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BER Performance of Lognormal Channel Model with Imperfect CSI for Free Space Optical Communication

Prof. Dhaval Shah**, Dr. Dilip Kothari
Electronics & Communication Dept., Nirma University

ABSTRACT

In this paper, Lognormal Channel model has discussed with the assumption of imperfect CSI. This channel model is used to describe atmospheric channel condition in mathematical form. BER performance of this channel model is shown according to turbulence scenario based on BER equation available. Here Gauss-Markov model is used for the case of imperfect CSI.

Keywords-*lognormal, Imperfect CSI, Gauss-Markov model, scintillation index,turbulence*

****Author for Correspondence:*** dhaval.shah@nirmauni.ac.in, dilip.kothari@nirmauni.ac.in

1. INTRODUCTION

Recently, the use of free space optics is growing rapidly compared to other transmission. Free-Space Optics (FSO) is a line-of-sight technology that uses LASER to provide optical bandwidth connections. FSO requires light, which can be focused by using either light emitting diodes (LEDs) or LASER. The use of LASER is a simple concept similar to optical transmissions using fiber-optic cables; the only difference is the medium. FSO technology is relatively simple. It's based on connectivity between FSO units, each consisting of an optical transceiver with a laser transmitter and a receiver to provide full duplex (bi-directional) capability. Each FSO unit uses a high-power optical source (i.e. LASER), plus a lens that transmits light through the atmosphere to another lens receiving the information.FSO technology requires no spectrum licensing [1].

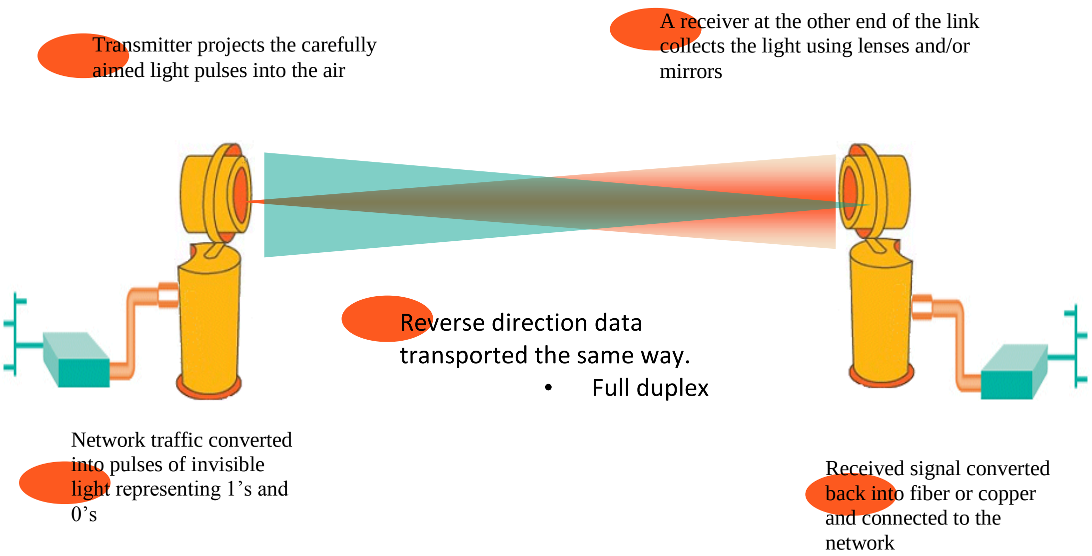

Fig. 1: *Principle of FSO.*

The following are the advantages of the FSO:

- High bit rate (10 Mbps to 2.5 Gbps)[2]
- No licensing required
- Installation cost is very low as compared to laying Fiber
- Easy to install
- Narrow light beam
- Highly secure transmission possible

In this type of communication, air acts as a guiding media for the signals from transmitter to the receiver. So BER performance is largely dependent on environmental conditions. To define these environmental conditions in mathematical terms four models are used, namely Lognormal, Gamma-Gamma, K, I-K. Each of these models is used for particular environmental condition.

Here in this paper, lognormal model is used. Log-normal distribution is the most widely used model for the probability density function (PDF) of the irradiance due to its simplicity, this PDF model is only applicable to weak turbulence conditions. As the strength of turbulence increases, multiple scattering effects must be taken into account. It has been observed that lognormal PDF underestimates the behavior in the tails as compared with measurement results. Because detection and fade probabilities are based on the tails of the PDF, underestimating this region affects the accuracy of performance analysis. Due to this, other channel models have been proposed to overcome this limitation.

Depending on the signal's fading statistics, it could be assumed either as fast or slow. When the fluctuations of the signal intensity are supposed to be very rapid, and thus there idifference from one symbol to another, the channel can be characterized as fast fading and when these fluctuations are very slow compared to the bit rate of the link, it may be considered as a slow fading. For the cases of high bit rate transmission, the channel can be characterized as slow fading.

Background noise, dark noise and thermal noise are among the most common noise, which are all modeled as an additive Gaussian noise with zero mean. The design and practical implementation of optimal detector is commonly based on ML function. Like any wireless communication link, the FSO channel faces fading of the signal due to propagation in free space.

Conventional model of this lognormal channel model is based on assumption that we have perfect CSI. In practical scenario it is hard to have perfect CSI. So result of this model does not agree with the practical results. So in this paper, it is discussed that how to combine this Lognormal model with Gauss-Markov model to have perfect result in the case of imperfect CSI. The principle of least squares has been known for almost two hundred years being invented independently by both Gauss (1809) and Legendre (1806). However, it was Gauss (1880) who gave the first unified development of the theory of least squares estimation and placed it on a sound theoretical basis. At the beginning of this century, Markov (1900) rediscovered the key theorem in the theory of least squares and published it in a text. The thrust of this monograph is to use this theorem as the foundation for the analysis of all regression and experimental design models. The theory of least squares and linear estimation has become the "corner stone" of much of modern statistics.

Finally the results of both the cases are compared. Here in the entire paper the effect of inter symbol interference has not been considered. It has also been assumed an on-off keying (OOK) modulation format at the transmitter and direct (incoherent) detection at the receiver side.

In following section II, various terms related to lognormal channel model are discussed. In section III, the equation for lognormal channel model for the condition of imperfect CSI is derived. Here the conventional equation of lognormal with Gauss-Markov model is combined. In section IV, there is a simulation result in form of BER vs SNR according to the equation derived in the previous section. Here comparison is given between the cases of imperfect CSI and perfect CSI. At last, there is a conclusion of this paper.

2. CHANNEL MODEL

A. *Modulation and Additive Noise*

B. *As stated in [1], the received signal i_d (t) by OOK modulation can be expressed by*

$$i_d(t) = h(t)s(t) + i_n(t) \tag{1}$$

where $s(t)$ is the transmitted signal,; $h(t)$ is the normalized channel fading intensity due to atmospheric turbulence and it is constant over a large number of transmitted bits; and $i_n(t)$ is total additive noise.

The averaged ML-based bit error rate for such a Gaussian channel is expressed as:

$$P_{e,G}(\sigma_1, \sigma_0, P_t) = \frac{1}{2} erfc\left(\frac{\sqrt{2}RP_t}{\sigma_1+\sigma_0}\right) \tag{2}$$

Where, σ_1 and σ_0 are the standard deviations of the noise currents for symbols '1' and '0', respectively [2]. R is the receiver's responsivity; P_t is the average of transmitted power; and $erfc(.)$ is the complementary error function.

For an FSO channel with only additive Gaussian noise, the average SNR can be expressed by [3]

$$\gamma_G = \frac{4R^2{P_t}^2}{(\sigma_1+\sigma_0)^2} \tag{3}$$

Then BER can be expressed in terms of the average SNR

$$P_{e,G}(\gamma_G) = \frac{1}{2} erfc\left(\sqrt{\frac{\gamma_G}{2}}\right) \tag{4}$$

C. *Fading Intensity*

A random variable B has a lognormal distribution if the random variable A=ln$B$ has a normal distribution. So, the fading channel coefficient is given by

$$h = \frac{I}{I_m} = e^{2X} \tag{5}$$

Here I_m is the signal light intensity at the transmitter, without turbulence; I is the signal light intensity at the receiver, with turbulence; and log-amplitude X is the identically distributed normal random variable with mean μ_x and standard deviation σ_x

$$(X) = \frac{1}{\sqrt{2\pi}\sigma_x} exp\left(-\frac{(X-\mu_x)^2}{2{\sigma_x}^2}\right) \tag{6}$$

Substituting (5) in (6), the distribution of light intensity fading induced by turbulence is a log-normal distribution, which is expressed by

$$f_I(h) = \frac{1}{\sqrt{8\pi}h\sigma_x} exp\left(-\frac{[\ln(h)-2\mu_x]^2}{8\sigma_x{}^2}\right) \qquad (7)$$

D. Scintillation Index

Performance of many laser communications systems is adversely affected by scintillation on bright sunny days. It is Beam spreading and wandering due to propagation through air pockets of varying temperature, density, and index of refraction. Through a large aperture receiver, widely spaced transmitters, finely tuned receive filtering and automatic gain control, downtime due to scintillation can be avoided.

In scintillation fading with scintillation index *S.I.*, one can generate average power loss due to atmospheric fading unity, such that the fading does not, on average, attenuate or amplify the optical power. Here

$$\mu_I = 1 \qquad (8)$$

that leads to $\mu_x=-\sigma_x^2$. Thus, the variance will be equal to

$$\sigma_I{}^2 = e^{4\sigma_x{}^2} - 1 \qquad (9)$$

This parameter is called *scintillation index*, S.I[1,4–7]. Here σ_x Is defined as

$$\sigma_x = \frac{\sqrt{\ln(S.I.+1)}}{2} \qquad (10)$$

Note that for a lognormal channel with additive Gaussian noise, the instantaneous SNR from (4) will be converted to

$$\gamma_L = \frac{4h^2R^2P_t{}^2}{(\sigma_1+\sigma_0)^2} \qquad (11)$$

E. Channel State Information (CSI)

CSI is the realization of the *instantaneous* fading state, i.e. fading coefficients *h*, at each symbol period. Some investigations assume perfect availability of CSI at the receiver. Here, perfect CSI mean exactly how good the channel is. It means the perfect value of h at any instant of time is known. If CSI is imperfect, it leads to increased BER at the receiver side. ML-based decision threshold is defined simply as

$$I_{D,With-CSI} = \frac{\sigma_0(I_0+2P_tRh)+\sigma_1 I_0}{\sigma_0+\sigma_1} \qquad (12)$$

A BER analysis will be presented in the following section, in which receiver has no knowledge of the instantaneous fading state will be discussed in following sections.

F. Gauss-Markov Model

Modern estimation theory can be found at the heart of many electronic signal processing systems designed to extract information. The majority of applications require estimation of an unknown parameter θ from a collection of observation data x[n] which also includes “artificacts” due to sensor inaccuracies, additive noise, signal distortion (convolutional noise), model imperfections, unaccounted source variability and multiple interfering signals. This section develops the fundamental results of linear estimation and least squares theory.

The Gauss–Markov theorem[8–10], named after Carl Friedrich Gauss and Andrey Markov, states that in a linear regression model in which the errors have expectation zero and are uncorrelated and have equal variances, the best linear unbiased estimator (BLUE) of the coefficients is given by the ordinary least squares estimator[11]. Here "best" means giving the lowest possible mean squared error of the estimate. The errors need not be normal, nor independent and identically distributed (only uncorrelated and homoscedastic)[12–15]. This model can be expressed by

$$h1=\delta h+\sqrt{1-\delta^2}w \qquad (13)$$

where δ is the correlation factor which varies between 0 to 1. 0 means totally uncorrelation between estimated value and actual value, while 1 means purely related to actual value. w is white Gaussian noise of channel.

Based on this model one can predict transmitted bit h1 from the received bit h based on correlation factor, which can be determined by correlating training sequences.

3. BER PERFORMANCE WITH IMPEFECT CSI

We are using atmosphere as a channel in free space optics. Due to atmospheric condition the received power will be changed. In presence of scintillation fading, the received power will be hP_t.so from Equation (2) and (5),BER can be expressed by[2]

$$P_{e,L}(\sigma_1,\sigma_0,P_t,f_x)=\frac{1}{2}\int_{-\infty}^{\infty} f_x(X)erfc\left(\frac{\sqrt{2RP_te^{2X}}}{(\sigma_1+\sigma_0)}\right)dX \qquad (14)$$

where $f_x(X)$ is defined in Equation (6).erfc(.)is complementary error function and can be expressed by erfc(x)= $2(\pi)^{-1/2}\int_{-\infty}^{\infty}\exp(-r^2)dr$ .By converting X into h, Equation (14) can be written as

$$P_{e,L}(\sigma_1,\sigma_0,P_t,\sigma_x)=\int_0^{\infty}\frac{1}{h}\exp\left(-\frac{\left[\ln(h)+2\sigma_x^2\right]^2}{8\sigma_x^2}\right)erfc\left(\frac{\sqrt{2}RP_th}{(\sigma_1+\sigma_0)}\right)dh \qquad (15)$$

Replcing h by h1 for imperfect CSI at receiver

$$P_{e,L}(\sigma_1,\sigma_0,P_t,\sigma_x)=\frac{1}{\sqrt{32\pi}\sigma_x}\int_0^{\infty}\frac{1}{h1}\exp\left(-\frac{\left[\ln(h1)+2\sigma_x^2\right]^2}{8\sigma_x^2}\right)erfc\left(\frac{\sqrt{2}RP_th1}{(\sigma_1+\sigma_0)}\right)dh \qquad (16)$$

Putting value of h1 according to Gauss-Markov mode

$$P_{e,L}(\sigma_1,\sigma_0,P_t,\sigma_x)=\frac{1}{\sqrt{32\pi}\sigma}\int_0^{\infty}\frac{1}{\delta h+\sqrt{1-\delta^2}w}\exp\left(-\frac{\left[\ln(\delta h+\sqrt{1-\delta^2}w)+2\sigma_x^2\right]^2}{8\sigma_x^2}\right)erfc\left(\frac{\sqrt{2}RP_t,(\delta h+\sqrt{1-\delta^2}w)}{(\sigma_1+\sigma_0)}\right)dh \qquad (17)$$

Above equation is true for lognormal channel model having imperfect CSI at the receiver. Simulation results achieved from above equation are shown in next section.

4. SIMULATION RESULTS

In this section, we have provided numerical simulation for the BER performance of lognormal channel model. It provides the analysis of mathematical computation in previous section. Figure 2 shows the bit error rate probability by(17)and (16),$P_{e,L}$ versus signal to noise ratio Υ_G,for different values of fading intensity σ_x for perfect CSI and imperfect CSI(with δ is equal to 0.50) both.

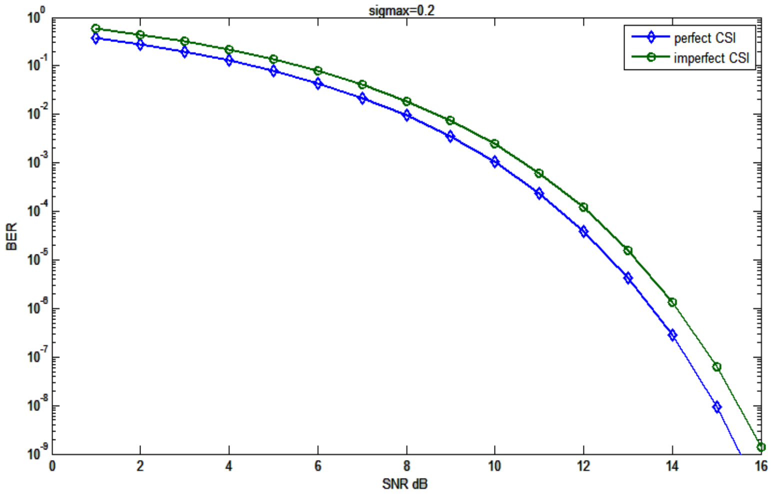

Fig.2: *The Bit Error Rate versus SNR for σ_x=0.2 for Lognormal Channel with Perfect and Imperfect CSI at Receiver.*

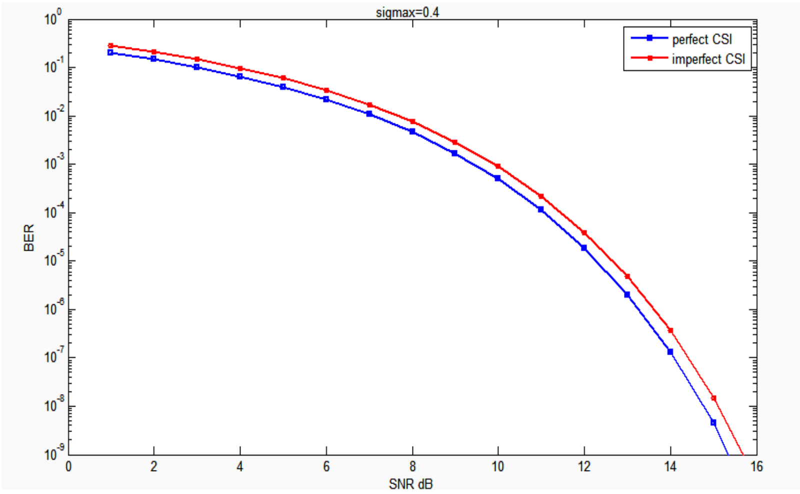

Fig.3: *The Bit Error Rate versus SNR for σ_x=0.4 for Lognormal Channel with Perfect and Imperfect CSI at Receiver.*

In Figures 2 and 3, we can observe that the BER performance for imperfect CSI is not as good as perfect CSI. We can observe constant error floor between BER performance at perfect and imperfect CSI. These results match with actual experimental results for weak turbulence only.BER performance for imperfect CSI matches better than perfect CSI to experimental results. At high turbulence, it shows large deviation compared to actual results.

5. CONCLUSION

In nearby future, FSO will become important medium of information exchange due to its advantages. In this type of communication environment condition plays an important role in transmission setup. Proper distribution model must be used while designing the channel model. For weak turbulence scenario lognormal distribution is used. In the case of imperfect CSI we have to combine this Lognormal distribution with Gauss-Markov model. Though for imperfect CSI BER performance is not as good as perfect CSI but this model gives the perfect agreement between theoretical and practical results.

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