A Simplified method for analysis and design of free standing stairs

By

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DEPARTMENT OF CIVIL ENGINEERING

INSTITUTE OF TECHNOLOGY

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Declaration

This is to certify that

i) The thesis comprises of my original work towards the Degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) at Nirma University and has not been submitted elsewhere for a degree.

ii) Due acknowledgement has been made in the text to all other material used.

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Certificate

This is to certify that the Major Project Report entitled **"A Simplified method for analysis and design of free standing stairs "** submitted by **Mr. Purohit Kishan V (Roll No: 11MCLC15)** towards the partial fulfillment of the requirements of Master of Technology (CIVIL Engineering) in the field of Computer aided structural analysis and design of Nirma University is the record of work carried out by him under our supervision and guidance.The work submitted has in our opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other University or Institution for award of any degree or diploma.

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Abstract

Straight free standing staircases have come into fairly wide use now-a-days. Its attractiveness coupled with the elimination of obstructing columns under the landing which enables designers to maximize the utilization of the floor area have encouraged its use in many modern buildings. Its use, however, has been restricted due to the lack of an adequate and simple method of analysis.

Fuchssteiner has proposed the simplification of the basic structure by substituting a space frame composed of linear bar elements, the frame consisting of two cantilevered joined by a bow girder. Analysis of free standing stair has also been studied by A.C. Liebenberg, A.R. Cusens & J.G.Kuang, Sauter & A. Siev, et al.

Analysis of free standing stair requires long & complex equations. The aim of present study is to develop a simplified method for analysis of free standing stair. Normally, free standing stair is an indeterminate to the six degree but in this report a simplification such as symmetrical loading condition is made in such a way that it reduces to two degree of freedom & also it is further assumed that torsional modulus by neglecting poisson's ratio of the material & Ix/Iy(Ratio of moment of inertia about X axis & Y axis) is neglected due to stair slab width is very wide as compare to its thickness based on above assumption a stair with equal flight under symmetrical loading, displacement are calculated using virtual work method.

Analysis and design of free standing stair is carried out using simplified method & A.Siev, W. Fuchssteiner and A.R. Cusens-J.G. Kuang. The comparison of simplified method and above methods is also presented.

Design example is presented & structural detailing is provided.

Acknowlegement

I would like to thank my guide **Prof.Krutik Trivedi**,whose keen interest and knowledge base helped me to carry out the major project work.His constant support and guidance during my project work equipped me with a great understanding of different aspects of the project work. He has shown keen interest in this work right from beginning and has been a great motivating factor in outlining the flow of my work.I am also thankful to **Shri Himat Solanki**,Department of Civil Engineering whose keen interest and excellent knowledge base helped me to carry out the dissertation work.His constant support and interest in the subject equipped me with a great understanding of different aspects of the required architecture for the project work.he has shown keen interest in this dissertation work right from beginning and has been a great motivating factor in outlining the flow of my work.

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-Purohit Kishan V

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Nomenclature

b..Width of the flight

t_f..Thickness of the flight

t_L..Thickness of the landing

L..Length of the flight

g..Width of the landing

h..Height of the flight

α..Angle of the flight

ψ..Horizontal angle of the landing

c..Spacing between two flights

f_{ck}..Characterisic cube compressive strength of the concrete

f_y..Characteristic strength of the steel

E..Modulus of elasticity in bending

G..Modulus of elasticity in shear

I_X..Moment of inertia of landing in horizontal axis

I_Y..Moment of inertia of landing in vertical axis

M_r..Bending moment about X axis

M_s..Bending moment about Y axis

M_t..Bending moment about Z axis

X_5..Bending moment redundant

X_6..Shearing force redundant

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Chapter 1

Introduction

1.1 General

A Stair is a set of steps leading from one floor to the other. It is provided to afford the means of ascent & descent between various floors of a building. The room or enclosure of the building, in which the stair is located, is known as staircase. The opening or space occupied by the stair is known as a stairway.

Stairs must be provided in building having more than one floor level, even if adequate numbers of elevators are provided between floor levels. Stairs consists of risers, treads & landing. Normally risers & treads are constructed on a waist slab. The riser and tread dimensions are kept such that an easy and comfortable access to a floor level is maintained.

The normal dimensions of the riser and tread in a building are related by some empirical rules and are governed by the building codes. For example:

$$Tread + Riser = 450mm \tag{1.1}$$

$$2 * Riser + Tread = 635mm \tag{1.2}$$

$$Riser * Tread = 50000mm^2 \tag{1.3}$$

The finishing on the stairs varies from building to building and its type of material used. The stair must be designed for a minimum live load governed by the building codes.

1.2 History

The stairs are one of the oldest buildings in architectural history, they have always played a central role in the history of humanity, although it is difficult to tell exactly in which year they were born, it is believed his appearance was by the year 6000 before Christ. The stairs seems to change shape with the change of architectural eras, reflecting the trends used in different ages and revealing the talent of those who designed them.The 1st stair in the history was wood trunk stair **Fig.1.1**, these kind of stairs were used to acquire strategic positions for survival.In a basic sense, the 1st use which was given to the stairs was to overcome the difficulties presented by the terrain, such as valleys or mountains **Fig.1.2**. In the history of the stairs they

Figure 1.1: Wood trunk stair

1st emerged as a solution to a problem, although, years later it was found in China the 1st granite staircase leading to the sacred mountain in Tai Shan.This indicates that one of the utilities that was given to the stairs in his story was for religious purposes. Confucius in one of his stories said to have gone up this ladder to the top

Figure 1.2: Stairs in mountain

in the year 55 BC. The ladder was used in a metaphoric way reach the divine height and establish a connection between earth and sky. Other examples of stairs built for religious purposes are: the biblical Jacob's ladder, the tower of Babel, which was a helical tower, the pyramids of Egypt that had stairs, the celestial ladder of Shantung in China, the stairs in India, a peculiarity of the stairs in India is that they had also scientific utility. All these stairs have something in common, they symbolize the rise of the light, the sun, and a way in to the gods path. Later in the history of stairs, spiral stairs were used in clastles for military reasons, the proliferation of spiral stairs in castles was not casual, they allow a strategic position to the soldier who defended the castle, these spiral staircases and railings were built in order to make the solder placed in top an advantage, this soldier would have his right hand full of space to move his sword, while the woldier placed on the bottom would constantly hit the wall whily fighting, because he would have blocked part of the range of motion of his right, besides, his head would be easy to reach for his opponent, the lack of handrails was not casual, the aim was to push the opponent over the edge of the stair.

1.3 Stairs in present

The end of the nineteenth century is regarded by many as the golden era of construction of stairs; Peter Nicholson developed a mathematical system for stairs and railings approaching the art of the stairs to the workers of wood and metal. By the end of 1980 Eva Jiricna in London started designing stairs in glass and stainless steel which gave the stairs a sleek and futuristic look. Today it is increasingly common to exit the conventional design of iron and wood and move on to different materials such as stainless steel, glass and titanium.

1.4 Common types of stairs

There are different type of stairs which depends on the type and function of the building. The following are most common types of stairs used in the buildings.

1.4.1 Straight stairs

Straight stairs runs straight between the two floors. It is used for small houses where there are restrictions in available width. The stair may consist of either one single flight or more than one flight with a landing.

The structural behaviour of a flight of stairs is similar to that of a one-way slab supporting at both ends. The thickness of the slab is referred as the waist (slab) in **Fig.1.3**. When the flight of stairs contains landing, it may be more economical to provide beams at B and C between landing as shown in **Fig.1.4**.

1.4.2 Two flight stairs

A two flight stairs with an intermediate landing is the most common type of stairs as shown in **Fig.1.6** When the flight span including landing are longer, central beams may be considered to support the flight slabs which may be cantilever out on either side.In some buildings, stairs can be built in double flight between floors. This type

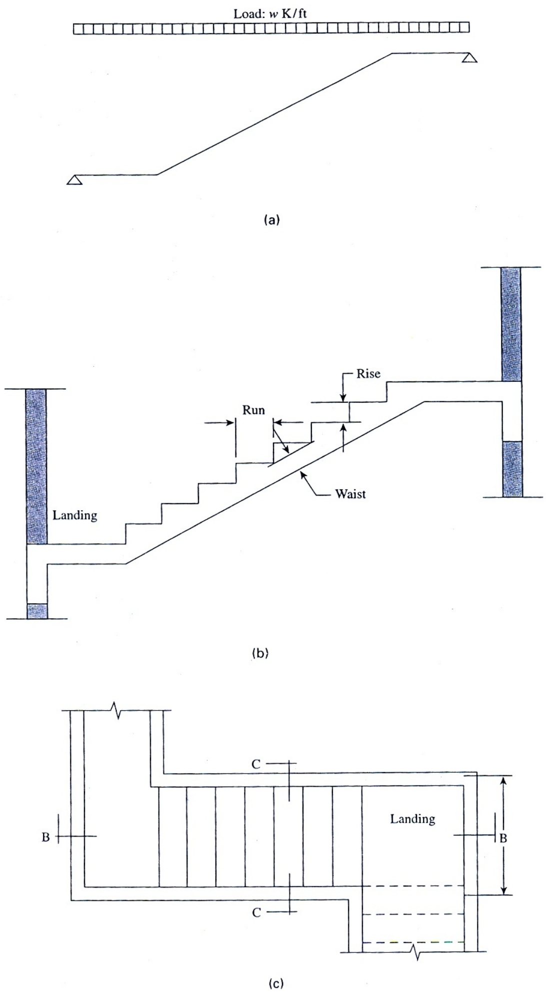

Figure 1.3: (a)Loads(b)Section at B-B(c)Plan

of stairs commonly used are quarter turn and closed or open well stair as shown in **Fig.1.5** The structural behavior of each flight of stair is designed as a one way slab supported at both ends.

1.4.3 Three or more flight stairs

Three or more flight stairs are used where the overall dimensions of the staircase are limited. In **Fig.1.7**,two long flights of span as a main flight and short flight of span as an intermediate flight can be considered. The intermediate short flight of span

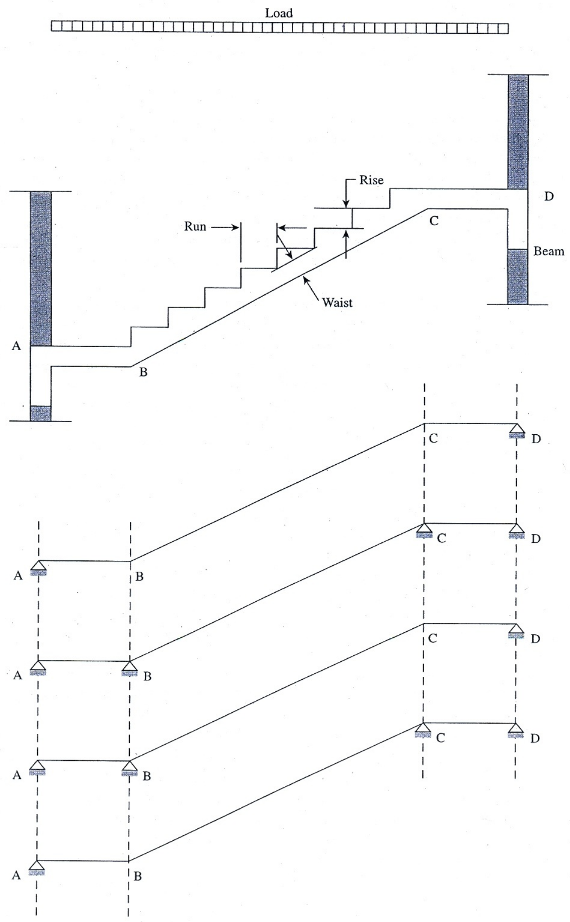

Figure 1.4: Supporting system of single flight

could be considered as a suspended flight supported on two main flights. In fig.1.7b , each flight may be considered as a simply supported one with flight span and half the landing loading in each direction. When the staircase area is rectangular in plan, long flights can be considered as main flights which support the short flight and is being regarded as suspended flight.

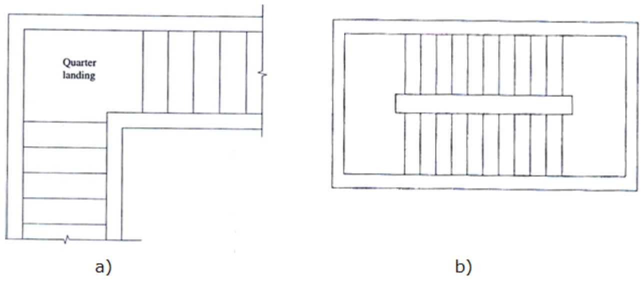

Figure 1.5: a) Quarter turn stair b) Closed Well Stair

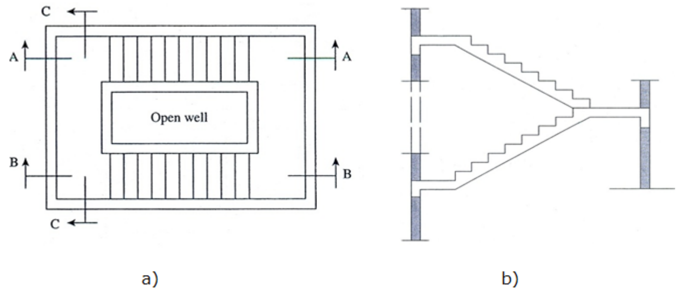

Figure 1.6: (a)Open Well Stair (b) Section at B-B

1.4.4 Cantilever stairs

Cantilever stairs are commonly used in fire escape stairs and they are supported by either concrete walls or beams. The tread and riser can be of the full flight type cantilever from one side of the wall since each step acts as a cantilever, the main reinforcement is placed in the tension side of the tread and rebars anchored within the concrete wall. Shrinkage and temperature reinforcement is provided in the transverse direction as shown in below **Fig.1.8**.

The cantilever stair can be opened riser steps supported by a central beam. The central beam has a similar function as flight stairs and receives the steps on its horizontal prepared portion. Normally, in this type of stairs precast steps are used with special provision of anchorage that fix the steps into the beam.

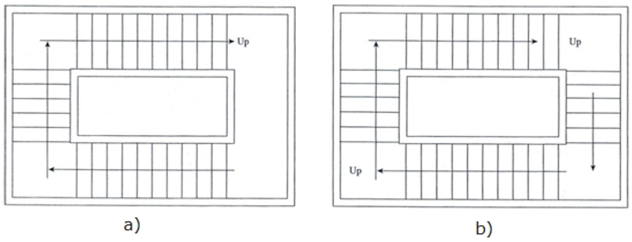

Figure 1.7: (a) Three flight (b) Four flight

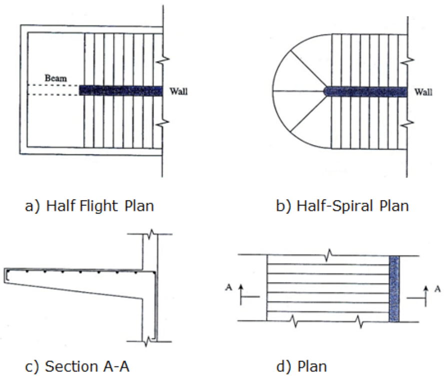

Figure 1.8: Steps projecting from one or two sides of the supporting wall

1.4.5 Precast flights of stairs

To accelerate the construction,use of precast flight of stair is beneficial.The flight can be cast separately and then fixed to cast-in-place landings or both flight and landing are cast and then placed in a position on their supporting walls or beams as shown in below **Fig.1.9**.They are designed as a simply supported one way slabs or beams with main reinforcement at the bottom of the waist slab or beam. Adequate reinforcement is required at the joints. Provision must be made for lifting and handling of stair units such as slab or beam, steps etc.

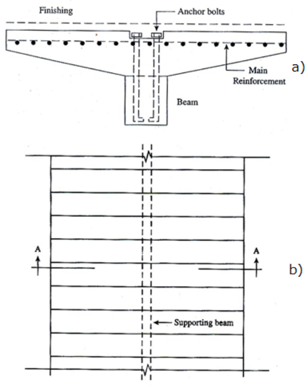

Figure 1.9: Precast cantilever stair supported by central beam (a)section A-A (b) Plan

1.4.6 slabless stairs

Stairs can be made slabless as shown in below **Fig.1.111.10**.In this stair waist slab can be eliminated and treads and risers are rigidly connected. This type of stairs has elegant appearance and is many times favored by the architects. The structural analysis of slab less stairs can be simplified by assuming the effect of axial force is negligible and can be neglected and that the load on each tread is by Cusens(1966)[3] [4], Saenz and Martin(1961),Benjamin (1962), Fuchststeiner(1965)[1] and Koseoglu(1980).

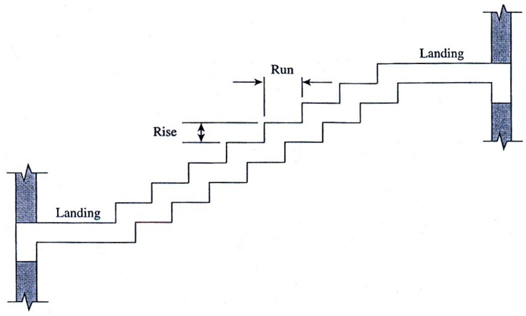

Figure 1.10: Cross Section of Run riser staircase

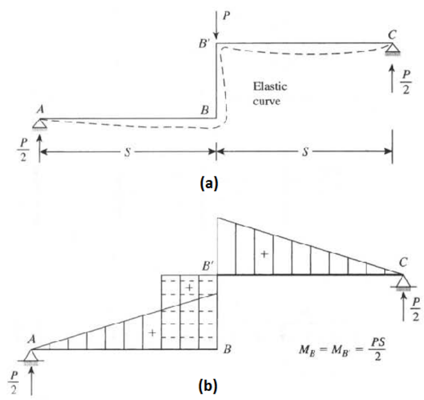

Figure 1.11: (a) Elastic Curve (b) Bending Moment Diagram

1.4.7 Free standing stairs

Free standing stairs has an elegance appearance. It is similar to a two flight common flight stairs with its landing remaining completely unsupported as shown in below **Fig.1.12** The stairs behave in a springboard manner. This type of stair is a triangular space structure and for safe analysis, its both ends can be assumed fixed. The structural analysis of this type of stairs has been given by A.R.Cusens and J.W.Kuang (1965,1966)[3][4],Sauter (1961)[7], Fuchssteiner (1953,1965)[1].

1.4.8 Helicoidal stairs

Helicoidal stairs as shown in **Fig.1.13**are elegant access way to floor and it is provided in prestigious building. The stair can be circular or elliptical in plan. The stair can be supported at same edges within the adjacent wall or it can be designed as a free standing(columnless) helicoidal staircase. The structural analysis of helicoidal staircase is complicated and discussed by A.R.cusens and Trirojna (1964), Cusens and Santathadaporn (1966) and Fuchssteiner (1953,1955)[1].

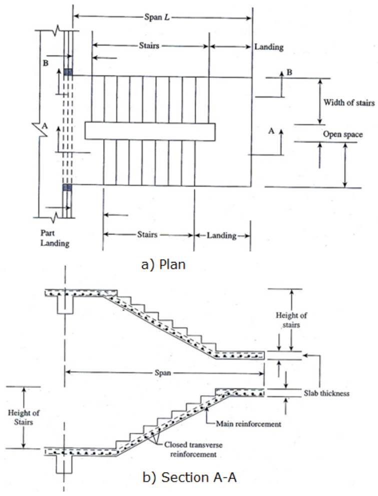

Figure 1.12: (a) Plan of free standing staircase (b) Section of free standing staircase

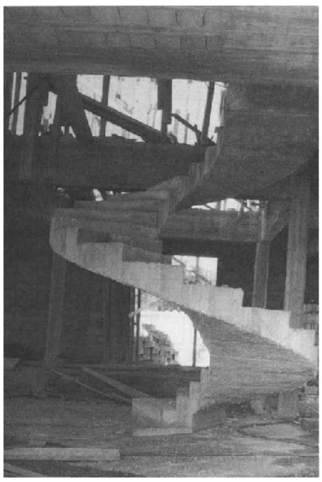

Figure 1.13: Helical Stair

1.5 Objective of study

- To study the Fuchssteiner's method, Cusens and Kuang's method and siev's method for analysis free standing stair
- To Simplify method by considering virtual work method for scissor, horse type and L-shape type of free standing stair
- To elaborate the simplify method for analysis of free standing stair
- Compare Fuchssteiner's method and simplified method for analysis of free standing stair
- To compare the results of simplified method with finite element analysis
- To design and detail of different type of free standing stair

1.6 Scope of work

- Study the Fuchssteiner's, Siev's and Cusens and Kuang's method for analysis of scissor type of free standing stair
- Develop the simplify method by considering virtual work method for different types of free standing stairs
- Evaluate methods and compare the results (Bending moment and Torsional moment) with results of simplified method for free standing stair
- Study the scissor type of stair with finite element analysis and compare the results with the simplified method
- Design and detailing for free standing stair

1.7 Organization of Major project

Chapter 2 covers the literature review from research paper and books. It gives overall idea about method of analysis.
Chapter 3 includes analysis of free standing stairs based on Siev's method,Cusens and Kuang's method,Fuchssteiner's method and Simplified method based on Fuchssteiner's assumptions and analysis example based on this method.It also covers analysis of Horse shoe type of stair and L shape type of stairs based on Fuchssteiner's method and find out the vertical,lateral and torsional moment. It also includes the analysis result of free standing stairs using finite element analysis.
Chapter 4 presents the analysis of free standing stairs using different types of support conditions using Gould's method.
Chapter 5 presents the design of free standing stairs based on analysis by simplified method.It also includes the design of Horse shoe type of stairs and L shape type of stairs.It also includes detail drawing.
Chapter 6 summarizes the work done in the major project. It consists summary of work done, various conclusions obtained from the study and future scope of work.

Chapter 2

Literature review

2.1 General

Recently, free standing staircase has been constructed, supported only at the top and bottom. Although they are dog legged in plan projection, in elevation their description is Scissor type. Various analyses are available to solve such a complicated problem. From each analysis torsional moment, bending moment is resulted. The geometry of each free standing staircase affects the application of load and hence the results. This subject has been thoroughly reviewed in depth by various researchers.

2.2 Literature review

Various papers have been referred for Analysis of free standing stairs. Some of the important Papers and books are summarized below.

W.Fuchssteiner[1] was first who has developed the method of analyzing the statically indeterminate staircase formed by a series of bar elements. He has considered free standing basic staircase as a rigid space frame. He has considered flights as sloping cantilever beams & landing as a semicircular bow girder. In his type of staircase, due to geometrical and loading symmetry, he has cut the whole frame at the mid point of landing into two equal halves, which will be act as a two separate cantilever beams. Then from the cantilever structure, the two unknown redundant can be found out. i.e., bending moment Mo and shear force H acting along at the cut section.

Avinadav Siev[2] has analyzed the free standing stair as similar to folded plate analysis. He has extended the theory to include the determination of the secondary stresses resulting from the compatibility condition at the intersection between the flights and the landing. He has concluded that the torsional moments were usually small and may be considered as secondary stresses; for most practical purposes it was sufficient to compute primary stresses.

A.R. Cusens and **J.W. Kuang** [3][4] has also developed method for analyze the basic free standing staircase as rigid space frame. They have analyzed staircase by reducing plates to beam elements. Thus the stair will be in the form of a space frame consisting of beams located in a position coincident with their longitudinal axes. They have used strain energy method for analyzing the staircase.

Phillip L Gould[5] has analyzed the staircase by considering torsional moment at the intermediate landing and the support condition of the upper leg.

A.C. Liebenberg[6] has introduced the concept of the space interaction of plates of basic staircase. He has made an analysis for an indeterminate structure. In his analysis, torsional moment was very small so he made an assumption to neglect the effect of torsional moment.

Franz Sauter[7] has analyzed basic staircase based on the method of Fuchssteiner's theory. He has converted the stair structure into a space frame composed of linear bar elements. He has determined the deformations from the work integral with the application of the principle of least work and redundant are determined by solving elastic equations.

Nazih J Taleb[8] has described for calculating the six bending moments and reactions at each support of reinforced concrete stairs comprising two flights and an unsupported indeterminate landing. His method was based on the principle of least work.

Chapter 3

Free standing stairs

3.1 General

Free standing stairs without a landing support are attractive, structurally and create architectural effects. The elimination of columns under the landing frequently has both structurally and aesthetic advantages. Lack of an adequate and simple method of analysis has restricted their use and has hindered architects and engineers from adopting more widely this impressive stair design. Methods of analyzing a concrete cantilever staircase comprising two straight flights and a landing and supported only on the upper and the lower floors are presented herein.

Theoretical analyses have been published by W.Fuchsteiner, G.Szabo, A.Siev, A.C. Liebenberg[6], P.L. Gould[5], A.R. Cusens and J.G. Kuang[3][4] and F. Sauter[7].

Siev has extended Liebenberg's theory to include the determination of the torsional restraining moment resulting from the compatibility of deformations between the flights and the landing. The restraining moment is usually small and may be considered as a secondary effect.

Cusens and Kuang's method is based on the strain energy method with the assumption that the flight plates can be reduced to bar elements which coincide with their longitudinal axes. The landing bar element will be a straight line to be located in a

position near the line of intersection.
W.Fuchsteiner is the 1st person who suggested the space bar method which is similar to that of Cusens and Kuang with the only difference being that the landing is replaced by a curved bar instead of a straight one.
This chapter also includes the analysis results of free standing stairs using SAP 2000 software.

3.2 Siev's analytical method

3.2.1 Introduction

Free straight multi flight as shown in **Fig.3.1** stairs without landing support are attractive. The elimination of columns under the landing frequently has both structural and aesthetic advantages.Siev approaches the problem of the free straight multi flight staircase in a procedure similar to that of a folded plate analysis. The stress analysis for this structure will be accomplished in stages and only the case of a symmetrical loading is considered.

3.2.2 Statically determinate structure

As shown in **Fig.3.1**, free standing staircase is considered as a truss system in which torsional rigidity is not considered.Elements A,C,E; B,D,N; J,G,M; K,H,F; C,D,G,H; each represent a single part, all connections being pin joints. It is imperative that these elements be rigid in the vertical planes.
Another element, a diagonal A,D must be added in order to supply resistance to any horizontal forces in the direction so under vertical load, the stress in this bar will be zero. Thus, the structure is statically determinate.
There are five unknown forces: X'j-g;X'k-h;X'b-d;X'a-c as shown in **Fig.3.2** and the

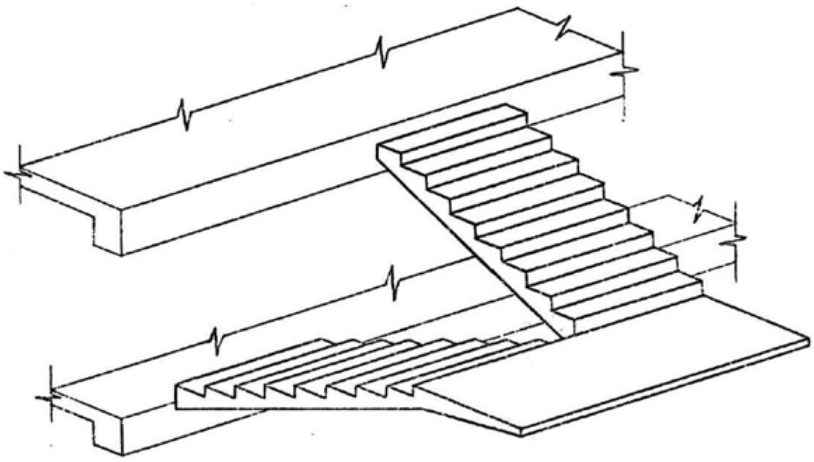

Figure 3.1: Free standing stair

force in the bar a-d. The forces are noted in accordance with the axes. The forces X and Z **Fig.3.2**[e.]and **Fig.3.2**[f], acting on the landing, are the horizontal and vertical components of X', respectively.(

$$X = X' * cos\alpha \, and \, Y = Y' * sin\alpha$$

).

On the other side, 5 equilibrium conditions for the landing must be satisfied. These conditions are as follows:

$$(1.)\Sigma Fx = 0(2.)\Sigma Mx = 0(3.)\Sigma Fy = 0(4.)\Sigma Fz = 0and(5.)\Sigma Mz = 0$$

. So, degree of externally redundancy

$$E = R - r = 5 - 5 = 0$$

. Thus the structure is statically determinate.

Where, R=Total number of reaction components and r=Total number of equations of equilibrium.

Finding the forces by writing the five equations is a tedious mathematical solution. So an easier approach will be used herein.

3.2.3 Symmetrical loading

The resultant R as shown in **Fig.3.2**[b] of the forces acting on beam C-H is at O as shown in **Fig.3.2**[d]. A vertical plane is introduced, perpendicular to line C-H through O. This plane intersects with the planes of the upper and lower flights at lines OL and OU as shown in **Fig.3.2**[a] The resultant R is now resolved into components X'_{ol} and X'_{ou}, in the direction of OL and OU, respectively as shown in **Fig.3.2**[a].

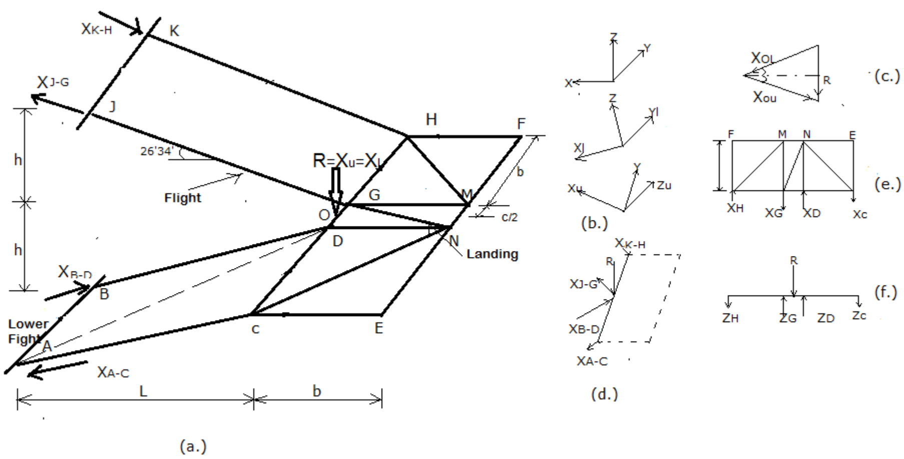

Figure 3.2: (a.)Truss system(b.)Notations of axes and forces(c.)Resultant(d.)Axonometric view of landing(e.) Equilibrium of landing in horizontal plane(f.)Equilibrium of beam C-H in vertical plane

$$X' = X'ol = X'ou = \frac{R}{2 * sin\alpha} \tag{3.1}$$

in which is the slope of a flight and the forces are noted in accordance with the axes in **Fig.3.2**[a]. The components X'_{ol} is again resolved into forces X' in bars A-C and

B-D yielding

$$X'_{AC} = \frac{X'_{OL} * c}{2 * b} \tag{3.2}$$

Substituting X'_{ol} from Eq.3.1 into Eq.3.2 yields

$$X'_{AC} = \frac{R * c}{4 * b * sin\alpha} \tag{3.3}$$

and

$$X'_{BD} = \frac{-R * (c + 2 * b)}{4 * b * sin\alpha} \tag{3.4}$$

In which the positive sign denotes tension. Similarly,

$$X'_{JG} = \frac{R * c}{4 * b * sin\alpha} \tag{3.5}$$

and

$$X'_{KH} = \frac{-R * (c + 2 * b)}{4 * b * sin\alpha} \tag{3.6}$$

An axonometric view of these and of the external forces acting on beam C-H is shown in **Fig.3.1**[d].

The forces X' are transmitted at joints A-B-C-D to the lower and upper floors, where as in joints C-D-G-H they act in the landing. Their horizontal component,

$$X = X' * cos\alpha$$

, acts on the horizontal truss C-E-F-H[Fig.3.2(e)], whereas the vertical components,

$$Z = X' * sin\alpha$$

Fig.3.2[f], act on the vertical beam C-H. **Fig.3.2**[e] and **Fig.3.1**[f] immediately show the equilibrium of the landing. Stress analysis of the beam C-H and of the truss C-E-F-H is elementary.

Remarks:-The calculation of stresses under any load may be accomplished (1.) By resolving the load into symmetrical components, using the given solutions, and superposing the results or (2.) By resolving the resultant R of the forces acting on beam C-H and then proceeding as before. The resultant R must not necessarily pass through O.

Beam C-H may be considered as a support of the cantilevered beams A-E, B-N, J-M and K-F and the moments in these beams are calculated accordingly having obtained the forces on beam C-H. The flight beams are often fixed in the vertical planes at points A-B-J-K. In this case, the indeterminate character of the structure must be considered.

3.2.4 Monolithic staircases of reinforced concrete

The stress analysis for this structure will be accomplished in stages. Initially as in the cases of hipped plates, a support is assumed at line C-H**Fig.3.2**[a] and the stairs is then analyzed as two separate slabs being fixed at one end and hinged at the intersection**Fig.3.3**. The degree of restraint at lines A-B and J-K is determined by specific conditions. The support moments of the slabs under various possible loading conditions can be easily obtained by using any classical method for solving statically indeterminate structure. This system subsequently will be referred to as a slab structure. The reactions at the imaginary supports due to external loads acting along the line of intersection of the slab structure.

The structure supporting the beam C-H will be referred to as a plate structure. The stresses in the plate structure may be considered as a superposition of two types of stresses :(1) Primary stresses caused by the determinate structure, with the only differences being that the forces X' shown in Eq.3.1, act as eccentric forces on the flight plates instead of on bars in the truss; and (2) Secondary stresses necessitated by the compatibility between stress and displacements. It may be assumed that part

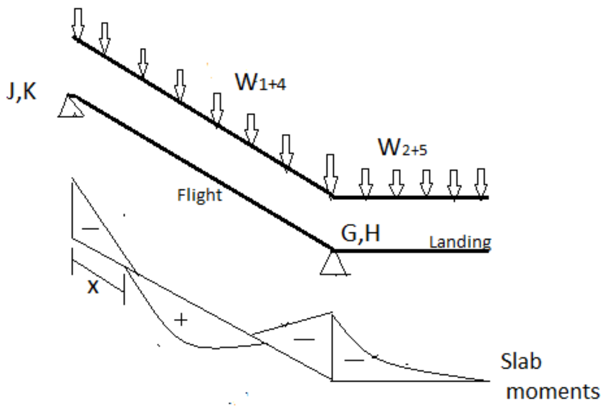

Figure 3.3: Moments due to slab action with imaginary support at C-H

of the load is resisted by the primary stresses, and another part by secondary stresses. The solution of the plate system will be as follows: First the system will be solved for primary stresses and the corresponding deformation under an arbitrary load R'. Next, the secondary moments are calculated, from which follows, in certain cases, a resistance to an additional load R". The combined resistance is

$$R = R' + R'' \tag{3.7}$$

However, as will be shown subsequently,

$$R'' \lll R'$$

and therefore

$$R \simeq R' \tag{3.8}$$

In this case, only symmetrical loading condition is given for calculating the stresses at each point but for deriving maximum stresses, superposition of symmetrical and asymmetrical loading is necessary. Moreover, the displacements of the system under each load, because of strains in the various elements, will be studied separately for the effect of each factor. Thus, strains will be considered in a single element with all others being assumed to have infinite rigidity. Thus, the flight deformation will be considered first with the landing and beam C-H **Fig.3.2**[a] being considered as having infinite rigidity, and so forth. The overall stresses and displacement are the sum of all effects.

There are 3 types of secondary moments or redundant at the junction with the lower and upper floors: Mx', Mz' and My. Mx' will be calculated by considering the effect of each factor on the differential deflections between points C and D or G and H. The same differential deflections must be obtained for beam C-H and the flights. $M_{z'}$ will be calculated by assuming that the distance between points D and G is constant. M_y will be calculated by considering the deflection of beam C-H**Fig.3.2**[a] as settlement of supports.

Symmetrical loading- Primary stresses Assume a load R' along C-D-G-H as

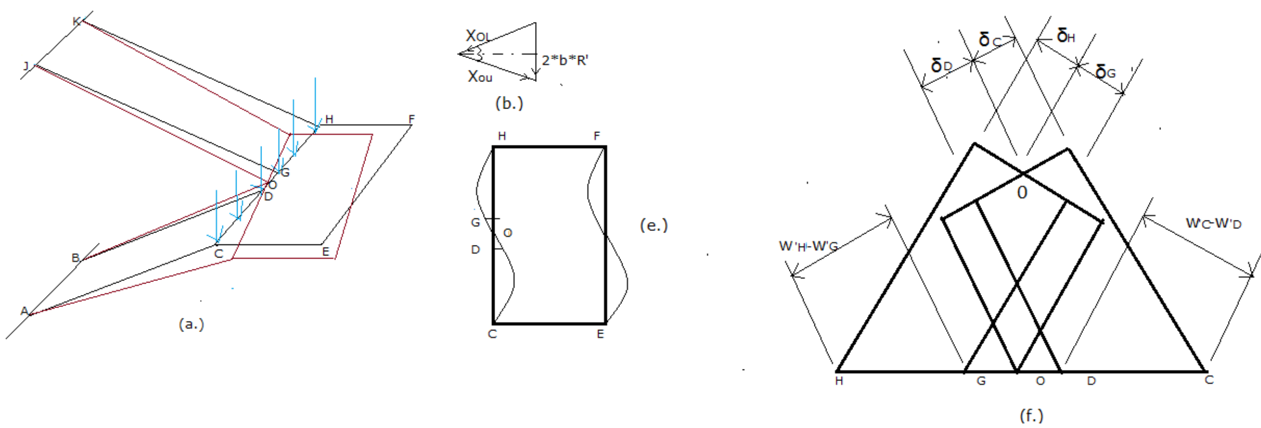

Figure 3.4: Stresses and displacements resulting from symmetrical loading due to strain in plates(a.)Stresses strains and deflections(b.)Resultant(e.)Deformation landing(f.)Williot diagram of displacements due to strains in plates

shown in **Fig.3.4**[b]. The resultant of this load, 2bR' is resolved, as previously, into two diagonal forces X' passing through point O.

$$X' = X'ol = X'ou = \frac{b * R'}{sin\alpha} \tag{3.9}$$

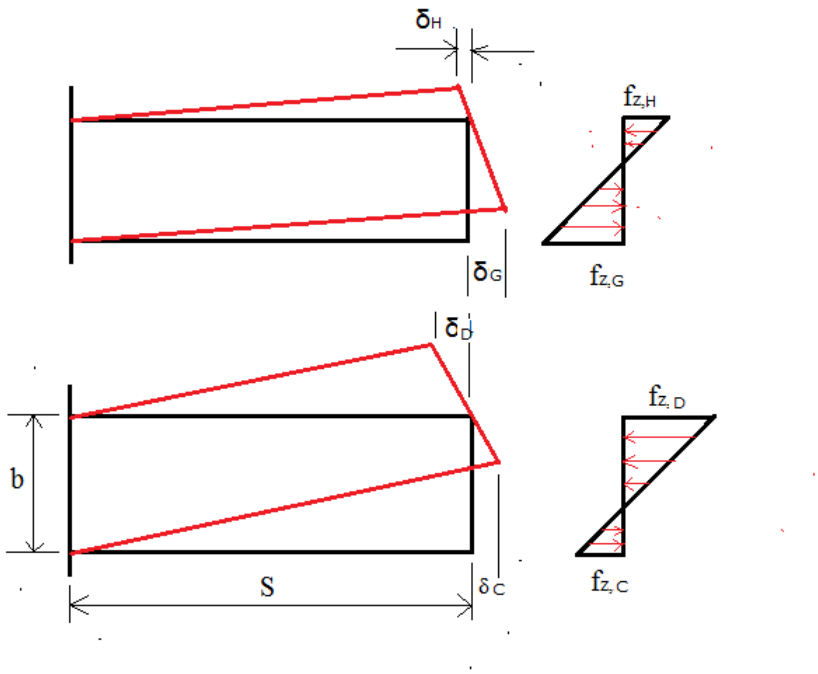

Figure 3.5: (c.)strains in upper flight(d.)strains in lower flight

Each component acts on the corresponding flight plate, through point O, as eccentric tension or compression, and the resulting stresses, f, are as shown in **Fig.3.5**[c][d], yielding

It is obivous that the upper flight is subjected to an axial force in addition to a bending moment about its own plane and the lower flight is subjected to a axial compression in addition to a bending moment to its own plane.

$$f'_{GJ} = -f'_{DB} = \frac{\overline{X}}{A} + \frac{\overline{X}*(b+c)*b}{4*Iz} = \frac{\overline{X}*(1+3*\frac{b+c}{b})}{b*t} = \frac{R'*(1+3*\frac{b+c}{b})}{t*sin\alpha} \quad (3.10)$$

$$-f'_{CA} = f'_{HK} = \frac{\overline{X}}{A} - \frac{\overline{X}*(b+c)*b}{4*Iz} = \frac{\overline{X}*(1-3*\frac{b+c}{b})}{b*t} = \frac{R'*(1-3*\frac{b+c}{b})}{t*sin\alpha} \quad (3.11)$$

In which t represents the overall depth of slab, A is the cross sectional area of the flight, and Iz' denotes the moment of inertia about the Z' axis.(The axis perpendicular to the plate surface). The resultant of the vertical components of these stresses gives the reaction on the landing C-E-F-H which can be considered as a beam(beam CH) because the slab is usually to designed that the section tapered to the end will have the centroid close to the inner edge. **Fig.3.7**.The reactions at points D and C, respectively, are

$$R'(1 + 3 * \frac{b+c}{b}) \; and \; R'(-1 + 3 * \frac{b+c}{b})$$

It is now necessary to consider the primary bending moments in beam C-H. As shown in **Fig.3.7**[b], in addition to the load R', the beam is subjected to the reactive forces from the flights. It is apparent that the resultant of these forces will pass through point O. Owing to the symmetrical forces on the beam, it is therefore possible to calculate the bending moment in beam C-H taking it as free at the ends and fixed at point O. The maximum (midspan) primary negative bending moment for beam C-H is

$$M'_O = -3 * R' * \frac{(b+c)}{b} * \frac{b}{4} * \frac{2*b}{3} = -b * R' * \frac{b+c}{2} \tag{3.12}$$

The horizontal components of $\bar{X}$ is $\bar{X}$cosα.

$$X = \overline{X} * cos\alpha = \frac{b * R' * cos\alpha}{sin\alpha} = b * R' * cot\alpha \tag{3.13}$$

Substituting X'h into Eq.3.10 and 3.11 and multiplying by t, the horizontal loads on the landing at points D and C, respectively, are

$$-t * f_D = R' * cot\alpha * (1 + 3 * \frac{b+c}{b}) \;\; and \;\; t * f_C = R' * cot\alpha * (-1 + 3 * \frac{b+c}{b})$$

Where a positive sign represents the tensile force and a negative sign the compressive force. It is seen that, from symmetry in loading, Mz and My are both equal to zero. At this stage, all primary moments have been known. Subsequently, the secondary moments will be calculated and shown to be small. Therefore, the calculations to the present stage are sufficient for most practical design use. The displacements caused by the primary stresses are produced by deformation of the flight and the landing. For simplification, the effect of shear deformation will be neglected.

Symmetrical loading- Displacement and Secondary stresses The displacement caused by the primary stresses is produced by deformation of the flights and the landing. For simplification, the effect of shear deformation will be neglected in the following calculations.

Flights ends undergo displacements in the x and y directions (directions of the longitudinal axis of the landing and each flight). From **Fig.3.4**[c][d], if the flights are equal in length, both the flight and have equal displacements in the Y direction, there will be no change in stresses in the plate system and therefore, the magnitude of the deformation is of no further interest to the discussion. It is worthwhile to note, however, the strains of the end fibers of each flight in the X direction. From Hooke's law,

$$\epsilon'_G = \epsilon'_D = \frac{f'_{GJ}}{E} \; and \; \epsilon'_C = \epsilon'_H = \frac{f'_{CA}}{E}$$

In which E is the Modulus of elasticity, hence the total elongations and contractions of the end fibers are

$$\delta'_G = -\delta'_D = \frac{f'_{GJ} * a}{E} = \frac{R' * a * (1 + 3 * \frac{b+c}{b})}{t * E * sin\alpha} \tag{3.14}$$

$$\delta'_C = -\delta'_H = \frac{f'_{CA} * a}{E} = \frac{R' * a * (-1 + 3 * \frac{b+c}{b})}{t * E * sin\alpha} \tag{3.15}$$

If the deformed lines C-D and G-H are extended to the central point, the additional extensions $\delta u, o$ and $\delta_{l,o}$ at point O can be determined by simple geometric relations, That is

$$\frac{|\delta'_H| + |\delta'_G|}{|\delta'_H| + |\delta'_{u,O}|} = \frac{b}{b + \frac{c}{2}} \tag{3.16}$$

Substituting the corresponding values of δ'_g and δ'_h in Eq. 3.16

$$\delta_{u,O} = -\delta_{l,O} = \frac{R' * a * (1 + 3(\frac{b+c}{b})^2)}{t * E * sin\alpha} \tag{3.17}$$

A williot diagram **Fig.3.6** is now drawn for the projection of all point displacements in the flights on the X-Z plane and the vertical deflection OO' of point O is found to be

$$\delta_{O,O'} = \frac{\delta_{u,O}}{sin\alpha} \tag{3.18}$$

Introducing the known value of $\delta_{u,O}$ in Eq. 3.17

$$\delta_{O,O'} = \frac{R' * a * (1 + 3(\frac{b+c}{b})^2)}{t * E * sin^2\alpha} \tag{3.19}$$

The final step in the analysis of the cantilever staircase is to calculate the torsional restraining moment by substituting all related displacements of the flights and the landing into the compatibility equation. As shown in **Fig.3.5**[c][d] the difference between the displacements of point C and point D normal to the flight is

$$W_C^I - W_D^I = (| - \delta'_D| + |\delta'_C|) * tan\alpha \tag{3.20}$$

Substituting Eq. 3.14 and 3.15 into Eq. 3.20,

$$W_C^I - W_D^I = \frac{6 * R' * a * (b + c)}{E * b * t * cos\alpha} \tag{3.21}$$

The difference between the vertical displacements of points C and D in the landing will now be considered, As previously stated, the beam C-H is subjected to the load R' and the vertical reactions from the flights; thus the beam will deflect as though it were fixed at mid span or point O and free at either end as shown in **Fig.3.6**. However, this clearly shows that the deflection of point C is greater than that of point D, the flight plate is therefore twisted and a torsional moment $M_{x'}$ is introduced therein. As a result of this effect, it can be visualized from the **Fig.3.7** beam C-H, in addition to the negative bending, is restrained at the centre by a positive moment M_x which tends to decrease the deflection of the beam shown in **Fig.3.4**[b].

Thus the difference between the vertical deflections of points C and D may be obtained as

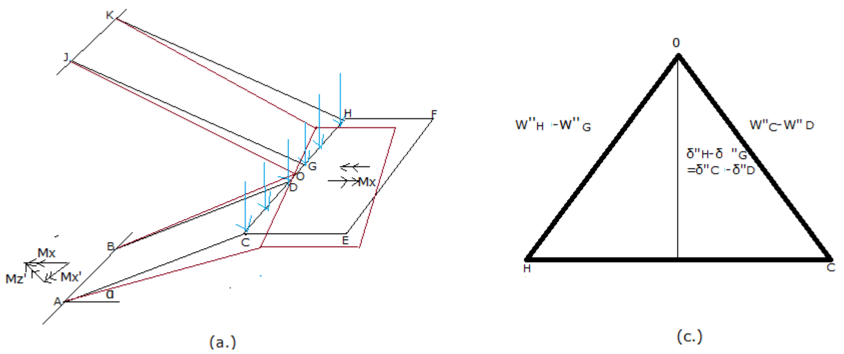

Figure 3.6: Displacement resulting from symmetrical loading due to bending of beam C-H(a.)Displacement due to bending of beam C-H(c.)Williot diagram

$$\Delta_C^{II} - \Delta_D^{II} = \frac{R'b^2 * (b + c) * (c + 0.7 * b)}{4 * E * I_b} - \frac{b * M_X * (3 * c + 2 * b)}{6 * E * I_b} \tag{3.22}$$

Where I_b is the moment of inertia of beam C-H. the relative displacement, W^{II}, in the direction normal to the flight plane **Fig.3.6**[a] is

$$W_C^{II} - W_D^{II} = \frac{\Delta_C^{II} - \Delta_D^{II}}{cos\alpha} \tag{3.23}$$

The third term of the relative displacement, W^{III}, caused by the torsional moment $M_{x'}$(component of M_x) in the flight, can be easily found by Castigliano's Theorem

$$W_C^{III} - W_D^{III} = \frac{M_{\overline{X}} * b * a}{G * J} = \frac{M_X * cos\alpha * b * a}{G * J} = \frac{M_X * b * l}{G * J} \tag{3.24}$$

In which G is the modulus of elasticity in shear, GJ is the torsional rigidity and L is the horizontal projection of the plate length.

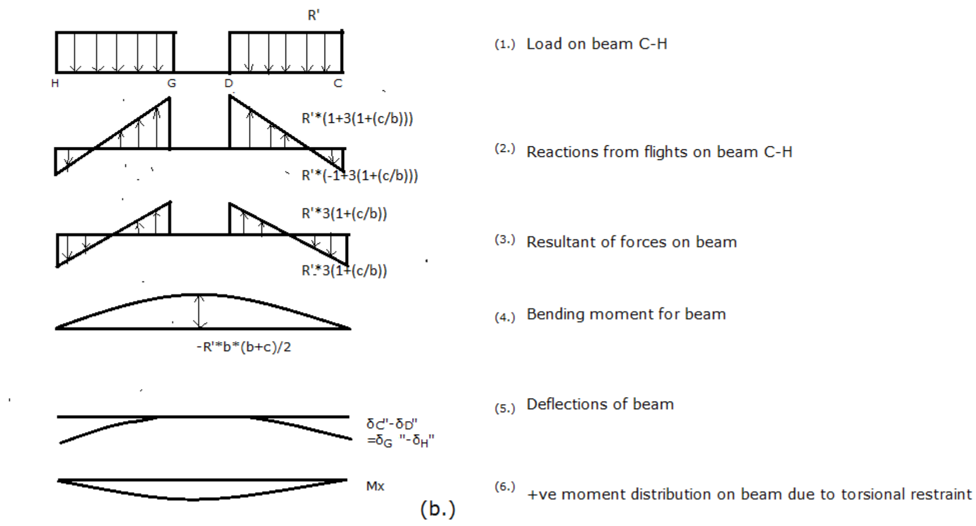

Figure 3.7: Loading and deflections of beam C-H

The last relative displacement W^{IV} in the flight plate caused by $M_{\overline{z}}$, the component of restraining moment Mx, will be obtained in a way similar to the first term, but in the opposite sense. The stresses due to $M_{\overline{z}}$ are

$$f_{CA}^{IV} = -f_{DB}^{IV} = f_{GJ}^{IV} = -f_{HK}^{IV} = -\frac{6 * M_{\overline{Z}}}{t * b^2} \tag{3.25}$$

and the total elongations and contractions of the end fibers are

$$\delta IV_{CA} = -\delta_{DB}^{IV} = \delta_{GJ}^{IV} = -\delta_{HK}^{IV} = -\frac{6 * M_{\overline{Z}}}{E * t * b^2} \tag{3.26}$$

The relative deflection W^{IV} is

$$W_C^{IV} - W_D^{IV} = (|-\delta'_D| + |\delta'_C|) * tan\alpha = -\frac{12 * M_{\overline{z}} * a * tan\alpha}{E * t * b^2} \tag{3.27}$$

but,since

$$M_{\overline{Z}} = M_X * sin\alpha \quad h = a * sin\alpha$$

Therefore,

$$W_C^{IV} - W_D^{IV} = -\frac{12 * M_X * h * tan\alpha}{E * t * b^2} \tag{3.28}$$

At this stage, all displacements in the same direction are known. All that must be done in the final step is to apply the compatibility condition which can be accounted for as follows: Along the line of intersection, the deflection of each flight and the landing, which are caused by both the primary and secondary stresses, should coincide with each other. Thus the compatibility equation will be of the form

$$W_C^I - W_D^I + W_C^{III} - W_D^{III} + W_C^{IV} - W_D^{IV} = W_C^{II} - W_D^{II} \tag{3.29}$$

As W^{II}, W^{III} and W^{IV} are represented in terms of the restraining moment M_x, solve the above equation and the moment M_x is then obtained.
The effect of the vertical deflection $\delta_{OO'}$, obtained from Eq. 3.19 is similar to that of the settlement of the supports and is governed by the specific conditions. If the flights are completely fixed at both floors, this effect may be considered by introducing an additional load R" acting on the line of intersection, producing the same amount of deflection $\delta_{OO'}$ in **Fig.3.4**[f] the cantilevered plates. Hence,

$$\delta_{OO'} = \frac{R'' * b * a^3}{3 * E * \frac{b*t^3}{12}} = \frac{4 * R'' * a^3}{E * t^3} \tag{3.30}$$

By equating Eq. 3.29 with Eq. 3.19 and rearranging,

$$R'' = \frac{R' * t^2 * (1 + 3(\frac{b+c}{b})^2)}{4 * a^2 * sin^2\alpha} \tag{3.31}$$

And the additional negative bending moment at the floor support is

$$M = R'' * b * l \tag{3.32}$$

At thickness of flight(t) is much smaller than inclined length of the flight(a), it can be concluded that the fraction t^2/a^2 in Eq. 3.30 will load R" to be only a very small portion of R'. Thus, Eq. 3.9 is a reasonably good approximation Eq. 3.8.

3.2.5 Example based on Siev's method

Fig.3.8 and**Fig.3.9** is to be displayed.

Step-1 Load calculation

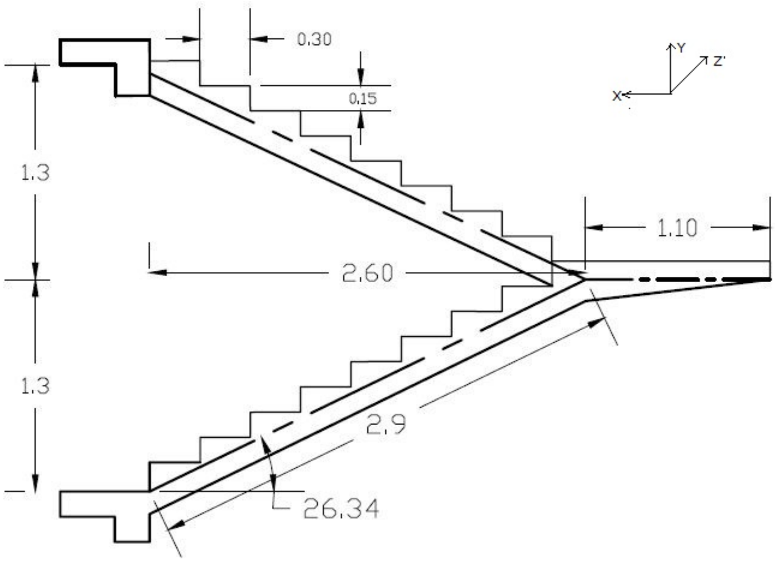

Figure 3.8: Elevation

W=Total weight of one flight

$$W = (2.6 * 0.110 * 1.2 * 25) + (8 * \frac{0.15}{2} * 0.3 * 1.2 * 25) = 13.98kN$$

Live Load=5 kN/m^2 for both flights and landing

DL of the flight =13.98/2.6 $*$ 1.2 =4.5 kN/m^2

DL of the landing =25*0.15=3.75 kN/m^2

Now, W_1=DL of the flight=4.5 kN/m^2

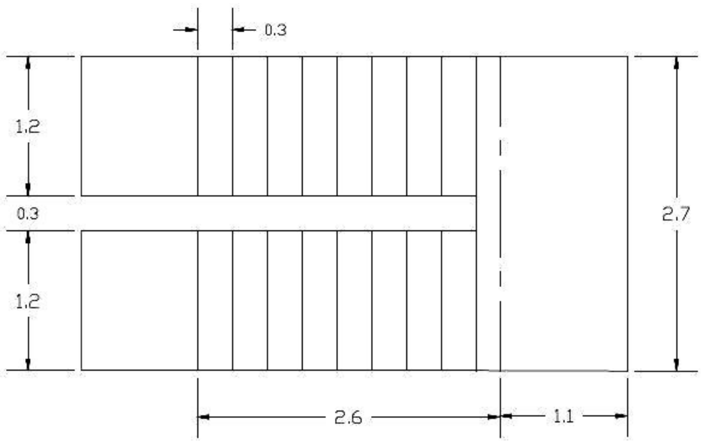

Figure 3.9: Plan

W_2=DL of the landing=3.75 kN/m^2

W_3=LL of the lower and upper flight=5 kN/m^2

W_5=LL of the landing=5 kN/m^2

W_{1+3}=DL+LL on the lower flight=9.5 kN/m^2

W_{1+4}=DL+LL on the upper flight=9.5 kN/m^2

W_{2+5}=DL+LL on the landing =8.75 kN/m^2

Step-2 M.I. of the inertia of the flights and the landing

I_1=M.I. of the inertia of the landing about horizonal axes =$(1.1 * 0.15^3)/12 = 3.1 * 10^{-4}m^4$

I_1'=M.I. of the inertia of the landing about vertical axes =$(1.1^3 * 0.15)/12 = 166.4 * 10^{-4}m^4$

I_2=M.I. of the inertia of the flight about horizonal axes =$(1.2 * 0.12^3)/12 = 1.33 * 10^{-4}m^4$

I_2'=M.I. of the inertia of the flight about vertical axes =$(0.12*1.2^3)/12 = 173*10^{-4}m^4$

Torsional rigidity of the rectangular section

Using Saint-Venant's forula, when $b/t \ggg 2.5$

GJ_1=Torsional rigidity of the landing

=$(b/3) * t^3 * (1 - 0.63 * (t/b)) * GN * m^2$

$=(1.1/3)*0.15^3*(1-0.63*(0.15/1.1))*G = 7.898GN*m^2$

GJ_2=Torsional rigidity of the flight

$=(b/3)*t^3*(1-0.63*(t/b))*G$

$=(1.2/3)*0.12^3*(1-0.63*(0.12/1.2))*G$=4.5G $N*m^2$

Step-3 Moments in slab structure

(a.)Max. cantilevered moment in the landing

$M_1 = -(W_{2+5}/2)*(b+(c/2))*g^2$

$= -(8.75/2)*(1.2+(0.3/2))*1.1^2 = -7.15kNm$

(b.)Min. cantilevered moment in the landing

$M_1 = -(W_2/2)*(b+(c/2))*g^2$

$= -(3.75/2)*(1.2+(0.3/2))*1.1^2 = -3.1kNm$

(c.)Max.-ve moment at the floor supports assuming that the flights are completely fixed

$M_3 = -(W_{1+3}/8)*(b)*l^2+(M_2/2)$

$= -(9.5/8)*(1.2)*2.6^2+(3.1/2) = -8.08kNm$

(d.)Max.-ve moment at the floor supports for full load in the landing

$M_4 = -(W_{1+3}/8)*(b)*l^2+(M_1/2)$

$= -(9.5/8)*(1.2)*2.6^2+(7.15/2) = -6.06kNm$

(e.)Max. positive moment in each flight

Fig.3.10 $x = V_1/W and M_5 = (V_1*x/2) - M_l$

1 For full load

$M_4 = M_l = 6.06kNm$

$M_r = M_1 = 7.15kNm$

$W = W_{1+3}*b = 9.5*1.2 = 11.4kNm$

$V_l = (W*l/2)+(M_l-M_r)/2$

$= (11.4*2.6/2)+(6.06-7.15)/2 = 14.4kN$

$V_r = 11.4*2.6-14.4 = 15.24kN$

$M_5 = (15.24^2/(2*11.4)) - 6.06 = 4.13kNm$

$x = V_1/W = 15.24/11.4 = 1.34m from\ left\ support.$

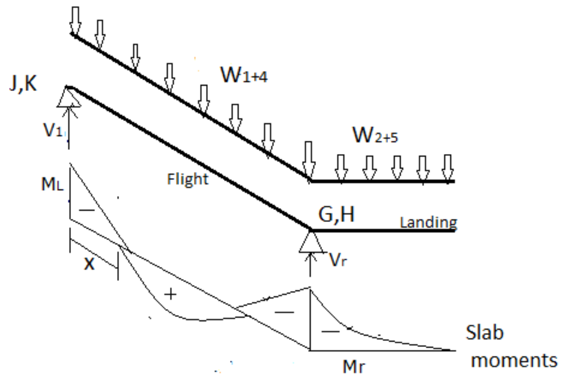

Figure 3.10: Notations for computing positive moments in the flights

2 For Dead Load

$M_3 = M_l = 6.06kNm$

$M_2 = M_1 = 7.15kNm$

$W = W_{1+3} * b = 9.5 * 1.2 = 11.4kNm$

$V_l = (W * l/2) + (M_l - M_r)/2$

$= (11.4 * 2.6/2) + (8.08 - 3.1)/2 = 16.73kN$

$V_r = 11.4 * 2.6 - 16.73 = 12.34kN$

$M_5 = (16.73^2/(2 * 11.4)) - 8.08 = 4.2kNm$

$x = V_1/W = 17.3/11.4 = 1.52m from\ left\ support.$

(f.)Max. reaction along the line of intersection

$b * R = V_r + W_{2+5} * g * (b + (c/2)) = 15.24 + 8.75 * 1.1 * (1.2 + (0.15/2))$

R=28.24/1.2 R=23.53kN/m Now, $R = R' + R''$ R'=The forces resisted by primary stresses R"=The forces resisted by secondary stresses

$$R'' = \frac{R' * t^2 * (1 + 3(\frac{b+c}{b})^2)}{4 * a^2 * sin^2\alpha}$$
$$= \frac{R' * 0.12^2 * (1 + 3(\frac{1.2+0.3}{1.2})^2)}{4 * 2.6^2 * sin^2 26'34'}$$

$$= \quad 0.011R'$$

$R''/R' = 0.011$

$R''/(R' + R'') = 0.011/1.011$

$R''/R = 0.011$

(g.)Min. reaction along the line of intersection

$b * R = V_r + W_2 * g * (b + (c/2))$

$= 12.34 + 3.75 * 1.1 * (1.2 + (0.15/2))$

R=17.9/1.2

R=14.92kN/m

(h.)Additional -ve moment at the floor support

1 Due to full load,

M_6=-0.011*R*b*l

=-0.011*23.53*1.2*2.6

=-0.805 kNm

2 Due to Live load

M_6'=-0.011*R*b*l

=-0.011*14.92*1.2*2.6

=-0.512 kNm

(i.)Total -ve moment at the floor support

$M_7 = M_6 + M_4 = -0.805 - 6.06 = -6.865kNm$

$M_7' = M_6' + M_3 = -0.512 - 8.08 = -8.592kNm$

Step 4 Computation of the torsional restraining moment M_x

$R'' \lll R$ *and* $R = R' + R''$ therefore, neglect R” so, R=R’ For full load, R=23.534 kN/m *(a.)Due to deformation of the flight plates caused by primary stresses*

$$\begin{aligned} E*(W_C^I - W_D^I) &= \frac{6*R'*a*(b+c)}{b*t*cos\alpha} \\ &= \frac{6*23.53*2.9*(1.2+0.3)}{1.2*0.12*0.8944} \\ &= 4768.3kN/m \end{aligned}$$

(b.)Due to twist of the flights caused by the torsional moment $M\bar{X}$

Take,$\nu = 0.15$

$E = 2*(1+\nu)*G = 2*(1+0.15)*G = 2.3G$

$$\begin{aligned} E*(W_C^{III} - W_D^{III}) &= \frac{b*l*M_X}{G*J_2} \\ &= \frac{1.2*2.9*6890*0.113*M_X}{0.435*E*4.5} \\ &= 1.384*M_X kN/m \end{aligned}$$

(c.)Due to bending of the flights caused by the moment $M\bar{Z}$

$$\begin{aligned} E*(W_C^{IV} - W_D^{IV}) &= \frac{-12*h*M_X*tan\alpha}{b^2*t} \\ &= \frac{-12*1.3*0.5*M_X}{1.2^2*0.12} \\ &= -5.1*M_X N/m \end{aligned}$$

(d.)Due to bending of the landing caused by the load R' and moment $M\bar{X}$

$$\begin{aligned} E*(W_C^{II} - W_D^{II}) &= \frac{R'*b^2*(b+c)*(c+0.7*b)}{4*E*I_b*cos\alpha} - \frac{b*M_X*(3*c+2*b)}{6*E*I_b*cos\alpha} \\ &= \frac{23.53*1.2^2*(1.2+0.3)*(0.3+0.7*1.2)}{4*3.1*10^{-4}*0.8944} - \frac{1.2*M_X*(3*0.3+2*1.2)}{6*3.1*10^{-4}*0.8944} \\ &= 52242870 - 342.34*M_X N/m \end{aligned}$$

By using compatibility equation

$(W_C^I - W_D^I) + (W_C^{III} - W_D^{III}) + (W_C^{IV} - W_D^{IV}) = (W_C^{II} - W_D^{II})$

$M_X = 3.12kNm$

The torsional moment

$M_T = M_X * cos\alpha = 3.12 * 0.8944 = 2.79kNm$

Saint Venant's formula for shear stress for less narrow section

The maximum torsional shear stress on the flight is,

$b/t = 1.2/0.12 = 10so, K2 = 0.212$

$$\begin{aligned}\tau &= \frac{M_T}{K2 * b * t^2} \\ &= \frac{2.79 * 1000}{0.212 * 1.2 * 0.12^2} \\ &= 762kN/m^2\end{aligned}$$

Step 5 Computation of the torsional restraining moment M_x in case of LL on flights only

(a.)Due to deformation of the flight plates caused by primary stresses

$$E * (W_C^I - W_D^I) = 3023.5kN/m$$

(b.)Due to twist of the flights caused by the torsional moment $M\bar{X}$

$$E * (W_C^{III} - W_D^{III}) = 1.384 * M_X kN/m$$

(c.)Due to bending of the flights caused by the moment $M\bar{Z}$

$$E * (W_C^{IV} - W_D^{IV}) \;=\; -5.1 * M_X N/m$$

(d.)Due to bending of the landing caused by the load R' and moment $M\bar{X}$

$$E * (W_C^{II} - W_D^{II}) \;=\; 33129200 - 342.34 * M_X N/m$$

By using compatibility equation

$(W_C^I - W_D^I) + (W_C^{III} - W_D^{III}) + (W_C^{IV} - W_D^{IV}) = (W_C^{II} - W_D^{II})$

$M_X = 1.976kNm$

Step 6 Computation of the bending moment in the landing

Total BM at the horizontal axis in the landing at point O=Primary moment-Torsional restraining moment,M_X

1 For full load condition

$M_O = -(R' * b * (b+c)/2) + M_X = -23.53 * 1.2 * (1.2+0.3)/2 + 3.12 = -18.1kNm$

2 For LL on the flights only

$M_O = -(R' * b * (b+c)/2) + M_X = -14.92 * 1.2 * (1.2+0.3)/2 + 1.976 = -11.452kNm$

Step 7 Computation of the bending moment in the flights about the vertical axis

Total BM at the vertical axis in the flight at point M=Primary moment-Torsional restraining moment,M_Z

Table I: Results for moments in kNm obtained from Siev's method

Method	M_r				M_s	M_t
Points	N	M	O	K	MN	MN
Siev's method	-6.865	-7.15	-18.1	4.13	46.34	-2.79

1 For full load condition

$$X = X_l = X_u = b * R'/sin\alpha = 1.2 * 23.72/0.4472 = 63.65kN$$

$$M_m = X * (b + c)/2 - M_{\overline{Z}} = 63.65 * (1.2 + 0.3)/2 - 3.12 * 0.4472 = 46.34kNm$$

2 For Live load condition

$$X = X_l = X_u = b * R'/sin\alpha = 1.2 * 14.92/0.4472 = 40kN$$

$$M_m = X * (b + c)/2 - M_{\overline{Z}} = 40 * (1.2 + 0.3)/2 - 1.976 * 0.4472 = 29.12kNm$$

3.3 Cusens and Kuang's method

3.3.1 Introduction

It is widely known that the principle of least work is a powerful tool in solving statically indeterminate structural problems.This is true when the structure is a 3-D frame of which the members are subjected to torsional members are subjected to torsional stresses in addition to the conventional bending and the axial stresses.

3.3.2 Method of analysis

According to Cusens and Kuang's assumptions, the staircase can be analyzed by reducing the plates to beam elements.Thus the stair will be in the form of a space frame consisting of the beam located in a position coincident with their longitudinal axes.The following analysis will be based on the application of these assumption and the method of least work.
In **Fig.3.11** shows the staircase with the beam elements represented by heavy lines.The beam is cut at point O and the horizontal forces and moments are applied to the two halves of the stair as shown in **Fig.3.13**.Notations and sign conventions for all members are shown in **Fig.3.12**.The bending and torsional moment along the members of the space frame are given below.

For the purpose of analysis the stair is simplified to the rigid frame in **Fig.3.11**; side and end elevation and a plan are given in **Fig.3.12**[a][b].The positive vectors for moments are given in **Fig.3.12**[c] and the usual right hand is applied. The frame is cut at O and the horizontal restraining forces H and the restraining moment M_O are applied to the two halves of the stairs as shown in **Fig.3.13**
The bending and torsional moments along the member forming the upper part of the frame are as follows:

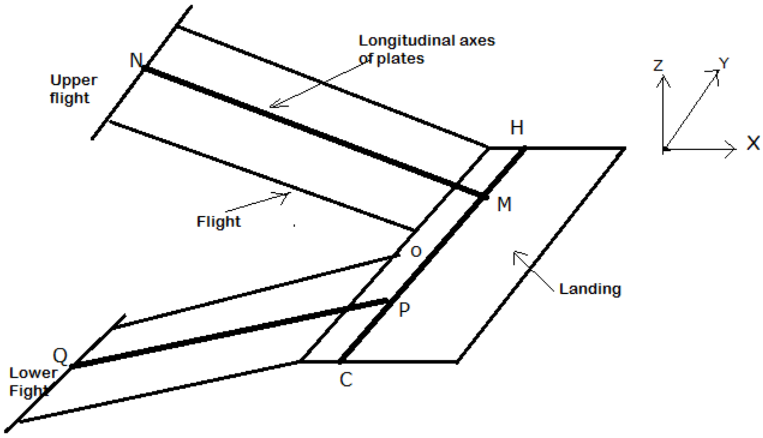

Figure 3.11: Skeletal rigid frame representing the cantilever staircase

For member OM,

$M_R = -M_O - (W_{2+5})/2 * y^2$

$M_S = -H * y$

$M_T = -(W_{2+5})/2 * y * g$

For member MH,

$M_R = -W_{2+5} * (2 * b1 + b2 - y)^2/2$

$M_S = 0$

$M_T = -W_{2+5} * (2 * b1 + b2 - y) * g/2$

For member MN,

$M_R = H * (sin\alpha * s) - (W_{1+3} * s^2)/2 * (cos\alpha)^2 - W_{2+5} * (b2 + 2 * b1) * (cos\alpha * s) - W_{2+5} * (b2 + 2 * b1) * g/2$

$M_S = -H*(cos\alpha*b2)/2 - M_O*sin\alpha + W_{2+5}*(sin\alpha)*(b2/2+b1)*1/2*((b2/2+b1)-b2)$

$M_T = -H*(sin\alpha*b2) + M_O*cos\alpha - W_{2+5}*(cos\alpha)*(b2/2+b1)*1/2*((b2/2+b1)-b2)$

Where, W_{1+3}=DL+LL on the lower flight in kN/m

W_{1+4}=DL+LL on the upper flight in kN/m

W_{2+5}=DL+LL on the landing in kN/m

s=distance measured along axis of flight

y=distance measured along y axis

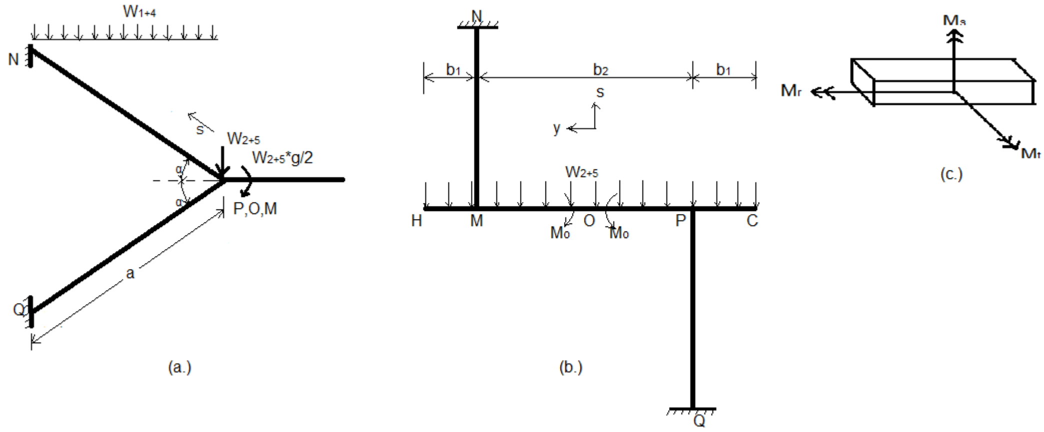

Figure 3.12: Side Elevation and End elevation of the staircase

α=angle of elevation of flight axis with the horizontal

g=width of landing

Since the floor supports are assumed to be perfectly rigid,it can be concluded that no deflections or rotations occur at the end support Q.Then from the theory of least work it follows that

$$\frac{\partial U}{\partial H} = 0 \frac{\partial U}{\partial M_O} = 0 \tag{3.33}$$

Where U is the total strain energy due to flexure and torsion in all members(the strain energy due to shearing and direct forces may be neglected). The complete expression for the total strain energy of the force will be

$$U = \int \frac{M_R^2}{2*E*I_2} ds + \int \frac{M_S^2}{2*E*I_2'} ds + \int \frac{M_T^2}{2*G*J_2} ds + \int \frac{M_R^2}{2*E*I_1} dy + \int \frac{M_S^2}{2*E*I_1'} dy + \int \frac{M_T^2}{2*G*J_1} dy \tag{3.34}$$

Where E,G,$I_1, I_2, I_1', I_2' and J_1, J_2$ have the same meaning as in the preceding method. Differentiating the total strain energy U in Eq.3.34 w.r.t.H, M_O respectively and substituting the corresponding moment expressions shows in above expressions into Eq.3.33; integrating and simplifying,2 linear equations will be obtained.

(1.)$\partial U/\partial H = \int_0^{\frac{b_2}{2}} \frac{M_s*(\partial M_s/\partial H)}{E*I_1'} dy + \int_0^a \frac{M_r*(\partial M_r/\partial H)}{E*I_2} ds + \int_0^a \frac{M_s*(\partial M_s/\partial H)}{E*I_2'} ds + \int_0^a \frac{M_t*(\partial M_t/\partial H)}{G*J_2} ds =$

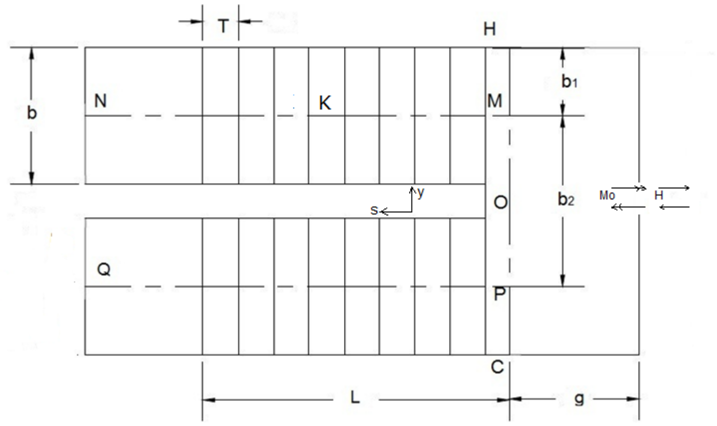

Figure 3.13: Plan of the staircase

0

(2.)$\partial U/\partial M_0 = \int_0^{\frac{b_2}{2}} \frac{M_r*(\partial M_r/\partial M_0)}{E*I_1} dy + \int_0^a \frac{M_s*(\partial M_s/\partial M_0)}{E*I_2'} ds + \int_0^a \frac{M_t*(\partial M_t/\partial M_0)}{G*J_2} ds = 0$

The details of the procedure will be illustrated in the following numerical example.

3.3.3 Numerical example based on Cusens and Kuang's method

The concrete cantilever as shown in **Fig.3.14** and **Fig.3.15** staircase will be analyzed by this method. The additional dimensions for the landing are:

$b1 = 0.6m \; and \; b2 = 1.5m$

(1.) Symmetrical loading

Step-1 Load calculation

W=Total weight of one flight

$$= (2.6 * 0.110 * 1.2 * 25) + (8 * \frac{0.15}{2} * 0.3 * 1.2 * 25) = 13.98kN$$

Live Load=5 kN/m^2 for both flights and landing

DL of the flight =13.98/2.6 $*$ 1.2 =4.5 kN/m^2

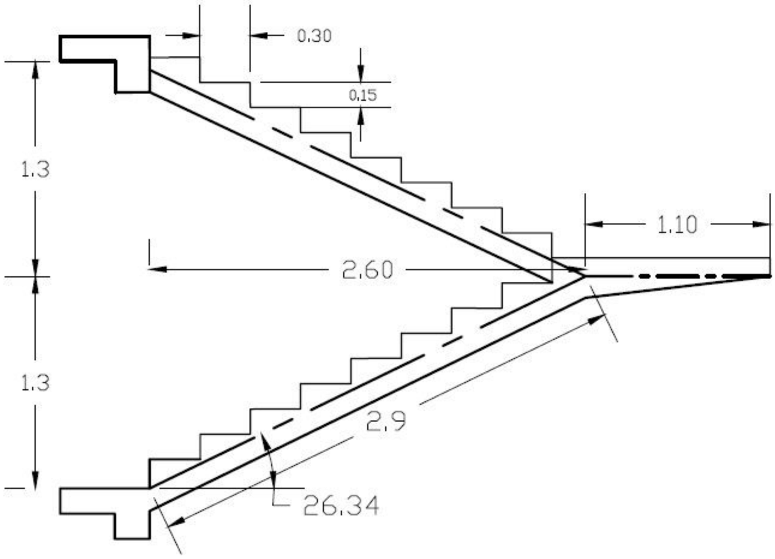

Figure 3.14: Elevation of the staircase

DL of the landing =25*0.15=3.75 kN/m^2

Now, W_1 =DL of the flight= $4.5kN/m^2 * 1.2 = 5.4kN/m$

W_2 =DL of the landing= $3.75kN/m^2 * 1.35 = 5.0625kN/m$

W_3 =LL of the lower and upper flight= $5kN/m^2 * 1.2 = 6kN/m$

W_5 =LL of the landing= $5kN/m^2 * 1.35 = 6.75kN/m$

W_{1+3} =DL+LL on the lower flight= $5.4 + 6 = 11.4kN/m$

W_{1+4} =DL+LL on the upper flight= $5.4 + 6 = 11.4kN/m$

W_{2+5} =DL+LL on the landing = $5.0625 + 6.75 = 11.81kN/m$

Step-2 M.I. of the inertia of the flights and the landing

I_1=M.I. of the inertia of the landing about horizonal axes

$=(1.1 * 0.15^3)/12 = 3.1 * 10^{-4}m^4$

I'_1=M.I. of the inertia of the landing about vertical axes

$=(1.1^3 * 0.15)/12 = 166.4 * 10^{-4}m^4$

I_2=M.I. of the inertia of the flight about horizonal axes

$=(1.2 * 0.12^3)/12 = 1.33 * 10^{-4}m^4$

I'_2=M.I. of the inertia of the flight about vertical axes

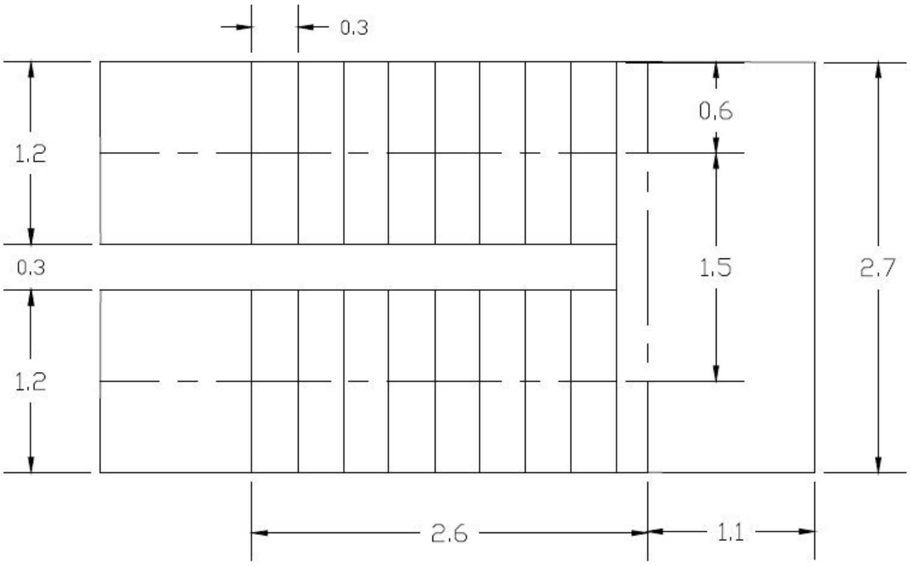

Figure 3.15: Plan of the staircase

$=(0.12 * 1.2^3)/12 = 173 * 10^{-4}m^4$

Torsional rigidity of the rectangular section

Using Saint-Venant's forula, when $b/t \ggg 2.5$

J_1=Polar M.I. of the inertia of the landing

$=(b/3) * t^3 * (1 - 0.63 * (t/b))N/m^2$

$=(1.1/3) * 0.15^3 * (1 - 0.63 * (0.15/1.1)) = 1.145 * 10^{-3}m^4$

J_2=Polar M.I. of the inertia of the flight

$=(b/3) * t^3 * (1 - 0.63 * (t/b))$

$=(1.2/3) * 0.12^3 * (1 - 0.63 * (0.12/1.2)) = 6.5 * 10^{-4}m^4$

Table II: Results for moments in kNm obtained from Cusens and Kuang's method

Method	M_r				M_s	M_t
Points	N	M	O	K	MN	MN
Cusens and Kuang's method	-20.71	-8.77	-14.3	7.04	25.4	-4.45

Step-3 Computation of the redundant in the structure substituting the appropriate values into Eq.(1) to (2), two simultaneous equation will be obtained as follows:

$(1.)3314.1 * H - 669.26 * M_O = 150 * 10^6 (2.)4866.13 * M_O - 1338.52 * H = -5.52 * 10^6$

$Solving,$

$H = 48.12kNM_O = 14.273kNm$

3.4 Fuchssteiner's method

3.4.1 Introduction

Free standing stairs without a landing support are attractive,structurally and create special architectural effects.Their light and slender form emphasizes the many possibilities offered and the advantages obtained by proper consideration of the space action of structural elements. Nevertheless,lack of an adequate and simple method of analysis has restricted their use and has hindered architects and engineers from adopting more widely this impressive stair design.Unfortunately, apparent complications derived from space action have compelled the use of empirical design methods and the introduction of undesirable and unnecessary simplification, with consequent loss of economy and slenderness.

3.4.2 Method of analysis

A rigorous method for analyzing the statically indeterminate space structure formed by the combination and joint action of stair and landing slabs, was developed by W.Fuchssteiner.The procedure developed by the author consists in assuming the stair supported along the intersection line between the landing and the flights **Fig.3.16**.Moments in the slabs are calculated for several loading conditions as for a continuous straight beam, fixed at one end and cantilevered at the other, and the corresponding reactions at the Edge B are determined. As the imaginary support is nonexistent, the effect of reaction B has to be counteracted by a force, equal in magnitude but of opposite sign, applied to the unsupported space frame.

The Theoretical Solution by Fuchssteiner is based on these assumptions:

a. The stair is made of a homogeneous and isotropic material which follows Hooke's law.

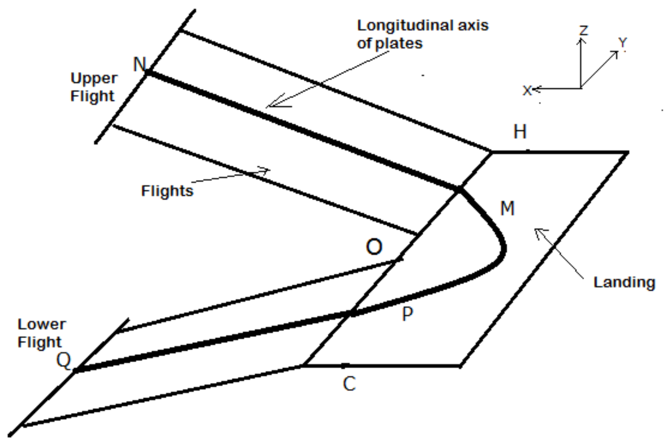

Figure 3.16: Fuchssteiner's assumed form for the cantilever staircase

b. The stresses across a section of the flights or landing vary in the same manner as they vary in a beam (linear variation due to moments, uniform stress across a section due to normal forces.)

c. The deformations due to normal and shear stresses are neglected.

d. Flights and landing have the same uniform rectangular cross section.

The staircase is indeterminate to the sixth degree, but geometric and load symmetry reduces the redundant to only 2 unknowns. To solve these, the stair is cut at the landing**Fig.3.17**, resulting in two cantilever beams acted on by reaction B, considered as an exterior load, and 2 unknown redundant, i.e., a bending moment X_6 and shearing force X_5 acting both along the cut section.
For this load condition the flexural and torsional moments are determined and the deformations due to the external load and the 2 unknown internal forces are calculated from the work integral, whose general expressions is as follows:

$$E*I_X*\delta_{ik} = \int M_{Xi}*M_{Xk}ds + \int M_{Yi}*M_{Yk}*\frac{I_X}{I_Y}ds + \int M_{Ti}*M_{Tk}*\frac{E*I_X}{G*I_T}ds \quad (3.35)$$

In this equation the differential element ds is:

$$ds = r * d\phi \; and \; ds = \frac{dx}{cos\alpha}$$

The torsional rigidity is taken as:

$$G * I_T = \frac{2 * E * I_X * I_Y}{I_X + I_Y}$$

Author has presented design formulas for the deformations as a function of the geometric properties of the stair and of reaction B. By substituting these in the elastic equation:

$$X_5 * \delta_{55} + X_6 * \delta_{56} + \delta_{50} = 0 \quad (3.36)$$

$$X_5 * \delta_{65} + X_6 * \delta_{66} + \delta_{60} = 0 \quad (3.37)$$

one obtains the desired unknowns $X_5 and X_6$.

With the known shear force X_5 and bending moment X_6 all the other moments

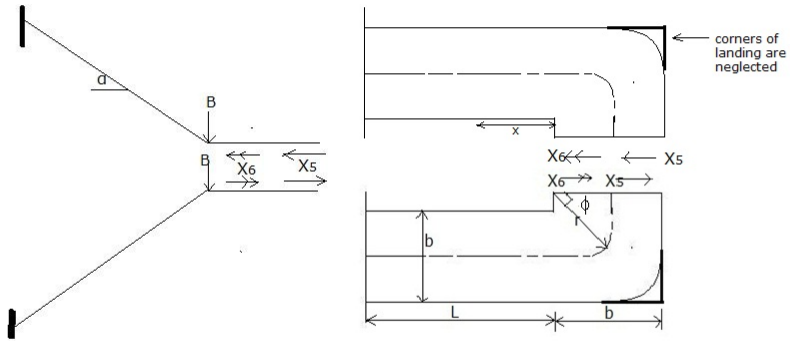

Figure 3.17: Cantilever beams as statically determinate system acted on by reaction B and redundant X_5 and X_6

and forces in the landing and flights slabs due to reaction B can be calculated for the various lading conditions, i.e., for the corresponding reactions at the imaginary

support B. These forces and moments have to be superimposed to those calculated for the continuous straight beam supported at edge B.
This method is based on the application of the principle of least work for the determination of deformations and the calculation of the redundant by solving the elastic equations, reduces the space structure composed by plates or slabs to a space frame composed of linear bar elements **Fig.3.17**. Thus Author represents a convenient way to solve an otherwise highly complex structure. Unfortunately, the method has the limitation that the assumed statically determinate system with the two cantilever beams analyzes only the case of fixed supports at the upper and lower floor levels. Actually the flight slabs are not always continuous with the floor slab, as in the case of precast floors, and are often supported by concrete edge beams which cannot provide fixity, resulting in a simple or elastic support condition.
Due to the redundant X_5 and X_6 and the reactive force B the following forces and moments are generated.

In the landing

$$\begin{aligned} Q_X &= 0 \\ Q_Y &= \frac{X_5 * cos\phi}{r} \\ Q_Z &= -\frac{X_5 * sin\phi}{r} \\ M_X &= X_6 * cos\phi \\ M_Y &= X_5 * sin\phi \\ M_Z &= -X_6 * sin\phi \end{aligned}$$

In the lower flight

$$\begin{aligned} Q_X &= \pm(B * cos\alpha + \frac{X_5 * cos\alpha}{r}) \\ Q_Y &= 0 \end{aligned}$$

$$
\begin{aligned}
Q_Z &= \pm(B * sin\alpha - \frac{X_5 * cos\alpha}{r}) \\
M_X &= -B * x - \frac{X_5 * tan\alpha * x}{r} \\
M_Y &= \pm(X_5 * cos\alpha + X_6 * sin\alpha) \\
M_Z &= \pm(X_5 * sin\alpha - X_6 * cos\alpha)
\end{aligned}
$$

Where Q_X and Q_Y are shear forces,N is a normal force, M_X and M_Y are constants.M_T is a torsional moment X_5 and X_6 are redundant.These quantities are shown in **Fig.3.17** as they act on an element of the stair of width b;all arrows in this Figure point in the positive direction.The redundant X_5 and X_6 are

$$
\begin{aligned}
X_5 &= -\frac{\delta_{05} * \delta_{66}}{(\delta_{55} * \delta_{66}) - \delta_{56}^2} \\
X_6 &= -\frac{\delta_{05} * \delta_{56}}{(\delta_{55} * \delta_{66}) - \delta_{56}^2}
\end{aligned}
$$

For the special case considered i.e.;$\phi = 0$ to $\phi = \frac{\pi}{2}$ for landing and X=0 to X=l for flight
$E * I_X * \delta_{ik} = (\int_0^{\pi/2}[M_{Xi} * M_{Xk} + M_{Yi} * M_{Yk} * (I_X/I_Y) + M_{Ti} * M_{Tk} * 1/2 * (1 + (I_X/I_Y))])d\phi + (\int_0^{l}[M_{Xi} * M_{Xk} + M_{Yi} * M_{Yk} * (I_X/I_Y) + M_{Ti} * M_{Tk} * 1/2 * (1 + (I_X/I_Y))])dx/r * cos\alpha$ By integrating,

$$
\begin{aligned}
\delta_{05} &= \frac{B * l^3 * tan\alpha}{3 * r^2 * cos\alpha} \\
\delta_{06} &= 0 \\
\delta_{55} &= \frac{\pi}{4} * \frac{I_X}{I_Y} + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * sin^2\alpha * (1 - \frac{I_X}{I_Y}) + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\
\delta_{56} &= -\frac{1}{2} * (1 - \frac{I_X}{I_Y}) * \frac{l}{r} * sin\alpha \\
\delta_{66} &= \frac{\pi}{8} * (3 + \frac{I_X}{I_Y}) + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * cos^2\alpha * (1 - \frac{I_X}{I_Y})]
\end{aligned}
$$

3.4.3 Numerical Example based on Fuchssteiner's method

The concrete cantilever staircase shown in **Fig.3.18** and **Fig.3.19** will be analyzed by this method. The additional dimensions for the landing are:
$\alpha = 26'34'$

$$R = \frac{b+m}{2} = \frac{1.2+0.3}{2} = 0.75m$$
$$\frac{I_X}{I_Y} = \frac{d^2}{b^2} = \frac{0.15^2}{0.12^2} = 0.02$$

Step 1 Deformation at section i in the direction of redundant X_i du to

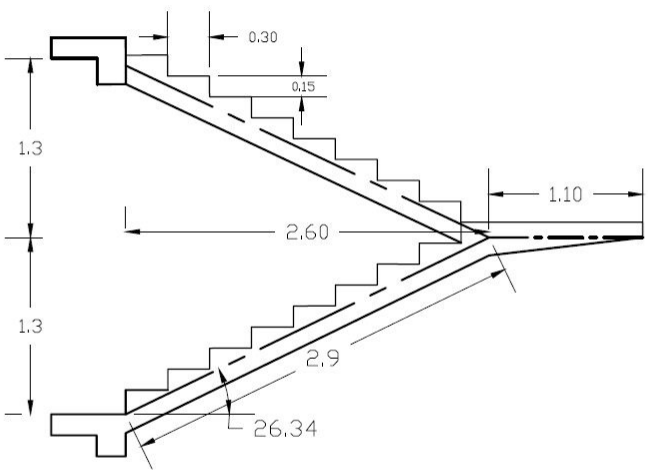

Figure 3.18: Elevation of the staircase

$X_k = 1$

$$\begin{aligned}\delta_{05} &= \frac{B * l^3 * tan\alpha}{3 * r^2 * cos\alpha} \\ &= \frac{B * 2.6^3 * tan26'34'}{3 * 0.75^2 * cos26'34'} \\ &= 5.823 * B\end{aligned}$$

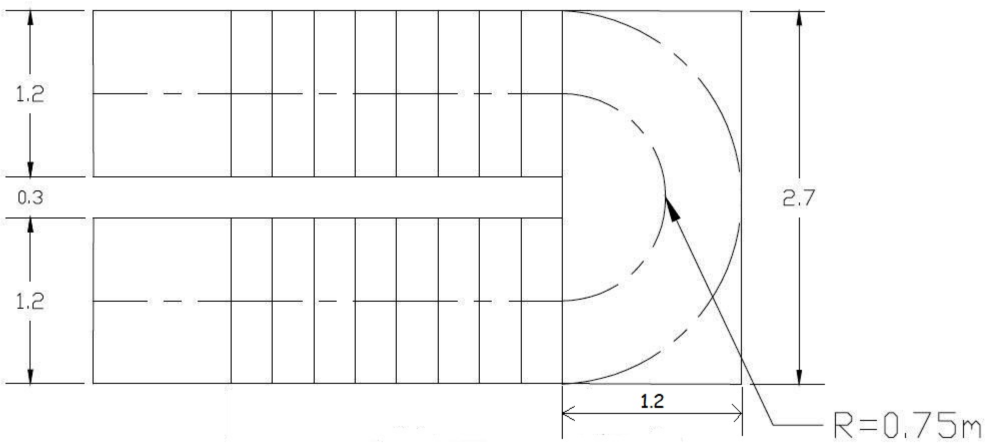

Figure 3.19: Plan of the staircase

$\delta_{06} = 0$

$$\begin{aligned}\delta_{55} &= \frac{\pi}{4} * \frac{I_X}{I_Y} + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * sin^2\alpha * (1 - \frac{I_X}{I_Y}) + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\ &= \frac{\pi}{4} * 0.02 + \frac{2.6}{0.75 * cos26'34'} * [0.02 + \frac{1}{2} * sin^2 26'34' * (1 - 0.02) + \frac{2.6^2 * tan^2 26'34'}{3 * 0.75^2}] \\ &= 4.355\end{aligned}$$

$$\begin{aligned}\delta_{56} &= -\frac{1}{2} * (1 - \frac{I_X}{I_Y}) * \frac{l}{r} * sin\alpha \\ &= -\frac{1}{2} * (1 - 0.02) * \frac{2.6}{0.75} * sin26'34' \\ &= -0.76\end{aligned}$$

$$\begin{aligned}\delta_{66} &= \frac{\pi}{8} * (3 + \frac{I_X}{I_Y}) + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * cos^2\alpha * (1 - \frac{I_X}{I_Y})] \\ &= \frac{\pi}{8} * (3 + 0.02) + \frac{2.6}{0.75 * cos26'34'} * [0.02 + \frac{1}{2} * cos^2 26'34' * (1 - 0.02)] \\ &= 2.783\end{aligned}$$

$$\delta_{55} * \delta_{66} - \delta_{56}^2 = 4.355 * 2.783 - 0.76^2 = 11.5$$

Step 2 Determine unknown redundant X_5 ***and*** X_6

$$\begin{aligned} X_5 &= -\frac{\delta_{05} * \delta_{66}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= -\frac{5.823 * B * 2.783}{11.5} \\ &= -1.41 * B \\ X_6 &= \frac{\delta_{05} * \delta_{56}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= \frac{5.823 * B * (-0.76)}{11.5} \\ &= -0.385 * B \end{aligned}$$

Step 3 Determine reactions and moments $B = 28.755kN$

a. In the landing

$Q_X = 0$

$$Q_Y = \frac{X_5 * cos\phi}{r} = -\frac{1.41 * B * sin\phi}{0.75} = -1.88 * B * cos\phi$$

$$N = -\frac{X_5 * sin\phi}{r} = -\frac{(-1) * 1.41 * B * sin\phi}{0.75} = 1.88B * sin\phi$$

$$M_X = -1.41 * B * cos\phi$$

$$M_Y = -1.41 * B * sin\phi$$

$$M_T = 0.385B * sin\phi$$

b. In the flight

$$Q_X = B * cos\alpha + \frac{X_5 * sin\alpha}{r} = B * cos\alpha - \frac{1.41 * B * sin\alpha}{0.75} = 0.054 * B$$

$$Q_Y = 0$$

$$N = B * sin\alpha - \frac{X_5 * cos\alpha}{r} = B * sin\alpha + \frac{1.41 * B * cos\alpha}{0.75} = 2.13 * B$$

Table III: Results for moments in kNm obtained from Fuchsstener's method

Method	M_r				M_s	M_t
Points	N	M	O	K	MN	MN
Fuchssteiner's method	-4.5	0	-11.45	2.25	42.61	-8.51

$$M_X = -B*x - \frac{x*X_5*tan\alpha}{r} = -B*x + \frac{x*1.41*tan26'34'}{0.75} = -1.7253*x$$

$$\begin{aligned} M_Y &= X_5*cos\alpha + X_6*sin\alpha \\ &= -1.41*B*cos26'34' - 0.385*B*sin26'34' \\ &= -1.433*B \\ M_T &= X_5*sin\alpha - X_6*cos\alpha \\ &= -1.41*B*sin26'34' + 0.385*B*cos26'34' \\ &= -0.286*B \end{aligned}$$

3.5 A Simplified method for free standing stairs based on Fuchssteiner's method

Where,The torsional rigidity is taken as:

$$G*I_T = \frac{2*E*I_X*I_Y}{I_X+I_Y}$$

$$\frac{E*I_X}{G*I_T} = \frac{1}{2}[1+\frac{I_X}{I_Y}]$$

Now, $I_X/I_Y \lll 1$.Therefore, neglect it.

$$\frac{E*I_X}{G*I_T} = \frac{1}{2}[1+\frac{I_X}{I_Y}] = \frac{1}{2}[1+0] = \frac{1}{2}$$

Equations of deformations

$$
\begin{aligned}
\delta_{05} &= \frac{B * l^3 * tan\alpha}{3 * r^2 * cos\alpha} \\
\delta_{06} &= 0 \\
\delta_{55} &= \frac{\pi}{4} * \frac{I_X}{I_Y} + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * sin^2\alpha * (1 - \frac{I_X}{I_Y}) + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\
&= \frac{\pi}{4} * 0 + \frac{l}{r * cos\alpha} * [0 + \frac{1}{2} * sin^2\alpha * (1 - 0) + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\
&= \frac{l}{r * cos\alpha} * [\frac{1}{2} * sin^2\alpha + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\
\delta_{56} &= -\frac{1}{2} * (1 - \frac{I_X}{I_Y}) * \frac{l}{r} * sin\alpha \\
&= -\frac{1}{2} * (1 - 0) * \frac{l}{r} * sin\alpha \\
&= -\frac{1}{2} * \frac{l}{r} * sin\alpha \\
\delta_{66} &= \frac{\pi}{8} * (3 + \frac{I_X}{I_Y}) + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} + \frac{1}{2} * cos^2\alpha * (1 - \frac{I_X}{I_Y})] \\
&= \frac{\pi}{8} * (3 + 0) + \frac{l}{r * cos\alpha} * [0 + \frac{1}{2} * cos^2\alpha(1 - 0)] \\
&= \frac{3 * \pi}{8} + \frac{l}{r * cos\alpha} * [\frac{1}{2} * cos^2\alpha]
\end{aligned}
$$

Equations of reactions and moments:

In the landing

$$
\begin{aligned}
Q_X &= 0 \\
Q_Y &= \frac{X_5 * cos\phi}{r} \\
Q_Z &= -\frac{X_5 * sin\phi}{r} \\
M_X &= X_6 * cos\phi \\
M_Y &= X_5 * sin\phi \\
M_Z &= -X_6 * sin\phi
\end{aligned}
$$

In the lower flight

$$Q_X = \pm(B * cos\alpha + \frac{X_5 * cos\alpha}{r})$$
$$Q_Y = 0$$
$$Q_Z = \pm(B * sin\alpha - \frac{X_5 * cos\alpha}{r})$$
$$M_X = -B * x - \frac{X_5 * tan\alpha * x}{r}$$
$$M_Y = \pm(X_5 * cos\alpha + X_6 * sin\alpha)$$
$$M_Z = \pm(X_5 * sin\alpha - X_6 * cos\alpha)$$

3.5.1 Numerical Example based on simplified method

Data are as same as Fuchssteiner's method.

Step 1 Deformation at section i in the direction of redundant X_i du to $X_k = 1$

$$\begin{aligned}\delta_{05} &= \frac{B * l^3 * tan\alpha}{3 * r^2 * cos\alpha} \\ &= \frac{B * 2.6^3 * tan26'34'}{3 * 0.75^2 * cos26'34'} \\ &= 5.823 * B\end{aligned}$$

$\delta_{06} = 0$

$$\begin{aligned}\delta_{55} &= \frac{l}{r * cos\alpha} * [\frac{1}{2} * sin^2\alpha + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\ &= \frac{2.6}{0.75 * cos26'34'} * [\frac{1}{2} * sin^2 26'34' + \frac{2.6^2 * tan^2 26'34'}{3 * 0.75^2}] \\ &= 4.27\end{aligned}$$

$$\begin{aligned}\delta_{56} &= -\frac{1}{2} * \frac{l}{r} * sin\alpha \\ &= -\frac{1}{2} * \frac{2.6}{0.75} * sin26'34' \\ &= -0.7751\end{aligned}$$

$$\begin{aligned}\delta_{66} &= \frac{3\pi}{8} + \frac{l}{r * cos\alpha} * [\frac{1}{2} * cos^2\alpha] \\ &= \frac{3\pi}{8} + \frac{2.6}{0.75 * cos26'34'} * [\frac{1}{2} * cos^2 26'34'] \\ &= 2.73\end{aligned}$$

$$\delta_{55} * \delta_{66} - \delta_{56}^2 = 4.27 * 2.73 - 0.7751^2 = 11.06$$

Step 2 Determine unknown redundant X_5 ***and*** X_6

$$\begin{aligned}X_5 &= -\frac{\delta_{05} * \delta_{66}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= -\frac{5.823 * B * 2.73}{11.06} \\ &= -1.44 * B \\ X_6 &= \frac{\delta_{05} * \delta_{56}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= \frac{5.823 * B * (-0.7751)}{11.06} \\ &= -0.408 * B\end{aligned}$$

Step 3 Determine reactions and moments

$B = 28.755kN$

a. In the landing

$$\begin{aligned}Q_X &= 0 \\ Q_Y &= \frac{X_5 * cos\phi}{r} = -\frac{1.44 * B * sin\phi}{0.75} = -1.92 * B * cos\phi \\ N &= -\frac{X_5 * sin\phi}{r} = -\frac{(-1) * 1.44 * B * sin\phi}{0.75} = 1.92B * sin\phi\end{aligned}$$

$$
\begin{aligned}
M_X &= -1.44 * B * cos\phi \\
M_Y &= -1.44 * B * sin\phi \\
M_T &= 0.408B * sin\phi
\end{aligned}
$$

b. In the flight $Q_X = B * cos\alpha + \frac{X_5 * sin\alpha}{r}$

$= B * cos\alpha - \frac{1.44 * B * sin\alpha}{0.75} = 0.036 * B$

$Q_Y = 0$

$N = B * sin\alpha - \frac{X_5 * cos\alpha}{r}$

$= B * sin\alpha + \frac{1.44 * B * cos\alpha}{0.75} = 2.165 * B$

$M_X = -B * x - \frac{x * X_5 * tan\alpha}{r}$

$= -B * x + \frac{x * 1.44 * tan26'34'}{0.75} = -B * x + 0.96x$

$M_Y = X_5 * cos\alpha + X_6 * sin\alpha$

$= -1.44 * B * cos26'34' - 0.408 * B * sin26'34' = -1.47 * B$

$M_T = X_5 * sin\alpha - X_6 * cos\alpha$

$= -1.44 * B * sin26'34' + 0.408 * B * cos26'34' = -0.28 * B$

Comparison of the analysis results

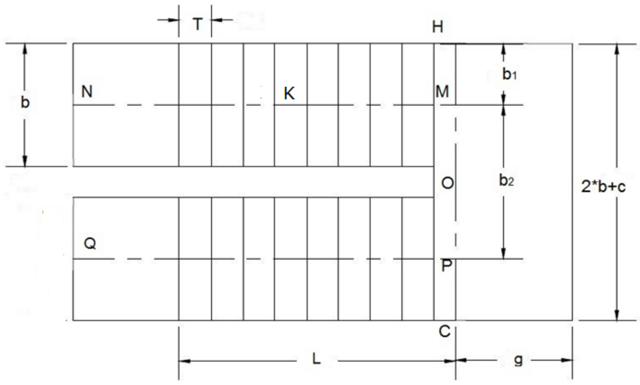

Figure 3.20: Plan of free standing stair

Table IV: Comparison of results of the different type of method of analysis

Method	M_r kNm				M_s kNm	M_t kNm
Points	N	M	O	K	MN	MN
Siev's method	-6.865	-7.15	-18.1	4.13	46.34	-2.79
Cusens and Kuang's method	-20.71	-8.77	-14.3	7.04	25.4	-4.45
Fuchssteiner's method	-4.5	0	-11.45	2.25	42.61	-8.51
Simplified method	-3.1	0	-12.13	1.55	43.71	-8.3
FEM using SAP2000	-5.86	-0.117	-11.9	0.905	1.2	-6.6

3.6 Analysis of Horse shoe type of stairs

Analysis of horse shoe type of stairs has been derived using Fuchssteiner's method. Fuchssteiner's method is based on virtual work method.Dimensions and **Fig.3.21** is given below.

3.6.1 Dimensions of the Horse shoe type of stairs

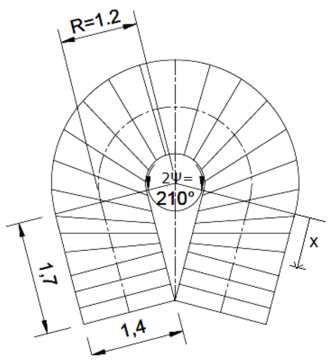

Figure 3.21: Plan of Horse shoe type of stair

h_L=4.2m

$2 * n = 26$

R=165mm and T=300 mm

α=28'36'

b=1.4 m

r=1.2m

l_l=2*n*T=26*300=7.8 m

$2 * \psi$=210'

$arclength = 2 * \pi * R * \frac{2*\phi}{360}$=4.4m

$l = \frac{7.8-4.4}{2} = 1.7m$

d=160mm

$\beta = \frac{I_X}{I_Y} = \frac{d^2}{b^2} = \frac{0.165^2}{1.4^2} = 0.014$

$k_e = 1 + \frac{e}{r} = 1 + \frac{b^2}{12*r^2} = 1 + \frac{1.4^2}{12*1.2^2} = 1.113$

$Q = \frac{l}{r} = \frac{1.7}{1.2} = 1.42$

3.6.2 Load calculation

W=$Selfwt. + Wt.ofthesteps$

=$0.165 * 25 + \frac{1}{2} * 0.165 * 25$

=$7.05kN/m^2$

Live Load=5 kN/m^2 for both flights and landing

Total load=$7.05 + 5 = 12.05kN/m^2$

$w = 12.05 * 1.4 = 16.87kN/m$

3.6.3 Determination of deformation

For 2ψ=210',

The results for the redundant are as follows:

$A = 0.1746$ $B = 1.4402$ $C = 1.0413$ $D = 0.7913$

$A' = 0.3343$ $B' = 0.3635$ $C' = 1.7799$ $D' = 0.2717$

The deformation can be derived as same as described in Fuchssteiner's method.

$\delta_{55} = \tan^2\alpha * C' + \frac{1+\beta}{2} * \sin^2\alpha(Q * B^2 + C - 2A' + D')$

$+ Q * \tan^2\alpha * \sin^2\psi_0 * [\psi_0 * (\psi_0 + Q) + \frac{Q^2}{3} * \cos^2\alpha]$

$+ (\beta) * [\cos^2\alpha * C + 2\sin^2\alpha * A' + \tan^2\alpha * \sin^2\alpha * D'$

$+ Q * (\cos^2\alpha * \sin^2\psi_0 + c^2\alpha * \psi_0 * \sin 2\psi_0 + \sin^2\alpha * \tan^2\alpha * \psi_0^2 * \cos^2\psi_0)$

$+ Q * \cos\psi_0 * (\sin\psi_0 + \tan^2\alpha * \psi_0 * \cos\psi_0) + \frac{Q^3}{3} * \frac{\cos^2\psi_0}{\cos^2\alpha}]$

$\delta_{55} = 3.515$

$\delta_{56} = -A' * \tan\alpha - \frac{1+\beta}{4} * \sin 2\alpha(Q * B\sin\psi_0 + C - A')$

$-\frac{Q}{2}\tan\alpha * \sin 2\psi_0(\psi_0 + \frac{Q}{2}\cos\alpha) + (\beta) * \sin\alpha[\cos\alpha(C + \tan^2\alpha * A')$

$+ Q * \cos\alpha * \sin\psi_0(\sin\psi_0 + \tan^2\alpha * \psi_0 * \cos\psi_0 + Q * \frac{\cos\psi_0}{2*\cos^2\alpha})]$

$\delta_{56} = -0.267$

$\delta_{66} = D + \frac{1+\beta}{2} * \cos^2\alpha(C + Q * \sin^2\psi_0) + Q * \cos^2\psi_0$

$+ (\beta) * \sin^2\alpha(C + Q * \sin^2\psi_0)$

$\delta_{66} = 2.622$

$\delta_{50} = qr^2[k_e(B - A') * \tan\alpha + Q^2 * \sin\alpha * \sin\psi_0 * (\frac{k_e}{2}(1 - \cos\psi_0) + \psi_0 * \frac{Q}{3} + \frac{Q^3}{8})$

$Q * \tan\alpha * \psi_0 * \sin\psi_0 * (k_e(1 - \cos\psi_0) + \psi_0\frac{Q}{2} + \frac{Q^2}{6})$

$+ \frac{1+\beta}{4}\sin 2\alpha(B - B' + Q * B(\psi_0 - k_e\sin\psi_0) + k_e(A' - C))]$

$-(\beta)*qr^2\sin\alpha[\cos\alpha(B-k_eC)+\tan\alpha*\sin\alpha(B-k_eA')+Q*(\psi_0-k_e\sin\psi_0)(\cos\alpha\sin\psi_0+$

$\frac{\sin^2\alpha}{\cos\alpha}\psi_0\cos\psi_0 + Q\frac{\cos\psi_0}{2*\cos\alpha})]$

$\delta_{50} = 6.306 * qr^2$

$\delta 60 = -qr^2[k_eA + \frac{1+\beta}{2}\cos^2\alpha * (B - k_eC + Q\sin\psi_0(\psi_0 - k_e\sin\psi_0)) + Q\cos\psi_0 * (k_e(1 -$

$\cos\psi_0) + \psi_0\frac{Q}{2} + \frac{Q^2}{6})]$

$-\beta * qr^2\sin^2\alpha(B - k_eC + Q\sin\psi_0(\psi_0 - k_e\sin\psi_0))$

$\delta_{60} = 0.4066qr^2$

- *Moments and reactions in the landing and flight*

(1.) In the landing

$Q_X = qr * X_5\cos\phi$

$Q_Y = -qr * [\cos\alpha * \psi + X_5 * \sin\alpha\sin\psi]$

$N = -qr * [\sin\alpha * \psi + X_5 * \cos\alpha\sin\psi]$

$M_X = -qr^2 * [k_e(1 - \cos\psi) + X_5\tan\alpha * \psi\sin\psi - X_6 * \cos\psi]$

$M_Y = qr^2 * [\sin\alpha(\psi - k_e\sin\psi) - X_5 * \cos\alpha(\sin\psi + \tan^2\alpha * \psi\cos\psi)$

$- X_6 * \sin\psi * \sin\alpha]$

Moment	M_X	M_Y	M_T
$X_5 = 1$	$-\tan\alpha\sin\psi_0(\psi_0 + Q\cos\alpha)$	$-(\sin\psi_0 + Q\cos\psi_0)$	$\tan\alpha\cos\psi_0(\psi_0 + Q)$
$X_6 = 1$	$\cos\psi_0$	0	$\sin\psi_0$
$qr^2 = 1$	$-[k_e(1-\cos\psi_0) + Q\psi_0 + \frac{Q^2}{2}]$	0	$-(\psi_0 - k_e\sin\psi_0)$

$$M_T = -qr^2 * [\cos\alpha(\psi - k_e\sin\psi) + X_5 * \sin\alpha(\sin\psi - \psi\cos\psi) - X_6 * \sin\psi * \cos\alpha]$$

(2.) In the flight

$Q_X = qr * X_5\cos\phi_0$

$Q_Y = -qr * [\cos\alpha * (\psi_0 + Q\xi) + X_5 * \sin\alpha\sin\psi_0]$

$N = -qr * [\sin\alpha * (\psi_0 + Q\xi) - X_5 * \cos\alpha\sin\psi_0]$

$M_X = -qr^2 * [k_e(1-\cos\psi_0) + \psi_0 Q\xi + \frac{Q^2}{2}\xi^2$

$+ X_5\tan\alpha\sin\psi_0(\psi_0 + \cos\alpha * Q\xi) - X_6 * \cos\psi_0]$

$M_Y = qr^2 * [\sin\alpha(\psi_0 - k_e\sin\psi_0)$

$- X_5 * \cos\alpha(\sin\psi_0 + \tan^2\alpha * \psi_0\cos\psi_0 + \frac{\cos\psi_0}{\cos^2\alpha}Q\xi)$

$- X_6 * \sin\psi_0 * \sin\alpha]$

$M_T = -qr^2 * [\cos\alpha(\psi_0 - k_e\sin\psi_0) + X_5 * \sin\alpha(\sin\psi_0 - \psi_0\cos\psi_0)$

$- X_6 * \sin\psi_0 * \cos\alpha]$

By putting the values of $X_5 = X_6 = qr^2 = \xi = 1$

These values can be put into above equations, From the above table,the deformation equations can be made:

The deformation equation are as follows:

$\delta^*_{55} = \delta_{55} + 2\theta_y * [\tan\alpha\sin\psi_0(\psi_0 + Q\cos\alpha)]^2$

$+ 2\theta_x * [\sin\psi_0 + Q\cos\psi_0]^2$

$+ 2\theta_z * [\tan\alpha\cos\psi_0(\psi_0 + Q)]^2$

$\delta^*_{55} = 3.515 + 5.256\theta_y$

$\delta^*_{56} = \delta_{56} - \theta_y[\tan\alpha\sin 2\psi_0(\psi_0 + Q\cos\alpha)]$

$+ \theta_x * \tan\alpha\sin 2\psi_0[\psi_0 + Q]$

$\delta^*_{56} = -0.267 + 0.839\theta_y$

$\delta^*_{66} = \delta_{66} + 2\theta_y[\cos^2\psi_0]$

$+ 2\theta_x * \sin^2 \psi$

$\delta^*_{66} = 2.622 + 0.134\theta_y$

$\delta^*_{50} = \delta_{50} + 2\theta_y qr^2[\tan\alpha \sin\psi_0(\psi_0 + Q\cos\alpha)] * [k_e(1 - \cos\psi_0)$

$+ Q\psi_0 + \frac{Q^2}{2}]$

$- 2\theta_x qr^2[\tan\alpha \cos\psi_0(\psi_0 + Q)](\psi_0 - k_e \sin\psi_0)$

$\delta^*_{50} = qr^2(6.306 + 16.24\theta_y)$

$\delta^*_{60} = \delta_{50} - 2\theta_y qr^2 \cos\psi_0 * [k_e(1 - \cos\psi_0) + Q\psi_0 + \frac{Q^2}{2}]$

$- 2\theta_x qr^2 \sin\psi_0(\psi_0 - k_e \sin\psi_0)$

$\delta^*_{60} = qr^2(0.4066 + 2.6\theta_y)$

3.6.4 Determination of redundant

$For\theta_y = 0.5$

$X_5 = \frac{\delta^*_{50} * \delta^*_{66}}{\delta^*_{55}\delta^*_{66} - \delta^{*2}_{56}} = -2.36$

$X_6 = \frac{\delta^*_{50} * \delta^*_{56}}{\delta^*_{55}\delta^*_{66} - \delta^{*2}_{56}} = 0.134$

3.6.5 Determination of moment and reactions in the landing and the flight

a. In the landing

Case (1.) For $\psi = \psi$

Q_X=14.430kN

Q_Y=-12.20kN

N=-68.13kN

M_X=0.2630kNm

M_Y=66.610kNm

M_T=34.130kNm

Case (2.) For $\psi = \psi/2$

Q_X=-33.93kN

Q_Y=2.3900kN

N=11.030kN

M_X=19.50kNm

M_Y=42.39kNm

M_T=11.50kNm

Case (3.) For $\psi = 0$

Q_X=-55.74kN

Q_Y=0

N=0

M_X=-32.4kNm

M_Y=0

M_T=0

b. In the flight

Case (1.) For $\xi = 1$

Q_X=14.500kN

Q_Y=-41.71kN

N=-84.10kN

M_X=39.88kNm

M_Y=33.83kNm

M_T=36.10kNm

Case (2.) For $\xi = 0$

Q_X=14.500kN

Q_Y=-12.20kN

N=-67.98kN

M_X=27.70kNm

M_Y=66.62kNm

M_T=36.10kNm

3.7 Analysis of L shape of type stairs

Analysis of L shape type of stairs can be done using Fuchssteiner's method as shown in **Fig.3.23**.In L shape stair case,the angle between 2 flights are having 90 degrees is as shown in the **Fig.3.22**.Analysis of L shape type of stair is given below:

3.7.1 Dimensions of the L type of stairs

Width of the flight =b=1.1m

Depth of the flight =Df=120mm Height of the one storey=H=1.3m

No. of risers =8

Riser=150mm and Tread =300mm

α =26'34'

Length of the flight =L=2.6m

$r = \frac{1.1}{2} + 0.2 = 0.75m$

Depth of the landing =Dl=150mm ***Step 1 Deformation at section i in the***

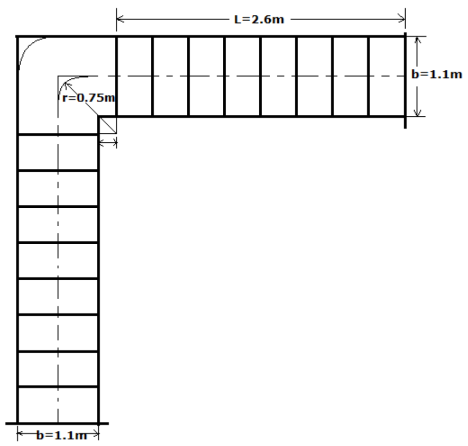

Figure 3.22: Plan of L shape type of stair

direction of redundant X_i ***du to*** $X_k = 1$

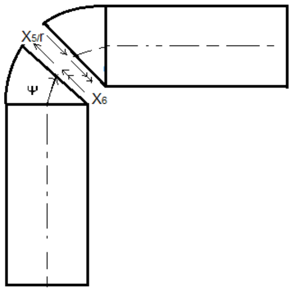

Figure 3.23: Redundant in the L shape type of stair

$$\begin{aligned}\delta_{05} &= \frac{B * l^3 * tan\alpha}{3 * r^2 * cos\alpha} \\ &= \frac{B * 2.6^3 * tan26'34'}{3 * 0.75^2 * cos26'34'} \\ &= 5.823 * B\end{aligned}$$

$\delta_{06} = 0$

$$\begin{aligned}\delta_{55} &= \frac{\pi}{8} * \frac{I_X}{I_Y} + \frac{l}{r * cos\alpha} * [\frac{I_X}{I_Y} * \frac{1}{2}(1 + cos^2\alpha) + \frac{l^2 * tan^2\alpha}{3 * r^2}] \\ &= 3.955\end{aligned}$$

$$\begin{aligned}\delta_{56} &= -\frac{1}{2} * (-1 + \frac{I_X}{I_Y}) * \frac{l}{r} * sin\alpha \\ &= -0.76\end{aligned}$$

$$\begin{aligned}\delta_{66} &= \frac{\pi}{4} * (0.75 + \frac{I_X}{I_Y}) + \frac{1}{8}[1 - \frac{I_X}{I_Y}] + \frac{l}{2rcos\alpha} * (\cos^2\alpha + \frac{I_X}{I_Y}(1 + \sin^2\alpha)) \\ &= 3.055\end{aligned}$$

$$\delta_{55} * \delta_{66} - \delta_{56}^2 = 4.355 * 2.783 - 0.76^2 = 11.5$$

B=19.634kN

Step 2 Determine unknown redundant X_5 ***and*** X_6

$$\begin{aligned} X_5 &= -\frac{\delta_{05} * \delta_{66}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= -\frac{5.823 * B * 2.783}{11.5} \\ &= -30.4 \\ X_6 &= \frac{\delta_{05} * \delta_{56}}{\delta_{55} * \delta_{66} - \delta_{56}^2} \\ &= \frac{5.823 * B * (-0.76)}{11.5} \\ &= -7.556 \end{aligned}$$

Step 3 Determine reactions and moments $B = 19.634kN$

a. In the landing

$Q_X = 0$

$$Q_Y = \frac{X_5 * cos\phi}{r} = -\frac{-30.4 * cos45}{0.75} = -28.66kN$$

$$N = -\frac{X_5 * cos\phi}{r} = -\frac{-30.4 * cos\phi}{0.75} = 28.66kN$$

$$M_X = -30.4 * cos\phi = -5.343kNm$$

$$M_Y = -30.4 * B * sin\phi = -21.5kNm$$

$$M_T = 7.556 * sin\phi = 5.343kNm$$

b. In the flight

$$Q_X = B * cos\alpha + \frac{X_5 * sin\alpha}{r} = B * cos\alpha - \frac{1.41 * B * sin\alpha}{0.75} = -0.567kN$$

$$Q_Y = 0$$

$$N = B * sin\alpha - \frac{X_5 * cos\alpha}{r} = B * sin\alpha + \frac{1.41 * B * cos\alpha}{0.75} = 45.034kN$$

$$M_X = -B * x - \frac{x * X_5 * tan\alpha}{r} = -B * x + \frac{x * 1.41 * tan26'34'}{0.75} = -1.7253 * x$$

$$\begin{aligned} M_Y &= X_5 * cos\alpha + X_6 * sin\alpha \\ &= -30.6kNm \\ M_T &= X_5 * sin\alpha - X_6 * cos\alpha \\ &= -6.838kNm \end{aligned}$$

Chapter 4

Design of free standing stairs

4.1 Introduction

In this chapter design of free standing stairs is derived using analysis result of Simplified method.The design and reinforcement detailing of free standing stair are done using limit state method.The design method is based on Indian standards.
This chapter also deals with the design and reinforcement detailing of L shape and Horse shoe type of staircase.The analysis result of Horse shoe and L shape type of staircase were taken from Fuchssteiner's method and the design was done by limit state method by using IS code.

4.2 Design of the free standing stairs

4.2.1 Dimensions of the free standing stairs

Elevation and Plan of the staircase are as shown in the **Fig.4.1** and **Fig.4.2**.

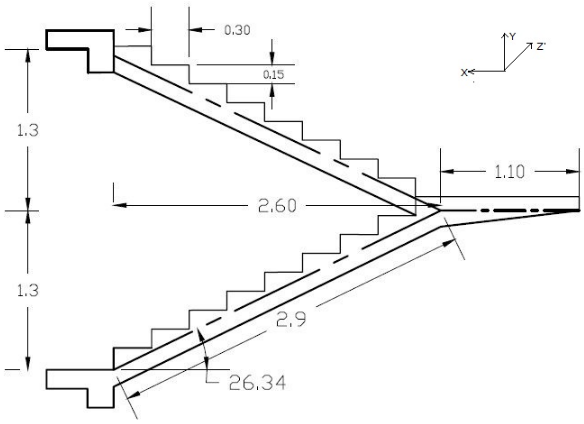

Figure 4.1: Elevation

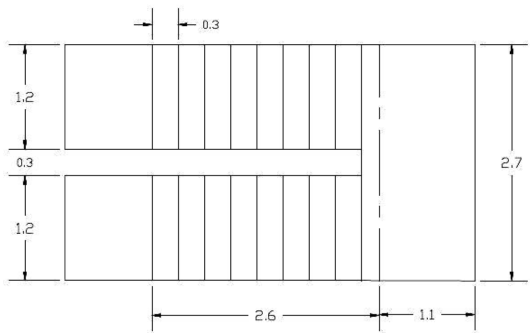

Figure 4.2: Plan

4.2.2 Load calculation

M-20 grade of concrete and Fe-415 grade of steel is to be used.

W=Total weight of one flight

$$W = (2.6 * 0.110 * 1.2 * 25) + (8 * \frac{0.15}{2} * 0.3 * 1.2 * 25) = 13.98kN$$

Live Load=5 kN/m^2 for both flights and landing

DL of the flight =$13.98/2.6 * 1.2$ =4.5 kN/m^2

DL of the landing =25*0.15=3.75 kN/m^2

W_1 =DL of the flight= $4.5kN/m^2 * 1.2 = 5.4kN/m$

W_2 =DL of the landing= $3.75kN/m^2 * 1.35 = 5.0625kN/m$

W_3 =LL of the lower and upper flight= $5kN/m^2 * 1.2 = 6kN/m$

W_5 =LL of the landing= $5kN/m^2 * 1.35 = 6.75kN/m$

W_{1+3} =DL+LL on the lower flight= $5.4 + 6 = 11.4kN/m$

W_{1+4} =DL+LL on the upper flight= $5.4 + 6 = 11.4kN/m$

W_{2+5} =DL+LL on the landing = $5.0625 + 6.75 = 11.81kN/m$

4.2.3 Design results of the Simplified method

(1.) Flight section

Flexural moment M_X=3.1 kNm

Torsional moment M_Z=8.3 kNm

Flexural moment M_Y=43.71 kNm

Shear force V_u=1.03 kN

M_X=1.5*3.1=4.65 kNm

M_Y=1.5*43.71=65.6 kNm

M_Z=1.5*8.3=12.45 kNm

V_u=1.5*1.03=1.545 kN

(2.) Landing section

Flexural moment M_X=12.13 kNm

M_X=1.5*12.13=18.2 kNm

4.2.4 Main reinforcement of the flight

Overall thickness of the flight,D=120 mm

cover d'=20 mm

Effective depth d=100 mm

Width of the flight,b=1200mm

Main reinforcement of the flight

For M_X=4.65 kNm

b=1200 mm and d=100 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 12.45 * \frac{1+\frac{120}{1200}}{1.7} = 8.055kNm$

$M_{ue} = M_X + M_t = 4.65 + 8.055 = 12.7kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1200*100}{2*415} * [1 - \sqrt{1 - \frac{4.6*12.7*10^6}{20*1200*100^2}}] = 376mm^2$

Provide 5 No.s of 10 mm dia bars.

For M_Y=65.6 kNm

D=1.2 m and b=120 mm

d'=50 mm and d=1150 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} M_t = 12.45 * \frac{1+\frac{1200}{120}}{1.7} = 80.56kNm$

$M_{ue} = M_X + M_t = 65.6 + 80.56 = 146.16kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{(1 - \frac{4.6*M_u}{f_{ck}*b*d^2})}]$

$= \frac{20*1200*100}{2*415} * [1 - \sqrt{(1 - \frac{4.6*146.16*10^6}{20*120*1150^2})}] = 373mm^2$

Provide 2 No.s of 12 mm and 2 No.s of 10 mm dia bars are provided on

each vertical face.

4.2.5 Transverse reinforcement of the flight

Equivalent shear force

$V_{ue} = V_u + 1.6 * \frac{T_u}{b} = 1.545 + 1.6 * \frac{12.45}{1.2} = 18.545kN$

Equivalent nominal shear stress

$\tau_{ve} = \frac{V_{ue}}{bd} = \frac{18.545*1000}{1200*100} = 0.151MPa$

$p_t = \frac{A_{st}*100}{bd} = \frac{393*100}{1200*100} = 0.33$

From Is 456-2000,pg.84,Table 23,

τ_c=0.2456 MPa

So,τ_c¿τ_{ve}

Hence, it is safe.

However, nominal torsion/shear reinforcement shall be provided in the form of closed stirrups with spacing not exceeding the smallest of the following.

x=120-20-20-10-10-10=50 mm

y=1200-50-50-10-10-10=1070 mm

Spacing should be less than

a. $(x + y)/4$=280mm

b. 300 mm

Provide 10 mm dia @ 280mm c/c spacing.

4.2.6 Main and distribution reinforcement of landing

M_u=18.2 kNm

Width of the landing, g=1.1 m

Depth of the landing, D=150 mm

Effective dept of the landing, d=130 mm

$\frac{M_u}{b*d^2} = \frac{18.2*10^6}{1200*130^2} = 0.9$

For M-20 grade concrete and Fe-415 grade of steel,

From SP-16,

$\%p_t = 0.264\%$

$A_{st} = \frac{1000*130*0.264}{100} = 343mm^2$

Provide 10 mm dia.@ 200 mm c/c spacing as main reinforcement.

Distribution steel

$A_{st} = 0.0012 * b * D = 0.0012 * 1100 * 150 = 198mm^2$

Provide 10 mm dia.@ 200 mm c/c spacing at near supports and 10 mm dia.@300 mm c/c as distribution reinforcement.

The reinforcement detailing is shown in the **Fig.4.3** and **Fig.4.4**.

4.3 Design of the Horse shoe type of stairs

4.3.1 Dimensions of the Horse shoe type of stairs

h_L=4.2m

$2 * n = 26$

R=165mm and T=300 mm

α=28'36'

b=1.4 m

r=1.2m

l_l=2*n*T=26*300=7.8 m

$2 * \psi$=210'

$arclength = 2 * \pi * R * \frac{2*\phi}{360}$=4.4m

$l = \frac{7.8-4.4}{2} = 1.7m$

d=160mm

4.3.2 Load calculation

W=Total weight of one flight

=$Selfwt. + Wt.ofthesteps$

=$0.165 * 25 + \frac{1}{2} * 0.165 * 25$

=$7.05kN/m^2$

Live Load=5 kN/m^2 for both flights and landing

Total load=$7.05 + 5 = 12.05kN/m^2$

$w = 12.05 * 1.4 = 16.87kN/m$

4.3.3 Design results of the Fuchssteiner's method

(1.) Flight section

At the intersection of flight and landing

Flexural moment M_X=27.7 kNm

Torsional moment M_Z=36.1 kNm

Flexural moment M_Y=66.62 kNm

Shear force V_u=21.61 kN

M_X=1.5*27.70=41.55 kNm

M_Y=1.5*66.62=99.93 kNm

M_Z=1.5*36.10=54.15 kNm

V_u=1.5*21.61=32.415 kN

At the flight supports

Flexural moment M_X=39.88 kNm

Torsional moment M_Z=36.1 kNm

Flexural moment M_Y=33.83 kNm

Shear force V_u=41.71 kN

M_X=1.5*39.88=59.82 kNm

M_Y=1.5*33.83=50.74 kNm

M_Z=1.5*36.10=54.15 kNm

V_u=1.5*41.71=62.66 kN

(2.) Landing section

Flexural moment M_X=32.4 kNm

M_X=1.5*32.4=48.6 kNm

4.3.4 Main and Transverse reinforcement of the flight

Overall thickness of the flight,D=165 mm

cover d'=20 mm

Effective depth d=145 mm

Width of the flight,b=1400mm

Main reinforcement of the flight at the intersection of the landing and the

flight

For M_X=41.55 kNm

b=1400 mm and d=145 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 54.15 * \frac{(1+\frac{165}{1400}}{1.7} = 35.61kNm$

$M_{ue} = M_X + M_t = 41.55 + 35.61 = 77.16kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1400*145}{2*415} * [1 - \sqrt{1 - \frac{4.6*77.16*10^6}{20*1400*145^2}}] = 1809mm^2$

Provide 10 No.s of 16 mm dia bars at top and bottom.

For M_Y=99.93 kNm

D=1.4 m and b=165 mm

d'=50 mm and d=1350 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 54.15 * \frac{1+\frac{1400}{165}}{1.7} = 302.12kNm$

$M_{ue} = M_X + M_t = 99.93 + 302.12 = 402.1kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{(1 - \frac{4.6*M_u}{f_{ck}*b*d^2})}]$

$= \frac{20*165*1350}{2*415} * [1 - \sqrt{(1 - \frac{4.6*402.1*10^6}{20*165*1350^2})}] = 900mm^2$

Provide 2 No.s of 20 mm and 2 No.s of 16 mm dia bars are provided on each vertical face.

Transverse reinforcement of the flight at the intersection of the landing and the flight

V_u=18.3 kN

Equivalent shear force

$V_{ue} = V_u + 1.6 * \frac{T_u}{b} = 18.3 + 1.6 * \frac{54.15}{1.4} = 80.2kN$

Equivalent nominal shear stress

$\tau_{ve} = \frac{V_{ue}}{bd} = \frac{80.2*1000}{1400*145} = 0.39MPa$

$p_t = \frac{A_{st}*100}{bd} = \frac{2011*100}{1400*145} = 1.00$

From Is 456-2000,pg.84,Table 23,

τ_c=0.395 MPa

So,$\tau_c \geq \tau_{ve}$

Hence, it is safe.

However, nominal torsion/shear reinforcement shall be provided in the form of closed stirrups with spacing not exceeding the smallest of the following.

x=165-20-20-10-10-16=89 mm

y=1400-50-50-10-10-16=1264 mm Should be less than

a. $(x + y)/4$=339mm

b. 300 mm

Provide 10 mm dia @ 300mm c/c spacing

Main reinforcement of the flight at the flight support

For M_X=59.82 kNm

b=1400 mm and d=145 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 54.15 * \frac{(1+\frac{165}{1400}}{1.7} = 35.61kNm$

$M_{ue} = M_X + M_t = 59.82 + 35.61 = 95.43kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1400*145}{2*415} * [1 - \sqrt{1 - \frac{4.6*95.43*10^6}{20*1400*145^2}}] = 2425mm^2$

Provide 13 No.s of 16 mm dia bars at top and bottom.

For M_Y=50.75 kNm

D=1.4 m and b=165 mm

d'=50 mm and d=1350 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 54.15 * \frac{1+\frac{1400}{165}}{1.7} M_t = 302.12kNm$

$M_{ue} = M_X + M_t = 50.75 + 302.12 = 352.87kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{(1 - \frac{4.6*M_u}{f_{ck}*b*d^2})}]$

$= \frac{20*165*1350}{2*415} * [1 - \sqrt{(1 - \frac{4.6*352.87*10^6}{20*165*1350^2})}] = 782mm^2$

Provide 1 No.s of 20 mm and 2 No.s of 16 mm dia bars are provided on each vertical face.

Transverse reinforcement of the flight at the flight support

V_u=62.565 kN

Equivalent shear force

$V_{ue} = V_u + 1.6 * \frac{T_u}{b} = 62.565 + 1.6 * \frac{54.15}{1.4} = 12.45kN$

Equivalent nominal shear stress

$\tau_{ve} = \frac{V_{ue}}{bd} = \frac{124.45*1000}{1400*145} = 0.613MPa$

$p_t = \frac{A_{st}*100}{bd} = \frac{2614*100}{1400*145} = 1.30$

From Is 456-2000,pg.84,Table 23,

τ_c=0.426 MPa

So,τ_{ve} τ_c

So, section is acceptable with shear reinforcement.

Consider transverse reinforcement considering 10 mm dia.($A_{sv} = 157mm^2$).

Minimum reinforcement to be provided is given by,

$A_{sv} = \frac{(\tau ve - \tau c)*S_v*b}{0.87*f_y}$

Thus,

$S_v = \frac{0.87*f_y*A_{sv}}{(\tau_{ve} - \tau c)*b} = \frac{0.87*415*157}{0.187*1400} = 217mm$

However, nominal torsion/shear reinforcement shall be provided in the form of closed stirrups with spacing not exceeding the smallest of the following.

x=165-20-20-10-10-16=89 mm and y=1400-50-50-10-10-16=1264 mm

Spacing should be less than

a. (x+y)/4=339mm

b. 300 mm

c. 89 mm

d. 0.75*d=0.75*145=109mm

Provide 10 mm dia @ 100 mm near support and increase the spacing to 300 mm c/c spacing from support

4.3.5 Main and distribution reinforcement in the landing

For M_X=48.6 kNm

b=1400 mm and d=145 mm

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1400*145}{2*415} * [1 - \sqrt{1 - \frac{4.6*48.6*10^6}{20*1400*145^2}}] = 1039mm^2$

Provide 10 No.s of 16 mm dia bars at top and bottom and 2 No.s of 20mm and 2 No.s of 16 mm dia. of bars at each vertical face.

The reinforcement detailing of the horse shoe type of stairs is as shown in **Fig.4.5**.

4.4 Design of the L type of stairs

4.4.1 Dimensions of the L type of stairs

Width of the flight =b=1.1m

Depth of the flight =Df=120mm Height of the one storey=H=1.3m

No. of risers =8

Riser=150mm and Tread =300mm

α =26'34'

Length of the flight =L=2.6m

$r = \frac{1.1}{2} + 0.2 = 0.75m$

Depth of the landing =Dl=150mm

4.4.2 Design moment and shear force of flight and landing

(1.) Flight section

Flexural moment M_X=1.6484 kNm

Torsional moment M_Z=6.838 kNm

Flexural moment M_Y=30.6 kNm

Shear force V_u=0.567 kN

M_X=1.5*1.6484=2.4726 kNm

M_Y=1.5*30.6 =45.9 kNm

M_Z=1.5*6.838 =10.257 kNm

V_u=1.5*0.567 =0.85 kN

(2.) Landing section

Flexural moment M_X=5.343 kNm

Torsional moment M_Z=21.5 kNm

Flexural moment M_Y=5.343 kNm

Shear force V_u=0 kN

M_X=1.5*5.343=8.0145 kNm

M_Y=1.5*21.5 =32.25 kNm

M_Z=1.5*5.343=8.0145 kNm

V_u=1.5*0 =0 kN

4.4.3 Main reinforcement of the flight

Overall thickness of the flight,D=120 mm

cover d'=20 mm

Effective depth d=100 mm

Width of the flight,b=1100mm

Main reinforcement of the flight

At the intersection of the landing and the flight

For M_X=2.4726 kNm

b=1100 mm and d=100 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 10.257 * \frac{1+\frac{120}{1100}}{1.7} = 6.7kNm$

$M_{ue} = M_X + M_t = 2.4726 + 6.7 = 9.2kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1100*100}{2*415} * [1 - \sqrt{1 - \frac{4.6*9.2*10^6}{20*1100*100^2}}] = 269mm^2$

Provide 4 No.s of 10 mm dia bars at top and bottom.

For M_Y=45.9 kNm

D=1.1 m and b=120 mm

d'=50 mm and d=1050 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 10.257 * \frac{1+\frac{1100}{120}}{1.7} = 61.34kNm$

$M_{ue} = M_X + M_t = 45.9 + 61.34 = 107.24kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{(1 - \frac{4.6*M_u}{f_{ck}*b*d^2})}] =$

$\frac{20*120*1050}{2*415} * [1 - \sqrt{(1 - \frac{4.6*107.24*10^6}{20*120*1050^2})}] = 298mm^2$

Provide 4 No.s of 10 mm bars are provided on each vertical face.

4.4.4 Transverse reinforcement of the flight

V_u=0.85 kN

Equivalent shear force

$$V_{ue} = V_u + 1.6 * \frac{T_u}{b} = 0.85 + 1.6 * \frac{10.257}{1.1} = 15.77kN$$

Equivalent nominal shear stress

$\tau_{ve} = \frac{V_{ue}}{bd} = \frac{15.77*1000}{1100*100} = 0.144MPa$

$p_t = \frac{A_{st}*100}{bd} = \frac{314*100}{1100*100} = 0.3$

From Is 456-2000,pg.84,Table 23,

τ_c=0.236 MPa

So,$\tau_c \quad \tau_{ve}$

Hence, it is safe.

However, nominal torsion/shear reinforcement shall be provided in the form of closed stirrups with spacing not exceeding the smallest of the following.

x=120-20-20-10-10-10=50 mm and y=1100-50-50-10-10-10=970 mm

Spacing should be less than

a. (x+y)/4=255mm

b. 300 mm

c. 0.75*d=75 mm

d. 50 mm

Provide 10 mm dia @ 100 mm c/c near support and increase the spacing to from 250 mm c/c spacing.

4.4.5 Main reinforcement of the landing

Overall thickness of the flight,D=150 mm

cover d'=20 mm

Effective depth d=130 mm

Width of the flight,b=1300 mm

Main reinforcement of the landing

For M_X=8.01 kNm

b=1300 mm and d=130 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 8.01 * \frac{(1+\frac{150}{1300}}{1.7} = 5.255kNm$

$M_{ue} = M_X + M_t = 8.01 + 5.255 = 13.265kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{1 - \frac{4.6*M_u}{f_{ck}*b*d^2}}]$

$= \frac{20*1300*130}{2*415} * [1 - \sqrt{1 - \frac{4.6*13.265*10^6}{20*1300*130^2}}] = 293mm^2$

Provide 4 No.s of 10 mm dia bars at top and bottom.

For M_Y=32.25 kNm

D=1.3 m and b=150 mm

d'=50 mm and d=1250 mm

Equivalent bending moment

$M_t = T_u * \frac{1+\frac{D}{b}}{1.7} = 8.01 * \frac{1+\frac{1300}{150}}{1.7} = 45.55kNm$

$M_{ue} = M_X + M_t = 32.25 + 45.55 = 77.8kNm$

$A_{st} = \frac{f_{ck}*b*d}{2*f_y} * [1 - \sqrt{(1 - \frac{4.6*M_u}{f_{ck}*b*d^2})}]$

$= \frac{20*150*1250}{2*415} * [1 - \sqrt{(1 - \frac{4.6*77.8*10^6}{20*150*1250^2})}] = 176mm^2$

Provide 3 No.s of 10 mm dia bars are provided on each vertical face.

4.4.6 Transverse reinforcement of the landing

V_u=0 kN

Equivalent shear force

$V_{ue} = V_u + 1.6 * \frac{T_u}{b} = 0 + 1.6 * \frac{8.01}{1.3} = 9.86kN$

Equivalent nominal shear stress

$\tau_{ve} = \frac{V_{ue}}{bd} = \frac{9.86*1000}{1300*130} = 0.06MPa$

$p_t = \frac{A_{st}*100}{bd} = \frac{314*100}{1300*130} = 0.2$

From Is 456-2000,pg.84,Table 23,

τ_c=0.2 MPa

So,τ_c ¿ τ_{ve}

Hence, it is safe.

However, nominal torsion/shear reinforcement shall be provided in the form of closed stirrups with spacing not exceeding the smallest of the following.

x=150-20-20-10-10-10=80 mm and y=1300-50-50-10-10-10=1170 mm

Spacing should be less than

a. $(x + y)/4$=313mm

b. 300 mm

Provide 10 mm dia @ 300mm c/c spacing

Reinforcement detailing of L shape type of stairs is as shown below **Fig.4.6**

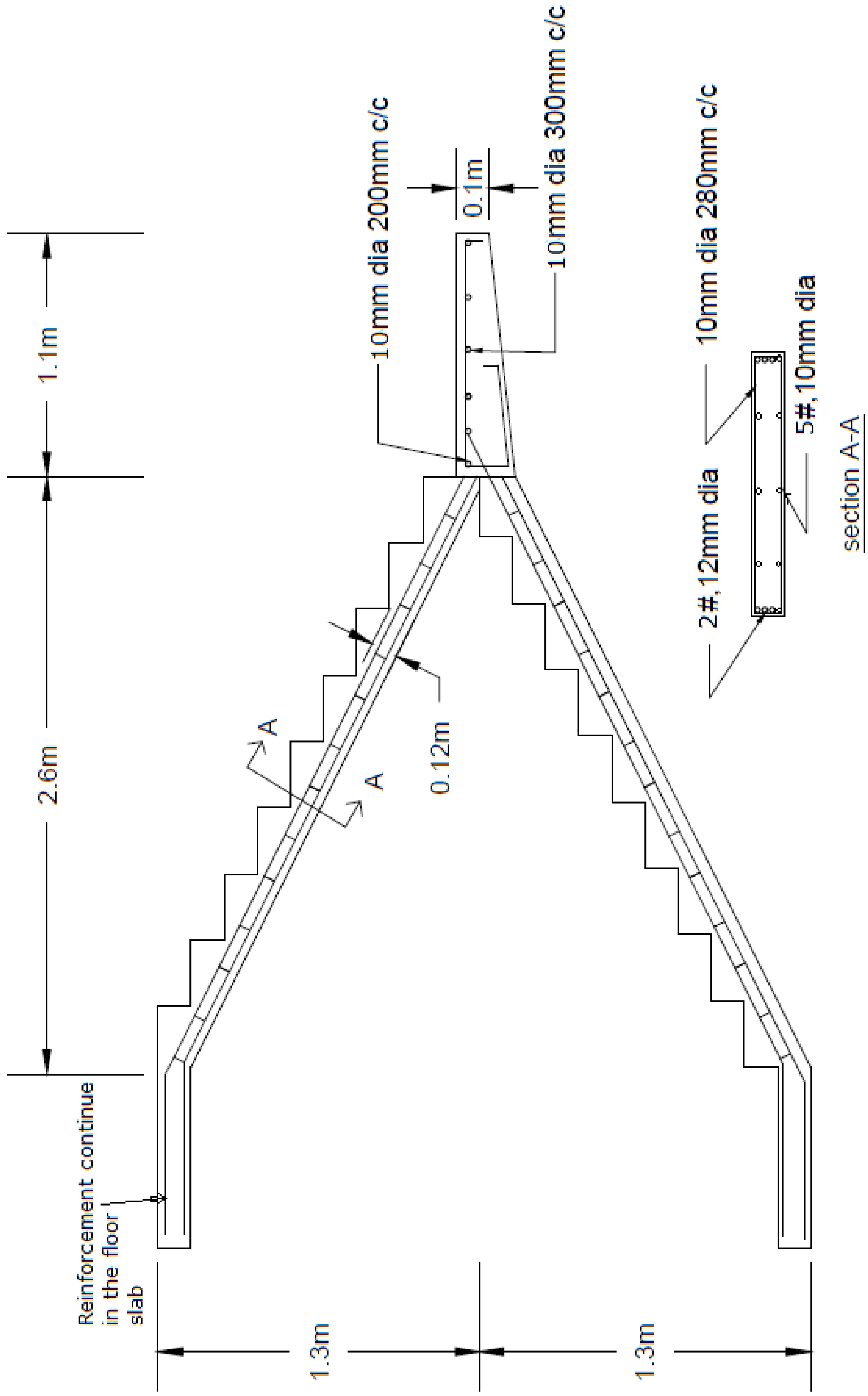

Figure 4.3: Reinforcement detailing of free standing stairs

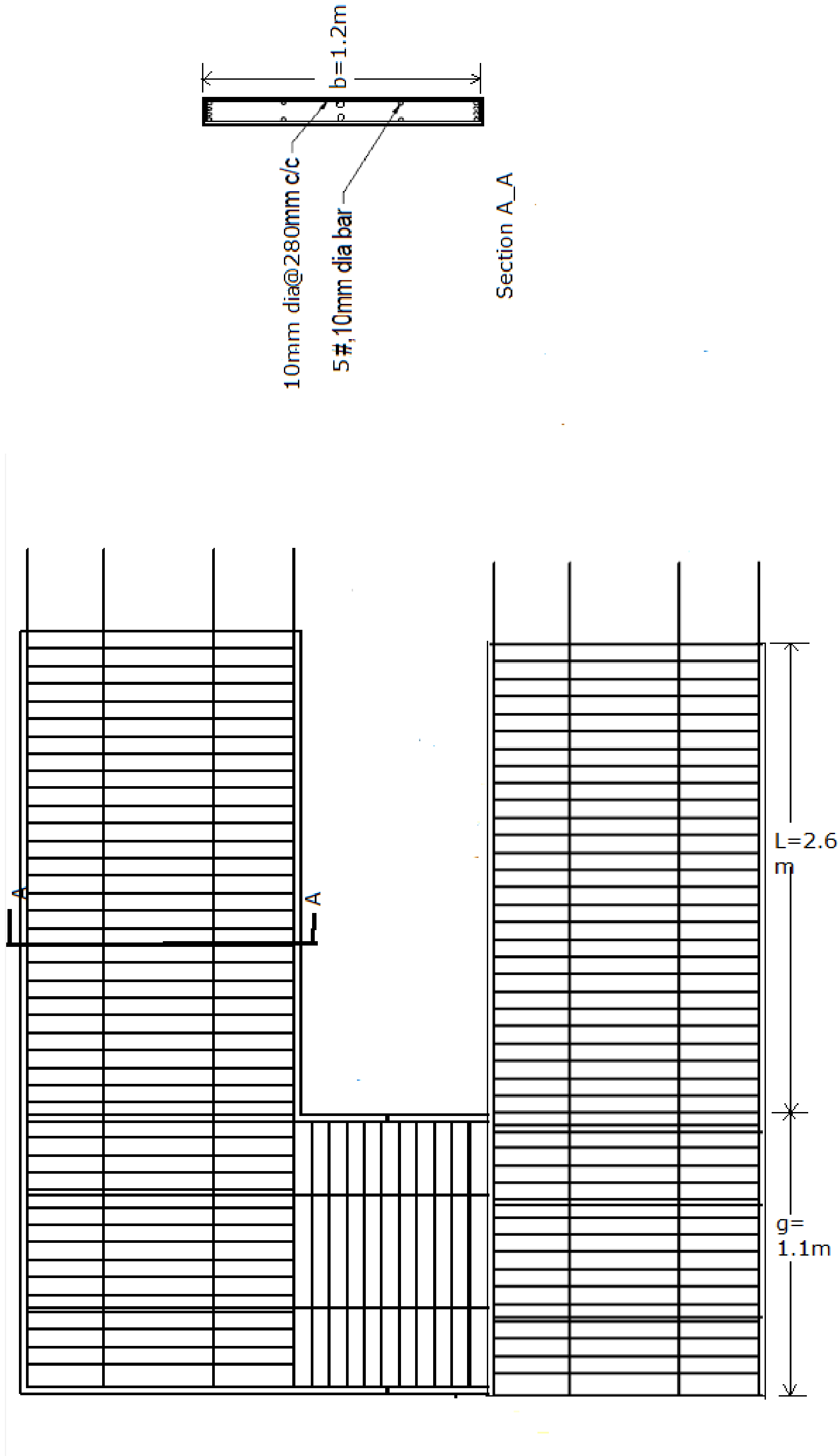

Figure 4.4: Reinforcement detailing of free standing stairs

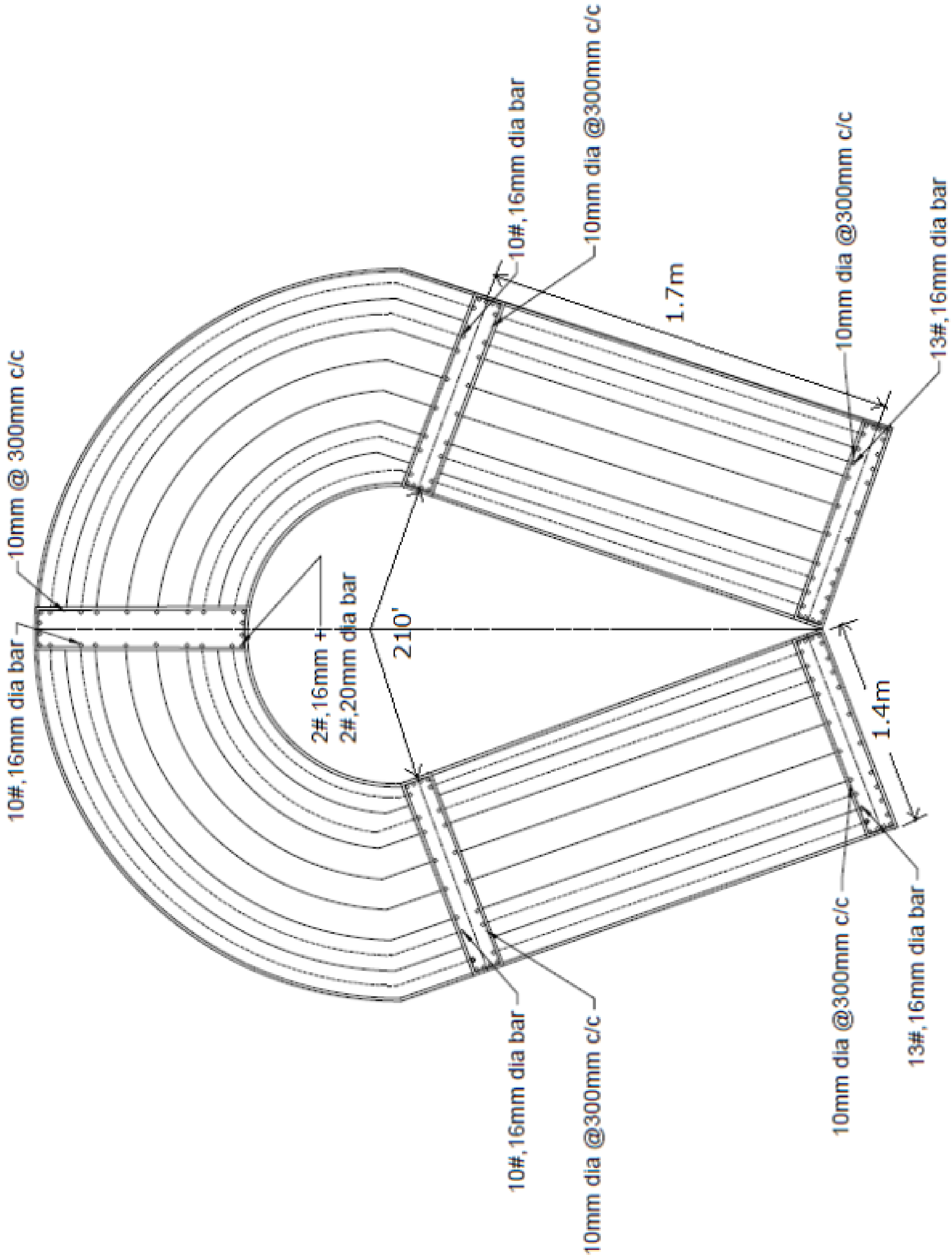

Figure 4.5: Reinforcement detailing of Horse shoe type of stairs

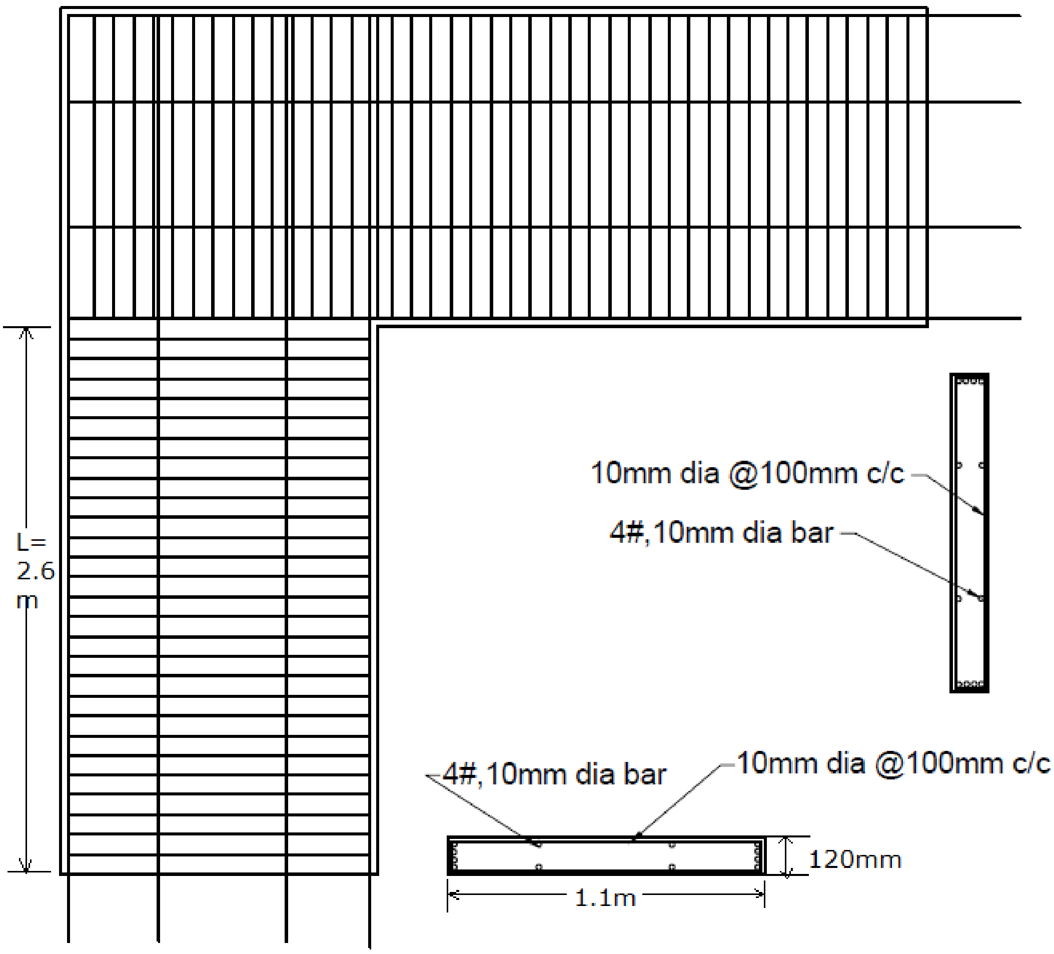

Figure 4.6: Reinforcement detailing of ELL type of stairs

Chapter 5

Analysis of FSS using different support conditions

5.1 General

In recent years the free standing staircase has become quite popular. Many variations of this type of staircase are possible and some will be described herein. A discussion of the factors that affect the behavior of the staircase is presented together with an abbreviated design example which illustrates only those calculations which are peculiar to this type of design.Once the forces and moments are determined, the calculations are generally routine.

5.2 Discussion of support conditions using Gould's method

The behavior of the staircase is greatly influenced by the support at point A in **Fig.5.1**.If a horizontal thrust can be developed at point A, the moment at the base is quite small;however, if only a vertical reaction can be developed, the moment at the base is greatly increased. The various possibilities are illustrated in **Fig.5.2**.If the horizontal reaction is to be developed, consideration must be given to the construction procedure as well as the final finished stair to insure that the structure is never free standing.The great variation in base moment between the two conditions is illustrated in the design example.

To reduce the moment at the base, it seems logical to take advantage of the large

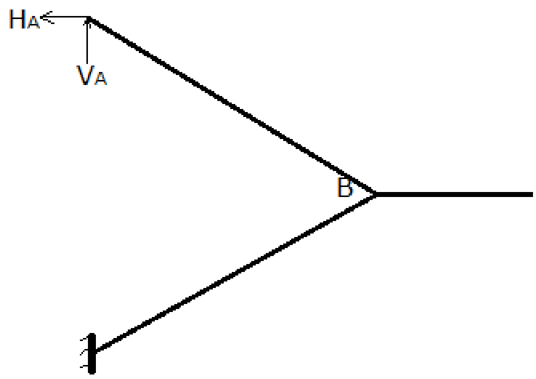

Figure 5.1: Vertical and Horizontal reaction at point A

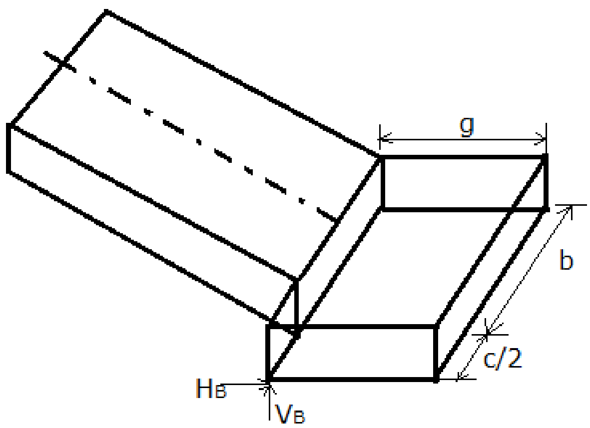

Figure 5.2: Torsion at the intermediate landing

lever arm afforded by the stair height and, thereby, reduce the overturning moment by developing the horizontal reaction.However, there are certain situations in which this is not practical such as the case when the upper end of the staircase rests on a spandrel beam which is designed to resist only vertical loads and is frequently restricted in size because of architectural considerations.The example illustrates the treatment of this type of support.

5.2.1 Analysis

For the analysis the staircase is considered as a frame with the moment at the intermediate landing being transferred between the legs by torsion developed through the landing.The method of analysis used depends again on the support conditions at the upper landing.If the horizontal reaction can be developed, classical moment distribution can be used with point B **Fig5.1** considered fixed against translation.On the other hand if only the vertical reaction can be developed, point B can translate.To avoid a deflection correction to the moment distribution procedure ,the problem can be solved by Castigliano's theorem of strain energy.This is the approach used in the example.For a fixed support at point A, moment distribution can again be used and for a completely free standing stair, the moment may be solved by statics.For the case where the upper support is flexible, the structure may be solved by Castigliano's theorem as illustrated in the examples.

5.2.2 Torsion at intermediate landing

As shown in **Fig.5.2**,the torsional moment in the landing at the junction of the upper and lower legs can be found by statics.The maximum torsional shear stress can be approximated by the formula

$$\omega = \frac{T}{\alpha * b * h^2}$$

The coefficient α is itself proportional to b/h but approaches a limit of 0.333 for large values of b/h.There is considerable doubt as to the distribution of shear stress on the landing may be regarded as effective. For the design example, the reinforcing will be proportioned on the basis of the entire cross section resisting the torsion.

5.2.3 Additional moments on upper and lower legs

For the staircase to behave as a frame, vertical and horizontal forces (H_B and V_B) must be transmitted between the legs of the staircase through the landing. Since these forces act on a section through the center of the landing parallel to the longitudinal axis of the legs, they are eccentric with respect to the legs and produce bending in the plane of the legs, as well as torsion of the legs. The staircase should be investigated for the effects of these additional torsional and bending moments as well as primary moment and axial force acting on the legs.
In as much as the legs are generally quite stiff in their own plane and have considerable torsional resistance, these additional moments should have only a minor effect on the design.

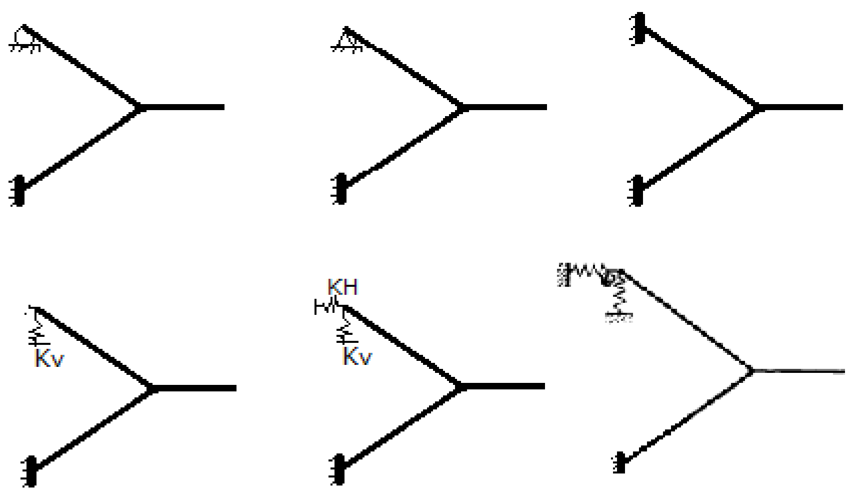

Figure 5.3: Different support conditions applied on the free standing stairs

5.3 Design examples

The staircase which is shown in **Fig.5.4**,will be studied for the various support conditions at point A mentioned in **Fig.5.3**.The dimensions of the stairs are given in **Fig.5.4**.

Case A: Vertical reaction at point A

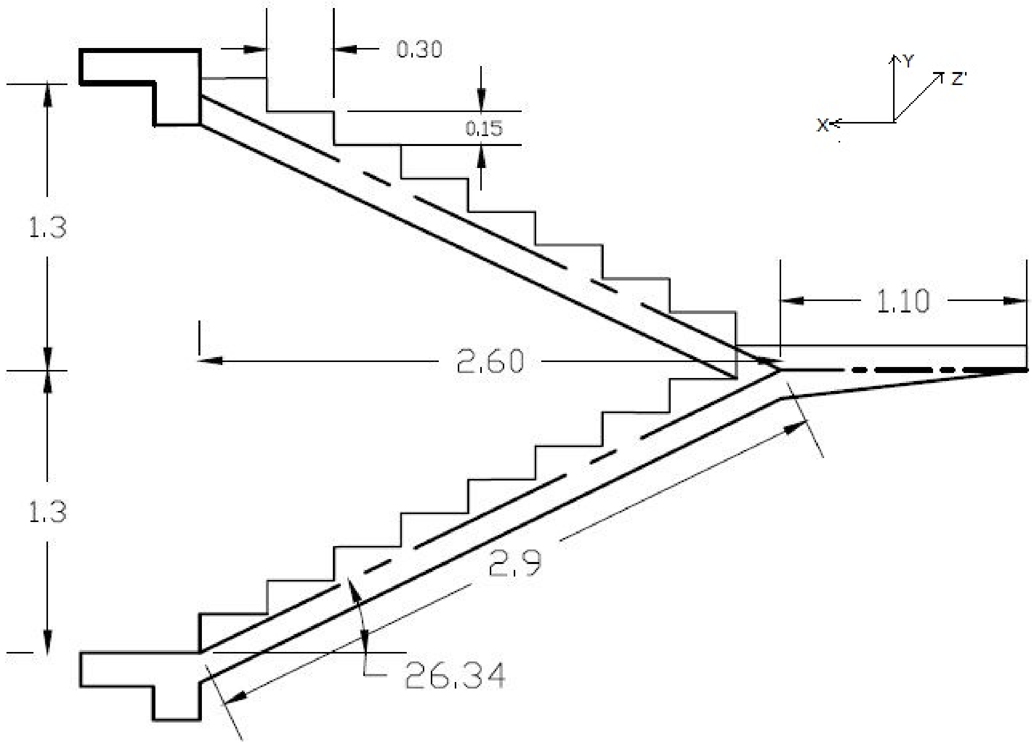

Figure 5.4: Elevation of the stair

The forces acting on the structure are shown in **Fig.5.5**.Castigliano's theorem states that

$$\frac{\partial U}{\partial V_A} = \Delta V_A \tag{5.1}$$

Where,V_A=Vertical reaction at point A, and

ΔV_A= Vertical deflection at point A

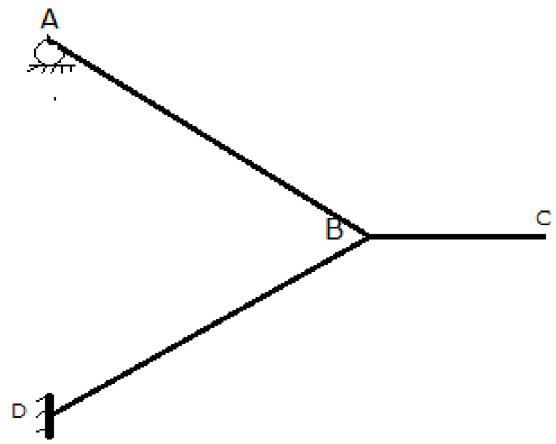

Figure 5.5: Vertical reaction at point A

Since the vertical deflection at point A equals zero:

$$\frac{\partial U}{\partial V_A} = 0 \tag{5.2}$$

Only the strain energy due to bending is considered.

$$U = \int \frac{M^2 dx}{2EI} \tag{5.3}$$

$$\frac{\partial U}{\partial V_A} = \int \frac{M dx}{EI} * \frac{\partial M}{\partial V_A} \tag{5.4}$$

To simplify the algebraic expressions the integration will be performed with the formulas for moment and the limits based on the horizontal dimensions as shown in **Fig.5.6**. After integration and simplification of the expressions, they will be multiplied by the ratio of the actual length of the member to the horizontal projection(F) to obtain the actual value of the integral. For the case A this is not necessary since the factor will not affect V_A.The value of F is computed below.
Integration factor

$$F_1 = \frac{2.9}{2.6} = 1.12 \quad F_2 = \frac{2.9}{2.6} = 1.12$$

Substituting the expressions for bending moment and the partial derivatives into

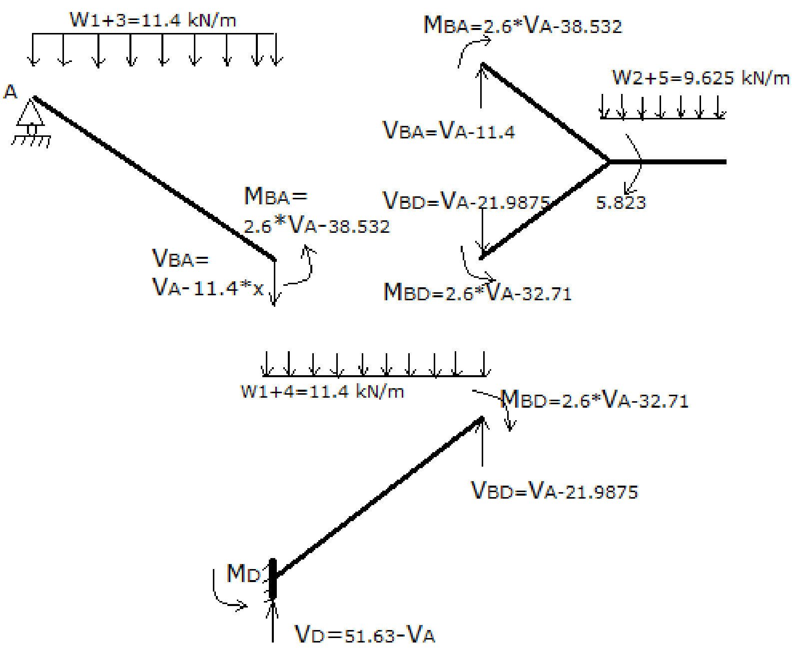

Figure 5.6: Moments and reactions

Eq.5.4.

$EI\frac{\partial U}{\partial V_A} = \int_0^{\frac{26}{10}}[(V_A-2.85x)(x)(x)+(V_A(2.6-x)-32.71+40.2275x+2.85x^2)(2.6-x)]dx$

$11.72 * V_A - 36.13 = 0$

$V_A = 3.083$

From the equations given in **Fig.5.6**.

$M_D = 63kNm(anticlockwise)$

$V_D = 69.87 - V_A = 69.87 - 3.083 = 66.8kN$

$M_{BA} = 2.6 * V_A - 38.532 = -25.423kNm(clockwise)$

$M_{BD} = 2.6 * V_A - 32.71 = -19.6kNm(anticlockwise)$

$T = \frac{25.423+19.6}{2} = 22.51kNm$

Case B:Horizontal and Vertical reaction at point A

The moment distribution procedure is used in this solution in which is illustrated in

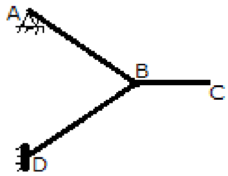

Figure 5.7: Hinged support at point A

Fig.5.3 and **Fig.5.8**.To solve for H_A and V_A moments are taken about Point B in **Fig.5.9**and about Point D in **Fig.5.10**.

$\sum M_B = 29.64 * 1.3 - 5.233 - 2.6 * V_A + 1.3 * H_A = 0$

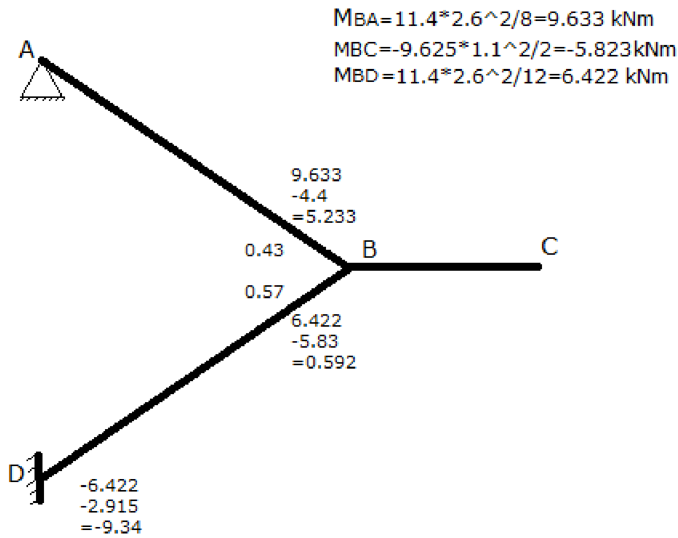

Figure 5.8: Moment distribution

$\sum M_D = 2 * 29.64 * 1.3 + 10.5875 * (\frac{1.1}{2} + 2.6) - 9.34 - 2.6 * H_A = 0$

By solving above equations,

$H_A = 38.875kN$

$V_A = 32.25kN$

From the **Fig.5.9** and **Fig.5.10**,

$M_D = 8.98kNm(anticlockwise)$

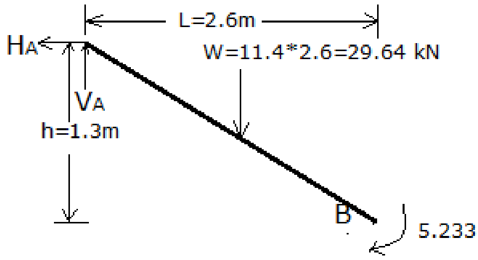

Figure 5.9: Moment at point B

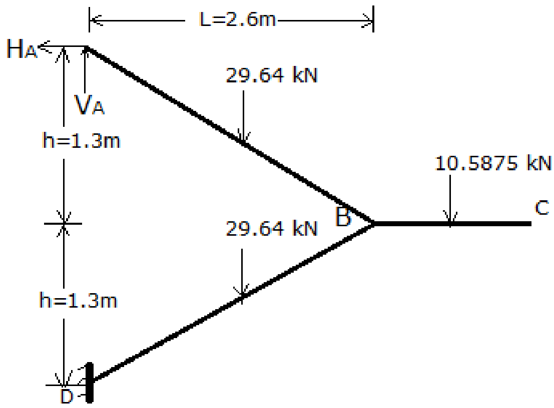

Figure 5.10: Moment at point D

$V_D = 69.87 - V_A = 69.87 - 32.25 = 37.62kN$

$M_{BA} = 5.233kNm(clockwise)$

$M_{BD} = 0.592kNm(clockwise)$

$T = \frac{5.233-0.592}{2} = 2.91kNm$

Case C: Fixed at point A

The moment distribution procedure is used in this solution in which is illustrated in **Fig.5.3** and **Fig.5.12**.To solve for H_A and V_A moments are taken about Point B in **Fig.5.13** and about Point D in **Fig.5.14**.

$\sum M_B = 29.64 * 1.3 - 2.911 - 2.6 * V_A + 1.3 * H_A + 6.423 = 0$

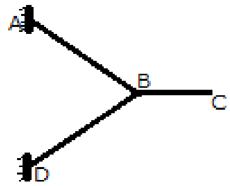

Figure 5.11: Fixed support at point A

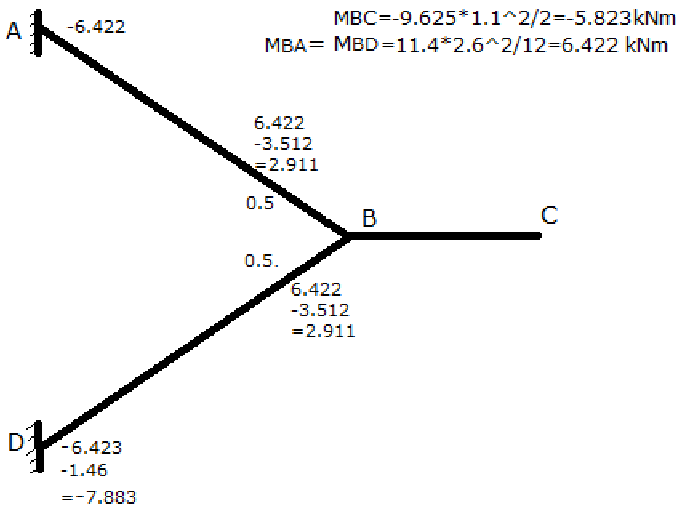

Figure 5.12: Moment distribution

$\sum M_D = 2 * 29.64 * 1.3 + 10.5875 * (\frac{1.1}{2} + 2.6) - 7.883 - 2.6 * H_A = 0$

By solving above equations,

$H_A = 39.43kN$

$V_A = 35.89kN$

From the **Fig.5.13** and **Fig.5.14**,

$M_D = 7.883kNm(anticlockwise)$

$V_D = 69.87 - V_A = 69.87 - 35.89 = 33.98kN$

$M_{BA} = 2.911kNm(clockwise)$

$M_{BD} = 2.911kNm(clockwise)$

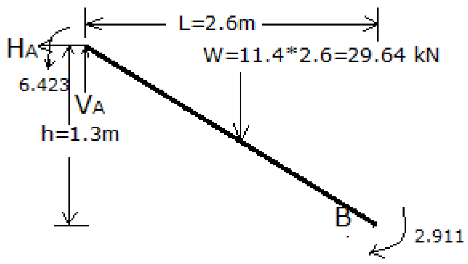

Figure 5.13: Moment at point B

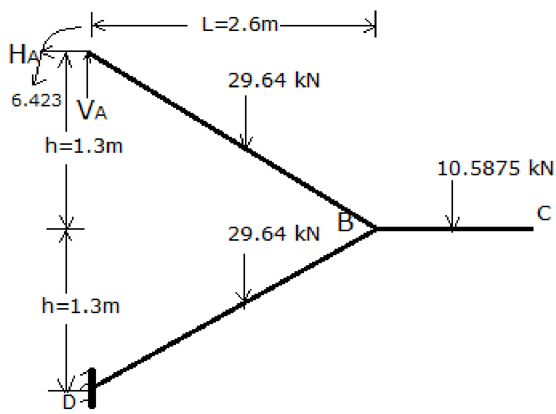

Figure 5.14: Moment at point D

$T = \frac{2.911-2.911}{2} = 0kNm$

Case D: Vertical reaction of flexible support at point A

This case is similar to case A except that the support at point A is flexible as shown in **Fig.5.3**. Assume that the support may be represented by a spring such that

$$K_V = \frac{V_A}{\Delta V_A} \tag{5.5}$$

$$\frac{\partial U}{\partial V_A} = -\frac{V_A}{K_V} \tag{5.6}$$

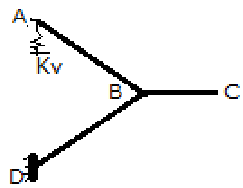

Figure 5.15: Vertical spring constant at point A

Where,

V_A=Vertical reaction at point A

ΔV_A= Vertical deflection at point A

K_V=Vertical spring constant

$$\frac{\partial U}{\partial V_A} = \int \frac{Mdx}{EI} * \frac{\partial M}{\partial V_A}$$

From the **Fig.5.6**,

$1.12 * (11.72 * V_A - 36.13) = -\frac{V_A * EI}{K_V}$

$V_A(13.13 + \frac{EI}{K_V}) = 40.5$

$V_A = \frac{40.5}{(13.13 + \frac{EI}{K_V})}$

Integration factor is multiplied to obtain the actual value of the integral.

Case E: Horizontal and Vertical reaction of flexible support at point A

This case is similar to case B.However, since the deflections are involved the solving method is different. The supports will be treated as K_H and K_V as shown in **Fig.5.16**.

As per Castigliano's theorem,

$$\frac{\partial U}{\partial V_A} = -\frac{V_A}{K_V} \tag{5.7}$$

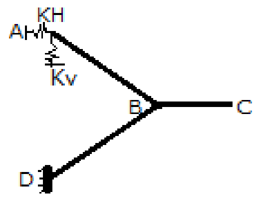

Figure 5.16: Horizontal and vertical spring constant at point A

$$\frac{\partial U}{\partial H_A} = -\frac{H_A}{K_H} \tag{5.8}$$

The expressions for bending moment and partial derivatives are as shown in **Fig.5.17**.

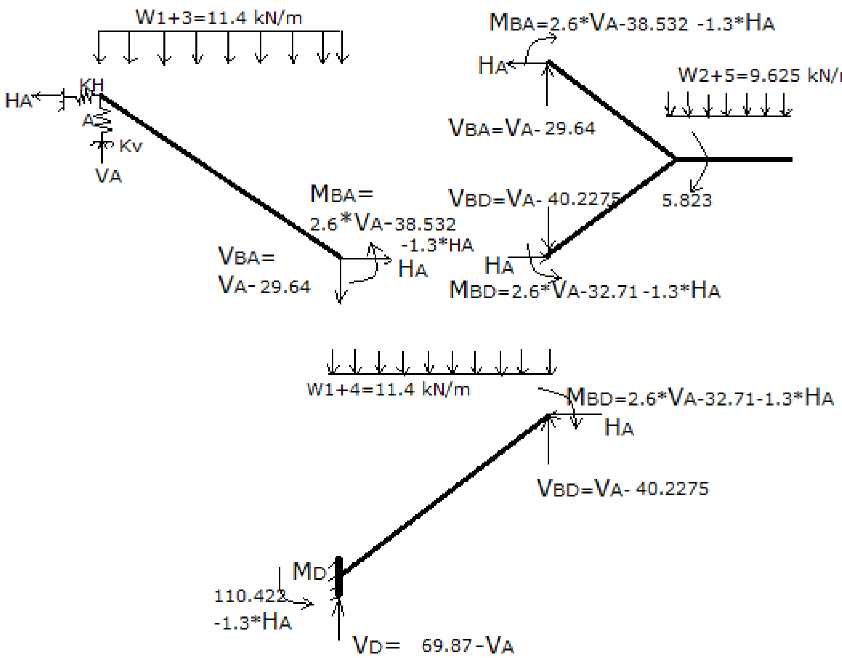

Figure 5.17: Moment and reactions due to spring constants at point A

From Eq. 4.7 and 4.8

$EI\frac{\partial U}{\partial V_A} = \int_0^{\frac{26}{10}}[(V_A - 0.5H_A - 2.85x)(x)(x) +$

$(V_A(2.6 - x) - 32.71 + 40.2275x + 5.7x^2 - 1.3H_A - 0.9H_Ax)(2.6 - x)dx]$

$EI\frac{\partial U}{\partial H_A} = \int_0^{\frac{26}{10}}[(V_A - 0.5H_A - 2.85x)(x)(-0.5x) +$

$(V_A(2.6-x)-32.71+40.2275x+5.7x^2-1.3H_A-0.9H_Ax)(-1.3-0.9x)]dx$

From the above Equations,

$$1.12*\frac{1}{EI}*(11.72*V_A-9.96*H_A-36.47)=-\frac{V_A}{K_V}$$

$$V_A*C1=11.15*H_A+40.85$$

$$1.12*\frac{1}{EI}*(18.515*V_A-9.96*V_A-248.31)=-\frac{H_A}{K_H}$$

$$H_A*C2=11.16*V_A+278.11$$

Where,

$$C1=13.13+\frac{EI}{K_V}\quad C2=20.74+\frac{EI}{K_H}$$

Form the Above equations,

$$H_A=\frac{278.11*C1-455.89}{C1C2-124.434}$$

$$V_A=\frac{40.85*C2+3100.93}{C1C2-124.434}$$

Case F:Partial fixity at point A

If a restraint to rotation proportional to the angle of twist is assumed at point

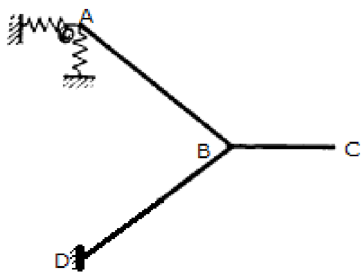

Figure 5.18: Partial fixity at point A

A($K_M=M/\phi$), the effect of the moment may be accounted for in a similar manner as the elastic deflections of the supports.The equations of case E **Fig.5.17**,may easily be modified by the addition of a $-M_A$ term to the moment expressions as shown in **Fig.5.19**.An additional equations is obtained from this condition.

$$\frac{\partial U}{\partial M_A} = -\frac{M_A}{K_M}$$

The three equations are then

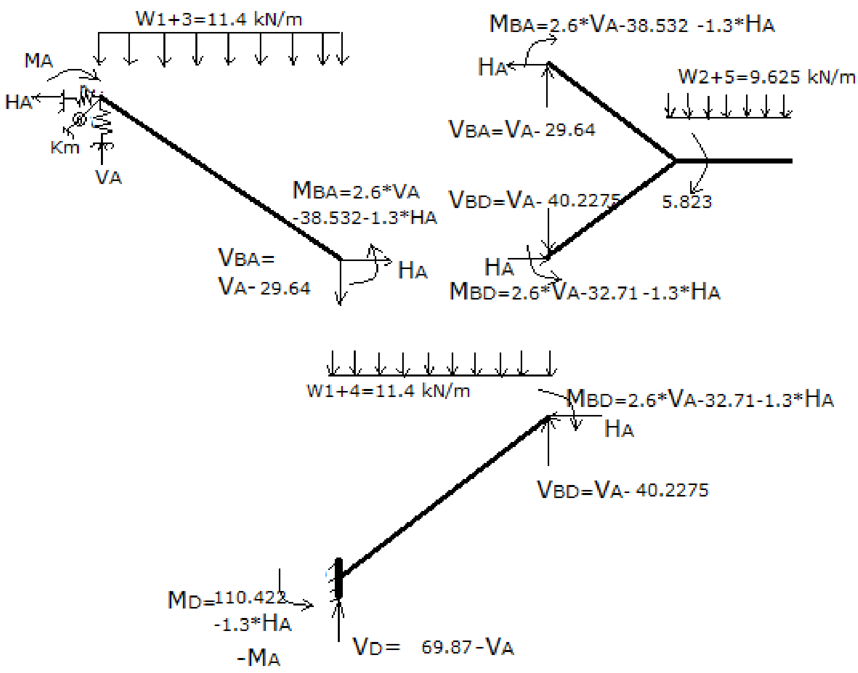

Figure 5.19: Moments and reactions due to partial fixity at point A

$EI\frac{\partial U}{\partial M_A} = \int_0^{\frac{26}{10}}[(V_A x - 0.5H_A x - 2.85x^2 - M_A)(-1) +$
$(V_A(2.6 - x) - 32.71 + 40.2275x + 5.7x^2 - 1.3H_A - 0.9H_A x - M_A)(-1)]dx$
$EI\frac{\partial U}{\partial V_A} = \int_0^{\frac{26}{10}}[(V_A x - 0.5H_A x - 2.85x^2 - M_A)(x) +$
$(V_A(2.6 - x) - 32.71 + 40.2275x + 5.7x^2 - 1.3H_A - 0.9H_A x - M_A)(2.6 - x)]dx$
$EI\frac{\partial U}{\partial H_A} = \int_0^{2.6}[(V_A x - 0.5H_A x - 2.85x^2 - M_A)(-0.5x) +$
$(V_A(2.6 - x) - 32.71 + 40.2275x + 5.7x^2 - 1.3H_A - 0.9H_A x - M_A)(-1.3 - 0.9x)]dx$
From the above Equations,

$$1.12 * \frac{1}{EI} * (11.72 * V_A - 9.96 * H_A - 6.76 * M_A - 36.47) = -\frac{V_A}{K_V} \tag{5.9}$$

$$V_A * C1 = 11.15 * H_A + 7.571 * M_A + 40.85$$

$$1.12 * \frac{1}{EI} * (18.515 * V_A - 9.96 * V_A + 8.112 * M_A - 248.31) = -\frac{H_A}{K_H} \tag{5.10}$$

$$H_A * C2 = 11.16 * V_A - 9.085 * M_A + 278.11$$

$$1.12 * \frac{1}{EI} * (11.72 * V_A - 9.96 * H_A - 36.47) = -\frac{M_A}{K_M} \tag{5.11}$$

$$M_A * C3 = 7.5712 * V_A - 9.085 * H_A + 57.043$$

Where,

$$C1 = 13.13 + \frac{EI}{K_V} \quad C2 = 20.74 + \frac{EI}{K_H} \quad C3 = 5.824 + \frac{EI}{K_M}$$

From the Above equations,

$$H_A = \frac{518.23 * C1 - 455.5 * C3 - 278.11 * C1C3 + 13935.36}{82.54 * C1 + 57.32 * C2 + 124.32 * C3 - C1C2C3 - 1534}$$

$$V_A = \frac{-40.85 * C2C3 - 3100.93C3 - 431.87 * C2 + 28728.62}{82.54 * C1 + 57.32 * C2 + 124.32 * C3 - C1C2C3 - 1534}$$

$$M_A = \frac{2526 * C1 - 309.2 * C2 - 57.043 * C1C2 - 12248.23}{82.54 * C1 + 57.32 * C2 + 124.32 * C3 - C1C2C3 - 1534}$$

Properties of supporting members

Supporting beam

For cases D, E and F a supporting beam is assumed with the following dimensions and section properties.

L=Length of the beam=5m

B=width of the beam=300mm

D=Depth of the beam=450mm

$E = 5000 * \sqrt{20} = 22360.7MPa \;\; G = \frac{E}{2*(1+\mu)} = \frac{22360.7}{2*(1+0.15)} = 9722MPa$

$I_{bx} = \frac{B*D^3}{12} = 2.278 * 10^9 mm^4 \;\; I_{by} = \frac{D*B^3}{12} = 1.0125 * 10^9 mm^4$

$K_H = \frac{48*E*I_{by}}{L^3} = 8693.84N/mm \;\; K_V = \frac{48*E*I_{bx}}{L^3} = 19560.06N/mm$

$K_M = \tau * \beta * D * B^3 * G$

$\tau = \frac{\phi}{L} = \frac{1}{5000}$

$\frac{D}{B} = 1.5$

$\beta = 0.196$

$K_M = 4630.4 * 10^6 N/mm$

g=Width of stair slab=1.1m

d=Depth of stair slab=0.15m

$I_L = \frac{g*d^3}{12} = 309.4 * 10^6 mm^4$

$C1 = 13.5m^3 \;\; C2 = 21.54m^3 \;\; C3 = 5.825m^3$

Case D

$V_A = \frac{40.5}{C1} = 3kN$

$V_D = 69.87 - V_A = 66.87kN$

$M_D = 63kNm(anticlockwise)$

$M_{BA} = 2.6 * V_A - 38.532 = -30.732kNm(clockwise)$

$M_{BD} = 2.6 * V_A - 32.71 = -24.91kNm(anticlockwise)$

$T = \frac{30.732+24.91}{2} = 27.821kNm$

Case E

$H_A = \frac{278.11*C1-455.89}{C1C2-124.434} = 25.31kN$

$V_A = \frac{40.85*C2+3100.93}{C1C2-124.434} = 23.93kN$

$V_D = 69.87 - V_A = 45.94kN$

$M_D = 110.422 - 1.3 * H_A = 77.519kNm(anticlockwise)$

$M_{BA} = 2.6 * V_A - 1.3 * H_A - 38.532 = -9.22(clockwise)kNm$

$M_{BD} = 2.6 * V_A - 1.3 * H_A - 32.71 = -3.395(anticlockwise)kNm$

$T = \frac{9.22+3.395}{2} = 6.31kNm$

Case F

$H_A = 23.21kN$

$V_A = 27.22kN$

$M_A = 8.96kNm$

Table I: Summary of Moment and Reactions

Case	V_A(kN)	H_A(kN)	M_A(kNm)	V_D(kN)	M_D(kNm)	M_{BA}(kNm)	M_{BD}(kNm)	T(kNm)
A	3.083	-	-	66.8	63	25.423	19.6	22.51
B	32.25	38.875	-	37.62	8.98	5.233	0.592	2.91
C	35.89	39.43	6.423	33.98	7.883	2.911	2.911	0
D	3	-	-	66.87	63	30.732	24.91	27.821
E	23.93	25.31	-	45.94	77.519	9.22	3.395	6.31
F	27.22	23.21	8.96	42.65	71.289	2.1	7.9	5

$V_D = 69.87 - V_A = 42.65kN$

$M_D = 110.422 - 1.3 * H_A - M_A = 71.289kNm(anticlockwise)$

$M_{BA} = 2.6 * V_A - 1.3 * H_A - 38.532 = 2.1kNm(clockwise)$

$M_{BD} = 2.6 * V_A - 1.3 * H_A - 32.71 = 7.9kNm(anticlockwise)$

$T = \frac{2.1+7.9}{2} = 5kNm$

In below table,the analysis results of Gould's method is above.

When the only vertical reaction is applied to the flight support at A **Fig.5.1**,the torsion in the landing is much more but in the case when the both horizontal and vertical reactions are to be applied to the flight support then the torsion can be reduced in larger no. of amount.It is to be because the lever arm effect afforded by the stair height so more of the stair height the more will be horizontal thrust so it will reduce the overturning moment of the stair.It is to be noted that in fully fixed condition,th effect of torsion is becoming zero it means that the it will be better option for support condition but that condition can not achieve as fully fixed support so it will be impractical condition.

Chapter 6

Summary and Conclusion

6.1 Summary

From the summary of results shown in **Table 3.1**, it is apparent that there were differences in values in the different type of methods.The main reason can be stated as the application of method is totally different from each other so the in every method there will be different assumptions were made.This type of structure is very highly complex structure which has six degree of indeterminacy and it seems to be impractical for solving therefore it is required to analyze the structure as for practical use, the application of a simple and approximate approach is necessary therefore the structure is required to be converted into statically determinate structure.

In Siev's method of analysis,the whole structure is assumed as two separate plates with one end is fixed and the other end has an imaginary support is considered.In this analysis, the loading condition is given as symmetrical.The slab moments are computed and the secondary stresses are computed from the result of the compatibility equations and also concluded as torsional moment is small.It is very difficult to solve when the two flights are having unequal flights.

In Cusens and Kuang's method,the whole structure is considered as a rigid frame and the flights and landing portion is considered as a beam and it is assumed that it is cut from the midsection of the whole structure and using strain energy method moment can be computed at specified point.In this analysis, the loading condition is given as symmetric condition.

In Fuchssteiner's method,the Whole structure is considered as a rigid frame except that the landing portion is considered as a horizontal bow girder and done the analysis using virtual work method.The loading condition is symmetrical.In this method,redundant are determined solving elastic equations and from the redundant the moment and reactions can be determined.

Simplified method is based on the Fuchssteiner's assumptions in which the ratio of moment of inertia in horizontal axis and M.I. in vertical axis of landing portion is considered as zero and analyze the structure and determined the moments and reactions.

Finite element analysis is done using SAP 2000 software,in which the structure is considered as a thick shell element and the loading are given as area load and do the analysis.

In Gould's method,the behavior of the staircase can be known as shown in **Table 5.1** using applying different support conditions.In which the torsion in the landing is to be considered as main objective.In which at different support condition the effect of torsion can be measured using Strain energy and moment distribution method is to be applied and finding out which is the most appropriate support condition.

6.2 Conclusion

Based on work carried out the following conclusions are to be made:

- Using simplified method the computation time can be saved as compare to other methods.

- The simplified method can be applied to calculate the bending moment,torsional moment and lateral moment and the forces for the design of the free standing stairs.
- The ratio of I_X/I_Y can be neglected without significant errors in calculating forces and moments.
- The torsional and bending moment, in Simplified method and finite element method are very close to each other.
- In the free standing stair,when the different types of support condition are to be applied then the torsion can be reduced by applying horizontal thrust at the flight support.In fixed condition at the flight support, the torsion is zero but is a impractical condition to achieve the fully fixed condition at the flight.
- In the design of the free standing stairs, at the intersection of the flight and landing the reinforcement bars spacing will be more to avoid the torsion of the landing.

6.3 Future Scope work

- To develop finite element analysis for Horse shoe and L shape type of structure.
- Testing can be carried out to know the actual behavior of the stairs.
- To know the behavior of the free standing stairs while applying the lateral loading.
- To know the behavior of the Horse shoe type of stair and l shape type of stair while applying the lateral loading.
- To do the parametric study of the Horse shoe type of stairs and know its actual behavior.

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