Performance Analysis of Alamouti Transmit Diversity with A Sub-Optimum Joint Transmit-Receive Antenna Selection Scheme

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***Abstract*—We consider a sub-optimum joint transmit-receive antenna selection (JTRAS) scheme in multiple input multiple output (MIMO) systems equipped with N transmit and two receive antennas. At the transmitter, we keep one antenna as fixed and select the best among the remaining $N-1$ antennas. After selecting two transmit antennas for each of the receive antennas, we select the receive antenna for which the signal to noise ratio (SNR) is maximum. We assume spatially independent flat fading channels with perfect channel state information (CSI) at receiver and an ideal feedback link. We use Alamouti transmit diversity and derive the exact closed-form expression for the pdf of received SNR, using which we obtain bit error rate (BER) for BPSK constellation. We have presented simulation results and compared them with the derived analytical expressions. We have discussed some special cases of the considered antenna selection scheme. We have compared performance of the considered scheme with the other available schemes in terms of number of feedback bits and BER. We conclude that the considered JTRAS scheme reduces number of feedback bits.**

***Index Terms*—Alamouti transmit diversity (ATD), bit error rate (BER), Rayleigh fading channel, joint transmit-receive antenna selection (JTRAS).**

I. Introduction

Multiple input multiple output (MIMO) systems with space time coding are used in wireless communications to reduce the effect of fading by providing diversity gain [1]- [3]. However, there are some bottlenecks of MIMO systems such as limited spacing between adjacent antennas at mobile station, power spreading between transmit antennas and the requirement of costly RF chains for every active transmit/receive antenna pair. One of the approaches to alleviate these problems, without loss of diversity gain, is selection of antenna or subset of antennas at the transmitter or at the receiver or at both the ends [4]- [12].

In practice for the case of Frequency Division Duplex (FDD), based on the available channel state information (CSI), receiver selects transmit antennas and send the index of the antenna to the transmitter via a dedicated feedback channel. However, execution of antenna selection (AS) algorithm at the receiver and requirement of ideal feedback channel increase complexity and overheads. Therefore, sub-optimum AS with less complexity and feedback channel with low data rate are of interest [6]- [9].

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In [11] and [12], joint transmit-receive antenna selection (JTRAS) has been considered. In [11], exact BER is derived for a $(N_t, 2; N_r, 1)$ Space Time Block Coding (STBC), where $(N_t, 2; N_r, 1)$ denotes two out of N_t transmit antennas and one out of N_r receive antennas are selected. In [12], exact SER are derived for a $(N_t, L_t; N_r, L_r)$ STBC. However, [11] and [12] have considered optimum antenna selection scheme.

In this paper, we have considered a sub-optimum JTRAS scheme in a $(N, 2; 2, 1)$ system. In the first step, for each receive antenna, we keep one transmit antenna fixed and select the best among the remaining $N-1$ antennas. In the second step, we select the receive antenna for which the signal to noise ratio (SNR) is maximum. Then, we have considered Alamouti transmit diversity (ATD) and derived the exact closed form expression for the pdf of received SNR. Finally, we have obtained expression of BER for BPSK constellation. We have also compared the considered scheme with [9], [10] and [11]. We have presented simulation results and verified the analytical expression. We have also discussed some special cases.

The remainder of the paper is organized as follows. The system under consideration and channel model are introduced in Section II. In Section III, we present closed form expression of received SNR and BER; and also discuss some special cases. In Section IV, we show simulation and analytical results. Finally, we conclude in Section V.

II. System and Channel Models

We consider a MIMO wireless link in a flat Rayleigh fading environment equipped with N transmit antennas and two receive antennas. We assume that the CSI is perfectly known at the receiver. A block diagram of the above mentioned system is shown in Fig. 1. The channel fading coefficients are denoted by $h_{j,i}$ (between receive

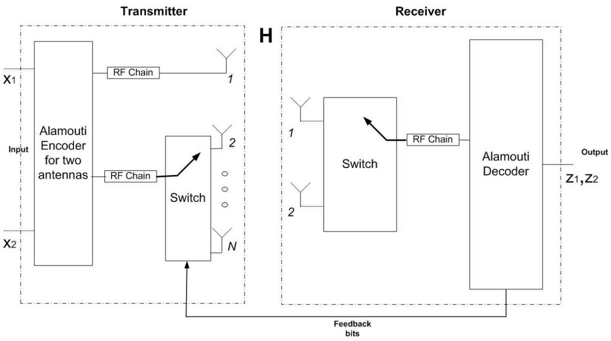

Fig. 1. Block diagram of an Alamouti transmit diversity system with sub-optimum $(N, 2; 2, 1)$ JTRAS scheme

antenna j and transmit antenna i), where $1 \leq j \leq 2$ and $1 \leq i \leq N$. We have made the following assumptions:

1) All $h_{j,i}$ are spatially independent and identically distributed (i.i.d.) as circularly symmetric complex Gaussian random variables with zero mean and unit variance.
2) All $h_{j,i}$ are quasi static and they remain constant for the duration of at least two consecutive symbols.
3) Perfect knowledge of $h_{j,i}$ is available at the receiver.
4) A dedicated perfect feedback link is available.

We use Alamouti transmit diversity [2] for which the received symbols y_1 and y_2 can be represented as

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{q,m} & h_{q,f} \\ h_{q,f}^* & -h_{q,m}^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2^* \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}, \quad (1)$$

where the superscript $*$ denotes the complex conjugate. x_1 and x_2 are transmitted data symbols for BPSK modulation scheme, i.e., $x_1, x_2 \in \{-\sqrt{E_s/2}, \sqrt{E_s/2}\}$ and $n_i \sim \mathcal{C}N(0, N_0)$ for $1 \leq i \leq 2$ is additive white Gaussian noise (AWGN).

In (1), $h_{q,f}$ is the channel between the fixed transmit antenna f and the selected receive antenna q, whereas $h_{q,m}$ is the channel between the selected antenna among the $N-1$ transmit antennas and the selected receive antenna q. The selection of transmit antenna is done in two steps, based on the maximization of the instantaneous SNR.

In the first step, for each receive antenna j where $1 \leq j \leq 2$, we select transmit antenna p_j as

$$p_j = \arg \max_{1 \leq i \leq N-1} \{|h_{j,i}|^2\}, \quad 1 \leq j \leq 2.$$

In the second step, one receive antenna q is selected out of two as

$$q = \arg \max_{1 \leq j \leq 2} \{|h_{j,p_j}|^2 + |h_{j,f_j}|^2\},$$

Now for brevity, we denote the selected transmit antenna as m instead of p_j and fixed transmit antenna as f, instead of f_j.

In this case, the decision variables can be represented as [2]

$$\begin{aligned} z_1 &= (|h_{q,m}|^2 + |h_{q,f}|^2)x_1 + h_{q,m}^* n_1 + h_{q,f} n_2^*, \\ z_2 &= (|h_{q,m}|^2 + |h_{q,f}|^2)x_2 + h_{q,m} n_2^* - h_{q,f}^* n_1^*, \quad (2) \end{aligned}$$

where z_1 and z_2 are the decision variables for data symbols x_1 and x_2 respectively.

III. Performance analysis

In this section, we derive the probability density function (pdf) of SNR, using which we obtain exact expression of bit error rate (BER).

We assume that both x_1 and x_2 are independent and equally distributed. Therefore, BER can be determined using any one of them. Hence, we consider the symbol x_1 and express the instantaneous SNR (γ) as

$$\gamma = \|\mathbf{h}\|^2 \gamma_c, \quad (3)$$

where $\mathbf{h} = \begin{bmatrix} h_{q,m} & h_{q,f} \end{bmatrix}$ and $\gamma_c = E_s/2N_o$. Now, we require the pdf of γ which can be determined as follows.

Let us denote $|h_{j,i}|^2$ as $X_{j,i}$. All $X_{j,i}$ are chi-squared distributed variables with two degrees of freedom. Since all $X_{j,i}$ are equally distributed, we can represent the pdf $p_X(x)$ and the Cumulative Distribution Function (CDF) $F_X(x)$ as [15]

$$\begin{aligned} p_X(x) &= e^{-x}, \quad x \geq 0 \\ F_X(x) &= 1 - e^{-x}. \end{aligned}$$

Furthermore, since all $X_{j,i}$ are independent, the pdf of $X_{j,m}$ can be expressed using order statistics [16] as

$$\begin{aligned} p_{X_{j,m}}(x_{j,m}) &= (N-1)[F_X(x_{j,m})]^{N-2} p_X(x_{j,m}), x_{j,m} \geq 0 \\ &= (N-1) \sum_{k=0}^{N-2} (-1)^k \binom{N-2}{k} e^{-x_{j,m}(1+k)} \\ &= (N-1) \Bigg[e^{-x_{j,m}} + \sum_{k=1}^{N-2} (-1)^k \binom{N-2}{k} \\ &\times \left\{ e^{-x_{j,m}(1+k)} \right\} \Bigg]. \quad (4) \end{aligned}$$

Let $X_{j,f} = |h_{j,f}|^2$ which is the instantaneous channel power gain between the receive antenna j and fixed transmit antenna f. The pdf for which is denoted by $p_{X_{j,f}}(x_{j,f}) = e^{-x_{j,f}}$. Let us take R_j, $1 \leq j \leq 2$, as

$$R_j = X_{j,m} + X_{j,f}.$$

The pdfs of R_1 and R_2 are identical and they can be expressed as the convolution of pdfs $p_{X_{j,m}}(x_{j,m})$ and

$p_{X_{j,f}}(x_{j,f})$ as [14]

$$p_{R_j}(r_j) = \int_0^{r_j} p_{X_{j,m}}(x_{j,m}) p_{X_{j,f}}(r_j - x_{j,m}) dx_{j,m}$$
$$= (N-1)\left[r_j e^{-r_j} + \sum_{k=1}^{N-2}(-1)^k \left(\frac{1}{k}\right)\binom{N-2}{k} \times \left\{e^{-r_j} - e^{-r_j(1+k)}\right\}\right]. \quad (5)$$

Similarly, the Cumulative Distribution Function (CDF) $F_R(r_j)$ can be represented as

$$F_{R_j}(r_j) = \int_0^{r_j} p_{R_j}(r_j) dr_j$$
$$= 1 - Y(r), \quad (6)$$

where $Y(r)$ is represented as

$$Y(r) = (N-1)\left[r_j e^{-r_j} + e^{-r_j} + \sum_{k=1}^{N-2}\left(\frac{(-1)^k}{k}\right) \times \binom{N-2}{k}\left\{e^{-r_j} - \frac{e^{-r_j(1+k)}}{1+k}\right\}\right]. \quad (7)$$

Now, we have to select one out of two receive antennas, hence again using order statistics [16], the pdf of R_q can be represented as

$$p_{R_q}(r_q) = 2[F_R(r_q)]p_{R_j}(r_q), \quad r_q \geq 0$$
$$= 2[1 - Y(r)]p_{R_j}(r_q)$$
$$= 2p_{R_j}(r_q) - 2Y(r)p_{R_j}(r_q)$$
$$= \alpha p_{R_j}(r_q) - \beta Y(r) p_{R_j}(r_q), \quad (8)$$

for $\alpha = 2$ and $\beta = 2$. It is to be noted that, $\alpha = 1$ and $\beta = 0$ represents $(N, 2; 1)$ sub-optimum system discussed in [9]. Substituting equation (5) and (7) in (8), we get

$$p_{R_q}(r_q) = \alpha(N-1)\left[re^r + \sum_{k=1}^{N-2}\frac{(-1)^k}{k}\binom{N-2}{k} \times \left\{e^{-r} - e^{-r(1+k)}\right\}\right] - \beta(N-1)^2\left[r^2 e^{-2r} + re^{-2r} + \sum_{k=1}^{N-2}\frac{(-1)^k}{k}\binom{N-2}{k}\left\{2re^{-2r} - \frac{r(2+k)}{(1+k)}e^{-r(2+k)} - e^{-r(2+k)} + e^{-2r}\right\} + \sum_{m=1}^{N-2}\sum_{n=1}^{N-2}\frac{(-1)^{(m+n)}}{mn}\binom{N-2}{m}\binom{N-2}{n} \times \left\{e^{-2r} - \frac{e^{-r(2+n)}}{(1+n)} - e^{-r(2+m)} + \frac{e^{-r(2+m+n)}}{(1+n)}\right\}\right]. \quad (9)$$

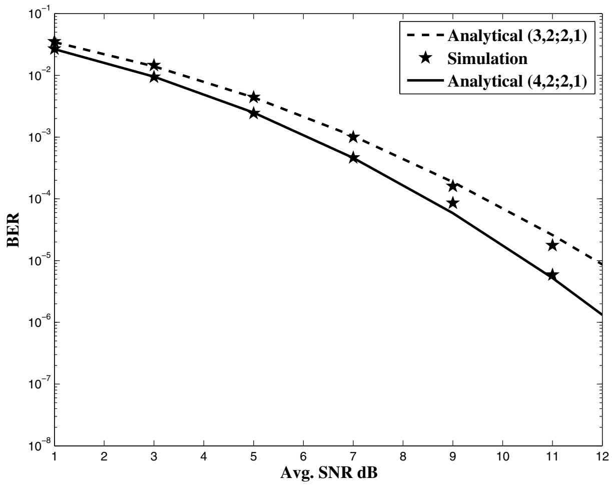

Fig. 2. BER Vs Avg. SNR

Finally, since $\gamma = \|\mathbf{h}\|^2\gamma_c$, the pdf $p_\gamma(\gamma)$ can be represented as a function of random variable as

$$p_\gamma(\gamma) = \frac{p_{R_q}(\mathbf{h} = \frac{\gamma}{\gamma_c})}{\frac{d\mathbf{h}}{dR_q}}$$
$$= \frac{\alpha(N-1)}{\gamma_c}\left[\frac{\gamma}{\gamma_c}e^{-\frac{\gamma}{\gamma_c}} + \sum_{k=1}^{N-2}\frac{(-1)^k}{k}\binom{N-2}{k} \times \left\{e^{-\frac{\gamma}{\gamma_c}} - e^{-\frac{\gamma(1+k)}{\gamma_c}}\right\}\right] - \frac{\beta(N-1)^2}{\gamma_c} \times \left[\frac{\gamma^2}{\gamma_c^2}e^{-\frac{2\gamma}{\gamma_c}} + \frac{\gamma}{\gamma_c}e^{-\frac{2\gamma}{\gamma_c}} + \sum_{k=1}^{N-2}\frac{(-1)^k}{k} \times \binom{N-2}{k}\left\{\frac{2\gamma}{\gamma_c}e^{-\frac{2\gamma}{\gamma_c}} - \frac{\gamma(2+k)}{\gamma_c(1+k)}e^{-\frac{\gamma(2+k)}{\gamma_c}} - e^{-\frac{\gamma(2+k)}{\gamma_c}} + e^{-\frac{2\gamma}{\gamma_c}}\right\} + \sum_{m=1}^{N-2}\sum_{n=1}^{N-2}\frac{(-1)^{(m+n)}}{mn} \times \binom{N-2}{m}\binom{N-2}{n}\left\{e^{-\frac{2\gamma}{\gamma_c}} - \frac{e^{-\frac{\gamma(2+n)}{\gamma_c}}}{(1+n)} - e^{-\frac{\gamma(2+m)}{\gamma_c}} + \frac{e^{-\frac{\gamma(2+m+n)}{\gamma_c}}}{(1+n)}\right\}\right]. \quad (10)$$

A. Bit Error Probability

Now, the BER for BPSK constellation can be derived as [15]

$$P_e = \int_{\gamma=0}^{\infty} P_e(\varepsilon|\gamma) p_\gamma(\gamma) d\gamma \quad (11)$$

where probability of error $P_e(\varepsilon|\gamma)$ is given by the Gaussian tail function

$$P_e(\varepsilon|\gamma) = Q(\sqrt{2\gamma}) = \int_{x=\sqrt{2\gamma}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx. \quad (12)$$

Hence, P_e can be derived by substituting (10) and (12) in (11)

$$P_e = \frac{\alpha(N-1)}{\gamma_c}\Bigg[-\frac{\sigma_0^3}{4} - \frac{\sigma_0\gamma_c}{2} + \frac{\gamma_c}{2} + \sum_{k=1}^{N-2}\frac{(-1)^k}{k} \times \binom{N-2}{k}\Bigg\{-\frac{\sigma_0\gamma_c}{2} - \frac{\sigma_{0k}\gamma_c}{2(1+k)} + \frac{\gamma_c}{2} - \frac{\gamma_c}{2(1+k)}\Bigg\}\Bigg] - \frac{\beta(N-1)^2}{\gamma_c}\Bigg[-\frac{3\sigma_{00}^5}{16\gamma_c} - \frac{\sigma_{00}^3}{4} - \frac{\sigma_{00}\gamma_c}{4} + \frac{\gamma_c}{4} + \sum_{k=1}^{N-2}\frac{(-1)^k}{k}\binom{N-2}{k} \times\Bigg\{-\frac{\sigma_{00}^3}{4} - \frac{\sigma_{00}\gamma_c}{2} + \frac{\gamma_c}{2} + \frac{\sigma_{0k}^3}{4(1+k)} - \frac{\gamma_c(1-\sigma_{0k})}{2(1+k)}\Bigg\} + \sum_{m=1}^{N-2}\sum_{n=1}^{N-2}\frac{(-1)^{(m+n)}}{mn} \times\binom{N-2}{m}\binom{N-2}{n}\Bigg\{\frac{\gamma_c(1-\sigma_{00})}{4} - \frac{\gamma_c(1-\sigma_{0n})}{2(1+n)(2+n)} + \frac{\gamma_c(1-\sigma_{mn})}{2(1+n)(2+m+n)} - \frac{\gamma_c(1-\sigma_{0m})}{2(2+m)}\Bigg\}\Bigg], \quad (13)$$

where

$$\sigma_i = \sqrt{\frac{\gamma_c}{i+1+\gamma_c}},$$
$$\sigma_{ij} = \sqrt{\frac{\gamma_c}{i+j+2+\gamma_c}}.$$

B. Special Cases

1) Case 1*:* For $N=2$, equation (13) corresponds to $(2,2;2,1)$ system. This equation can be further reduced to

$$P_e = \frac{1}{2} - \sigma_0 - \frac{\sigma_0^3}{2\gamma_c} + \frac{3\sigma_{00}^5}{8\gamma_c^2} + \frac{\sigma_{00}^3}{2\gamma_c} + \frac{\sigma_{00}}{2}. \quad (14)$$

where

$$\sigma_0 = \sqrt{\frac{\gamma_c}{1+\gamma_c}},$$
$$\sigma_{00} = \sqrt{\frac{\gamma_c}{2+\gamma_c}}.$$

Equation (14) is same as equation (53) in [6] for $N=4$. In [6], $(N,2;1)$ subset selection scheme (Scheme 3) is considered where transmit antennas are divided into subsets consisting of two adjacent antennas.

2) Case 2*:* Substituting $\alpha=1$ and $\beta=0$ in equations (10) and (13) we get the pdf of γ and BER respectively for the $(N,2;1)$ sub-optimum scheme given in [9].

IV. Results

In this section, we present simulation results of the considered system in quasi-static Rayleigh fading channel. We denote the system as $(N,2;2,1)$, which indicates that two transmit antennas are selected from N antennas and one receive antenna is selected from two receive antennas. We compare the simulation results with the analytical results derived in Section III. We also compare the considered scheme with prevailing schemes in terms of BER and feedback bit requirement. In all the figures average SNR is E_s/N_0 in dB.

Fig. 2 shows BER for $N=3$ and 4. It can be seen that simulation results are closely matching with their analytical counter-parts. In Fig. 3, we compare the performance of our $(N,2;2,1)$ Sub-Optimum JTRAS system with $(N,2;1)$ Optimum TAS scheme [10] and $(N,2;1)$ Sub-Optimum TAS scheme [9]. In $(N,2;1)$ Optimum TAS scheme, two best antennas are selected out of N transmit antennas. In $(N,2;1)$ Sub-Optimum TAS scheme, out of N transmit antennas one antenna is fixed and the other is selected out of the remaining $N-1$ antennas. It is observed that the performance of $(3,2;2,1)$ scheme is better than the performance of $(N,2;1)$ Optimum and Sub-Optimum TAS scheme with $N=3$ and 4. The reason is that, the selection complexity increases with two receive antennas, which results in selection of better antennas. In Table I, we have shown number of feedback bits required in different antenna selection schemes. The first two schemes are $(N,2;1)$, whereas remaining two are $(N,2;2,1)$. It can be seen that difference of number of bits is increasing between optimum $(N,2;2,1)$ and sub-optimum $(N,2;2,1)$ schemes with increasing N. We have seen in Fig. 3 that the considered sub-optimum $(N,2;2;1)$ scheme outperforms $(N,2;1)$ scheme, however number of feedback bits in the considered $(N,2;2,1)$ is almost half of the feedback bits in the optimum $(N,2;1)$.

Furthermore, the considered scheme requires same number of feedback bits as $(N,2;1)$ Sub-Optimum TAS scheme in [9], however the BER performance of $(N,2;2,1)$ is better.

In Fig. 4, we compare the performance of the considered $(N,2;2,1)$ Sub-Optimum JTRAS system with $(N,2;2,1)$ Optimum JTRAS scheme in [11]. It can be seen that, compared to the optimum scheme, the performance of the considered scheme suffers an SNR loss of only 0.2 dB and 0.5 dB for $N=3$ and $N=4$ respectively, however number of feedback bits is reduced substantivally in the considered $(N,2;2,1)$ Sub-Optimum scheme as shown in Table I.

V. Conclusion

We have considered a joint transmit and receive antenna selection (JTRAS) scheme in a special case of MIMO systems, equipped with N transmit antennas and

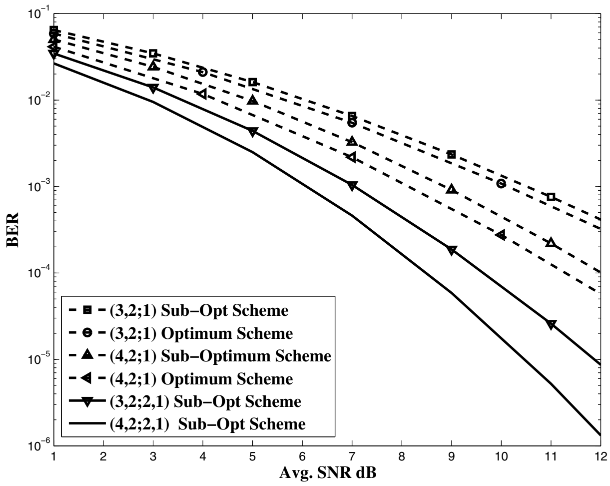

Fig. 3. BER Vs Avg. SNR

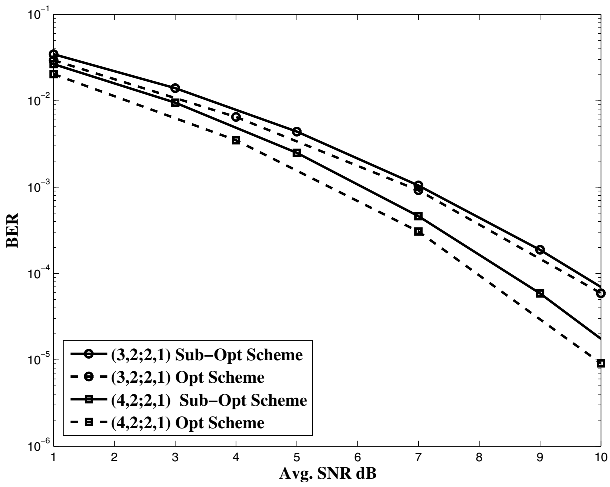

Fig. 4. BER Vs Avg. SNR

two receive antennas. At the transmitter, we keep one antenna as fixed and select the best among the remaining $N-1$ antennas. At the receiver, we select the best antenna out of two antennas. The pdf of SNR is derived for the considered $(N,2;2,1)$ scheme using which we obtain the closed-form expression of BER for Alamouti transmit diversity. We compare the analytical results with simulations and find a close matching between them. We have also compared performance of the considered sub-optimum system with optimum scheme. We conclude that the considered JTRAS scheme requires less number of feedback bits.

TABLE I
NUMBER OF FEEDBACK BITS REQUIREMENT

N	**3**	**4**	**5**	**6**	**9**	**10**
$(N,2;1)$ **Opt**	4	4	6	6	8	8
$(N,2;1)$ **Sub-Opt**	1	2	2	3	3	4
$(N,2;2,1)$ **Opt**	4	4	6	6	8	8
$(N,2;2,1)$ **Sub-Opt**	1	2	2	3	3	4

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