

Optimal Design of UPFC Based Damping Controller

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Optimal Design of UPFC Based Damping Controller

Major Project Report

*Submitted in Partial Fulfillment of the Requirements for the
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MASTER OF TECHNOLOGY IN ELECTRICAL ENGINEERING (Electrical Power System)

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Undertaking for Originality of the Work

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This is to certify that the Major Project Report entitled “**Optimal Design of UPFC Based Damping Controller**” submitted by **Mr.Vinod B. Prajapati (11MEEE13)**, towards the partial fulfillment of the requirements for the degree of **Master of Technology (Electrical Engineering)** in the field of **Electrical Power System** of Nirma University is the record of work carried out by him under our supervision and guidance. The work submitted has reached a level required for being accepted for examination. The results embodied in this major project to the best of my knowledge have not been submitted to any other University or Institution for award of any degree or diploma.

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- Vinod B.Prajapati

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Abstract

The problem of designing damping controller for low frequency oscillations in power systems under dynamic uncertainty. Power systems must typically perform over a wide range of operating conditions. The existence of such dynamic uncertainties requires good robustness of the control systems. H_∞ mixed sensitivity technique is apply to the design of robust damping controllers for Unified Power Flow Controllers (UPFCs) to damp power system oscillations under uncertain conditions. Detailed investigations have been carried out by previous researchers considering four alternative (m_{se} , m_{sh} , δ_{se} , and δ_{sh}) UPFC parameter based damping controllers.

Single Machine Infinite Bus (SMIB) system incorporating a UPFC is considered. The objective of the UPFC is to provide damping to the low frequency oscillations in the system. Weighted mixed-sensitivity H_∞ design approach has been used to design dual damping controller based on δ_{sh} (Phase angle of the shunt inverter) and m_{se} (modulating index of series inverter). The controller design is aimed at providing adequate damping to oscillations over a range of operating conditions. The function of the damping controller is to provide auxiliary stabilizing signals in phase with speed deviation ($\Delta\omega$).The optimal design and Effectiveness of the proposed controller design over a wide range of operating conditions for the SMIB system with the help of simulation using MATLAB.

Abbreviations

UPFC	Unified Power Flow Controller
SMIB	Single Machine Infinite Bus
FACTS	Flexible AC Transmission system
STATCOM	Static Synchronous Compensator
SSSC	Static Synchronous Series Compensator
POD	Power Oscillation Damping
SVC	Static Var Compensator
TCSC	Thyristor Controlled Series Capacitor
TCPAR	Thyristor Controlled Phase Angle Regulator

Nomenclature

H	inertia constant ($M=2H$)
ω_n	natural frequency of oscillation. $\text{rad } S^{-1}$
X_e	equivalent reactance of the system
X_T	reactance of transmission line 1
X_{Bv}	reactance of transmission line 2
X_{tE}	reactance of transformer
X_E	reactance of excitation transformer
X_B	reactance of boosting transformer
X_d	direct axis Steady-state synchronous reactance of the generator
X_q	quadrature axis steady-state synchronous reactance of the generator
X'_d	direct axis transient synchronous reactance of the generator
T'_{do}	direct axis open-circuit time-constant of the generator
P_e	electrical power of the generator
P_m	mechanical power input to the generator
$P_{e2(ref)}$	reference power on transmission line 2
$P_{e2(ref)_m}$	modified reference power on transmission line 2
V_t	generator terminal voltage
V_b	infinite bus voltage

V_o	voltage at UPFC bus
V_{Bo}	initial value of series-injected voltage
V_{Eo}	initial value of shunt-injected voltage
I_{t1}	current through transmission line 1
I_B	current through transmission line 2
I_E	current through shunt converter
V_{dc}	voltage at DC link
C_{dc}	DC link capacitor
m_E	modulation index of shunt converter
m_B	modulation index of series converter
K_{dc}	gain of damping controller
T_1, T_2	time constants of phase compensator
δ_E	phase angle of shunt-converter voltage
δ_B	phase angle of series-converter voltage

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Chapter 1

Introduction

Operational reliability and financial profitability along with more efficient utilization and control of existing transmission system infrastructure is required in open access. When large power systems are interconnected by relatively weak tie lines, low frequency oscillations are observed. Power system oscillations can influence the power system locally, partially or thoroughly unnoticeably as operating conditions change. These oscillations may sustain and grow, may cause system separation if adequate damping devices are not available. An approach to reduce risk of instability and thus to increase power transfer capability is to rapidly diminish the power system oscillations by increasing system damping. FACTS devices can also be used to enhance the damping of low frequency power oscillations. These devices are installed mainly for reasons, other than improved damping of low frequency oscillations, such as power flow control or voltage enhancement. However, FACTS devices are very expensive, a supplementary controller may be designed for the FACTS device to increase the damping of electromechanical oscillatory modes (both local and inter-area modes), while meeting the primary goal of the device on power system. FACTS controllers enhance both dynamic and static performance of power system and thus improvement in overall stability .

FACTS based stabilizers such as UPFC, STATCOM and SSSC can be effectively used for damping oscillations . Among all this, UPFC is versatile and has emerged as

a strong candidate for power control in electrical power transmission systems. It combines beautifully the features of Static Synchronous Compensator (STATCOM) and Static Synchronous Series Compensator (SSSC) . UPFC is multifunctional FACTS device which has the capability to control all three parameters that dictate power flow over power transmission line, i.e., it can control transmission line voltage, transmission line impedance and phase angle, sequentially or concurrently, with internal reactive power generation, in real time. Hence it can be used for effective and efficient power flow control, enhancement of transient stability, mitigation of low frequency power system oscillations and voltage (reactive power) regulation. UPFC can improve power oscillation damping effectively. The damping capability of UPFC is required to be investigated thoroughly for proper on line applications in changing operating conditions. Different approaches based on modern control theory have been applied to UPFC based POD controller design. Wang et. al. have presented a modified linearized Phillips-Heffron model of a power system installed with UPFC and addressed basic issues pertaining to design of UPFC based power oscillation damping controller along with selection of input parameters of UPFC to be modulated in order to achieve desired damping.

An approach to design an optimal POD controller which places the eigenvalue corresponding to mode of oscillation at desired location such that eigenvalues get placed within a vertical strip on LHS in complex plane and the system will have a desired degree of stability. In order to show the effectiveness of proposed optimal POD controller, for damping of both local and inter-area mode power oscillations, the response of system states and eigenvalue analysis technique has been used to demonstrate that the proposed UPFC based optimal POD controller can effectively damp power oscillations and hence significantly improve the performance of power system over wide range of operating conditions.

1.1 Objective of the project

To simulate unified power flow controller(UPFC) based damping controller to damp power system oscillation and observe its performance as power flow controller and dc voltage regulator for SMIB system. Based on modeling of SMIB system installed with UPFC Controller, using H_∞ mixed sensitivity technique is applied to the design of robust damping controllers with considering four alternative (m_{se} , m_{sh} , δ_{se} , and δ_{sh}) UPFC parameter as control signal to damp power system oscillations. To verify effectiveness of the simulated controller design over a wide range of operating conditions for the SMIB system with the help of simulation using MATLAB.

1.2 Literature Survey

Power systems must typically perform over a wide range of operating conditions. The existence of such dynamic uncertainties requires good robustness of the control systems. A novel method of utilizing the statistical approach is discussed in [1]. H ∞ mixed sensitivity technique is applied in [2] to the design of robust damping controllers for Unified Power Flow Controllers (UPFCs) to damp power system oscillations under uncertain conditions. In this paper, a Single Machine Infinite Bus (SMIB) system incorporating a UPFC is considered. The objective of the UPFC is to provide damping to the low frequency oscillations in the system.

References[1] a comprehensive approach for the design of UPFC controllers (i.e. power flow controller, DC voltage regulator and damping controller) for a multimachine system. UPFC controllers have been designed in the presence of conventional PSS. The interaction between the UPFC controllers and PSS has been studied. Investigations reveal that the system damping gets adversely affected with the incorporation of DC voltage regulator. Investigations have been carried out to understand relative effectiveness of modulation of the UPFC control signals (m_{se} , m_{sh} , δ_{se} , and δ_{sh}) on damping of the system oscillations using controllability index. Studies reveal that the UPFC based damping controller considering modulation of control parameter m_B is most effective in damping the oscillations.

The Unified Power Flow Controller (UPFC) was proposed for the Flexible AC Transmission Systems (FACTS), which is a multiple-functional FACTS controller with primary duty to be power flow control. The secondary functions of the UPFC can be voltage control, transient stability improvement, oscillation damping [11] and [12]. In this research paper it is demonstrated by examples that the UPFC can be very effective to damp power system oscillations. Also derive the linearized Phillips-Heffron model of a power system installed with a UPFC is derived which turns out

to be of the exactly same form as that of the unified model presented in [3] and [4] for SVC, TCSC and TCPAR. The major contributions of this paper are

- Establishment of the linearized Phillips-Heffron model of single-machine and multi-machine power systems installed with a UPFC, which adds the UPFC into the category of FACTS controllers for their unified model.
- The applications of the Phillips-Heffron model are demonstrated by studying the effect of UPFC DC voltage regulator on power system oscillation stability; and selecting the most effective damping control signals for the design of the UPFC damping controller.

This paper[6] delivers information how to design UPFC based damping controller for robust stabilization of power system low frequency oscillations using LMI technique. The motivation of using this control is the flexibility of the synthesis procedure. Normally, H_∞ synthesis problem is formulated as weighted mixed sensitivity design and solved by Riccati approach analytically. The analytical approach is relatively straightforward as it involves a non iterative solution. However, an analytical solution to the H_∞ control design based on the Riccati approach generally produces a controller that suffers from pole-zero cancellations between the plant and the controller. Also Riccati based designs depends heavily on the proper selection of weighting functions for conditioning of the plant, and no clear method for selecting these weighting functions exists. LMIs provide more flexibility for combining various design objectives in a numerically tractable manner, and can even cope with those problems to which analytical solution is out of question. Therefore, linear matrix inequality (LMI) formulation can produce the desired result as it gives the robust controller in damping control design.

The UPFC Based Damping controller model has been referred from IEEE paper titled “Damping of power system oscillation with Unified Power Flow Controller” by N. Tambey and M.L. Kothari, IEE Proc. Gener. Transm. Distrib. vol.150, no.2, March 2003.

and has been used for the project work .The results are obtained by introducing some changes in the reference model and parameters.The author doesn't claim for originality of this work.

1.3 Outline Of the Thesis

Since this thesis is basically concerned on optimal design of UPFC based damping controller to mitigate the low frequency oscillation in power system.

In the 1st chapter gives the details of necessity of the FACTS controller in power system and motivation behind the project is briefly mentioned.

The 2nd chapter mentions the basic model of UPFC ,operating modes of and control system of UPFC .A comprehensive approach to designing UPFC controller (power flow controller,DC voltage regulator and damping controller) has been presented.A new robust design of UPFC controller applying H_∞ Mixed Sensitivity technique is proposed to mitigate low frequency oscillations. The motivation to apply this control strategy is flexibility of the synthesis procedure.

In the 3rd chapter considering a SMIB system with UPFC .It includes the mathematical modeling of the SMIB System with UPFC and calculation of the constant of the transfer function model.

In the 4th chapter simulation and results were given .

Chapter 2

UPFC MODEL

2.1 Basic of UPFC Model

The UPFC consists of two voltage sourced converters, which operate from a common dc circuit consisting of a dc-storage capacitor [2] and [7]. The UPFC could be described as consisting of a shunt and a series branch as shown in the fig. 1. Each converter can independently generate or absorb reactive power. This arrangement enables free flow of active power in either direction between the ac-terminals of the two converters. The function of shunt converter is to supply or absorb the active power demanded by the series branch. This converter is connected to the ac terminal through a parallel-connected transformer. If required it may also generate or absorb reactive power, which can provide independent parallel reactive compensation of the line. The second series connected converter provides the main function of UPFC by injecting an ac voltage with the controllable magnitude and phase angle. The transmission line current flows through this voltage source resulting in an active and reactive exchange with the ac-system. The active power exchange at the ac-terminal is provided by the shunt branch, while the reactive power exchange generated internally by the converter. Neglecting losses, during steady state operation UPFC neither absorbs nor injects active power with respect to the system i.e. the voltage of the dc

capacitor remains constant at the pre-specified value. The Series branch of the UPFC model connected between the input (shunt side) and output (series side) bus consists of controllable voltage source and series impedance, which represents the inductance and losses of the series transformers. The Shunt controllable voltage source behind shunt impedance represents the shunt transformers. By varying magnitude and phase angle of the series voltage source, the line power flow at the output bus of the device can be controlled. Power oscillation damping is also achieved by modulating the power flow in the series branch.

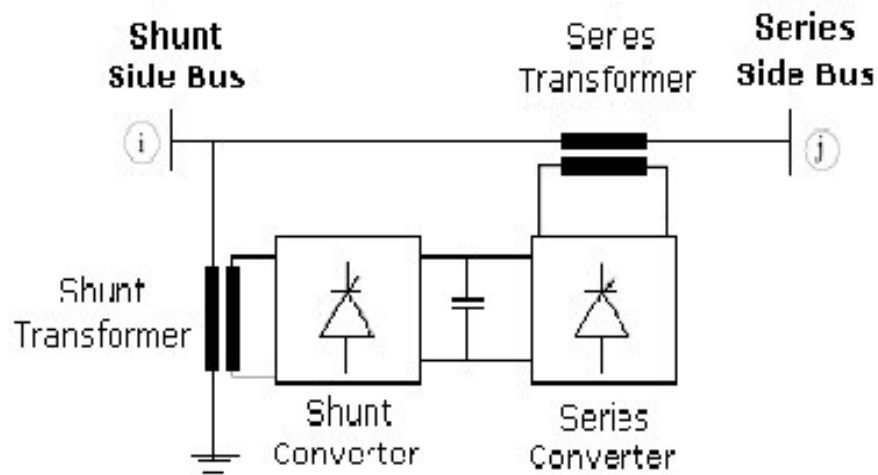


Figure 2.1: Basic Scheme of upfc model

2.2 Single machine infinite bus system with upfc model

Fig. 2.2 is a single-machine infinite-bus power system installed with a UPFC which consists of an excitation transformer (ET), a boosting transformer (BT), two three-phase GTO based voltage source converters (VSC's) and a DC link capacitor. In Fig. 2.2, $(m_{se}, m_{sh}, \delta_{se}, \text{ and } \delta_{sh})$ and are the amplitude modulation ratio and phase angle of the control signal of each VSC respectively, which are the input control signals to the UPFC.

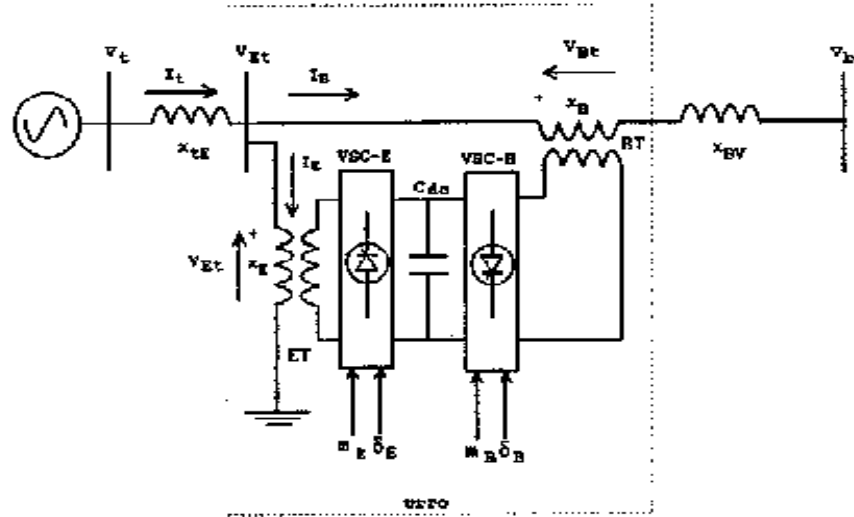


Figure 2.2: A UPFC installed in a single-machine infinite-bus power system.

By applying Park's transformation on the three-phase dynamic differential equations of the UPFC and ignoring the resistance and transients of the transformers, the dynamic model of the UPFC is derived in [3] using park's transformation .Hence derive the PhillipsHeffron model for studying the effect of UPFC DCvoltage regulator on power system oscillation stability; and selecting the most effective damping control signals for the design of the UPFC damping controller.

2.3 Control System of UPFC

The shunt converter operates as a STATCOM. The shunt converter controls the AC voltage at its terminals and the voltage of the DC bus. It uses a dual voltage regulation loop: an inner current control loop and an outer loop regulating AC and DC voltages. The series converter can operate either in power flow control (automatic mode) or in manual voltage injection mode. In power control mode, the measured active power and reactive power are compared with reference values to produce P and Q errors. The P error and the Q error are used by two PI regulators to compute respectively the V_q and V_d components of voltage to be synthesized by the VSC. (V_q in quadrature with V_1 controls active power and V_d in phase with V_1 controls reactive power). In manual voltage injection mode, regulators are not used. The reference values of injected voltage V_{dref} and V_{qref} are used to synthesize the converter voltage.

The UPFC block is a phasor model which does not include detailed representation of the power electronics. You must use it with the phasor simulation method, activated with the Powergui block. It can be used in three-phase power systems together with synchronous generators, motors, dynamic loads and other FACTS and DR systems to perform transient stability studies and observe impact of the UPFC on electromechanical oscillations and transmission capacity at fundamental frequency.

2.3.1 Operating Modes of UPFC

The UPFC has many possible operating modes. In particular, the shunt inverter is operating in such a way to inject a controllable current, into the transmission line. This current consists of two components with respect to the line voltage: the real or direct component, which is in phase or in opposite phase with the line voltage, and the reactive or quadrature component, which is in quadrature. The direct component is automatically determined by the requirement to balance the real power of the

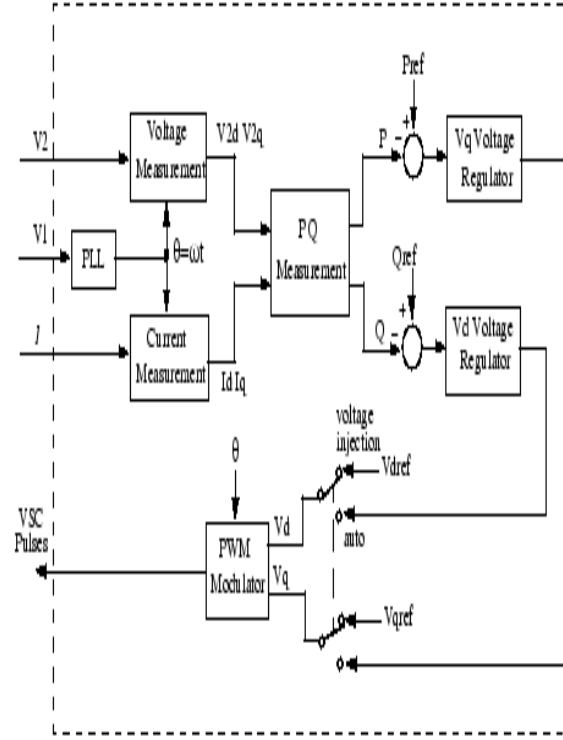


Figure 2.3: Simplified Block Diagram of the Series Converter Control System

series inverter. The quadrature component, instead, can be independently set to any desired reference level (inductive or capacitive) within the capability of the inverter, to absorb or generate respectively reactive power from the line. The shunt inverter can be controlled in two different modes:

- **VAR Control Mode:** The reference input is an inductive or capacitive VAR request. The shunt inverter control translates the Var reference into a corresponding shunt current request and adjusts gating of the inverter to establish the desired current. For this mode of control a feedback signal representing the dc bus voltage, V_{dc} , is also required.
- **Automatic Voltage Control Mode:** The shunt inverter reactive current is automatically regulated to maintain the transmission line voltage at the point of connection to a reference value. For this mode of control, voltage feedback

signals are obtained from the sending end bus feeding the shunt coupling transformer. The series inverter controls the magnitude and angle of the voltage injected in series with the line to influence the power flow on the line. The actual value of the injected voltage can be obtained in several ways.

- Direct Voltage Injection Mode: The reference inputs are directly the magnitude and phase angle of the series voltage.
- Phase Angle Shifter Emulation mode: The reference input is phase displacement between the sending end voltage and the receiving end voltage.
- Line Impedance Emulation mode: The reference input is an impedance value to insert in series with the line impedance
- Automatic Power Flow Control Mode: The reference inputs are values of P and Q to maintain on the transmission line despite system changes.

2.4 UPFC Controller

The UPFC control system comprises three controller:

- a. power flow controller
- b. DC voltage regulator
- c. power system oscillation damping controller

Fig.2.4.shows a schematic diagram of a UPFC control system.

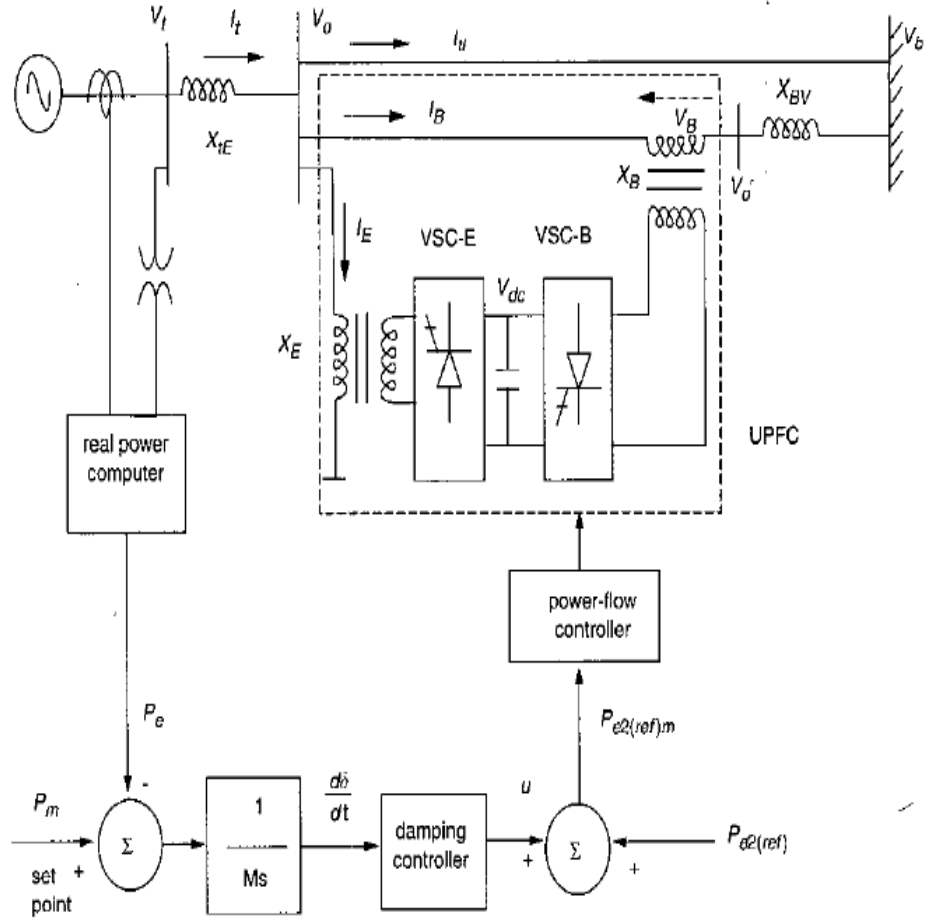


Figure 2.4: Schematic diagram of a UPFC control system

(1)**power flow controller:** The UPFC is installed in one of the two lines of the SMIB system. Fig.2.5. shows the transfer function of the P-I type power-flow controller. The power-flow controller regulates the power flow on this line. K_{pp} and K_{pi} are the proportional and integral gain settings of the power-flow controller.

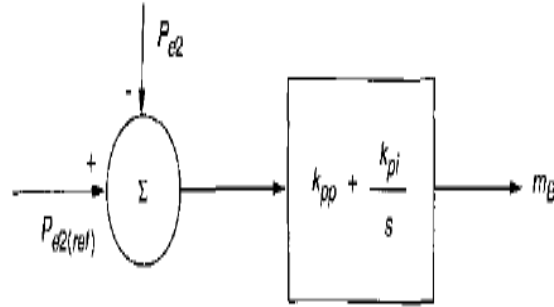


Figure 2.5: Structure of power flow controller

(2)DC voltage regulator: The real power output of the shunt converter must be equal to the real power input of the series converter or vice versa. In order to maintain the power balance between the two converters, a DC-voltage regulator is incorporated. DC-voltage is regulated by modulating the phase angle of the shunt-converter voltage. Thus, the DC-voltage regulator forms part of the power-flow controller. A P-I type DC voltage regulator is considered (Fig.2.6.). K_{dp} and K_{di} are the proportional and integral gain settings of the DC regulator.

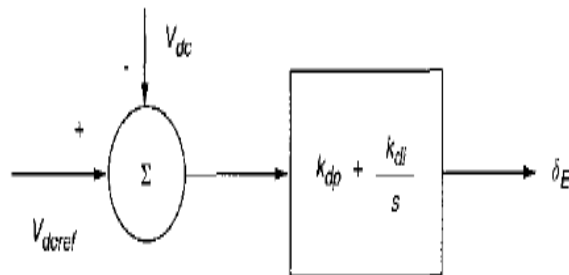


Figure 2.6: Structure of DC voltage regulator

(3)power system oscillation damping controller: A damping controller is provided to improve the damping of power system-oscillations. The damping con-

troller may be considered as comprising two cascade-connected blocks. Block I is provided to derive a speed-deviation signal from the electrical power P_e . The total electrical power is measured at the UPFC location. It is then compared with the set point (mechanical power). The error is integrated and multiplied by $\frac{1}{M}$ to derive a speed-deviation signal. It may be noted that the speed-deviation signal derived is used instead of the speed-deviation signal which has been measured, since the speed-deviation signal, in general, may not be available at the UPFC location. The second block comprises a lead-lag compensator. An electrical torque in phase with the speed deviation is to be produced in order to improve the damping of the system oscillations. The parameters of the lead-lag compensator are chosen so as to compensate for the phase shift between the control signal and the resulting electrical power deviation. In this way an additional electrical power output is obtained in phase with the speed deviation. The gain setting of the damping controller is chosen so as to achieve the desired damping ratio of the electromechanical mode. The output of the damping controller modulates the reference setting of the power-flow controller (Fig.2.4).

2.5 H_∞ Mixed Sensitivity Roboust Control Technique

Zames originally formulated the H_∞ optimal control theory . Based on state-space methods, Doyle introduced the solution to a general rational MIMO H_∞ optimal control problem and there has been a considerable amount of interest towards this advanced H_∞ control design and its applications. The performance of the controller designed by using such techniques outweighs that of the conventional methods as it addresses the issue of worst-case controller design for linear plants subject to unknown disturbances and plant uncertainties, including problems of disturbance attenuation, model matching and tracking. The H_∞ design techniques are closely related to the

problem of minimizing the ∞ norm of a combination of closed loop transfer function of a system. The robust controller design process using H_∞ Mixed Sensitivity methods is iterative between controller design and performance evaluation. If desired damping characteristic through the use of the controller is not achieved then the weighting factors used to synthesize the controller is adjusted. Eventually this ensures a fixed structure and fixed parameter yet robust controller. Stability and damping effect are the criterions that are adjudged for the closed loop in terms of performance. Robust performance generally implies how the close loop system behaves if some parts of the system gets changed or perturbed. In the one degree of freedom controller, the closed loop transfer functions such as sensitivity function 'S', the complementary sensitivity function 'T' and the input signal which is dependent on 'KS', the input sensitivity function are shaped by applying weights which are bounds of the aforesaid closed loop transfer functions to satisfy performance and robustness objectives. The weighting function $W1(s)$ to shape S, is a scalar low pass filter with a bandwidth equal to that of disturbance could be selected. This weight can be interpreted in terms of robust stability with respect to feedback perturbation. To shape T for tracking problems and noise reduction, the weighting function $W2(s)$ is adopted and can be interpreted as robust stability to multiplicative uncertainty. The weighting function $W3(s)$ to shape KS, a scalar high pass filter with a cross over frequency more or less equal to the desired closed loop band width is selected. Normally, in the mixed sensitivity design, bounds (weights) are applied on S / KS. To bound KS at high frequency, it is better to put a bound on T. Applying weights on T, sensitivity to noise and uncertainty is reduced. This gives another procedure of mixed sensitivity damping controller design by shaping the sensitivity function S and complementary sensitivity function T. This approach is called S / T mixed sensitivity approach. In this paper, combination of both the approaches i.e. shaping of S, T and KS simultaneously through appropriate weights, the UPFC damping controller has been designed. The H_∞ optimal damping controller here is found by meeting the following mixed sensitivity design objective:

weights are adjusted and each time the system damping response has been studied.

The final weights for the system under study are as follows: For damping controller related to power flow controller,

$$w_1(s) = \frac{10}{s + 0.0097}$$

$$w_2(s) = 1.1$$

$$w_3(s) = 0$$

For damping controller related to DC voltage regulator,

$$w_1(s) = \frac{50}{s + 0.05}$$

$$w_2(s) = 1.0$$

$$w_3(s) = 1.0$$

2.6 Advantages of H_∞ Mixed Sensitivity Robust Control Technique

- A new robust design of UPFC controller applying H_∞ Mixed Sensitivity technique is proposed to mitigate low frequency oscillations.
- The motivation to apply this control strategy is flexibility of the synthesis procedure.
- The reduction in the controller complexity is averted by reducing system size keeping in view suitability for practical implementation.
- The controller designed uses only speed deviation of the generator as the feed-

back signal.

- The time domain simulation results show that it has better performance on damping low frequency oscillation and improves stability under wide range of operating conditions and disturbances.

Chapter 3

Mathematical Modeling of UPFC and Calculation

3.1 System Investigated

We consider a single machine infinite bus (SMIB) system with UPFC installed [13]. The UPFC is installed in one of the two parallel transmission lines (Fig. 3.1). This configuration, comprising two parallel transmission lines, permits the control of real and reactive power flow through a line. The static excitation system, model type IEEE-STIA, has been considered. The UPFC is assumed to be based on pulse width modulation (PWM) converters. The conventional PSS is considered. The nominal loading condition and system parameters are given in Appendix A.

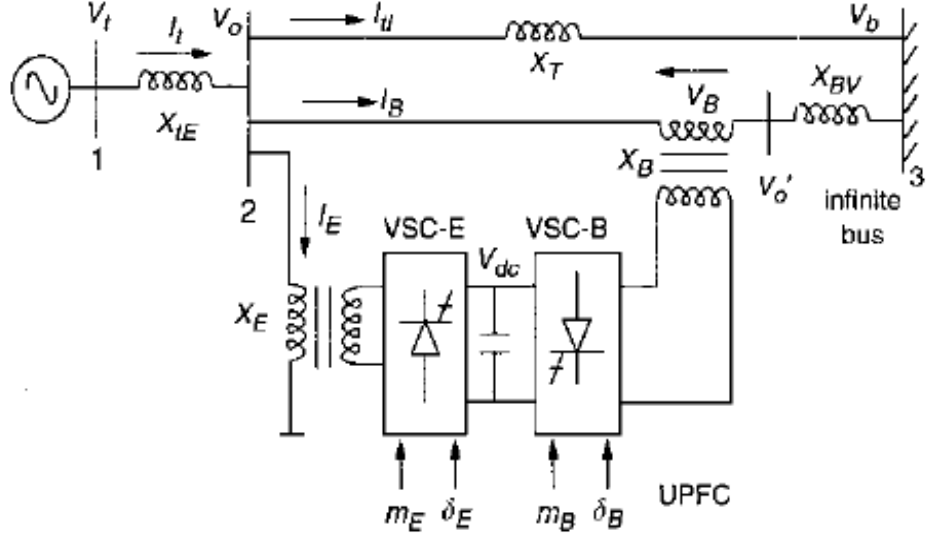


Figure 3.1: A SMIB system installed with an UPFC in one of the lines

3.2 Dynamic model of the system with UPFC

3.2.1 Non-linear dynamic model

A non-linear dynamic model of the system is derived by disregarding the resistances of all the components of the system (generator, transformier, transmission lines, shunt and series converter transformers) and the transients of the transmission lines and transformers of the UPFC. The nonlinear dynamic model of the system using UPFC is given below:

$$\dot{\omega} = \frac{P_m - P_e - D\Delta\omega}{M} \quad (3.1)$$

$$\dot{\delta} = \omega_0 * (\omega - 1) \quad (3.2)$$

$$\dot{E}_q = \frac{(-E_q + E_{fd})}{T'_{do}} \quad (3.3)$$

$$\dot{E}_{fd} = \frac{-E_{fd} + K_a(V_{ref} - V_t)}{T_a} \quad (3.4)$$

$$\dot{V}_{dc} = \frac{3m_E}{4C_{dc}}(\sin(\delta_e) * I_{Ed} + \cos(\delta_e) * I_{Eq}) + \frac{3m_B}{4C_{dc}}(\sin(\delta_B) * I_{Bd} + \cos(\delta_B) * I_{Bq}) \quad (3.5)$$

The equation for the real power balance between the series and shunt converters is given as:

$$R_e(V_B I_B^* - V_E I_E^*) = 0 \quad (3.6)$$

3.2.2 Linear dynamic model (modified Heffron- Phillips model of an SMLB system including UPFC)

A linear dynamic model is obtained by linearising the nonlinear model around an operating condition. The linearised model is given below:

$$\Delta\dot{\omega} = \frac{\Delta P_m - \Delta P_e - D\Delta\omega}{M} \quad (3.7)$$

$$\Delta\dot{\delta} = \omega_0 * \Delta\omega \quad (3.8)$$

$$\Delta\dot{E}_q = \frac{(-\Delta E_q + \Delta E_{fd})}{T'_{do}} \quad (3.9)$$

$$\Delta\dot{E}_{fd} = \frac{-\Delta E_{fd} + K_a(\Delta V_{ref} - \Delta V_t)}{T_a} \quad (3.10)$$

$$\Delta\dot{V}_{dc} = K_7\Delta\delta + K_8\Delta\dot{E}_q - K_9\Delta V_{dc} + K_{ce}\Delta m_E + K_{c\delta e}\Delta\delta_E + K_{cb}\Delta m_B + K_{c\delta b}\Delta\delta_B \quad (3.11)$$

where,

$$\Delta P_e = K_1\Delta\delta + K_2\Delta\dot{E}_q + K_{pe}\Delta m_E + K_{p\delta e}\Delta\delta_E + K_{pb}\Delta m_B + K_{p\delta b}\Delta\delta_B + K_{pd}\Delta V_{dc}$$

$$\Delta E_q = K_4\Delta\delta + K_3\Delta\dot{E}_q + K_{qe}\Delta m_E + K_{q\delta e}\Delta\delta_E + K_{qb}\Delta m_B + K_{q\delta b}\Delta\delta_B + K_{qd}\Delta V_{dc}$$

$$\Delta V_t = K_5\Delta\delta + K_6\Delta\dot{E}_q + K_{ve}\Delta m_E + K_{v\delta e}\Delta\delta_E + K_{vb}\Delta m_B + K_{v\delta b}\Delta\delta_B + K_{vd}\Delta V_{dc}$$

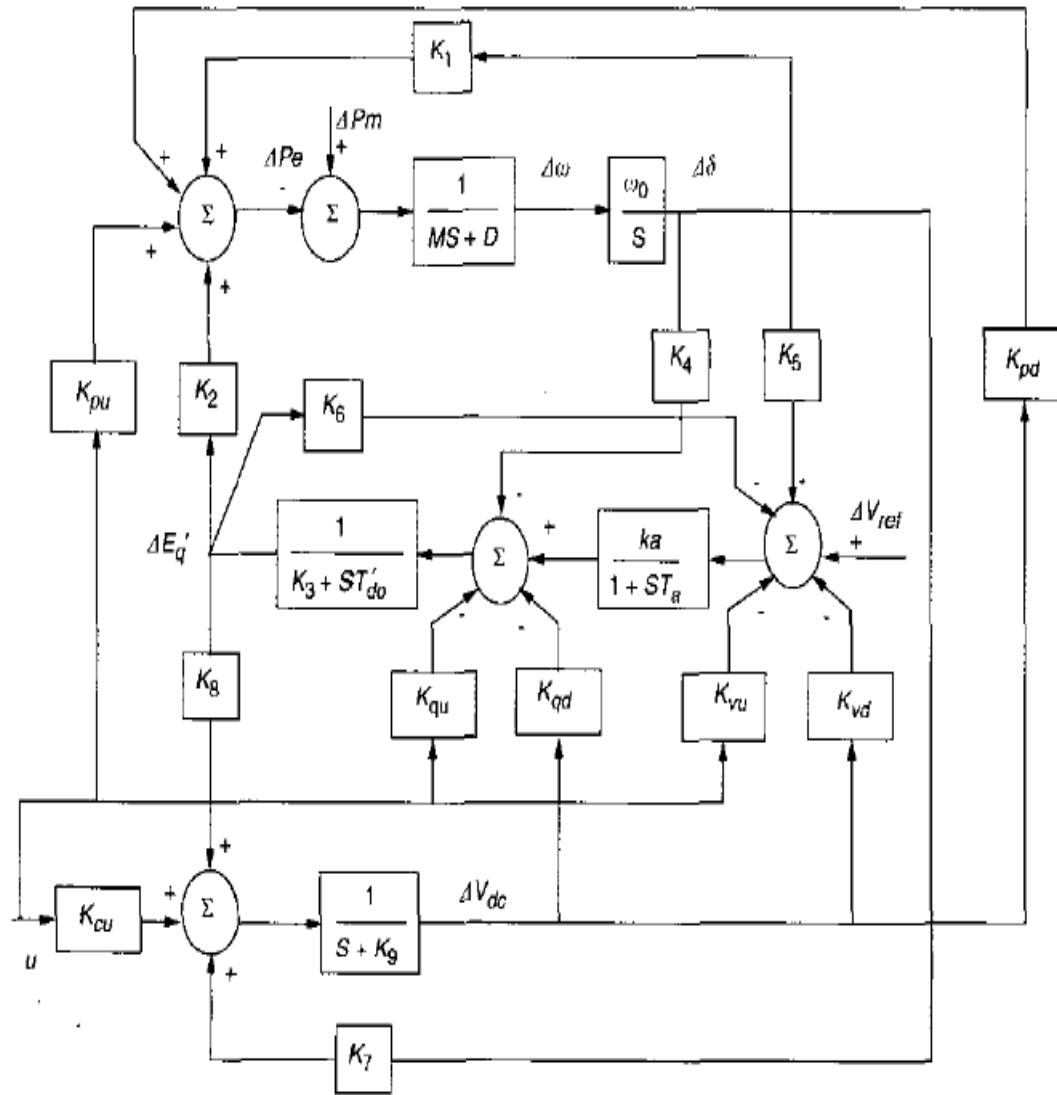


Figure 3.2: Modified Heffron-Phillips model of an SMIB system with UPFC

Fig.3.2. shows the modified Heffron-Phillips transfer-function model of the system including UPFC. The modified Heffron-Phillips model has 28 constants as opposed to 6 constants in the Heffron-Phillips model. These constants are functions of the system parameters and the initial operating condition. The equations for computing the constants of the model are given in Appendix B. The control vector U is defined as follows:

$$U = [\Delta m_B \quad \Delta m_E \quad \Delta \delta_B \quad \Delta \delta_E]^T \quad (3.12)$$

where

Δm_B =deviation in pulse width modulation index m_B of series inverter.

By controlling m_B , the magnitude of series-injected voltage can be controlled.

Δm_E =deviation in phase angle of the injected voltage

$\Delta \delta_B$ =deviation in pulse-width-modulation index m_E of the shunt inverter. By controlling m_E the output voltage of the shunt converter is controlled.

$\Delta \delta_E$ =deviation in phase angle of the shunt-inverter voltage. The series and shunt converters are controlled in a coordinated manner to ensure that the real power output of the shunt converter is equal to the real power input to the series converter. The fact that the DC voltage remains constant ensures that this equality is maintained.

It may be noted that K_{pu} , K_{qu} , K_{vu} , and K_{cu} in Fig. 2 are the row vectors defined below:

$$K_{pu} = [K_{pe} \quad K_{p\delta e} \quad K_{pb} \quad K_{p\delta b}]$$

$$K_{qu} = [K_{qe} \quad K_{q\delta e} \quad K_{qb} \quad K_{q\delta b}]$$

$$K_{vu} = [K_{ve} \quad K_{v\delta e} \quad K_{vb} \quad K_{v\delta b}]$$

$$K_{cu} = [K_{ce} \quad K_{c\delta e} \quad K_{cb} \quad K_{c\delta b}]$$

3.3 Dynamic Model In State-space Form

The dynamic model of the system in state-space form is obtained from the transfer-function model as

$$\dot{X} = AX + Bu \quad (3.13)$$

where,

$$X = [\Delta\delta \quad \Delta\omega \quad \Delta\dot{E}_q \quad \Delta E_{fd} \quad \Delta V_{dc}]^T$$

$$u = [\Delta m_B \quad \Delta m_E \quad \Delta\delta_B \quad \Delta\delta_E]^T$$

$$A = \begin{pmatrix} 0 & \omega_0 & 0 & 0 & 0 \\ \frac{-K_1}{M} & 0 & \frac{-K_2}{M} & 0 & \frac{-K_{pd}}{M} \\ \frac{-K_4}{T'_{do}} & 0 & \frac{-K_3}{T'_{do}} & \frac{1}{T'_{do}} & \frac{-K_{qd}}{T'_{do}} \\ \frac{-K_a K_5}{T_a} & 0 & \frac{-K_a K_6}{T_a} & \frac{-1}{T_a} & \frac{-K_a K_{td}}{T_a} \\ K_7 & 0 & K_8 & 0 & -K_9 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{-K_{pe}}{M} & \frac{-K_{p\delta e}}{M} & \frac{-K_{pb}}{M} & \frac{-K_{p\delta b}}{M} \\ \frac{-K_{qe}}{T'_{do}} & \frac{-K_{q\delta e}}{T'_{do}} & \frac{-K_{qb}}{T'_{do}} & \frac{-K_{q\delta b}}{T'_{do}} \\ \frac{-K_a K_{ve}}{T_a} & \frac{-K_a K_{v\delta e}}{T_a} & \frac{-K_a K_{vb}}{T_a} & \frac{-K_a K_{v\delta b}}{T_a} \\ K_{ce} & K_{c\delta e} & K_{cb} & K_{c\delta b} \end{pmatrix}$$

3.4 Analysis of the Mathematic Modeling of SMIB System with UPFC

The nominal parameters and the operating conditions of the system are given below:

Generator	$M = 2H$		
	8.0 MJ/MVA		
	$D = 4$	$T'_{do} = 5.04 \text{ sec}$	
	$X_d = 1.0 \text{ p.u.}$	$X_q = 0.6 \text{ p.u.}$	$X'_d = 0.3 \text{ p.u.}$
Excitation system	$K_a=50.0$	$T_a=0.05 \text{ sec}$	
Transformers	$X_{tE}=0.1 \text{ p.u.}$	$X_E= X_B=0.1 \text{ p.u.}$	
Transmission lines	$X_{T1}=1.0 \text{ p.u.}$	$X_{T2}=1.3 \text{ p.u.}$	
Operating condition	$P=0.9115 \text{ p.u.}$	$Q=0.2765 \text{ p.u.}$	$V_t=1.032 \text{ p.u.}$
	$V_b=1.0 \text{ p.u.}$	$f=60 \text{ Hz}$	
UPFC parameters	$m_E=1.0$	$m_B=0.1$	
	$\delta_E = 28.1^\circ$	$\delta_B = -21.1^\circ$	
DC Link parameters	$V_{dc}=2 \text{ p.u.}$	$C_{dc}=3 \text{ p.u.}$	

3.4.1 Computation of constants of the transfer function model

The initial d-q axes voltage and current components and torque angle are computed for the nominal operating condition ($P_e=0.912 \text{ p.u.}$, $Q=0.277 \text{ p.u.}$, $V_t= 1.032 \text{ p.u.}$, $V_b= 1 \text{ P.u.}$) These data are needed for computing the constants of the system model and are given below:

$$E_{do}=0.4010 \text{ p.u.}$$

$$E_{qo}=0.9493 \text{ p.u.}$$

$$V_{Eo}=1.0\angle 28.1^\circ$$

$$I_{do}=0.6735 \text{ p.u.}$$

$$I_{qo}=0.6736 \text{ p.u.}$$

$$E_{bdo}=0.8644 \text{ p.u.}$$

$$E_{bqo}=0.4890 \text{ p.u.}$$

$$V_{Bo}=0.1\angle -21.1^\circ$$

$$\delta_0=58.1^\circ$$

The constants of the transfer-function model (Fig.3.2.), computed for the (nominal operating condition and system parameters using expressions given in Appendix B, are as in follows:

$K_1=0.1849$	$K_{pe}=0.3800$	$K_{qb}=0.0821$
$K_2=1.0896$	$K_{qe}=-1.0858$	$K_{vb}=-0.0258$
$K_3=2.4422$	$K_{ve}=0.5486$	$K_{cb}=-0.0582$
$K_4=0.1792$	$K_{ce}=-0.0696$	$K_{p\delta b}=-0.0089$
$K_5=-0.0476$	$K_{p\delta e}=0.5803$	$K_{q\delta b}=-0.5388$
$K_6=0.3514$	$K_{q\delta e}=-0.0036$	$K_{v\delta b}=-0.0029$
$K_7=-0.2697$	$K_{v\delta e}=-0.0036$	$K_{c\delta b}=0.0175$
$K_8=0.2061$	$K_{c\delta e}=0.6206$	$K_{pd}=0.1931$
$K_9=0.0397$	$K_{pb}=0.0615$	$K_{qd}=-0.5388$
		$K_{vd}=0.2730$

3.5 Optimisation of UPFC controllers

The UPFC power-flow and DC-voltage regulators are designed independently. A brief description of the techniques used for the optimisation of UPFC power-flow controller, DC-voltage regulator and damping controller are described in the following section.

3.5.1 Optimisation of power flow controller and DC voltage regulator

- The parameters of the power flow controller (k_{pp} and k_{pi}) are optimised using a gradient-type Newton algorithm. A brief description of the gradient-type Newton algorithm is presented in Appendix C. The parameters of the power-flow controller are optimised while neglecting the DC-voltage regulator. Optimum values of the proportional and integral gain settings of the power flow controller are obtained as $k_{pp} = 2$ and $k_{pi} = 10$.
- The parameters of the DC-voltage regulator are now optimised using the gradient-type Newton algorithm. When optimising the DC-voltage regulator, power-flow controller parameters are set at their optimum values. The optimum gain settings of the P-I type DC-voltage regulator are $k_{dp} = 0.25$ and $k_{dj} = 0.35$.

Chapter 4

Simulation and Results

The dynamic performance of the SMIB system including UPFC controller is obtained with following model:

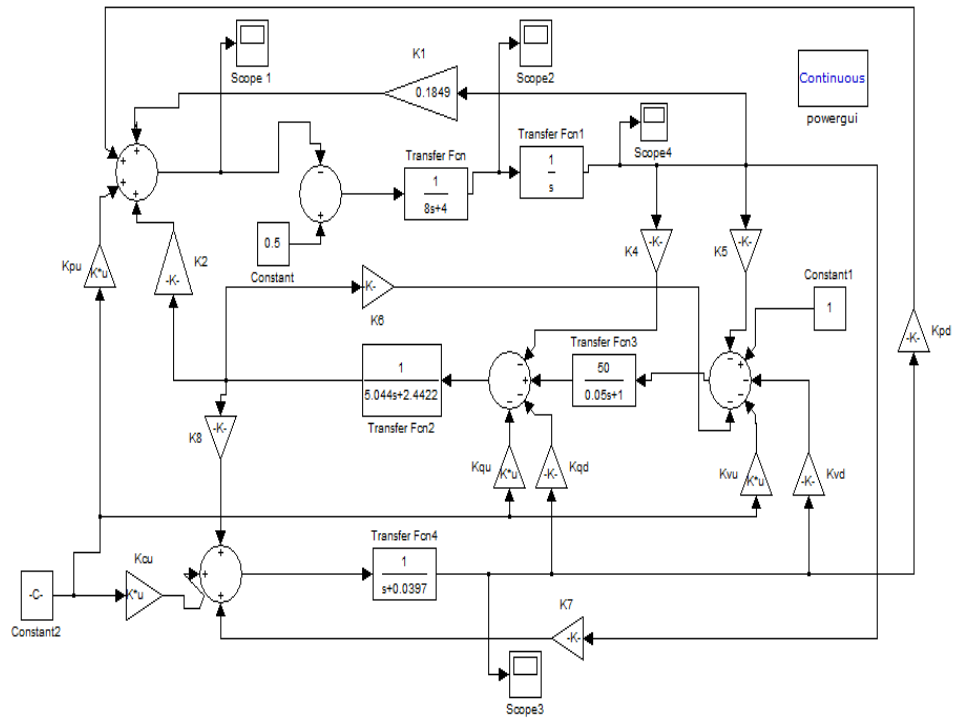


Figure 4.1: Modified Heffron- Phillips model of an SMIB system including UPFC

4.1 Results

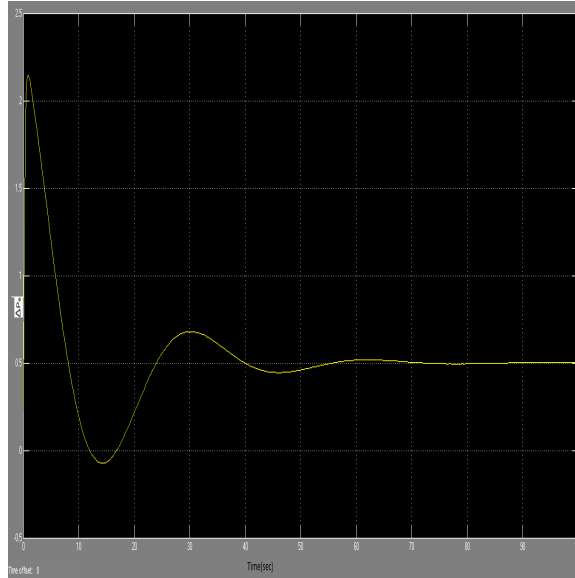


Figure 4.2: ΔP_e vs Time

Fig.4.2. shows the dynamic response for ΔP_e , i.e., transient deviation in power flow with UPFC controller. It shows that power flow is regulated to the desired value. The results are based on the data described in Appendix.

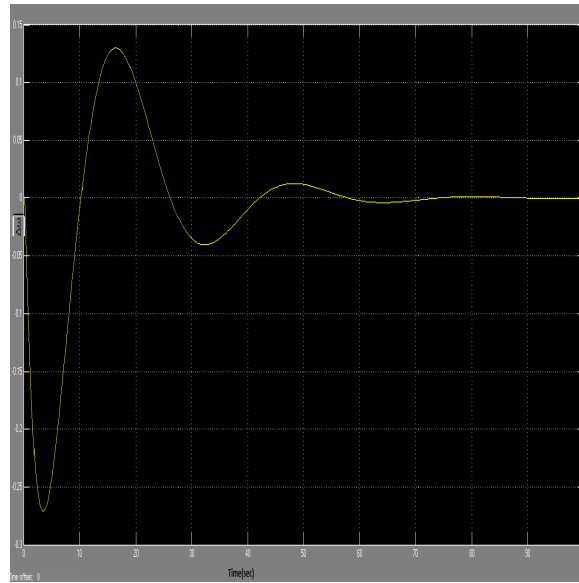
Figure 4.3: $\Delta\omega$ vs Time

Fig.4.3. shows the dynamic response for $\Delta\omega$ with UPFC controller is well damped.

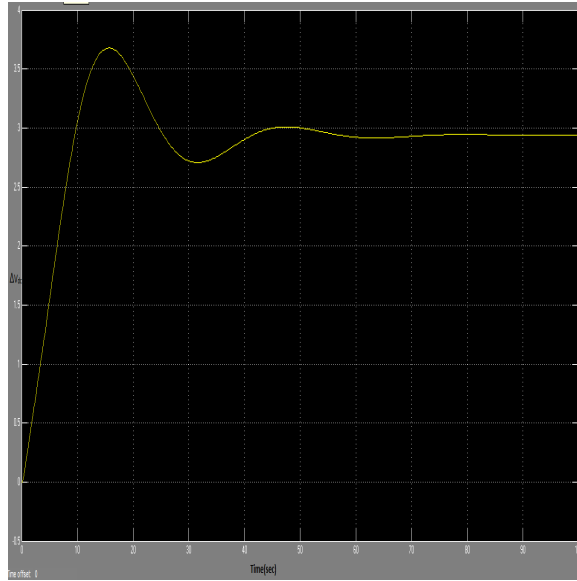
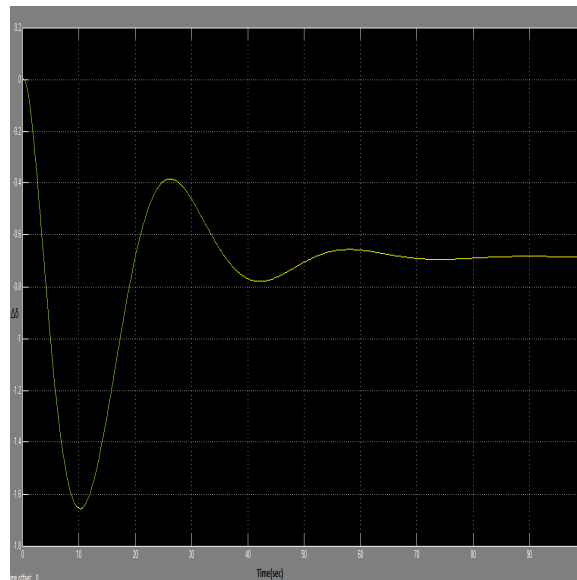
Figure 4.4: ΔV_{dc} vs Time

Fig.4.4. shows the dynamic response for ΔV_{dc} with UPFC controller. It shows that the deviation in dc link voltage ΔV_{dc} is regulated to zero.

Figure 4.5: $\Delta\delta$ vs Time

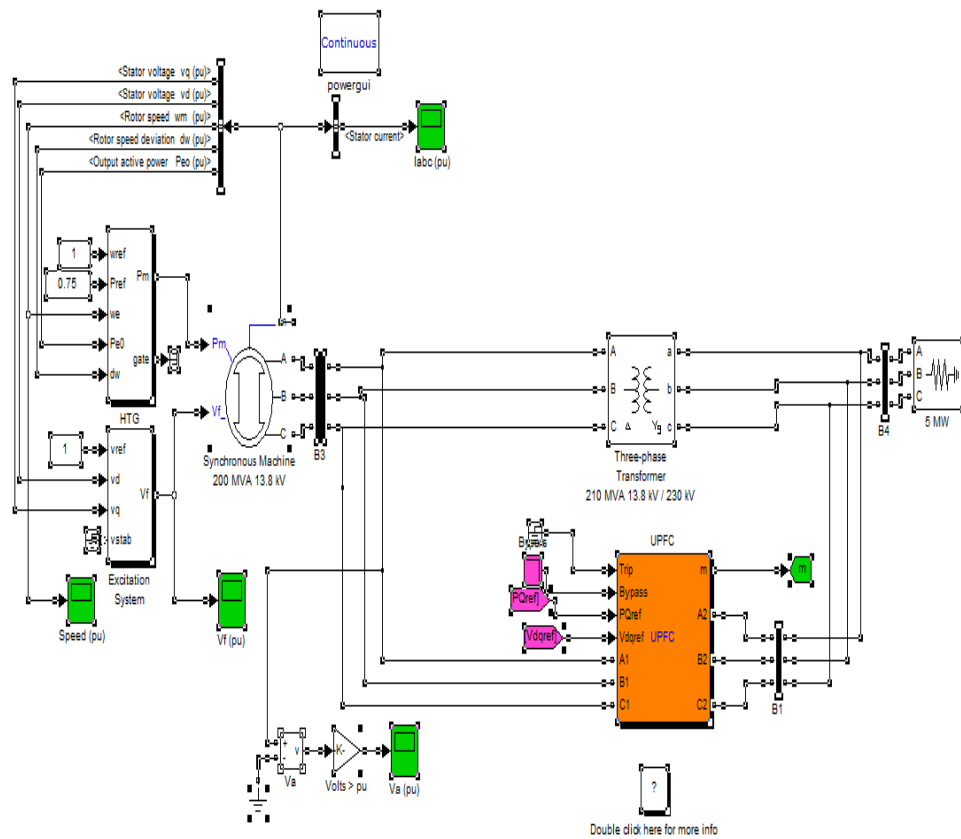


Figure 4.6: Matlab model of an SMIB system including UPFC

Chapter 5

Conclusion and Future Scope

5.1 Conclusions

- A new robust design of UPFC controller applying H_∞ Mixed Sensitivity technique is proposed to mitigate low frequency oscillations..The controller designed uses only speed deviation of the generator as the feedback signal.
- It has better performance on damping low frequency oscillation and improves stability under wide range of operating conditions and disturbances.
- The damping controller based on control parameter m_B and δ_E co-operate with each other.The alternative damping controller (damping controller m_B ,damping controller δ_E and dual damping controller) provide robust dynamic performance under wide variation loading condition .

5.2 Future Scope

- TO Design the UPFC based damping controller for multi-machine system.
- To study the effect of UPFC DC voltage regulator on power system oscillation stability.
- Selection of the most effective damping control signals for the design of the UPFC damping controller for multi-machine system.

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Appendix A

Nominal parameters and Operating condition

The nominal parameters and the operating conditions of the system are given below:

Generator	$M = 2H$ 8.0 MJ/MVA $\mathbf{D} = 4$	$T'_{do} = 5.044 \text{ sec}$	
	$X_d = 1.0 \text{ p.u.}$	$X_q = 0.6 \text{ p.u.}$	$X'_d = 0.3 \text{ p.u.}$
Excitation system	$K_a=50.0$	$T_a=0.05 \text{ sec}$	
Transformers	$X_{tE}=0.1 \text{ p.u.}$	$X_E= X_B=0.1 \text{ p.u.}$	
Transmission lines	$X_{T1}=1.0 \text{ p.u.}$	$X_{T2}=1.3 \text{ p.u.}$	
Operating condition	$P=0.9115 \text{ p.u.}$	$Q=0.2765 \text{ p.u.}$	$V_t=1.032 \text{ p.u.}$
	$V_b=1.0 \text{ p.u.}$	$f=60 \text{ Hz}$	
UPFC parameters	$m_E=1.0$	$m_B=0.1$	
	$\delta_E = 28.1^\circ$	$\delta_B = -21.1^\circ$	
DC Link parameters	$V_{dc}=2 \text{ p.u.}$	$C_{dc}=3 \text{ p.u.}$	

Note:Used by author to modify the system behaviour

Appendix B

Computation of constant of the model

The constants of the modified heffron-phillips model are computed from the expression given below.

$$K_1 = r_2 n_2 + r_3 o_1; K_2 = r_1 + r_2 n_1; K_3 = (1 - t_1 n_1);$$

$$K_4 = -t_1 n_2$$

$$K_5 = (\frac{V_{td}}{V_t})p_1 + (\frac{V_{tq}}{V_t})q_2; K_6 = \frac{V_{tq}}{V_t}q_1;$$

$$K_7 = y_5 b_2 + y_6 f_1 + y_7 d_2 + y_8 h_1;$$

$$K_8 = y_5 b_1 + y_7 d_1; K_9 = -(y_5 b_7 + y_6 f_6 + y_7 d_7 + y_8 h_6);$$

$$K_{pe} = r_2 n_3 + r_3 o_2; K_{p\delta e} = r_2 n_4 + r_3 o_3; K_{pd} = r_2 n_5 + r_3 o_4;$$

$$K_{p\delta b} = r_2 n_6 + r_3 o_5 ;$$

$$K_{qe} = -t_1 n_3; K_{q\delta e} = -t_1 n_4; K_{qb} = -t_1 n_5; K_{q\delta b} = -t_1 n_6;$$

$$K_{ve} = (\frac{V_{td}}{V_t})p_2 + (\frac{V_{tq}}{V_t})q_3; K_{v\delta e} = (\frac{V_{td}}{V_t})p_3 + (\frac{V_{tq}}{V_t})q_4$$

$$K_{vb} = (\frac{V_{td}}{V_t})p_4 + (\frac{V_{tq}}{V_t})q_5; K_{v\delta b} = (\frac{V_{td}}{V_t})p_5 + (\frac{V_{tq}}{V_t})q_6$$

$$K_{ce} = y_5 b_3 + y_6 f_2 + y_7 d_3 + y_8 h_2 + y_1;$$

$$K_{c\delta e} = y_5 b_4 + y_6 f_3 + y_7 d_4 + y_8 h_3 + y_2;$$

$$K_{cb} = y_5 b_5 + y_6 f_4 + y_7 d_5 + y_8 h_4 + y_3;$$

$$K_{c\delta b} = y_5 b_6 + y_6 f_5 + y_7 d_6 + y_8 h_5 + y_4;$$

$$K_{pd} = r_2 n_7 + r_3 o_6; K_{qd} = -t_1 n_7;$$

$$K_{vd} = \left(\frac{V_{td}}{V_t}\right)p_6 + \left(\frac{V_{tq}}{V_t}\right)q_7;$$

Where,

$$\begin{aligned} a_1 &= \frac{x_{BB}}{x_{dE}}; a_2 = \frac{x_{dE} + x_{BB}x_{b3}}{x_{dE}}; \\ a_3 &= -\frac{x_{dT} + x_{BB}x_{b2}}{x_{dE}}; a_4 = -\frac{x_{dT}}{x_{dE}}; \\ b_1 &= a_1; b_2 = -a_2 V_b \cos(\delta); b_3 = a_3 V_{dc} \cos\left(\frac{\delta_E}{2}\right); \\ b_4 &= -a_3 m_E V_{dc} \sin\left(\frac{\delta_E}{2}\right); \\ b_5 &= a_4 V_{dc} \cos\left(\frac{\delta_E}{2}\right); b_6 = -a_4 m_B V_{dc} \sin\left(\frac{\delta_B}{2}\right); \\ b_7 &= \frac{a_3 m_E \cos(\delta_E) + a_4 m_B \cos(\delta_E)}{2}; \end{aligned}$$

$$\begin{aligned} c_1 &= \frac{x_E}{x_{dE}}; c_2 = \frac{x_{b3}x_E - x_{b1}}{x_{dE}}; \\ c_3 &= \frac{x_{b1} - x_E x_{b2}}{x_{dE}}; a_4 = \frac{x_{b1}}{x_{dE}}; \\ d_1 &= c_1; d_2 = -c_2 V_b \sin(\delta); d_3 = c_3 V_{dc} \cos\left(\frac{\delta_E}{2}\right); \\ d_4 &= -c_3 m_E V_{dc} \sin\left(\frac{\delta_E}{2}\right); \\ d_5 &= c_4 V_{dc} \cos\left(\frac{\delta_B}{2}\right); d_6 = -c_4 m_B V_{dc} \sin\left(\frac{\delta_B}{2}\right); \\ d_7 &= \frac{c_3 m_E \cos(\delta_E) + c_4 m_B \cos(\delta_B)}{2}; \end{aligned}$$

$$\begin{aligned} e_1 &= \frac{x_{qT} + x_{a3}x_{BB}}{x_{qE}}; e_2 = -\frac{x_{qT} + x_{a2}x_{BB}}{x_{qE}}; \\ e_3 &= -\frac{x_{qT}}{x_{qE}}; \\ f_1 &= e_1 V_b \cos(\delta); f_2 = e_2 V_{dc} \sin\left(\frac{\delta_E}{2}\right); \\ f_3 &= e_2 m_E V_{dc} \cos\left(\frac{\delta_E}{2}\right); \\ f_4 &= e_3 V_{dc} \sin\left(\frac{\delta_B}{2}\right); f_5 = e_3 m_B V_{dc} \cos\left(\frac{\delta_E}{2}\right); \\ f_6 &= \frac{e_2 m_E \sin(\delta_E) + e_3 m_B \sin(\delta_E)}{2}; \end{aligned}$$

$$\begin{aligned} g_1 &= \frac{x_{a3} - x_{a1}}{x_{qE}}; g_2 = \frac{x_{a1} - x_E x_{a2}}{x_{qE}}; \\ g_3 &= \frac{x_{a1}}{x_{qE}}; \end{aligned}$$

$$\begin{aligned}
h_1 &= g_1 V_b \cos(\delta); h_2 = g_2 V_{dc} \sin \frac{\delta_E}{2}; \\
h_3 &= g_2 m_E V_{dc} \cos \frac{(\delta_E)}{2}; \\
h_4 &= g_3 V_{dc} \sin \frac{(\delta_B)}{2}; h_5 = g_3 m_B V_{dc} \cos \frac{(\delta_B)}{2}; \\
h_6 &= \frac{g_2 m_E \sin(\delta_E) + g_3 m_B \sin(\delta_B)}{2};
\end{aligned}$$

$$\begin{aligned}
i_1 &= \frac{V_{dc}}{2x_T} \cos(\delta_E); i_2 = \frac{m_E V_{dc}}{2x_T} \sin(\delta_E); \\
i_3 &= \frac{m_E}{2x_T} \cos(\delta_E); i_4 = \frac{V_b}{x_T} \sin(\delta);
\end{aligned}$$

$$\begin{aligned}
j_1 &= \frac{x_E}{x_T} b_1; j_2 = \frac{x_E}{x_T} b_2; j_3 = \frac{x_E}{x_T} b_1 + i_1; j_4 = \frac{x_E}{x_T} b_4 + i_2 \\
j_5 &= \frac{x_B}{x_T} b_5; j_6 = \frac{x_E}{x_T} b_7; j_7 = \frac{x_E}{x_T} b_7 + i_3;
\end{aligned}$$

$$\begin{aligned}
l_1 &= \frac{V_{dc}}{2x_T} \sin(\delta_E); l_2 = -\frac{m_E V_{dc}}{2x_T} \cos(\delta_E); \\
l_3 &= \frac{m_E}{2x_T} \sin(\delta_E); l_4 = \frac{V_b}{x_T} \cos(\delta);
\end{aligned}$$

$$\begin{aligned}
m_1 &= \frac{x_E}{x_T} f_1 + l_4; m_2 = \frac{x_E}{x_T} f_2 + l_1; m_3 = \frac{x_E}{x_T} f_3 + l_2; m_4 = \frac{x_E}{x_T} f_4 \\
m_5 &= \frac{x_B}{x_T} f_5; m_6 = \frac{x_E}{x_T} f_6 + l_3; m_7 = \frac{x_E}{x_T} f_6 + l_3;
\end{aligned}$$

$$\begin{aligned}
n_1 &= b_1 + d_1 + j_1; n_2 = b_2 + d_2 + j_2; n_3 = b_3 + d_3 + j_3; \\
n_4 &= b_4 + d_4 + j_4; n_5 = b_5 + d_5 + j_5; n_6 = b_6 + d_6 + j_6; \\
n_7 &= b_7 + d_7 + j_7;
\end{aligned}$$

$$\begin{aligned}
o_1 &= f_1 + h_1 + m_1; o_2 = f_2 + h_2 + m_2; o_3 = f_3 + h_3 + m_3; \\
o_4 &= f_4 + h_4 + m_4; o_5 = f_5 + h_5 + m_5; o_6 = f_6 + h_6 + m_6; \\
o_7 &= f_7 + h_7 + m_7;
\end{aligned}$$

$$\begin{aligned}
p_1 &= x_q o_1; p_2 = x_q o_2; p_3 = x_q o_3; p_4 = x_q o_4; \\
p_5 &= x_q o_5; p_6 = x_q o_6;
\end{aligned}$$

$$q_1 = 1 - x'_d n_1; q_2 = -x'_d n_2; q_3 = -x'_d n_3; q_4 = -x'_d n_4;$$

$$q_5 = -x'_d n_5; q_6 = -x'_d n_6; q_7 = -x'_d n_7;$$

$$r_1 = I_{tq}; r_2 = V_{td} - x'_d I_{tq}; r_3 = V_{tq} + x_q I_{td}; t_1 = x_d - x'_d;$$

$$y_1 = \frac{3}{4C_{dc}} \sin(\delta_E) I_{Ed} + \cos(\delta_E) I_{Eq};$$

$$y_2 = \frac{3m_E}{4C_{dc}} \cos(\delta_E) I_{Ed} - \sin(\delta_E) I_{Eq};$$

$$y_3 = \frac{3}{4C_{dc}} (\sin(\delta_B) I_{Bd} + \cos(\delta_B)) I_{Bq};$$

$$y_4 = \frac{3m_B}{4C_{dc}} (\cos(\delta_E) I_{Bd} - \sin(\delta_B)) I_{Bq};$$

$$y_5 = \frac{3m_E}{4C_{dc}} (\sin(\delta_E)); y_6 = \frac{3m_E}{4C_{dc}} (\cos(\delta_E));$$

$$y_7 = \frac{3m_B}{4C_{dc}} (\sin(\delta_B)); y_8 = \frac{3m_B}{4C_{dc}} (\cos(\delta_B))$$

Appendix C

Gradient-type Newton algorithm

The gradient type Newton algorithm can be given as

The cost function C is defined as:

$$C = \int_0^{\infty} (\Delta\omega)^2 dt$$

The Dynamic model of the closed loop system with a P-I controller in state-space form is given below:

$$\dot{X} = AX + Bp$$

where the state vector x and perturbation vector p are defined as:

$$X = [\Delta\delta \quad \Delta\omega \quad \Delta\dot{E}_q \quad \Delta E_{fd} \quad \Delta V_{de} \quad \int \Delta P_e]^T$$

$$p = [\Delta P_m \quad \Delta V_{ref} \quad \Delta P_{e2(ref)}]^T$$

A and B are matrices of compatible dimensions.

A vector λ is defined as $\lambda = [K_p \quad K_i]^T$, where K_p is the proportional gain setting and K_i is the integral gain setting .

The algorithm is given as:

1. Initialize λ , i.e.,

$$\lambda = \lambda_0 = [K_{p0} \ K_{i0}]^T$$

2. Solve the system (2) to obtain x :
3. Obtain the gradient vector of cost function as:

$$\nabla C(\lambda) = \left[\frac{\partial C}{\partial K_p} \ \frac{\partial C}{\partial K_i} \right]$$

where,

$$\frac{\partial C}{\partial K_p} = 2 \int_0^\infty \Delta\omega \frac{\partial \Delta\omega}{\partial K_p} dt$$

$$\frac{\partial C}{\partial K_i} = 2 \int_0^\infty \Delta\omega \frac{\partial \Delta\omega}{\partial K_i} dt$$

4. Compute the Hessian of the cost function as:

$$H_c(\lambda) = \begin{pmatrix} \frac{\partial^2 C}{\partial k_{p,2}} & \frac{\partial}{\partial K_p} \left(\frac{\partial C}{\partial k_i} \right) \\ \frac{\partial}{\partial K_i} \left(\frac{\partial C}{\partial k_p} \right) & \frac{\partial^2 C}{\partial k_{i,2}} \end{pmatrix}$$

The Hessian is computed from the gradient vector by numerical differentiation.

5. Update the parameter vector λ using Newton iterations:

$$\lambda_{k+1} = \lambda_k - H_c^{-1} \times \nabla C(\lambda_k)$$

6. If $\| \lambda_{k+1} - \lambda_k \| \leq \varepsilon$ go to step 7. If not go to step 2. ε is the small positive number which defines the convergence criterion.
7. END.

Appendix D

Procedure for Calculating Controllability index

The procedure for calculating the controllability index is given below:

The state-space equation $\dot{X} = AX + Bu$ can be rearranged in the following manner:

$$\begin{pmatrix} \dot{\Delta\delta} \\ \dot{\Delta\omega} \\ \dot{x} \end{pmatrix} = \begin{pmatrix} 0 & \omega_0 & 0 \\ -k_j & -d_j & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} * \begin{pmatrix} \Delta\delta \\ \Delta\omega \\ x \end{pmatrix} + \begin{pmatrix} 0 \\ B_{2k} \\ B_{3k} \end{pmatrix} * \Delta u_k$$

The controllability index K_{bki} can be calculated as

$$K_{bki} = (\lambda_i) = B_{2k} + A_{23}(\lambda_i I - A_{33})^{-1} B_{3k}$$