Review On Relationship Between Pressure & Normal Shocks For Convergent-Divergent Nozzle

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Abstract:

Objective of the present study is to analyze experimentally the pressure variation in a varying area circular passage at different supply pressures and compare these with the theoretical solutions. It has been observed that flow of stream of the normal shock is always supersonic, while in downstream is always sub-sonic, thus shocks slow down the flow rates by sudden increase in pressure ratios. Shock strength is determined uniquely by the mach no. higher the mach no. of upstream of supersonic flow , greater is the shock strength & lower is downstream subsonic mach no. Theoretically minimum pressure is always present at the throat during subsonic condition. But experimentally the position of minimum pressure might be varied because of variation in the stagnation properties of the fluid. This project also includes flow phenomenon over the entire length of nozzle for varying back pressure.

the relationship between pressure & normal shocks for Convergent-Divergent Nozzle so that a better design can be proposed.

When the incompressible fluid flows through a pipe of varying cross-section, then the velocities simply vary with the area & can be calculated by $Q = A1V1 = A2V2$. But, when the compressible fluid flows through varying cross-section, the variations in the velocity is affected by the area as well as the density & can be further treated by modified continuity eqn. i.e. $\rho1 A1V1 = \rho2 A2V2$. Flow is considered as steady, one-dimensional, isentropic & compressible.

The effect of variation of back pressure on flow pattern and mach no. (Mach no. is defined as the ratio of velocity of flow at the point to the velocity of sound in the same medium) has been studied in the present experiment. The location of shocks for super sonic flow has been projected for various back pressures. Flow through a nozzle is variant of internal flow with added features of compressible flow & shocks. Such situation arises when there is a constriction across a pressure difference.[3]

1. Introduction:

The normal shock is an abnormal phenomena taking place in the nozzle when supersonic flow is abruptly converted into subsonic flow. Normal shock may leads to large amount of energy losses & intense vibrations. In past many hazardous problems has occurred due to formation of nozzle shocks in supersonic aircrafts. In the paper the review has been presented for

1.1 Comparison of Isentropic & Adiabatic process

FIGURE 4.1 (*a*) : Temperature-entropy diagram for expansion processes (Shaft work ≈ 0)
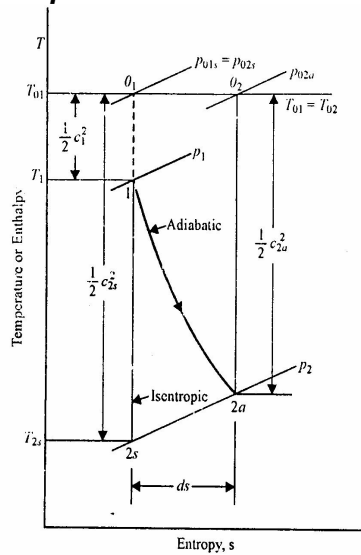

FIGURE 4.1 (*b*) : *p-v* diagram for expansion processes (Shaft work = 0)

Figure shows isentropic & adiabatic expansion of a gas between two state 1 & 2. The initial stagnation pressure is $P_{01}s$ & the kinetic energy $\frac{1}{2}C_1^2$. The stagnation & static temperature are T_{o1} & T1.

As per as the Adiabatic process is concerned, there would be increase in entropy & the final stagnation pressure is lower than its initial value. Final kinetic energy is also lower than its corresponding isentropic value. Stagnation temperature would be same in both in isentropic & adiabatic process. Volume in Adiabatic process would be greater than in Isentropic process.[1][2]

2. Expansion in Nozzle:

Gases & vapors are expanded in nozzle by providing a pressure ratio across them. It is shown by first equation of mathematical formulation. since the purpose of nozzle is to accelerate the flow by providing a pressure drop.

Following three conditions are considered:

- For m<1, area decreases, pressure decreases & velocity increases.
- For m=1, dA =0 , it implies there is no change of passage area at the point where mach no. is unity. This section is refered as the throat of the passage
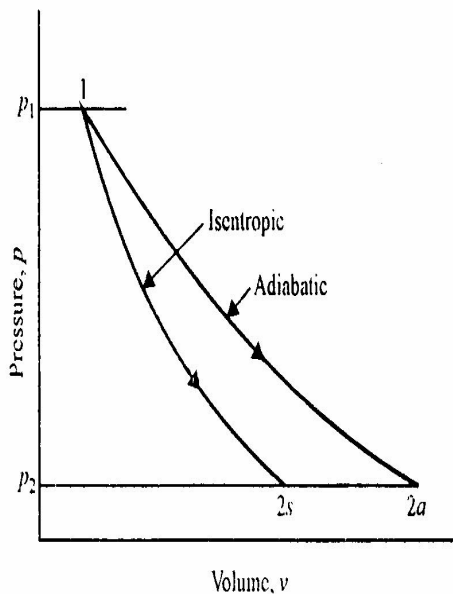- For m>1, area increases ,pressure decreases , velocity increases[1].

2.1 Some Relations between throat conditions & conditions for corresponding Mach number

$$\frac{To}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{Po}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$$

$$\frac{\rho o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

lower it far enough & we eventually get it to the situation shown in figure 3b. the flow pattern is exactly the same as in sub-sonic flow, except that the flow speed at the throat has just reached mach1. flow through the nozzle is now chocked since further reductions in the back pressure can't move the point of M =1 away from the throat. However the flow pattern in the diverging section does change as you lower the back pressure further. As pb lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a sub-sonic flow, the super-sonic flow accelerates as the area gets bigger. This region of supersonic acceleration is terminated by normal shock wave. The shock wave produces a near – instantaneous deceleration of the flow to sub-sonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhaust as a subsonic jet. In this regime if you lower or raise the back pressure, you increase or decrease the length of super-sonic flow in the diverging section before the shock wave.

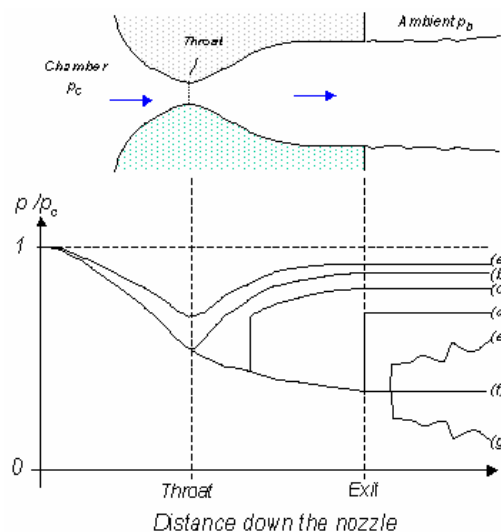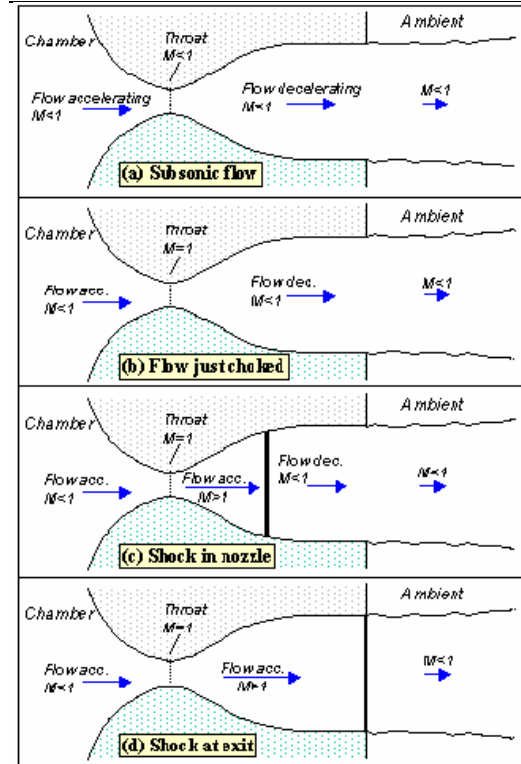### 2.2 Flow pattern in Convergent divergent nozzle:[3]

Figure 3a shows the flow through the nozzle when it is completely subsonic (i.e. the nozzle isn't choked). The flow accelerates out of the chamber through the converging section, reaching its maximum (subsonic) speed at the throat. The flow then decelerates through the diverging section and exhausts into the ambient as a subsonic jet. Lowering the back pressure in this state increases the flow speed everywhere in the nozzle.

Lower it far enough and we eventually get to the situation shown in figure 3b. The flow pattern is exactly the same as in subsonic flow, except that the flow speed at the throat has just reached Mach 1. Flow through the nozzle is now choked since further reductions in the back pressure can't move the point of M=1 away from the throat. However, the flow pattern in the diverging section does change as you lower the back pressure further.

As p_b is lowered below that needed to just choke the flow a region of supersonic flow forms just downstream of the throat. Unlike a subsonic flow, the supersonic flow accelerates as the area gets bigger. This region of supersonic acceleration is terminated by a normal shock wave. The shock wave produces a near-instantaneous deceleration of the flow to subsonic speed. This subsonic flow then decelerates through the remainder of the diverging section and exhausts as a subsonic jet. In this regime if you lower or raise the back pressure you increase or decrease the length of supersonic flow in the diverging section before the shock wave.

If you lower p_b enough you can extend the supersonic region all the way down the nozzle until the shock is sitting at the nozzle exit (figure 3d). Because you have a very long region of acceleration

(the entire nozzle length) in this case the flow speed just before the shock will be very large in this case. However, after the shock the flow in the jet will still be subsonic.[3]

2.3 Mach number Downstream Of The Normal Shock Wave

Generally the upstream Mach number M_x in a given problem is known & it is desired to determine the Mach no. My downstream of the shock wave.

It will be seen that the only independent parameter required for a given gas to determine the downstream Mach no. is the upstream Mach no.

$$M_Y^2 = \frac{\dfrac{2}{\gamma-1}+M_X^2}{\dfrac{2\gamma}{\gamma-1}M_X^2 -1}$$

2.4 Static pressure ratio across the shock

$$\frac{P_Y}{P_X} = \frac{2\gamma}{\gamma-1}M_X^2 - \frac{\gamma-1}{\gamma+1}$$

2.5 Density Ratio Across the Shock (The Rankine-Hugoniot Equation):

$$\frac{\rho_Y}{\rho_X} = \frac{\left(\dfrac{\gamma+1}{2}\right)M_X^2}{1+\dfrac{\gamma-1}{2}M_X^2} = \frac{1+\dfrac{\gamma+1}{\gamma-1}\dfrac{P_Y}{P_X}}{\dfrac{\gamma+1}{\gamma-1}+\dfrac{P_Y}{P_X}}$$

2.6 Strength of Shock Wave:

A parameter is defines the strength of a shock wave is often used in shock wave analysis.

$$\xi = \frac{\dfrac{2\gamma}{\gamma-1}\left(\dfrac{\rho_y}{\rho_x}-1\right)}{\dfrac{2}{\gamma-1}-\left(\dfrac{\rho_y}{\rho_x}-1\right)}$$

3. Conclusion:

It has been observed that increasing the stagnation pressure displaces the shock position in the downstream, away from the throat. This shows that if the length of the divergent portion is reduced than the designed condition, shock will travel towards the throat thereby making the entire flow subsonic, with high pressure waves in the divergent portion.

4. Future work:

In future we will make project setup & it contains one convergent and divergent nozzle, one piezometer for measuring the pressure at different point over the nozzle so that we can find out the position of normal shock wave inside the nozzle and one compressor to vary the inlet pressure. From this setup we will find out the relationship between pressure & normal shock wave for convergent & divergent nozzle of varying pressure.

5. Nomenclature:

A_1 =Area at inlet

A_2 =Area at outlet

V_1 =Velocity of fluid at inlet

V_2 =Velocity of fluid at outlet

P_X =Pressure at inlet

P_Y =Pressure at outlet

M =Mach number

M_X =Upstream Mach number

M_Y =Downstream Mach number

To = Stagnation temperature

T =Temperature at corresponding Mach no.

Po =Stagnation pressure

P =Pressure at corresponding Mach no.

Q =Flow rate

K =Bulk modulus of elasticity

R =Gas constant

E=Modulus of elasticity

P_b=Back pressure

ρ = Density at corresponding Mach no.

ρ_o = Stagnation density

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2.   John J. Bertin & Michael L. Smith "Aerodynamics For Engineers", 3rd edition.[346-350]

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