the expression for the moments from Eq. (3) into equilibrium condition, following characteristic equation is obtained.

$$Q_{22}s^4 + 4Q_{26}s^3 + 2(Q_{12} + 2Q_{66})s^2 4Q_{16}s + Q_{11} = 0. \qquad (4)$$

Taking stress functions as $\phi(z_1) = \frac{dF_1}{dz_1}, \psi(z_2) = \frac{dF_2}{dz_2}$, the deflection can be represented by

$$w(x,y) = 2\,\mathrm{Re}[F_1(z)_1 + F_2(z)_2] \qquad (4)$$

By substituting derivatives of deflection from Eq. (4) into Eq. (3), following relations can be derived.

$$M_x = \frac{-h^3}{6}\mathrm{Re}[p_1\phi'(z_1) + q_1\psi'(z_2)],$$
$$M_y = \frac{-h^3}{6}\mathrm{Re}[p_2\phi'(z_1) + q_2\psi'(z_2)], \qquad (5)$$
$$M_{xy} = \frac{-h^3}{6}\mathrm{Re}[p_3\phi'(z_1) + q_3\psi'(z_2)],$$

Where,

$$p_1 = D_{11} + D_{12}s^2 + 2D_{16}s_1, q_1 = D_{11} + D_{12}s_2^2 + 2D_{16}s_2,$$
$$p_2 = D_{11} + D_{22}s_1^2 + 2D_{26}s_1, q_2 = D_{12} + D_{22}s_2^2 + 2D_{26}s_2,$$
$$p_3 = D_{16} + D_{26}s_1^2 + 2D_{66}s_1, q_3 = D_{16} + D_{26}s_2^2 + 2D_{66}s_2, \qquad (6)$$
$$p_4 = \frac{D_{11}}{s_1} + 3D_{16} + (D_{12} + 2D_{66})s_1 + D_{26}s_1^2,$$
$$q_4 = \frac{D_{11}}{s_2} + 3D_{16} + (D_{12} + 2D_{66})s_2 + D_{26}s_2^2$$

3 Stress Functions

The Super ellipse, shown in Fig. 1, is represented by following equation.

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 1 \qquad (7)$$

where, a is semi major axis and b is semi minor axis.

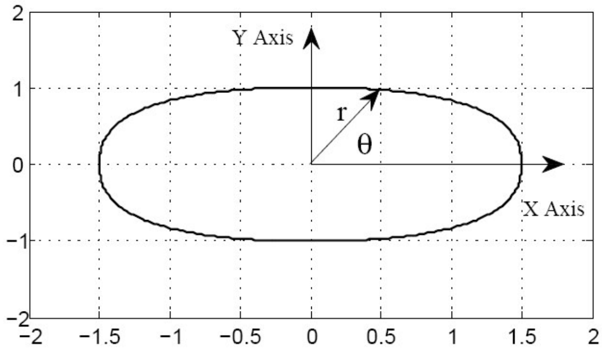


Figure 1: Super Ellipse

Taking polar form as $x = r\cos\theta$ and $y = r\sin\theta$ and substituting into Eq. (7), solution of radius r is obtained as follows.

$$\frac{1}{r} = \left[\left|\frac{\cos\theta}{a}\right|^n + \left|\frac{\sin\theta}{b}\right|^n\right]^{\frac{1}{n}} \qquad (8)$$

Taking $z = x + iy$,mapping function as, $z = w(\xi) = re^{i\theta}$ and incorporating constant of anisotropy s_j and applying affine transformation, the mapping function becomes,

$$z_j = \omega_j(\xi) = \frac{R}{2}\left[a_j\frac{1}{\xi} + b_j\xi\right] \qquad (9)$$

where, $a_j = 1 + s_j$ and $b_j = 1 - s_j$, $j = 1,...4$

Super ellipse is obtained by taking n = 2.5 in Eq. (8). By arranging values of n and dividing 2π into number of sector many other shapes can be obtained.

The hole free plate subjected to remotely applied moment $M_x^\infty, M_y^\infty, M_{xy}^\infty$ at infinity is considered initially. Taking derivatives of first stage stress functions as $\phi'(z_1) = (B^* + iC^*), \psi'(z_2) = (B'^* + iC'^*)$ in Eq. (5), solution for B^*, B'^*, C'^* can be obtained.

Boundary conditions f_1, f_2 on fictitious hole can be obtained by integrating bending moment and bending force over the hole boundary and applying tensor transformation for moment and shear force.

As hole is traction free, negative boundary conditions $-f_1, -f_2$ are considered in absence of moments at infinity. Introducing mapping function and solving by Schwarz formula, second stage stress functions are obtained as follows.

$$\phi_0(\xi) = \left[\frac{a_9}{\xi} + \frac{b_9}{\xi}\right], \qquad \psi_0(\xi) = \left[\frac{a_{10}}{\xi} + \frac{b_{10}}{\xi}\right] \tag{10}$$

where,

$$\begin{aligned} a_9 &= \frac{s_1}{(p_1 q_2 s_2 - p_2 q_1 s_1)}\left[q_1(K_7 + \overline{K_8}) - q_2 s_2 (K_5 + \overline{K_6})\right] \\ b_9 &= \frac{s_1}{(p_1 q_2 s_2 - p_2 q_1 s_1)}\left[q_1(K_8 + \overline{K_7}) - q_2 s_2 (K_6 + \overline{K_5})\right] \\ a_{10} &= \frac{-s_2}{(p_1 q_2 s_2 - p_2 q_1 s_1)}\left[p_1(K_7 + \overline{K_8}) - p_2 s_1 (K_5 + \overline{K_6})\right] \\ b_{10} &= \frac{-s_2}{(p_1 q_2 s_2 - p_2 q_1 s_1)}\left[p_1(K_8 + \overline{K_7}) - p_2 s_1 (K_6 + \overline{K_5})\right] \end{aligned} \tag{11}$$

and

$$\begin{aligned} K_5 &= \frac{R}{2}\left[\frac{p_1}{s_1} B_1^* a_1 + \frac{q_1}{s_2}(B_1'^* + iC_1'^*) a_2.\right], \\ K_6 &= \frac{R}{2}\left[\frac{p_1}{s_1} B_1^* b_1 + \frac{q_1}{s_2}(B_1'^* + iC_1'^*) b_2.\right], \\ K_7 &= \frac{R}{2}\left[p_2 B_1^* a_1 + q_2 (B_1'^* + iC_1'^*) a_2.\right] \\ K_8 &= \frac{R}{2}\left[p_2 B_1^* b_1 + q_2 (B_1'^* + iC_1'^*) b_2.\right] \end{aligned} \tag{12}$$

The final solution is given by,

$$\begin{aligned} \phi(z_1) &= \phi_1(z_1) + \phi_0(z_1) \\ \psi(z_2) &= \psi_1(z_2) + \psi_0(z_2) \end{aligned} \tag{13}$$

The moments obtained about the Cartesian coordinate system are transformed into an orthogonal curvilinear coordinate system by transformation.

4 Results and Discussion

The general formulation obtained as above can be used to get results for different material and loading conditions. The values are obtained for Graphite/epoxy (E_1=181GPa, E_2=10.3GPa, G_{12}=7.17GPa and ν_{12}=0.28) and Glass/epoxy (E_1=47.4GPa, E_2=16.2GPa, G_{12}=7.0GPa and ν_{12}=0.26) and Boron/Epoxy (E_1=282.77 GPa, E_2=23.79 GPa, G_{12}=10.35 GPa and ν_{12}=0.27).
The flexural modulii is obtained for given material and stacking sequence and using Eq. (4), constants of anisotropy are obtained. By selecting proper values of n, dimensions for super ellipse can be obtained by Eq. (8). Using conformal mapping and values of B^*, B'^*, C'^*, first stage solution $\phi_1(z_1), \psi_1(z_2)$ is obtained. Further, applying Eq. (10, 11, 12), second stage stress functions $\phi_0(z_1), \psi_0(z_2)$ is obtained.
The symmetric cross ply and angle ply are considered for solution with 16 plies. The ply group of 4, 8 and 16 is considered. The thickness of each ply taken as 125 μm.

4.1 Effect of stacking sequence

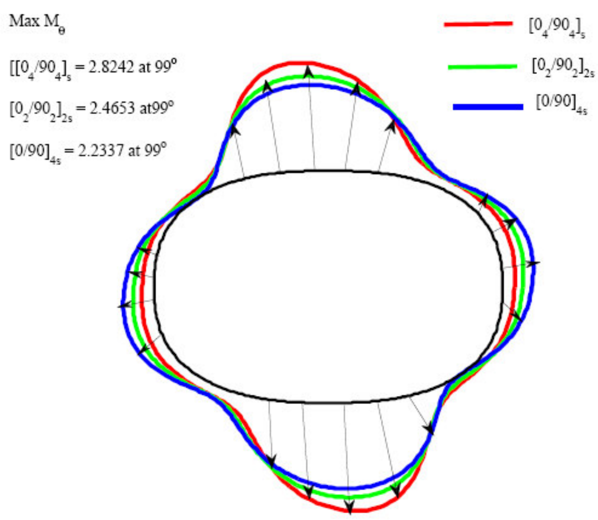


Figure 2: M_θ/M for moment about X - Direction

Fig. 2 shows the moment distribution M_θ/M for applied moment about X – Direction for $[0_4/90_4]_s$, $[0_2/90_2]_{2s}$ and $[0/90]_{4s}$ laminates. The maximum values are 2.8242, 2.4653 and 2.3337 at 99° respectively.

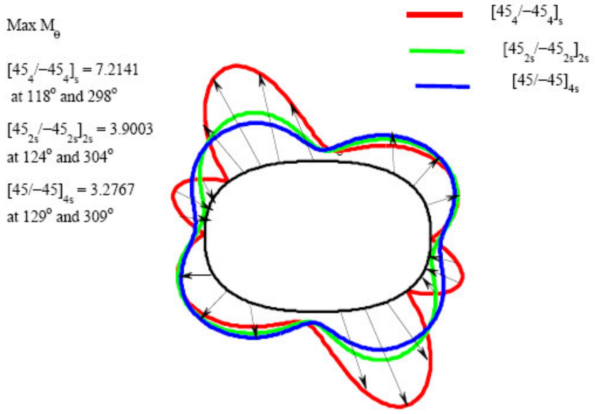


Figure 3: M_θ/M for Biaxial moment

Fig. 3 shows the effect of stacking of angle ply for biaxial moment. Symmetric angle ply laminate with 16 plies is taken in 4, 8 and 16 ply group. The maximum value of M_θ/M is in decreasing order respectively.

To study effect of material, Graphite/Epoxy, Glass/Epoxy and Boron/Epoxy is taken for comparison.

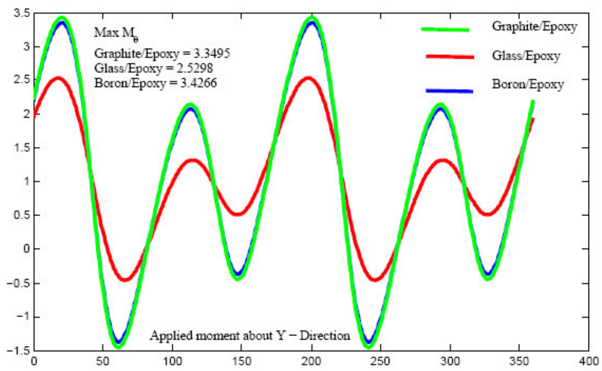
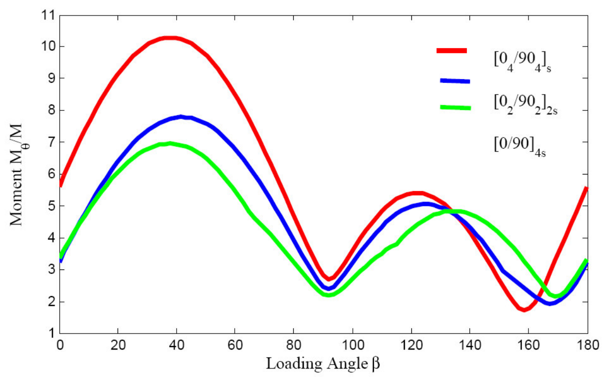


Figure 4: Effect of material

As shown in Fig. 4, the maximum values of M_θ for Graphite/Epoxy, Glass/Epoxy and Boron/Epoxy are 3.3495, 2.5298 and 3.4266 are respectively at 22° and 204°



.

Figure 5: Effect of loading angle for cross ply

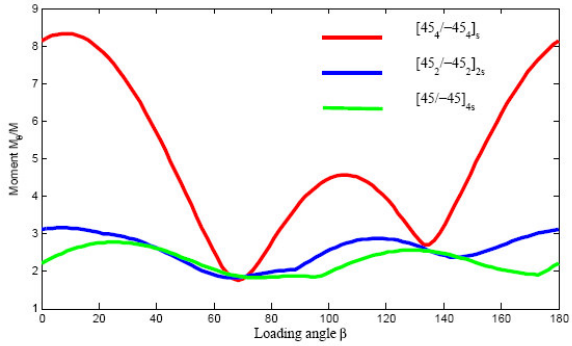


Figure 6: Effect of loading angle angle ply

Effect of change of loading angle for moment applied about X Direction is shown in Fig. 5 and Fig. 6 for cross ply and angle ply respectively.

The maximum value is occurred for 4 ply group in both cross and angle ply arrangement. For cross ply, maximum value is 10.2972 at 22 ° and for angle ply, maximum value is 8.3265 at 6°.

5 Conclusion

The stacking sequence, load angle and material have significant effect on moment distribution.

The maximum value of moment M_θ /M is decrease as we increase number of ply group for both cross and angle ply.

High value of M_θ /M is observed for Boron/Epoxy in comparison with Graphite/Epoxy and Glass/Epoxy for loading in Y direction.

References

[1] Savin G.N., *Stress Concentration around Hole,*. Pregamon Press, New York, 1961

[2] Lekhnitskii S.G., *Anisotropic plates*, Gordon and Breach science publishers, New York, 1968

[3] Goodier J., The influence of circular and elliptical holes on the transverse flexure of elastic plates, *Philosophical Magazine(Series 7)*, Vol. 22, p. 69-80, 1936

[4] Prasad C. B. and Shuart M. J., Moment distributions around holes in symmetric composite Laminates Subjected to Bending Moments. 1989

[5] Ukadgaonker V.G., Rao D.K.N., A general solution for stresses around holes in symmetric laminates under inplane loading, *Composite Structures*, Vol. 49, p. 339-354, 2000

[6] Ukadgaonker V.G., Kakhandki V., Stress analysis for an orthotropic plate with an irregular shaped hole for different in-plane loading conditions-part-1, *Composite Structures*, Vol.70, p. 255-274, 2005

[7] Muskhelishvili N.I., *Some Basic Problems of Mathematical Theory of Elasticity.* P.Noordhoo Ltd., The Netherlands, 1963

[8] Gao X. L., A general solution of an infinite elastic plate with an elliptic hole under biaxial loading, *International Journal of Pressure Vessels & Piping*, Vol. 67, p. 797-807, 1996

[9] Ukadgaonker V.G. and Rao D.K.N., A general solution for moments around holes in symmetric laminates, *Composite Structures*, Vol. 49, p. 41-54, 2000

[10] Sharma D. S. and Patel N., Moment Distribution Around Circular /Elliptical / Triangular Cutouts In Infinite Symmetric Laminated Plate, *Fourth International Conference on Structural Stability and Dynamics (ICSSD 2012),* Malaviya National Institute of Technology, Jaipur & Texas A&M University, USA, p. 817 – 824, 4–6 January, 2012