

Performance Evaluation of Energy Detectors in Cognitive Radio

Major Project Report

Submitted in partial fulfillment of the requirements

Master of Technology

in

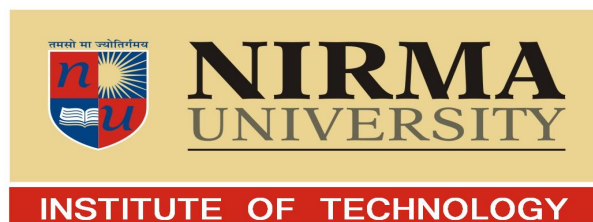
Electronics & Communication Engineering

(Communication Engineering)

By

Rina S. Parikh

11MECC51



Electronics & Communication Engineering Branch

Electrical Engineering Department

Institute of Technology

Nirma University

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Under the Guidance of

Dr. Y. N. Trivedi



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May 2014

Declaration

This is to certify that

- I) The thesis comprises my original work towards the Degree of Master of Technology in Communication Engineering at Nirma University and has not been submitted elsewhere for a degree.
- II) Due Acknowledgment has been made in the text to all other material used.

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Certificate

This is to certify that the Major Project entitled “**Performance Evaluation of Energy detectors in Cognitive Radio**” submitted by **Ms. Rina S. Parikh (11MECC51)**, towards the partial fulfillment of the requirements for the degree of Master of Technology in Electronics & Communication Engineering (Communication Engineering) of Nirma University, Ahmedabad, is the record of work carried out by her under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination. The results embodied in this major project, to the best of my knowledge, haven’t been submitted to any other university or institution for award of any degree or diploma.

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Abstract

Wireless networks are characterized by a fixed spectrum assignment policy. With the advancement of wireless communications, the problem of bandwidth scarcity has become more prominent. As per Federal Communications Commission (FCC), large portion of the spectrum lies vacant most of the time and that portion is the licensed spectrum band; which is utilized by licensed users only. The temporal and geographical variation in the usage of the assigned spectrum ranges from 15% to 85%. So, to solve this problem of spectrum under-utilization, FCC allowed secondary users to utilize the licensed band when it is not in use and named it as Cognitive Radio.

To sense the existence of licensed users or in other words, to utilize the unused spectrum, spectrum sensing techniques are used. Energy detection, Matched filter detection and Cyclo-stationary feature detection are the three conventional methods used for spectrum sensing. Each technique has its own advantages and drawbacks. Matched filter spectrum sensing technique requires a priori information about each primary user and a dedicated cognitive radio receiver is required for every primary user. Cyclostationary feature Detection is computationally complex and requires significantly long observation time to extract the features of primary user signal. Energy detection is the most simplest to implement, but the performance of energy detector is susceptible to uncertainty in noise power. This report discusses the conventional energy detection method in case of AWGN channel and Rayleigh fading channel. The performance is improved by introducing diversity in fading channels. The generalized energy detection method is discussed where squaring operation of conventional energy detection is replaced by any positive power constant, which is known as generalized energy detector. Also, effect of noise uncertainty is studied in this generalized energy detector. The performance of energy detector degrades significantly under low SNR circumstances and detection becomes impossible below certain critical values called SNR Walls. To improve the detection probability under such case, Stochastic Resonance (SR) based energy detection approach is used. It significantly reduces the SNR wall. A novel combination of generalized energy detector and SR phenomenon

is presented here and improved results have been obtained. Mathematical Analysis has been illustrated for all these cases. Simulation and analytical results have also been included in this report.

Abbreviation Notation and Nomenclature

FCC	Federal Communications Commission
AWGN	Additive White Gaussian Noise
ADC	Analog to Digital Converter
AP	Access Point
BPSK	Binary Phase Shift Keying
CR	Cognitive Radio
CRN	Cognitive Radio Network
CU	Cognitive User
CED	Conventional Energy Detector
CSR	Chaotic Stochastic Resonance
CLT	Central limit Theorem
DARPA	Defence Advanced Research Projects Agency
DSP	Digital signal Processing
DSSS	Direct Sequence Spread Spectrum
DFH	Dynamic Frequency Hopping
ED	Energy detector
EGC	Equal gain Combining
FCC	Federal Communications Commission
FPGA	Field Programmable Gate Array
FHSS	Frequency Hopping Spread Spectrum
FSK	Frequency shift Keying
GED	Generalized Energy Detector
IEEE	Institute of Electrical and Electronic Engineering
IID	Independent and Identically Distributed
LTE	Long Term Evolution
MAC	Medium Access Control
MIMO	Multiple Input Multiple Output
MF	Matched Filter

NP	Neyman Pearson
NPRM	Notice for Proposed Rule Making
OFDM	Orthogonal Frequency Division Multiplexing
PHY	Physical layer
PU	Primary User
PUE	Primary User Emulation
PDF	Probability Density Function
PSK	Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
ROC	Receiver Operating Characteristics
RF	Radio Frequency
SNR	Signal-to-Noise Ratio
SR	Stochastic Resonance
SC	Selection Combining
SSC	Switch and Stay Combining
USRP	Universal Software Radio Peripheral
WRAN	Wireless Regional Area Network
WLAN	Wireless Local Area Network
xG	NeXt Generation

Symbols

P_d	Probability of Detection
P_f	Probability of False Alarm
P_m	Probability of miss Detection
T	Obsevation Interval
f_c	Carrier Frequency
W	Bandwidth

Y	Output of integrator
λ	Threshold
h	Channel
r	Received Signal
s	Transmitted Signal
n	noise
H_0	Hypothesis 0
H_1	Hypothesis 1
2γ	Noncentrality parameter of non central Chi-square distribution
N	Degrees of Freedom
$I(.)$	Modified Bessel function of the first kind
$\Gamma(.)$	Gamma Function
$\Gamma(.;.)$	Incomplete gamma Function
$Q_m(a,b)$	Generalized Marcum Q function
${}_1F_1(.;.;.)$	<i>Confluent Hypergeometric function</i>
μ	Mean
σ^2	Variance
β	Noise Uncertainty factor
L	Noise Uncertainty factor in dB
η	Noise in Stochastic Resonance System
$U(x)$	Double-well potential
$Q_{\chi^2_N}$	<i>Chi - square pdf</i>
$E[h^2]$	Second Order moment of h

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Chapter 1

Introduction

The need for a flexible and robust wireless communication is becoming more evident in recent times. Conventionally, the policy of spectrum licensing and utilization lead to inefficient usage of the available spectrum. The requirement of different technologies and market demand leads to spectrum scarcity and non uniform utilization of frequencies.

It has become essential to introduce new licensing policies to enable dynamic way of utilizing the available spectrum efficiently. One promising solution is the Cognitive Radio. Cognitive radio is an intelligent wireless communication system that is aware of its surrounding environment (i.e., outside world) and dynamically adapts the environment by making changes in certain operating parameters (e.g., transmit-power, carrier-frequency, and modulation strategy) in real-time.

The limited available spectrum and the inefficiency in the spectrum usage necessitate use of existing wireless spectrum opportunistically [1]. Dynamic spectrum access is proposed to solve these current spectrum underutilization problems. Dynamic Spectrum Access network exploits NeXt Generation (xG) networks that aim to implement the policy based intelligent radios known as cognitive radios. Cognitive radio techniques provide the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access techniques allow the cognitive radio to operate in the best

available channel. The cognitive radio technology will enable the users to ,

- determine which portions of the spectrum is available and detect the presence of licensed users when a user operates in a licensed band (**spectrum sensing**),
- select the best available channel (**spectrum management**),
- coordinate access to this channel with other users (**spectrum sharing**), and
- vacate the channel when a licensed user is detected (**spectrum mobility**).

Among these operations, Spectrum Sensing is the most crucial operation to establish a Cognitive Radio. Sensing spectrum holes which are also referred to as White Space and vacant them as licensed user is detected requires binary decision for fast spectrum sensing.

1.1 Overview of Cognitive Radio

Cognitive Radio is a new paradigm that has been proposed so that the frequency spectrum can be efficiently utilized. The formal definition for Cognitive Radio is given as :

Cognitive Radio is a radio for wireless communications in which either a network or a wireless node changes its transmission or reception parameters based on the interaction with the environment to communicate effectively without interfering with the licensed users. [3]

In Fig. 1.1, the signal strength distribution over a large portion of the wireless spectrum is shown. The spectrum usage is concentrated on certain portions of the spectrum while a significant amount of the spectrum remains unutilized [2].

Dynamic spectrum access is proposed to solve these current spectrum inefficiency problems. DARPA's approach on Dynamic Spectrum Access network, the so-called

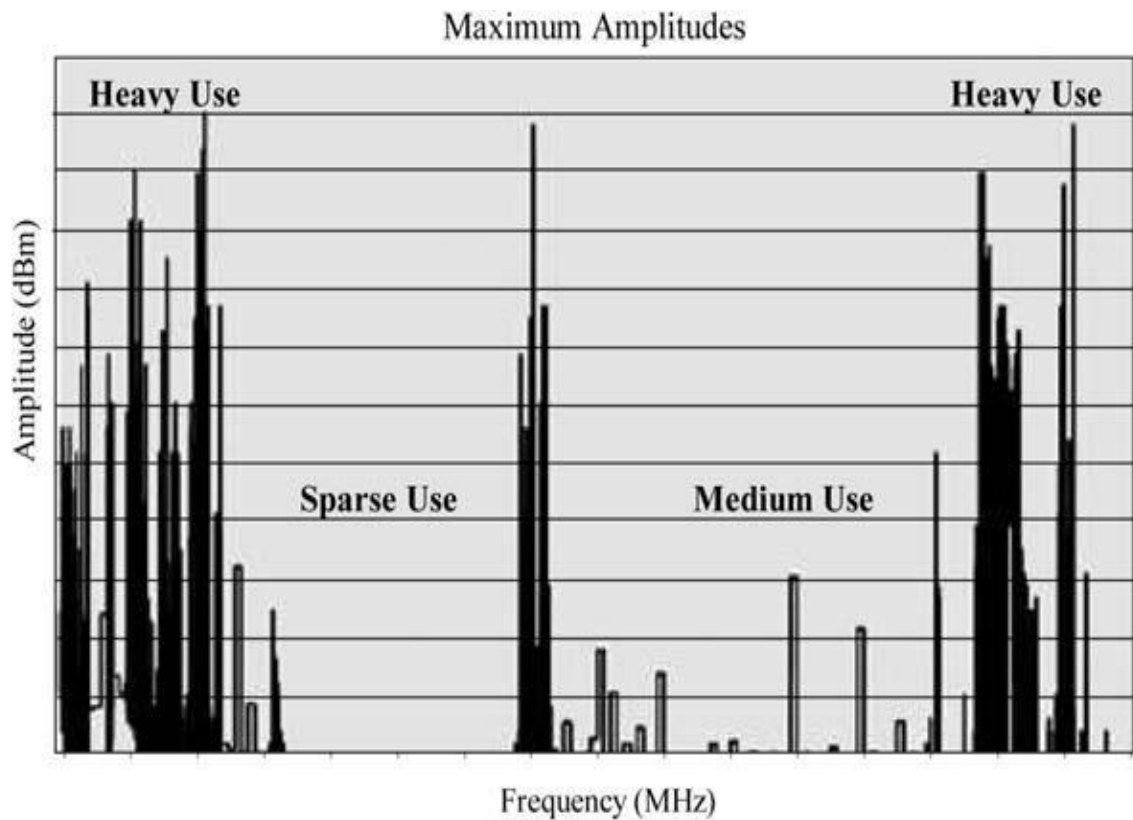


Figure 1.1: Spectrum Utilization

NeXt Generation (xG) program aims to implement the policy based intelligent radios known as cognitive radios. The key enabling technology of xG networks is the cognitive radio [1]. Cognitive radio provides the capability to use or share the spectrum in an opportunistic manner. Dynamic spectrum access allows the cognitive radio to operate in the best available channel.

1.1.1 Evolution of Cognitive Radio

The evolution of cognitive radio has been presented here:

In 1999, Joseph Mitola coined the term Cognitive Radio for the first time. In 2002, the Defense Advanced Research Projects Agency (DARPA) funded the NeXt Generation (DARPA-XG) program whose purpose was to define a policy based spectrum management framework such the radios can make use of the spectrum holes. This drew the attention of the Federal Communications Commission (FCC) which then found the underutilization of the bands based on the research conducted by it. Later the FCC issued a Notice for Proposed Rule Making (NPRM) , whose main purpose was to explore the cognitive radio technology to improve spectrum utilization. In 2004, the Institute of Electrical and Electronic Engineers (IEEE) formed the IEEE 802.22 working group for defining the Wireless Regional Area Network (WRAN) Physical (PHY) and Medium Access Control (MAC) layer specifications. By end 2005, IEEE launched the Project 1900 standard task group for next generation radio and spectrum management. It was related to giving standard terms and formal definitions for spectrum management,spectrum utilization, interference and co-existence analysis and policy architecture, dynamic spectrum access radio systems. In 2006, IEEE organized the first conference on cognitive radio CROWNCOM so as to bring together new ideas regarding the cognitive radio from a diverse set of researchers around the world. By 2008 end, the FCC established rules to allow cognitive devices to operate in TV White Spaces on a secondary basis. In 2010, FCC released a Memorandum Opinion and Order that defined the final rules for the use of spectrum holes by unlicensed wireless users. In July, 2011, the IEEE published IEEE 802.22 (WRAN) as an official standard.Currently, IEEE is working on the standard for recommended practice for installation and deployment of 802.22 systems.

1.1.2 Cognitive Radio Features

Cognitive Radio Characteristics:

Reconfigurability: This property of cognitive radios means their ability to modify their configuration dynamically. Reconfigurability can be realized through the use of

elements that can dynamically alter the performance parameters of their operation to improve the Quality of Services. Reconfigurations are software-defined, that is, they can be accomplished by activating the appropriate software at the transceiver. Followings are the capabilities of Reconfigurability:[1]

- **Frequency Agility:** It is the ability of a radio to change its operating frequency. This ability is combined with a method to dynamically select the appropriate operating frequency based on the sensing of signals from other transmitters or on other method.
- **Dynamic Frequency Selection:** It is defined as a mechanism that dynamically detects signals from other radio frequency systems and avoids co- channel operation with those systems.
- **Adaptive Modulation/Coding:** A cognitive radio could select the appropriate modulation type for use with a particular transmission system to permit interoperability between systems.
- **Transmit Power Control:** Transmit power control is a feature that enables cognitive radio to dynamically switch between several transmissions power levels in the data transmission process.

Cognition: The stochastic nature of the environment conditions raises the need for the second main attribute of cognitive radio systems, namely, cognition. Cognition refers to the process of knowing through perception, reasoning, knowledge and intuition with a focus on information available from the environment . Thus, Cognitive capability includes the features of spectrum sensing, spectrum sharing, location identification, network and service discovery.

Self-management: Each transceiver should be able to do self-adaption to its environment without the need to be instructed by a central management entity. This

concept provides significant reduction of system complexity because it does not call for a centralized management entity. It has the following features:

- **Spectrum/Radio Resource Management:** To efficiently manage and organize spectrum holes information among cognitive radios, good spectrum management scheme is necessary.
- **Mobility and Connection Management:** Due to the heterogeneity of Cognitive Radio Networks (CRNs), routing and topology information is more complex. Good mobility and connection management can help neighbourhood discovery, detect available Internet access and support handoffs, which can help cognitive radios to select route and networks.
- **Trust or Security Management:** AS CRNs are heterogeneous networks in nature, various heterogeneities (i.e. wireless access technologies, system or network operators) introduce lots of security issues. Trust among the users is one of the requirement for secured operations in CRNs .

1.2 Spectrum Hole

Since most of the spectrum is already assigned, the most important challenge is to share the licensed spectrum without interfering with the transmission of other licensed users as illustrated in Fig. 1.2. The cognitive radio enables the usage of temporally unused spectrum, which is referred to as spectrum hole or white space. If this band is further used by a licensed user, the cognitive radio moves to another spectrum hole or stays in the same band, altering its transmission power level or modulation scheme to avoid interference as shown in Fig. 1.2 [1].

1.3 Cognitive Cycle

The cognitive ability of a cognitive radio allows real time interaction with its environment to determine appropriate communication parameters and adapt to the dynamic

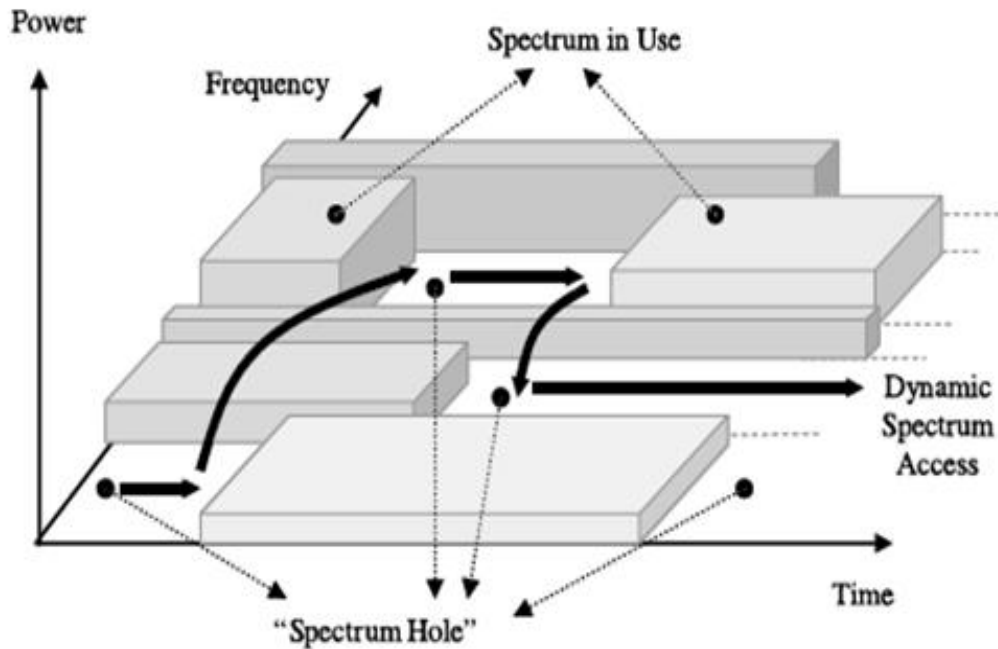


Figure 1.2: Spectrum Hole or White Space

environment. The main tasks required for dynamic operation in spectrum are shown in Fig. 1.3 [1] that is referred to as the cognitive cycle.

The steps of the cognitive cycle as shown in Fig. 1.3 are as follows:

- **Spectrum sensing:** A cognitive radio continuously monitors the available spectrum bands, captures their information, and detects the spectrum holes or white space.
- **Spectrum analysis:** The characteristics of the spectrum holes that are detected through spectrum sensing are estimated.
- **Spectrum decision:** A cognitive radio determines the data rate, the trans-

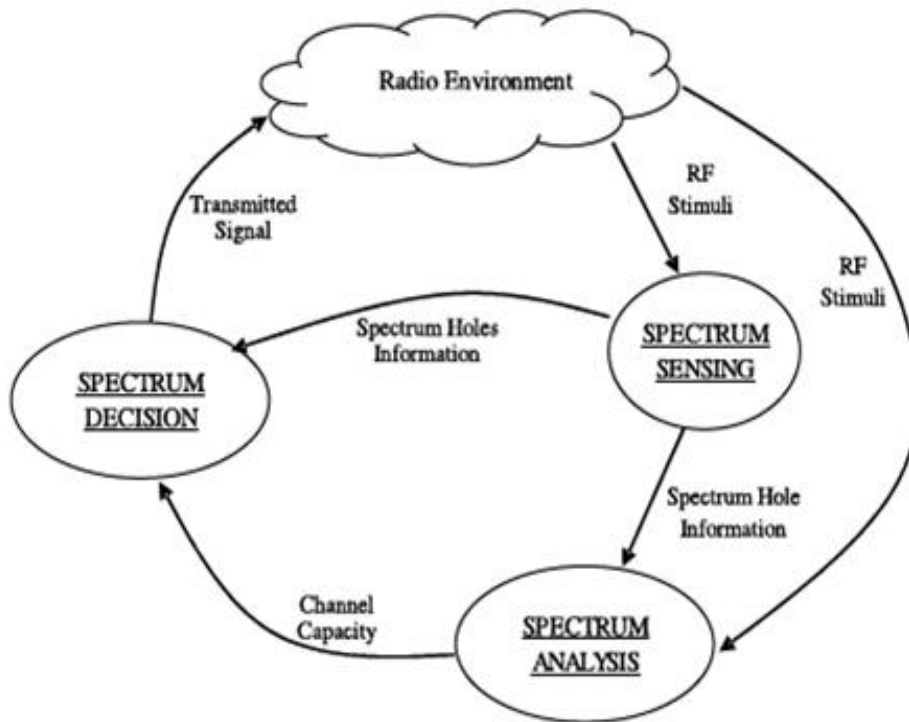


Figure 1.3: Cognitive Radio

mission mode, and the bandwidth of the transmission. Then, the appropriate spectrum band is chosen according to the spectrum characteristics and user requirements.

Spectrum sensing is the most essential requirement for the establishment of cognitive radio. Spectrum sensing is to be aware of about the spectrum usage and existence of primary users (PUs) in a geographical area of interest. This awareness can be obtained by using geolocations and database, by using beacons, or by local spectrum sensing at cognitive radios. Once the operating spectrum band is determined, the communication can be performed over this spectrum band. However, since the radio environment changes, the cognitive radio should keep track of the changes in

the radio environment. If the current spectrum band in use becomes occupied, the spectrum mobility function is performed to provide a seamless communication. Any environmental change during the transmission such as presence of primary user, user movement, or variation of traffic can start these adjustments. Different methods are proposed for identifying the presence of signal transmissions. The most commonly used methods are energy detection, matched filter detection, cyclostationary feature detection and cooperative detection.

There are certain Challenges associated with spectrum sensing techniques. One of the main requirements of cognitive networks is the detection of licensed users in a very short time. It is very necessary to develop an interference detection model by effectively measuring the interference temperature. The required SNR for detection might be very low. Multipath fading and time dispersion of the wireless channel complicate the sensing problem. Multipath fading may cause the signal power to fluctuate around 20 dB, and the time dispersion in wireless links may turn the coherent detection unreliable. The noise or interference level may have temporal and geographical variations which yields the noise power uncertainty issue for detection. For selecting a sensing technique some tradeoffs should be considered such as accuracy, computational complexity, sensing duration requirements, network requirements.

1.4 Motivation

Cognitive radio (CR) technique has been proposed to solve the conflicts between spectrum scarcity and spectrum under utilization. It allows the CR users to share the spectrum with primary users (PUs) by opportunistic spectrum accessing. The CR can use the spectrum only when it does not cause interference to primary users. Therefore, spectrum sensing is the most critical issue of cognitive radio technology as it requires to detect the presence of primary users accurately and immediately. In many wireless applications, it is of great interest to check the presence and avail-

ability of an active communication link when the transmitted signal is unknown. There are number of techniques for spectrum sensing like matched filter detection, cyclostationary feature detection and energy detection. Matched filter maximizes the received signal to noise ratio (SNR) but it requires a priori knowledge of the Primary User signal. Cyclostationary feature detection is computationally complex and requires significantly long observation time to extract the features of the signal to be detected. In such scenarios, one appropriate choice consists of using an energy detector which measures the energy in the received waveform over an observation time interval. Energy Detection is the most common type of spectrum sensing because of its low computational as well as implementation complexities. It is a more generic method as the receivers do not need any knowledge of the primary users signal.

This leads to the implementation of energy detectors in wireless communication. Performance of various energy detectors is found to be susceptible to noise and deteriorates under the circumstances of low SNR. Thus, further improvement in the performance in energy detection is required which leads to improved energy detection technique. The thesis presents mathematical analysis for implementation of energy detector in wireless fading channels, improvement in performance by introducing diversity, the generalized energy detector, the effect of noise uncertainty which is present in actual practice and Stochastic Resonance based energy detector as well as reduction in sample complexity by using generalized energy detection in SR based energy detection.

1.5 Thesis Organization

The rest of the thesis is organized as follows.

Chapter 2 describes various spectrum sensing techniques which is the most essential task of cognitive radio. It focuses on Transmitter Detection techniques of spectrum

sensing. This chapter describes advantages and disadvantages of various techniques. It also gives significance of performance parameters of detection like Probability of detection and Probability of false alarm. It discusses challenges of spectrum sensing in cognitive radio in detail.

Chapter 3 focuses on energy detection technique as it is simple to implement and presents the mathematical analysis to evaluate the performance criteria over AWGN channel, wireless fading channel and by incorporating diversity in wireless fading channel. It also represents analytical and simulation results showing Receiver operating characteristics (ROC) curves for the above mentioned cases. It also explains the concept of Improved Energy Detection technique which is also known as Generalized Energy Detection to further improve the performance in the case of low SNR. It also shows the effect of adding noise uncertainty in the above case. Analytical and simulation results are presented in this chapter.

Chapter 4 represents a study of a novel spectrum sensing technique in cognitive radio based on Stochastic Resonance phenomenon. It explains the SR phenomenon in detail. The distinguished performance in comparison with other techniques is represented in terms of SNR wall. This chapter introduces a new energy detection technique based on the combination of generalized energy detector explained in chapter 3 and SR based energy detector. It claims great reduction in sample complexity which is one of the limiting factors of energy detection.

Chapter 5 includes the conclusion on the basis of work done and scope for further studies to mitigate challenges in energy detection technique.

Chapter 2

Spectrum Sensing Techniques

An important requirement of the xG network is to sense the spectrum holes. A cognitive radio is designed to detect and adapt the changes in its surrounding. The spectrum sensing function enables the cognitive radio to adapt to its environment by detecting spectrum holes. The most efficient way of detecting spectrum holes is to detect the primary users (PUs) that are receiving data within the communication range of an secondary user. However, in practice it is difficult for a cognitive radio to have a direct measurement of a channel between a primary receiver (user) and a transmitter. Thus, the most recent work focuses on primary transmitter detection based on local observations of xG users. Generally, the spectrum sensing techniques can be classified as transmitter detection, cooperative detection, and interference-based detection, as shown in Fig. 2.1 [1].

2.1 Transmitter detection

The cognitive radio should distinguish between used and unused spectrum bands. Thus, the cognitive radio should have capability to determine if a signal from primary transmitter is locally present in a certain spectrum. Transmitter detection approach is based on the detection of the weak signal from a primary transmitter through the local observations of xG users. Basic hypothesis model for transmitter:

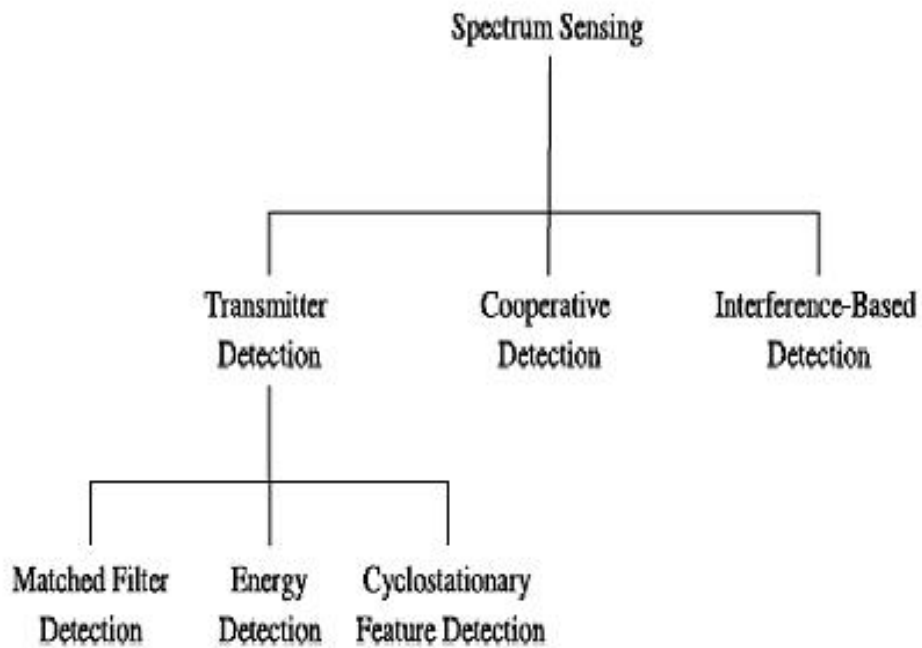


Figure 2.1: Spectrum Sensing techniques

$$x(t) = \begin{cases} n(t), H_0 \\ h.s(t) + n(t), H_1 \end{cases}$$

Where

H_0 : Primary user is absent.

H_1 : Primary user is present.

The goal of spectrum sensing is to decide between the following two hypotheses.

There are two basic hypothesis testing methods in spectrum sensing: the Neyman-

Pearson (NP) test and the Bayes test. In an NP test, the objective is to maximize the detection probability P_d given the constraint of $P_f < \alpha$, where α is the maximum false alarm probability. In a Bayes test, the objective is to minimize the expected cost called the Bayes Risk. the Bayes risk to be minimized is the sum of all possible costs weighted by the probabilities of two incorrect detection cases (false alarm and miss detection) and two correct detection cases.

Performance Parameters are:

- **Probability of detection (P_d):** Determines the level of interference-protection provided to the primary licensed user.
- **Probability of false-alarm (P_f):** The percentage of white spaces falsely declared occupied (i.e. the percentage of missed opportunities).
- **Probability of miss detection (P_m):** $P_m=1-P_d$

To achieve the goal of CR, it is a fundamental requirement that the cognitive user performs spectrum sensing to detect the presence of Primary User (PU) signal. The spectrum sensing is often considered as a detection issue where the CUs have to scan a vast range of frequencies to observe available spectrum white spaces or holes that are temporarily and spatially out of service.

2.1.1 Challenges of Spectrum Sensing

Hardware Requirements:

The requirements of Spectrum sensing in cognitive radio applications are high sampling frequency, high resolution analog to digital converters (ADCs), and high speed signal processors. It requires complex signal processing algorithms for ex., noise variance estimation, variable information generation, channel estimation as well as for improved handoff, power control, and channel allocation techniques to implement on

hardware. It is easier to estimate the noise/interference as receivers are tuned to receive signals that are transmitted over a desired bandwidth. Further, receivers are able to process the narrowband baseband signals with reasonably low complexity and low power processors. But, in cognitive radio, receivers are required to process transmission over a very wide band to search for any opportunity. Hence, cognitive radio should be able to capture and analyze a quite larger band for identifying spectrum holes. The large operating bandwidths impose additional requirements on the radio frequencies (RF) components such as antennas, power amplifiers etc. Also, high speed signal processing units like DSPs, FPGAs are needed for performing computationally demanding signal processing tasks at relatively small time delay. There are very few hardware and software platforms are available for the cognitive radio. GNU Radio, Universal Software Radio Peripheral (USRP) and Shared Spectrums XG Radio are some.

Hidden Primary Terminal Problem:

It can be caused by many factors including severe multipath fading or shadowing observed by secondary users while scanning for primary users transmissions. Fig.2.2 [1] shows the problem of hidden node where the circles show the operating ranges of the primary user and the cognitive radio terminal. Here, cognitive radio device causes unwanted interference to the primary user (receiver) as the primary transmitters signal could not be detected because of the locations of devices. Cooperative sensing is used to solve hidden primary user problem.

Detecting Spread Spectrum Primary Users:

There are two types of technologies: Fixed Frequency and Spread Spectrum. The two main spread spectrum technologies are frequency hopping spread-spectrum (FHSS) and direct sequence spread spectrum (DSSS). Fixed frequency devices operate at a single frequency or channel. FHSS devices may change their operational frequencies dynam-

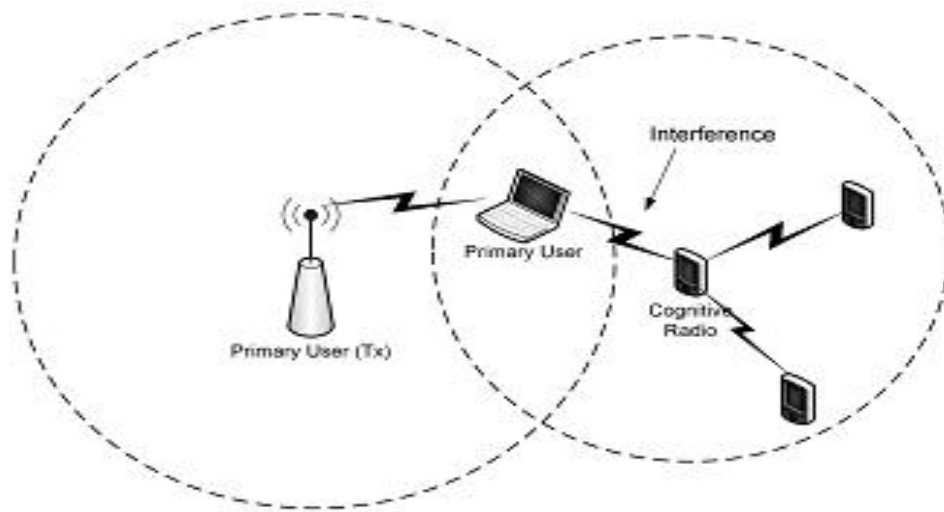


Figure 2.2: Hidden Terminal problem

ically towards multiple narrowband channels. This is called hopping. It is performed as per the sequence that is known by both transmitter and receiver. DSSS devices are similar to FHSS devices, but, they use a single band to spread their energy. Primary users which are using spread spectrum signaling are difficult to detect as the power of the PU is distributed over a wide frequency range.

Sensing Time (Duration) and Frequency:

Whenever primary user is detected, secondary user has to vacate the frequency band in which it is operating. In order to prevent interference to primary users, cognitive radio should be able to identify the presence of primary users as fast as possible and should vacate the band immediately. Hence, sensing methods should be such that it can identify the presence of primary users within a certain short duration. This requirement imposes a limitation on the performance of spectrum sensing algorithm and generates a challenge for the design of cognitive radio. There is a trade off between selection of sensing parameters: the speed (sensing time) and reliability of sensing.

Sensing frequency, i.e. rate at which cognitive radio should perform spectrum sensing, is one of the design parameter which should be chosen carefully. The optimum value of sensing frequency depends on the capabilities of cognitive radio and the temporal characteristics of primary users in that network. In addition to sensing frequency, the channel detection time, channel move time and other temporal parameters are also defined in the standard.

Quick detection:

It is a very crucial challenge, when the conditions are more dynamic. The challenge is to detect the beginning of a primary users transmission as quickly as possible after it happens. Similar issues with unknown parameters also occur in this detection problem.

Security:

In cognitive radio, a malicious user can modify its air interface to mimic a primary user. Hence, it can mislead the CR. Such a behavior or attack is known as primary user emulation (PUE) attack. Its harmful effects on the cognitive radio network are investigated. A more challenging problem is to develop effective countermeasures once an attack is identified. Public key encryption based primary user identification can be used to prevent secondary users mimicking as primary users. Legitimate primary users are required to transmit an encrypted value (signature) along with their transmissions which is generated using a private key. This signature is, then, used for validating the primary user. This method, however, can only be used with digital modulations. Furthermore, secondary users should have the capability to synchronize and demodulate primary users signal. [5]

2.1.2 Matched Filter Detection

When the information of the primary user signal is known to the secondary user, the optimal detector in stationary Gaussian noise is the matched filter since it maximizes the received signal-to-noise ratio (SNR).

The main advantage of the matched filter is that it requires less time to obtain high processing gain due to coherency. It requires a priori knowledge of the primary user signal such as the modulation type and order, the pulse shape, and the packet format. Hence, if this information is not accurate, then the matched filter performs poorly. However, since most wireless network systems have pilot, preambles, synchronization word or spreading codes, these can be used for the coherent detection. Block diagram of Matched Filter Detection is given in Fig.2.3 [32].

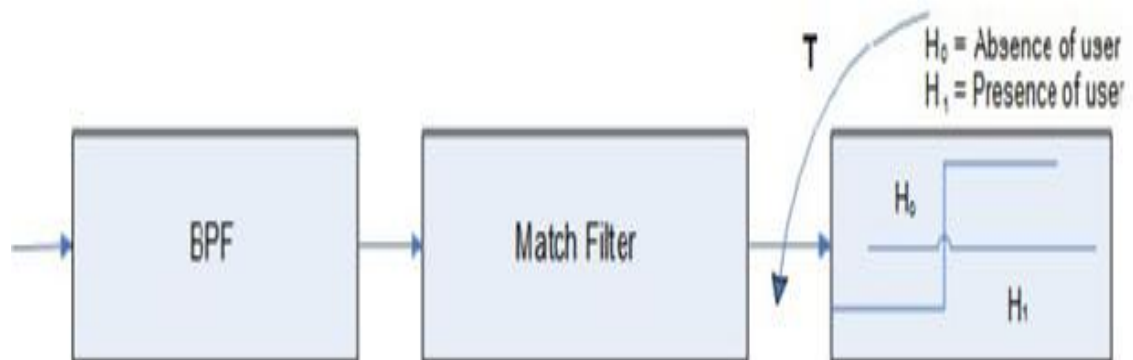


Figure 2.3: Block Diagram of Matched Filter Detection

Matched filter is the optimum detector of a known signal in the presence of Gaussian noise. It is the linear filter that maximizes the output signal-to-noise ratio (SNR).

The matched filter requires explicit knowledge of the transmitted signal and the noise.

Hence, the usability of the matched filter is limited to cases where explicit information about the waveform such as pilot signals or preambles is known. In addition, the performance may severely deteriorate with synchronization errors.

Advantages: Matched filter detection needs less detection time. When the information of the primary user signal is known to the cognitive radio user, matched filter detection is optimal detection in stationary gaussian noise.

Disadvantages: Matched filter detection requires a priori knowledge of every primary signal. If the information is not accurate, MF performs poorly. Also the most significant disadvantage of MF is that a CR would need a dedicated receiver for every type of primary user.

2.1.3 Cyclostationary Feature Detector

Another detection method is the cyclostationary feature detection. Modulated signals are in general associated with sine wave carriers, pulse trains, hopping sequences, repeating spreading or cyclic prefixes, which result in built-in periodicity. These modulated signals are characterized as cyclostationarity since their mean and auto-correlation exhibit periodicity. These features are detected by analyzing the spectral correlation function. The purpose to obtain the spectral correlation function is that it differentiates the noise energy from modulated signal energy, because the noise is a wide-sense stationary signal which has no correlation, while modulated signals have cyclostationary characteristics with spectral correlation due to the embedded redundancy of signal periodicity. The block diagram is shown in Fig. 2.4 [32]. Therefore, a cyclostationary feature detector can perform well in discriminating against noise due to its robustness to the uncertainty in noise power. The main drawback is that computationally complex and requires significantly long observation time. Distinct features of the received signal are extracted using cyclic spectral analysis and represented by both spectral coherent function and spectral correlation density function.

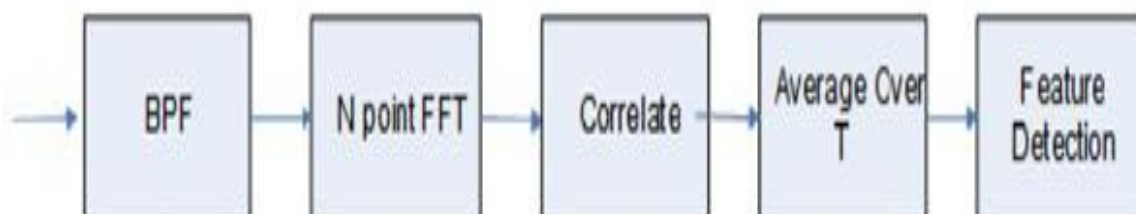


Figure 2.4: Block Diagram of Cyclostationary Feature Detection

It exploits the periodicity in the received primary signal to identify the presence of primary users (PU). The periodicity is commonly embedded in sinusoidal carriers, pulse trains, spreading code, hopping sequences or cyclic prefixes of the primary signals. Due to the periodicity, these cyclostationary signals exhibit the features of periodic statistics and spectral correlation, which is not found in stationary noise and interference.

Thus, cyclostationary feature detection is robust to noise uncertainties and performs better in low SNR regions. Although it requires a priori knowledge of the signal characteristics, cyclostationary feature detection is capable of distinguishing the CR transmissions from various types of PU signals. This eliminates the synchronization requirement of energy detection in cooperative sensing. Moreover, CR users may not be required to keep silent during cooperative sensing and thus improving the overall CR throughput. This method has its own shortfalls like its high computational complexity and long sensing time. Due to these issues, this detection method is less common than energy detection in cooperative sensing.

2.1.4 Energy Detector

Energy Detector simply measures the energy in the received waveform over an observation time window. Energy Detection is the most common way of spectrum sensing because of its low computational and implementation complexities. It is a more generic method as the receivers do not need any knowledge of the primary users signal. Urkowitz [7] has derived both the probability of detection (P_d) and the probability of false alarm (P_f). This energy detection problem has been revisited recently by Kostylev [8] for signals operating over a variety of fading channels.

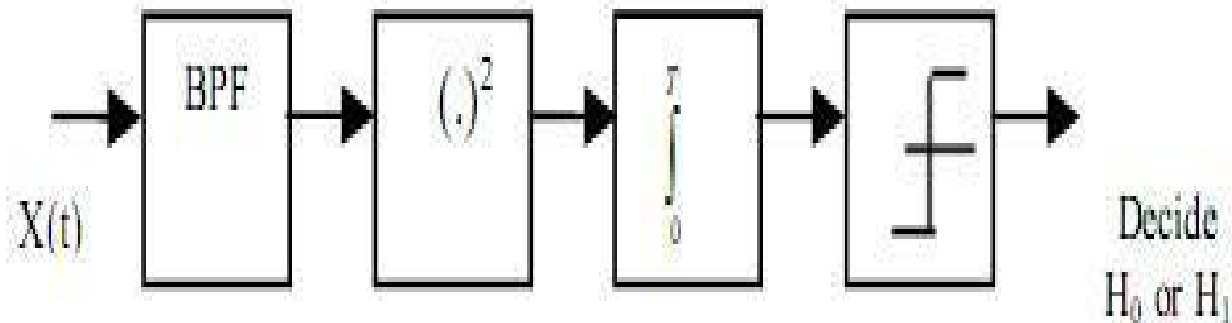


Figure 2.5: Block Diagram of Energy Detection

The input band pass filter selects the center frequency f_c , and bandwidth of interest, W . This filter is followed by a squaring device to measure the received energy and an integrator which determines the observation interval, T . Finally, output of the integrator, Y , is compared with a threshold, λ to decide whether signal is present or not.

2.2 Literature Review

I. F. Akyildiz et al., 2006 [1] presented a survey on xG networks which are also known as Cognitive Radio networks to solve wireless network problems resulting from the limited available spectrum and the inefficiency in the usage of spectrum by exploiting the current wireless spectrum opportunistically. xG networks, which are equipped with the inbuilt capabilities of the cognitive radio, will provide an ultimate spectrum-aware communication paradigm in wireless communications. In this survey, intrinsic properties and current research challenges of the xG networks are presented. They have also given overview and comparison of various spectrum sensing techniques.

A survey of spectrum sensing methodologies for cognitive radio is presented by Yucek and Arselan, 2009 [5]. Various aspects of spectrum sensing problem are studied from a cognitive radio perspective and multi-dimensional spectrum sensing concept is introduced. Challenges associated with spectrum sensing are given and enabling spectrum sensing methods are reviewed. The paper also explains the cooperative sensing concept and its various forms.

Urkowitz, 1967 [7] has discussed the detection of a deterministic signal of unknown structure in the presence of band-limited Gaussian noise. Urkowitz derived both the probability of detection (P_d) and the probability of false alarm (P_f).

This energy detection problem has been revisited recently by Kostylev, 2002 [8] for signals operating over a variety of fading channels. In this paper, signal with random (Rayleigh, Rice, Nakagami, and other) amplitude is considered. For such amplitude a distribution of a decision statistic of an energy detector is retrieved and expressions for a detection probability are obtained.

F. F. Digham et al., 2003 and 2007 [9], [10] present another look at the problem of energy detection of unknown signals over different fading channels. The analysis

has been started with the no diversity and presents alternative closed form equations for the probability of detection (P_d). The system performance when different diversity schemes are employed is analyzed. Also, receiver operating characteristic (ROC) curves to compare the performance of equal gain combining (EGC), selection combining (SC), and switch and stay combining (SSC) are presented. For example, EGC diversity introduces a gain of two orders of magnitude from the probability of miss perspective compared to the no diversity case while both SC and SSC introduce a gain of about one order of magnitude.

Y. Chen et al., 2009 [15] proposed new and improved energy detector for random signals in Gaussian noise by replacing the squaring operation of the signal amplitude in the conventional energy detector with an arbitrary positive power operation. Numerical results show that the best power operation depends on the probability of false alarm, the probability of detection, the average signal-to-noise ratio or the sample size. By choosing the optimum power operation according to different system settings, new energy detectors with better detection performances can be derived. These results give useful guidance on how to improve the performances of current wireless systems using the energy detector.

Performance of energy detector is susceptible to noise uncertainty. Sanket Kamalakar and Adrish Banerjee, 2013 [16] presents study of generalized energy detector (GED), obtained by replacing squaring operation of amplitude of the received signal in conventional energy detector (CED) with an arbitrary positive power operation p under noise uncertainty. For the worst case of noise uncertainty, SNR wall is not dependent on the value of p . The detection performance of GED for different values of p under uniformly distributed noise uncertainty is investigated and it has been shown that CED is the best ED under noise uncertainty. When noise uncertainty is greater than 0.5 dB, the performance gap between different EDs almost vanishes and the detection performances of all EDs almost become the same for all values of p .

Detection performance of energy detector (ED) deteriorates in low signal-to-noise ratio (SNR) conditions, specifically for $\text{SNR} < -10$ dB. Di He et al., 2010 [20] proposed a novel spectrum-sensing method for cognitive radio (CR) based on stochastic resonance (SR). The spectral power of primary users (PUs) can be amplified, and the signal-to-noise ratio (SNR) of a received signal can be increased using SR. This ensures that the detection probability of the proposed approach is higher than that of the traditional energy detector. Performance analyses and computer simulation results indicate that the effectiveness of the proposed SR-based spectrum-sensing approach, particularly under low SNR circumstances, is better than that of the traditional energy-detection method. This approach is helpful in enhancing the spectrum utility in CR networks basically with acceptable computational complexity.

F. Chapeau-Blondeau, 2000 [21], [25] explains Stochastic Resonance phenomenon, which is a nonlinear effect wherein the noise turns out to be beneficial to the transmission or detection of an information-carrying signal. Stochastic resonance can take place under various forms, according to the types considered for the noise, for the information-carrying signal, for the nonlinear system realizing the transmission or detection, and for the quantitative measure of performance receiving improvement from the noise. S.M. Kay, Pramod Varshney et al., 2007 [22] also presented the mathematical framework to analyze the stochastic resonance (SR) effect in binary hypothesis testing problems. The mechanism for SR noise enhanced signal detection is explored. The detection performance of a noise modified detector is derived in terms of the probability of detection P_d and the probability of false alarm P_f . Bruce McNamara and Kurt Wiesenfeld, 1989 [26] introduced a general theory for stochastic resonance in bistable systems subject to both periodic and random forcing. The theory has been applied to the two important cases of the double-well and two-state systems and it has been shown that signal power reaches a maximum when there is a matching of the signal frequency and the rate of hopping between the two states, which in turn is a function of the noise strength, which leads to an increase in SNR.

Di He, 2010 [29] has demonstrated that there is a significant decrease in SNR wall by introducing the received signal into the chaotic stochastic resonance (CSR) system. According to the properties of linear response theory and optimum stochastic resonance, the SNR of the received signal can be increased, thus will break the SNR wall and decrease the sample complexity under the same false alarm rate and detection probability requirements.

2.3 Objectives

Primary objectives of this thesis are:

- To study detailed mathematical analysis of energy detector and simulate the performance curve (ROC) for AWGN channel and wireless fading channel (Rayleigh fading channel).
- To obtain improved performance of energy detection by incorporating diversity in wireless fading channel.
- To show that the performance of energy detector can be further improved by Improved Energy Detector (Generalized Energy Detector).
- To study and analyze the impact of noise uncertainty in case of Generalized Energy detector.
- To study the Stochastic Resonance phenomenon and obtain the improved detection probability under low SNR circumstances by performing energy detection on the signal received through SR system.
- To propose a novel spectrum sensing technique that can mitigate the important challenges of spectrum sensing like SNR wall and sample complexity while obtaining improved detection probability and to validate the effectiveness of this method by simulation results.

2.4 Summary

Initially, introduction and classification of various spectrum sensing techniques are given. In section 2.1, various transmitter detection (non cooperative detection) techniques are briefly discussed. Also, various challenges of spectrum sensing are discussed in detail, which can be motivation factor for people interested in cognitive radio. Literature review is given in section 2.2. Finally, the objectives of the thesis are discussed in section 2.3.

Chapter 3

Performance Analysis of Energy Detector

Energy Detection is the most simplest method of spectrum sensing because of its low computational and implementation complexities. It is a more generic method as the receivers do not need any knowledge of the primary users signal. The signal is detected by comparing the output of the energy detector with a threshold which depends on the noise floor. The important challenge with the energy detector based sensing is the selection of the threshold for detecting primary users. The other challenges include inability to differentiate interference from primary users and noise and poor performance under low signal-to-noise ratio values.

Probability of detection (P_d) and Probability of false alarm (P_f) are the important factors for energy based detection which gives the information of the availability of the spectrum.

3.1 System Model :

The received signal $r(t)$ takes the form [9],

$$r_k = h.s_k + n_k \quad k=1,2,,N$$

where $h=0$ or 1 under hypothesis H_0 or H_1 , respectively. s is primary signal, n is modeled as a zero-mean white Gaussian random process. The received signal is first pre-filtered by an ideal band pass filter. The output of the filter is then squared and integrated over a time interval T to finally produce the measure of the energy of the received wave. The output of the integrator denoted by, Y , will act as the test statistic to test the two hypotheses H_0 and H_1 .

When primary signal is not present, only noise is received through channel. So, test statistics can be expressed as

$$Y = \sum_{k=1}^N n_k^2 \quad (3.1)$$

Y can be viewed as the sum of the squares of N standard Gaussian variates with zero mean and unit variance. Therefore, Y follows a central chi-square distribution with N degrees of freedom.

When signal S is present, we can replace each n_i by $n_i + s_i$. The decision statistic Y will have a noncentral chi-square distribution with N degrees of freedom and a non centrality parameter 2γ .

Thus, decision statistics can be represented as,

$$Y \sim \begin{cases} X_N^2, H_0 \\ X_N^2(2\gamma), H_1 \end{cases} \quad (3.2)$$

The probability density function (PDF) of Y can then be written as [13],

$$f_Y(y) = \begin{cases} \frac{1}{2^{N/2}\Gamma(N/2)} y^{N/2-1} e^{-y/2}, & H_0 \\ \frac{1}{2} \left(\frac{y}{2\gamma}\right)^{N/2-1} e^{-\frac{2\gamma+y}{2}} I_{N/2-1}(\sqrt{2\gamma y}), & H_1 \end{cases} \quad (3.3)$$

where $\gamma = N\gamma_a$. γ_a is SNR of one sample of the received signal and γ is the total SNR of the received signal. $\Gamma(\cdot)$ is the Gamma function and $I(\cdot)$ is the modified Bessel function of the first kind [13].

3.2 Detection and False Alarm probabilities over AWGN channels

The probability of detection and false alarm can be generally computed by

$$P_d = P(Y > \lambda/H_1) \quad (3.4)$$

Probability of detection is obtained when hypothesis H_1 is true, i.e., the signal is present and the

$$P_f = P(Y > \lambda/H_0) \quad (3.5)$$

Probability of false alarm is obtained when hypothesis H_0 is true, i.e., the signal is not present and the test statistics is greater than threshold λ .

Probability of False Alarm given by,

$$P_f = \int_{\lambda}^{\infty} f(y/H_0) dy \quad (3.6)$$

Substituting pdf of Y under H_0 from eq. (3.3),

$$Pf = \frac{1}{2^{N/2}\Gamma(u)} \int_{\lambda}^{\infty} y^{N/2-1} \exp(-y/2) dy \quad (3.7)$$

This integration can be transformed into incomplete Gamma function [13].

$$\Gamma(a, x) = \int_x^{\infty} \exp(-t)t^{a-1} dt \quad (3.8)$$

Using this,

$$\Gamma(N/2, \frac{\lambda}{2}) = \int_{\frac{\lambda}{2}}^{\infty} \exp(-y)y^{N/2-1} dy \quad (3.9)$$

Dividing and multiplying RHS of the eq. (3.7) by $2^{N/2-1}$,

$$Pf = \frac{2^{N/2-1}}{2^{N/2-1}\Gamma(N/2)} \int_{\lambda}^{\infty} e^{-y/2} (y/2)^{N/2-1} dy \quad (3.10)$$

Let $y/2=x$, $dy=2dx$ and accordingly changing the limits,

$$P_f = \frac{\Gamma(N/2, \lambda/2)}{\Gamma(N/2)} \quad (3.11)$$

Now, Probability of detection P_d ,

$$P_d = \int_{\lambda}^{\infty} f_y(y/H1) dy \quad (3.12)$$

Substituting pdf of Y under H_1 from eq. (3.3),

$$P_d = \int_{\lambda}^{\infty} \frac{1}{2} (y/2\gamma)^{\frac{N/2-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{N/2-1}(\sqrt{2\gamma y}) dy \quad (3.13)$$

There is no closed form expression for this integral. CDF can be expressed in terms of the generalized Marcum Q-function which is given as [13],

$$Q_m(a, b) = \int_b^{\infty} \left(\frac{x}{a}\right)^{m-1} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax) dx \quad (3.14)$$

Now,

$$P_d = \int_{\lambda}^{\infty} \frac{1}{2} (y/2\gamma)^{\frac{N/2-1}{2}} e^{-\frac{2\gamma+y}{2}} I_{N/2-1}(\sqrt{2\gamma y}) dy \quad (3.15)$$

Let $y = x^2$, and $dy=2x dx$, $a^2 = 2\gamma$ and $m=N/2$,

$$P_d = \int_{\sqrt{\lambda}}^{\infty} \frac{1}{2} \left(\frac{x}{a}\right)^{m-1} e^{-\frac{x^2+a^2}{2}} I_{m-1}(ax) 2x dx \quad (3.16)$$

$$P_d = Q_m(a, b) = Q_{N/2}(\sqrt{2\gamma}, \sqrt{\lambda}) \quad (3.17)$$

3.2.1 Simulation for AWGN channel

For simulation to obtain ROC (Pd vs. Pf), Monte Carlo simulation is used. Define no. of Monte carlo simulations, say M= 5000, 10000 etc. Define Range of Pf. Define SNR of received signal. ($\gamma=-2$ dB) Define no. of samples of received signal (N=10). For AWGN channel, we take h=1 i.e. channel gain=1. Generate a BPSK signal s

having total N no. of samples. Then, generate AWGN signal n with zero mean and unit variance. Received signal is $r_k = \sqrt{\gamma_a} \cdot s_k + n_k$. Now, apply the received signal r to squaring device and do summation of all samples (i.e. square of received signal). This is called Test Statistics. Detection Threshold can be calculated for each specific value of P_f . Then compare Test Statistics with detection Threshold. If Test Statistics is greater than Detection Threshold, Detection count can be incremented (initially declare detection count is zero).

The entire process have been repeated a large number of times (5000 times) and then the average value of P_d and P_m have been estimated.

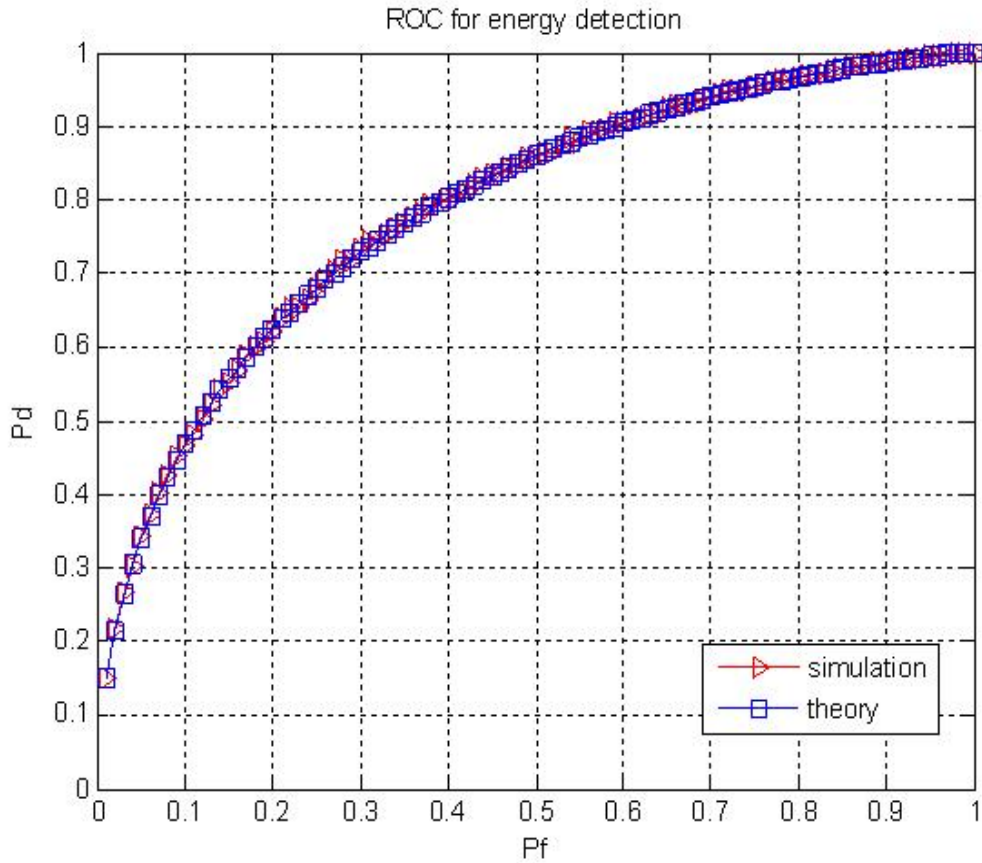


Figure 3.1: ROC for AWGN Channel: SNR=-2 dB, $N=10^3$

Pf versus SNR curve for AWGN channel:

The decision threshold setting procedure is very crucial as it directly affects the performance of the detector. The threshold should be chosen such that the probability of detection is maximized and the probability of false alarm is minimized. Achieving both these criteria cannot be realized in practice. Also, this requires the knowledge of signal and noise powers. The noise power can be estimated while the estimation of signal power is difficult. Thus the threshold is normally selected to satisfy a fixed Pf, which depends only on noise power. Thus the performance improvement of any detection method should not be achieved at the greater expense of degradation in the false alarm. However, it can be verified from the Pf versus SNR curves presented Fig

3.2. This curve is obtained for $N=50$. For different values of number of samples, the curve may vary.

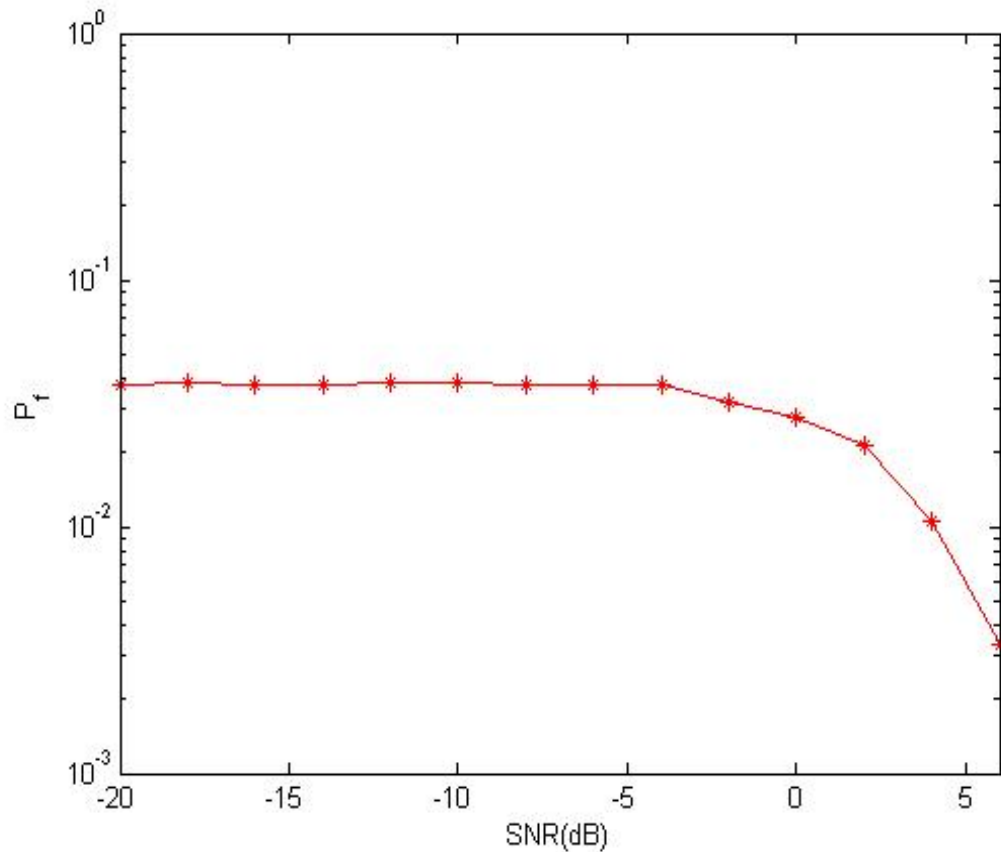


Figure 3.2: P_f versus SNR curve for AWGN channel, $N=50$

3.3 Average Detection Probabilities over Rayleigh Fading channel

3.3.1 Significance of Fading:

Fading is the significant effect in any wireless communication design and is important to predict about it. There are two different types of fading: small scale fading and large scale shadowing. Small scale fading is often handled in a wireless system

with diversity schemes, redundancy, or even retransmit. Large scale shadowing on the other hand is very dependent on location with respect to obstacles, and cannot always be fixed; its modeling consists in predicting the likelihood of outage.

Small Scale Fading: this effect of multipath causes deep fades within small distances and is referred to as small-scale fading. Another important yet different cause of small scale fading is that of small frequency variations such as Doppler effect.

Multipath Fading: Multipath fading is significant for both mobile and fixed wireless systems. Intuitively that type of fading varies with surrounding scatterers which reflect differently the wavefront between transmitter and receiver. In reality, it is very important to quantify this aspect of the propagation environment, and to draw out the standard to perform well in such kind of environment.

Doppler Spread: Another aspect of wireless communication, different from the above, is the concept of how fast things are changing in the wireless channel. In the time domain, that aspect is referred to as time dispersion and is measured by coherence time; the coherence time describes how fast the wireless channel is changing. In the frequency domain the effect is best described by the Doppler spread: it describes how fast transmitter, receiver, and scatterers in-between are moving; the faster they are moving, the faster the wireless channel changes, and the more Doppler shift will be present.

Small-scale fading is caused by different reflections of the signal (delayed, frequency shifted, constructive or destructive) and is usually modeled by a random variable with a certain probability distribution, which may be given as Rayleigh fading, Nakagami fading, Rician fading etc.

3.3.2 Rayleigh Fading Channel:

If the signal amplitude follows a Rayleigh distribution, then the SNR γ follows an exponential PDF given by

$$f(\gamma) = \frac{1}{\bar{\gamma}} e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)} \quad (3.18)$$

Where $\bar{\gamma}$ is the average SNR. The probability of detection for Rayleigh channel is obtained by averaging its pdf over probability of detection over AWGN channel.

$$P_{d, Ray} = \int_{\lambda}^{\infty} Q_{N/2}(\sqrt{2\gamma}, \sqrt{\lambda}) \frac{1}{\bar{\gamma}} e^{-\left(\frac{\gamma}{\bar{\gamma}}\right)} d\gamma \quad (3.19)$$

Substituting $\sqrt{\gamma} = x$ and $d\gamma = 2x dx$ and using [11],

$$\begin{aligned} \bar{P}_{d, Ray} = e^{-\frac{\lambda}{2}} \sum_{k=0}^{N/2-2} \frac{1}{k!} (\lambda/2)^k + \left(\frac{1+\bar{\gamma}}{\bar{\gamma}}\right)^{N/2-1} \left(e^{-\frac{\lambda}{2(1+\bar{\gamma})}} - \right. \\ \left. e^{-\frac{\lambda}{2}} \sum_{k=0}^{N/2-2} \frac{1}{k!} \left(\frac{\lambda\bar{\gamma}}{2(1+\bar{\gamma})}\right)^k \right) \end{aligned} \quad (3.20)$$

3.3.3 Simulation for Rayleigh Fading channel:

For simulation to obtain ROC (Pd vs. Pf), Monte Carlo simulation is used. Define no. of Monte Carlo simulations, say M= 5000, 10000 etc. Define Range of Pf. Define SNR of received signal. ($\gamma=-2$ dB) Define no. of samples of received signal (N=10). Channel $h \sim CN(0, 1)$ is Rayleigh fading channel. Generate a BPSK signal is having total N no. of samples. Received signal is $r_k = \sqrt{\gamma_a} \cdot s_k + n_k$. Now, apply the received signal r to squaring device and do summation of all samples (i.e. square of received

signal). This is called Test Statistics. Detection Threshold can be calculated for each specific value of P_f . Then compare Test Statistics with detection Threshold. If Test Statistics is greater than Detection Threshold, Detection count can be incremented (initially declare detection count is zero).

The entire process have been repeated a large number of times (5000 times) and then the average value of P_d and P_m have been estimated.

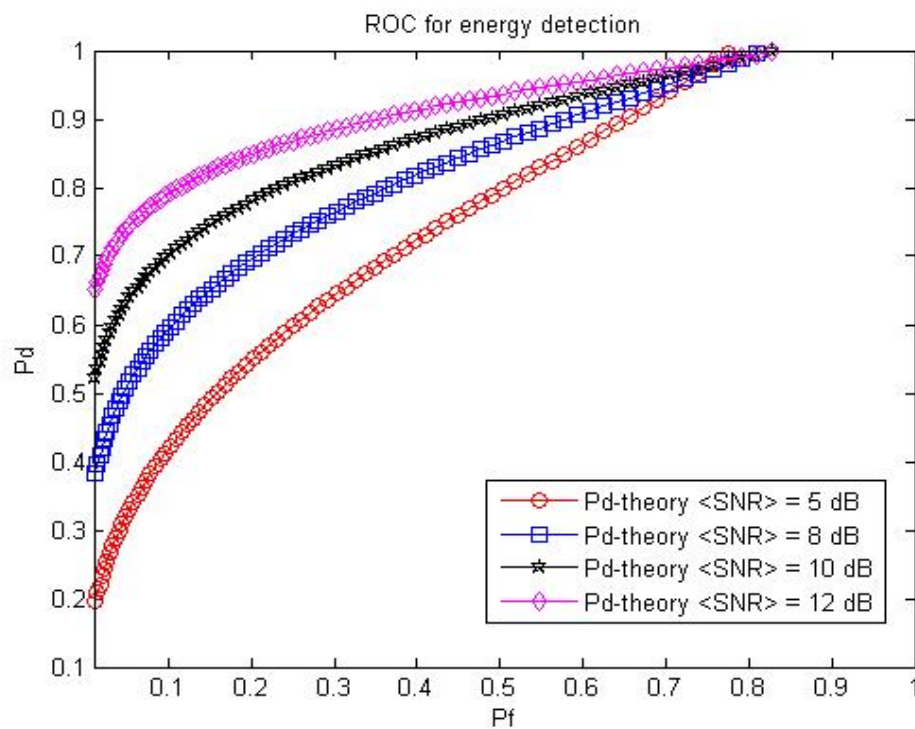


Figure 3.3: ROC for Rayleigh fading Channel: $N=10$

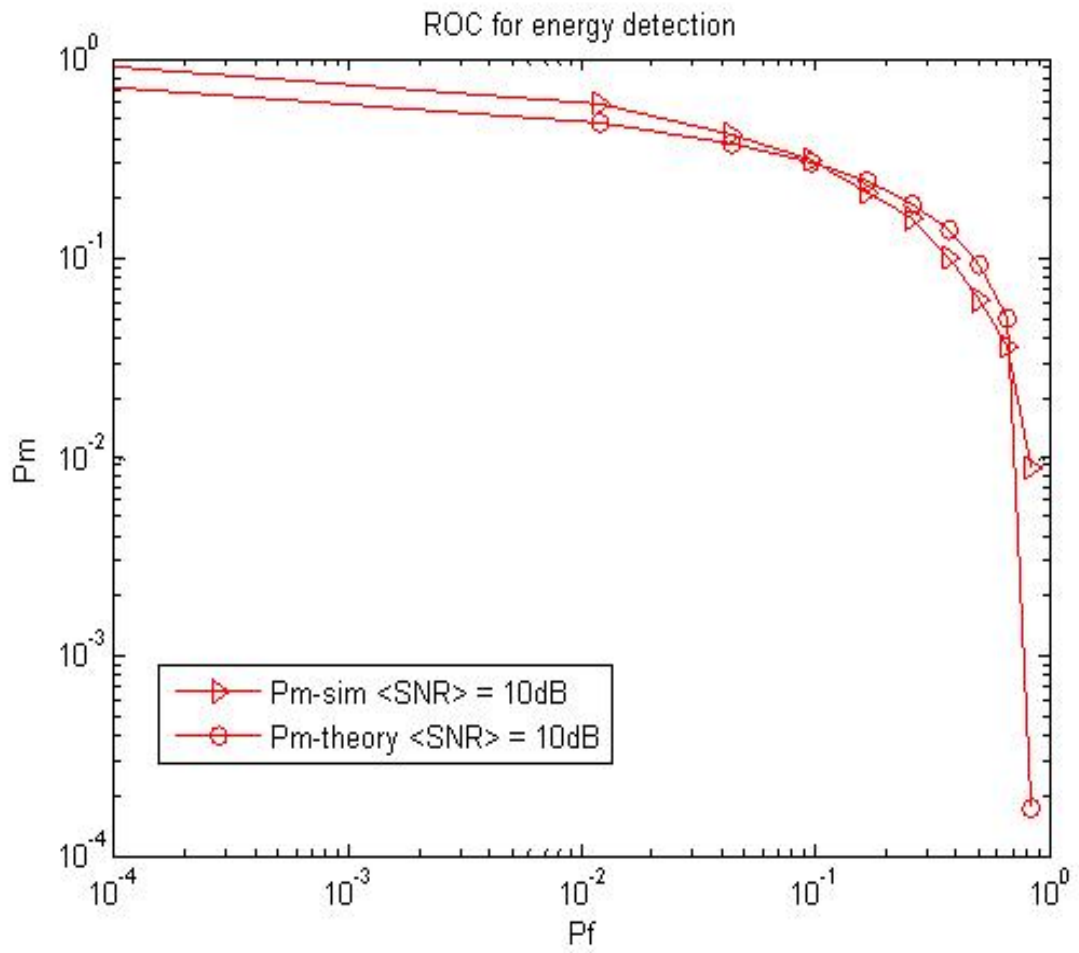


Figure 3.4: Analytical and Simulation Result for ROC of Rayleigh fading Channel: $N=10$

Pf versus SNR curve for Rayleigh channel:

The importance of the Pf versus SNR curve is mentioned in section 3.2. The same curve has been obtained for Rayleigh fading channel also in Fig 3.5.

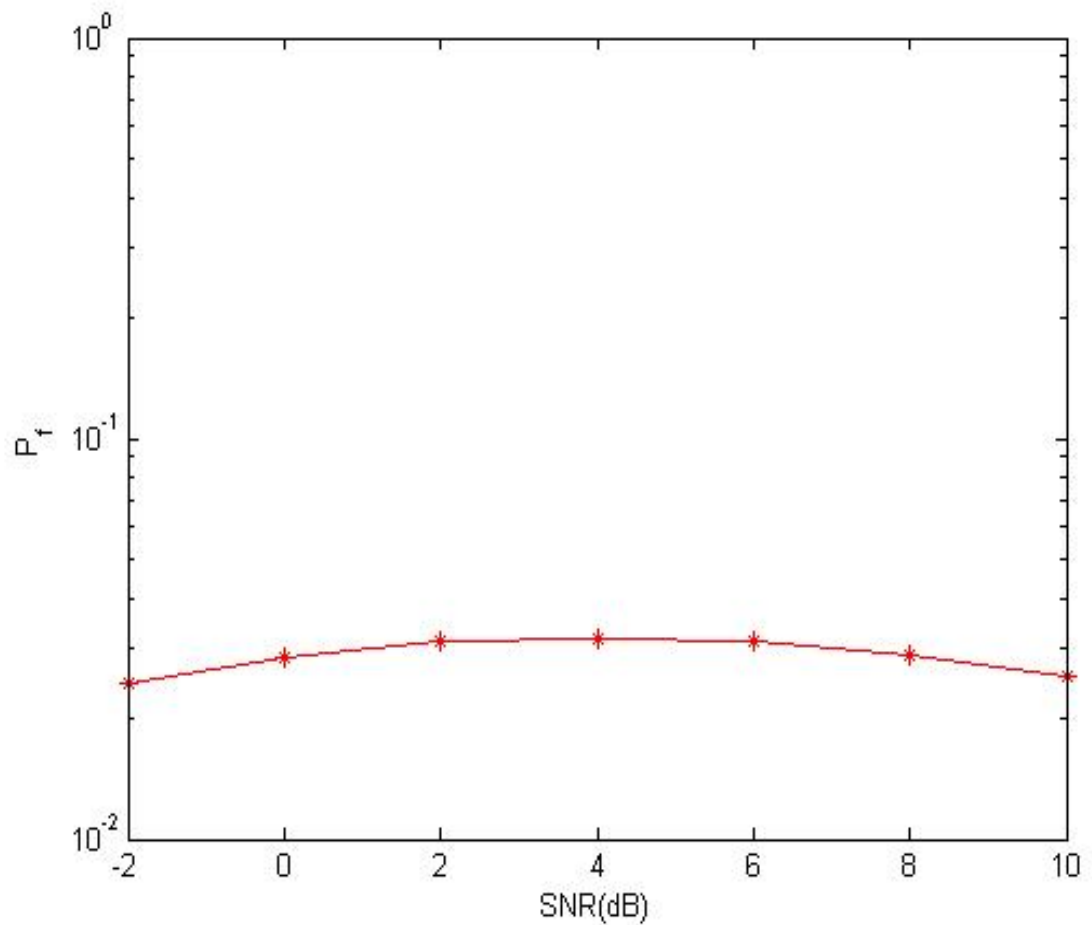


Figure 3.5: Pf versus SNR curve for Rayleigh fading Channel: N=50

3.4 Average Detection Probability with Diversity Reception

3.4.1 Significance of Diversity in Wireless Fading Channel:

Diversity is a powerful technique to combat the wireless fading impairment. Diversity refers to transmitting and/or receiving the same information via different independent ways. In such a system, multiple copies of the same information signal are being transmitted to the receiver over two or more real or virtual communication channels. Thus the basic idea of diversity is repetition or redundancy of information. There are various diversity schemes like time diversity, frequency diversity, space diversity etc.

Diversity Combining Techniques:

Equal Gain Combining (EGC): In Equal Gain Combining (EGC), all the received signals simply added together at the receiver.

Selection Combining (SC): From the number of antennas, the branch that receives the signal with the largest signal-to-noise ratio is selected.

Switch and Stay Combining (SSC): Stay with the signal branch until the envelop drops below a predefined threshold. Only one receiver is needed.

Probability of Detection can be improved when different diversity schemes like Equal gain Combining (EGC), Selection Combining (SC) and Switch and Stay Combining (SSC) are employed. For simplicity, it is considered that the diversity paths are independent and identically distributed (IID) and are subjected to Rayleigh fading. Here, EGC scheme has been considered.

3.4.2 Equal Gain Combining (EGC):

The output SNR of the EGC combiner is the sum of the SNRs on all branches, i.e.

$$\gamma_t = \sum_{l=1}^L \gamma_l$$

where L is the number of diversity branches.

Adding L IID noncentral χ^2 variates with N degrees of freedom and non-centrality parameter $2\gamma_l$ each results in another noncentral χ^2 variate with LN degrees of freedom and non-centrality parameter $\sum_{l=1}^L \gamma_l$. Hence, the P_d at the EGC output for AWGN channels can be evaluated as,[9]

$$P_{d,EGC} = Q_{LN/2}(\sqrt{2\gamma_t}, \sqrt{\lambda}) \quad (3.21)$$

The PDF of γ_t for IID Rayleigh branches is known to be given by,

$$f(\gamma_t) = \frac{1}{(L-1)!\gamma^L} \gamma_t^{L-1} e^{-\left(\frac{\gamma_t}{\gamma}\right)} \quad (3.22)$$

The average P_d for the EGC diversity scheme can be obtained by averaging this pdf over $P_{d,EGC}$. This probability of detection is similar to probability of detection for Nakagami channel, which is given by,

$$P_d = \alpha[G_1 + \beta \sum_{n=1}^{N/2-1} \frac{(\lambda/2)^n}{2(n!)} {}_1F_1(L; n+1; \frac{\lambda}{2} \frac{\gamma}{L+\gamma})] \quad (3.23)$$

Where ${}_1F_1(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function,

$$\alpha = \frac{1}{\Gamma(L)2^{L-1}} \left(\frac{L}{\gamma}\right)^L \quad (3.24)$$

$$\beta = \Gamma(L) \left(\frac{2\gamma}{L+\gamma}\right)^L e^{-(\lambda/2)} \quad (3.25)$$

and

$$G_1 = \frac{2^{L-1}(L-1)!}{(L/\gamma)^L} \frac{\gamma}{L+\gamma} e^{-\frac{\lambda}{2}} \frac{L}{L+\gamma} \left[\left(1 + \frac{L}{\gamma}\right) \left(\frac{L}{L+\gamma}\right)^{L-1} \times A \right] \quad (3.26)$$

$$A = L_{m-1} \left(-\frac{\lambda}{2} \frac{\gamma}{L+\gamma} \right) + \sum_{n=0}^{L-2} \left(\frac{L}{L+\gamma}\right)^n L_n \left(-\frac{\lambda}{2} \frac{\gamma}{L+\gamma} \right) \quad (3.27)$$

3.4.3 Simulation for Diversity reception

Generate two branches of diversity reception by generating two received signals which are independent of each other. i.e., r_1 and r_2 . The received signal is addition of two signals generated. The SNR will be total of two branches. Now, apply the received signal r to squaring device and do summation of all samples (i.e. square of received signal). This is called Test Statistics. Detection Threshold can be calculated for each specific value of P_f . Then compare Test Statistics with detection Threshold. If Test Statistics is greater than Detection Threshold, Detection count can be incremented (initially declare detection count is zero).

The entire process have been repeated a large number of times (5000 times) and then the average value of P_d and P_m have been estimated.

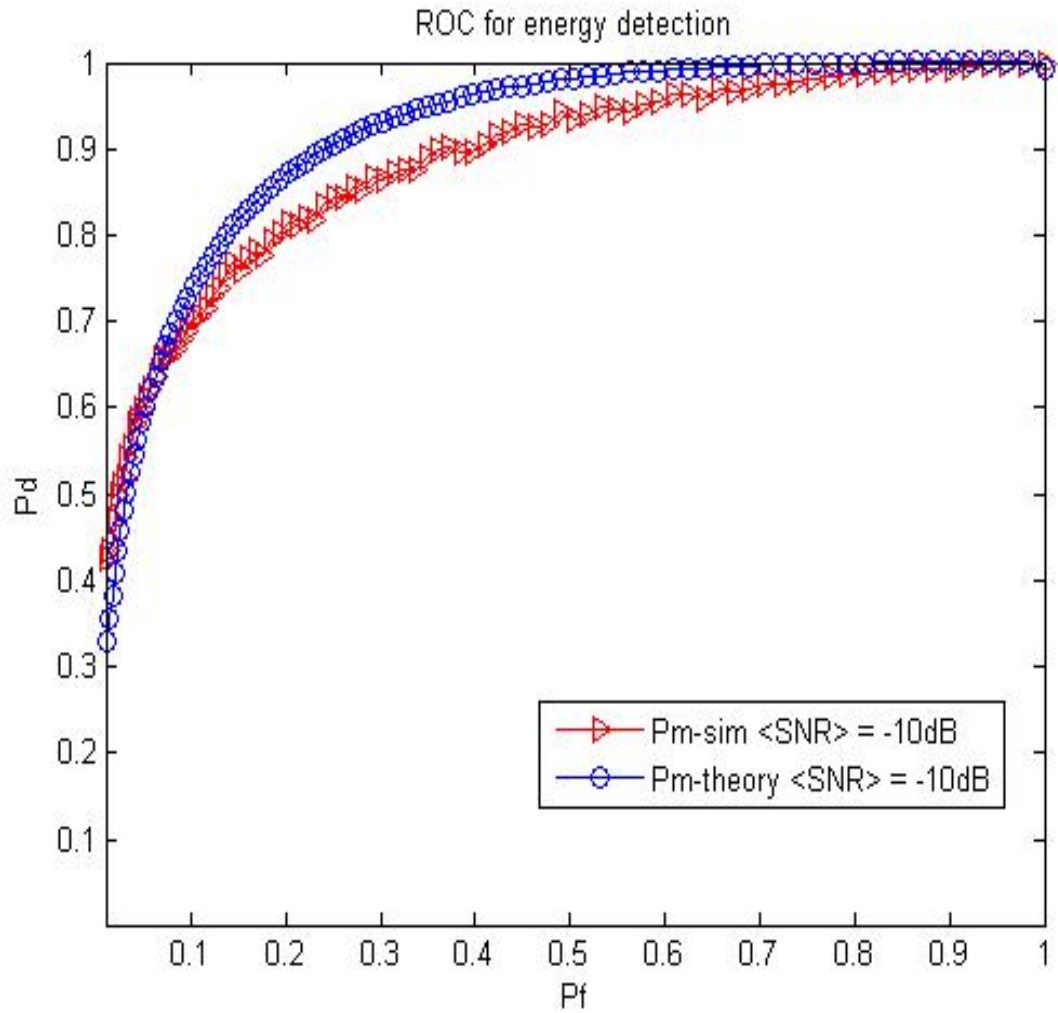


Figure 3.6: ROC for Rayleigh fading Channel with Diversity: $N=5000$

The curve obtained in Fig 3.7 indicates that the performance of energy detection in terms of P_f is highly improve with diversity reception.

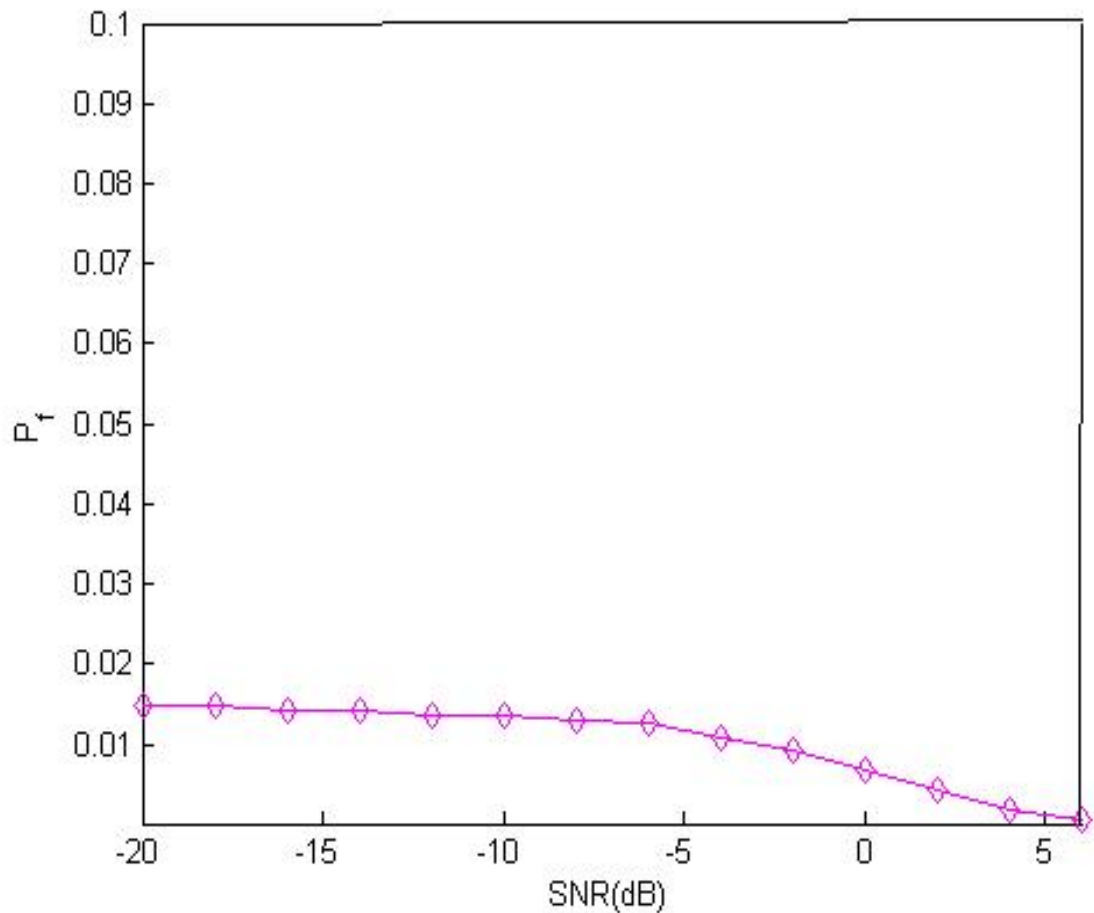


Figure 3.7: Pf versus SNR curve for Rayleigh fading Channel: N=50

3.5 Improved Energy detector

Energy detection is a popular spectrum sensing technique. But detection performance of energy detector (ED) deteriorates in low signal-to-noise ratio (SNR) conditions and under noise uncertainty.

Conventional energy detector (CED) can be generalized by replacing squaring operation of received signal amplitude by an arbitrary positive power operation constant p . This modified ED is known as generalized energy detector (GED). i.e., CED becomes a special case of GED with $p = 2$. [15],[16]

In conventional energy detector (CED), the received signal samples are first squared, then summed over the number of samples collected and then compared with a pre-determined threshold to take decision regarding presence or absence of Primary user (PU). The test statistic T_{CED} for conventional energy detector is given as

$$T_{CED} = \frac{1}{N} \sum_{n=1}^N |y(n)|^2 \quad (3.28)$$

Where N is the number of samples.

We can transform conventional energy detector to generalized energy detector by replacing squaring operation by an arbitrary positive operation p. Then the test statistic for GED is given as

$$T_{GED} = \frac{1}{N} \sum_{n=1}^N |y(n)|^p \quad (3.29)$$

where $p > 0$ is an arbitrary constant. It can be seen that CED is a special case of GED with $p = 2$. For large N and thus invoking central limit theorem (CLT), Probability of Detection and Probability of False Alarm can be expressed as [16]:

$$P_d = P_r(T_{GED} > T | H_1) = Q\left(\frac{T - \mu_1}{\sigma_1/\sqrt{N}}\right) \quad (3.30)$$

$$P_f = P_r(T_{GED} > T | H_0) = Q\left(\frac{T - \mu_0}{\sigma_0/\sqrt{N}}\right) \quad (3.31)$$

Where,

$$Q(t) = \frac{1}{\sqrt{2\pi}} \int_t^\infty e^{-\frac{x^2}{2}} dx \quad (3.32)$$

and T is the predetermined threshold which can be obtained by fixing probability of false alarm, μ_1 and μ_0 are means of T_{GED} under H_1 and H_0 respectively, and σ_1^2 and σ_0^2 are variances of T_{GED} under H_1 and H_0 respectively, which can be given as [16] and as given in [13, eq. 3.462.9]:,

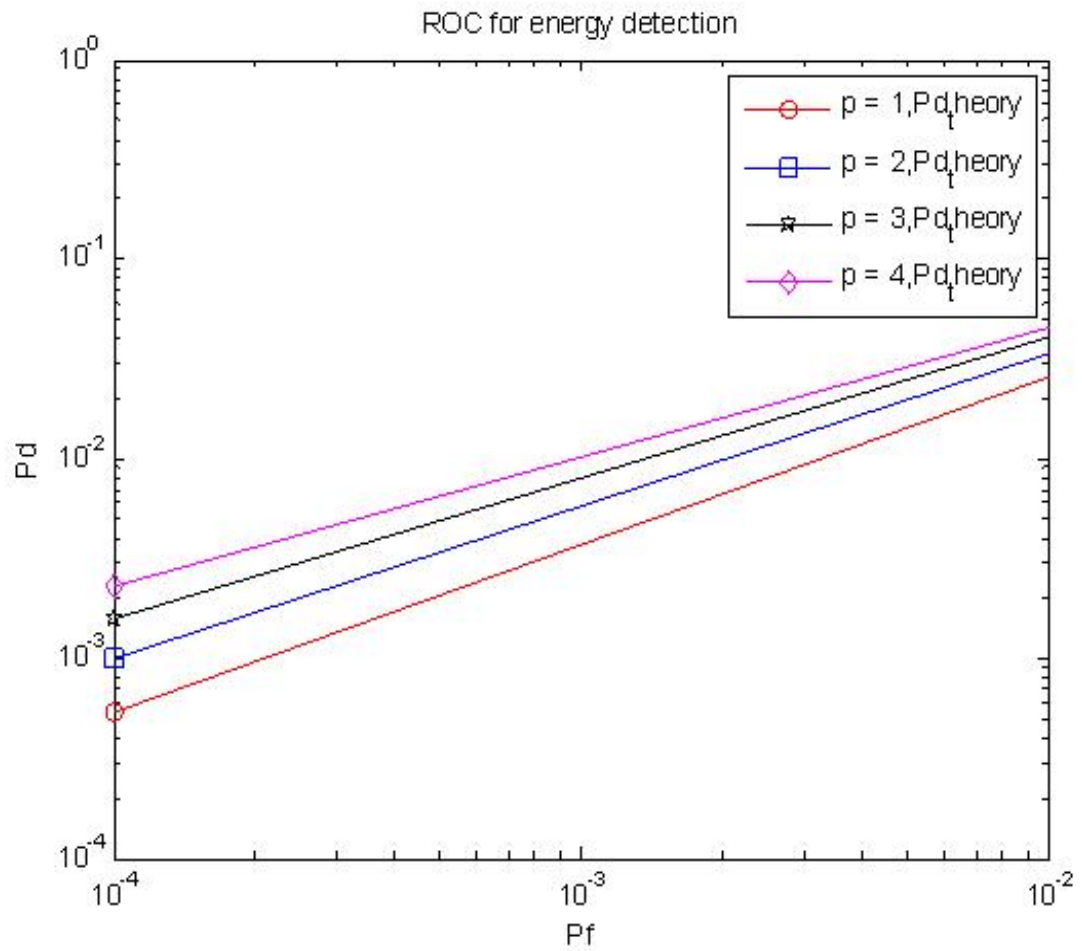
$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma^p \quad (3.33)$$

$$\sigma_0^2 = \frac{2^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \sigma^{2p} \quad (3.34)$$

$$\mu_1 = \frac{2^{p/2}(1+\gamma)^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma^p \quad (3.35)$$

$$\sigma_1^2 = \frac{2^p(1+\gamma)^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \sigma^{2p} \quad (3.36)$$

where γ is average received signal-to-noise ratio.

Figure 3.8: ROC for different p values: $N=1000$

However, it can be shown that the best power operation depends on the probability of false alarm, the probability of detection, the average signal-to-noise ratio or the sample size [15].

3.6 Noise Uncertainty Model for Generalized Energy Detector

3.6.1 Introduction of Noise Uncertainty

Probability of detection and probability of false alarm depend on the threshold T and noise variance σ^2 , and to set the threshold one needs exact knowledge of noise power. In general it is assumed that noise power is known a priori. But in practical scenario this is not the case. Variance/power of white noise is the only parameter on which noise distribution is dependent. In practice there exists noise uncertainty since noise power may change with time and location and is not known exactly. The presence of noise uncertainty makes it very difficult to obtain exact noise power at a particular time and location [18].

In practice, the average noise power is known. Let the average noise power be $\hat{\sigma}_\omega^2$. At a fixed time and location, let the actual noise power be σ_ω^2 which may be different from that of the average noise power $\hat{\sigma}_\omega^2$, which gives rise to the noise uncertainty. So we can define the noise uncertainty factor as ,

$$\beta = \frac{\hat{\sigma}_\omega^2}{\sigma_\omega^2} \quad (3.37)$$

Let the upper bound on noise uncertainty factor in dB be L which is defined as [16], where $L = 10 \text{Log}_{10} \beta$.

Assume that noise uncertainty factor β in dB is uniformly distributed in the range $[-L, L]$.

Let $k\hat{\sigma}_\omega^2$ be the threshold for GED, where k is constant and $\hat{\sigma}_\omega^2$ is average noise power as defined earlier.

For this noise uncertainty case, means and variances under H_0 and H_1 for T_{GED} can be obtained by replacing σ with σ_ω and adding noise uncertainty factor β , and are given as [16],

$$\mu_{0,nu} = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma_\omega^p \quad (3.38)$$

$$\sigma_{0,nu}^2 = \frac{2^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \sigma_\omega^{2p} \quad (3.39)$$

$$\mu_{1,nu} = \frac{2^{p/2}(1+\beta\gamma)^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma_\omega^p \quad (3.40)$$

$$\sigma_{1,nu}^2 = \frac{2^p(1+\beta\gamma)^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \sigma_\omega^{2p} \quad (3.41)$$

Define,

$$G_p = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \quad (3.42)$$

$$K_p = \frac{2^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \quad (3.43)$$

Then the probability of detection PD and probability of false alarm PFA for xed

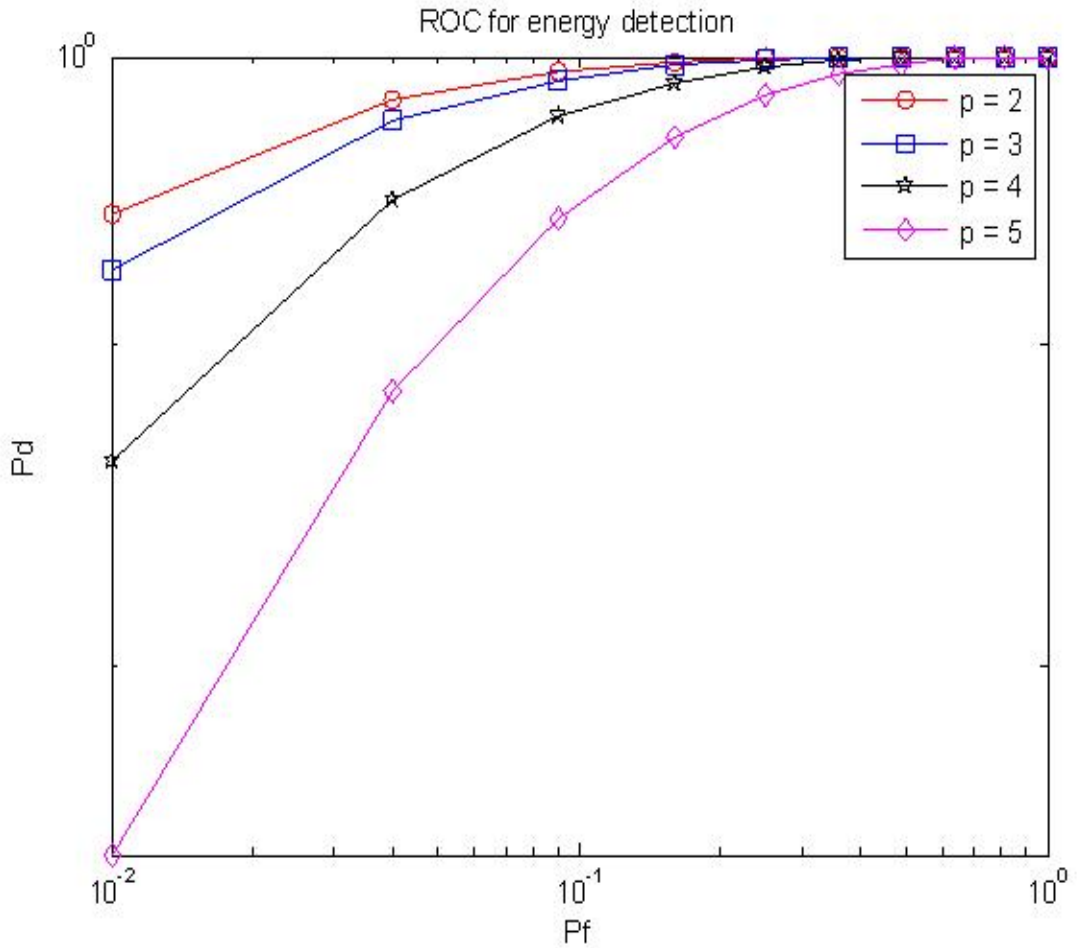
can be given as [16]:

$$P_D = P(T_{GED} > k\hat{\sigma}_\omega^2 \mid H_1) \quad (3.44)$$

$$P_D = Q\left(\left(\frac{k\beta^{p/2} - G_p(1 + \beta\gamma)^{p/2}}{(1 + \beta\gamma)^{p/2}}\right)\sqrt{\frac{N}{K_p}}\right) \quad (3.45)$$

$$P_F = P(T_{GED} > k\hat{\sigma}_\omega^2 \mid H_0) \quad (3.46)$$

$$P_F = Q\left((k\beta^{p/2} - G_p)\sqrt{\frac{N}{K_p}}\right) \quad (3.47)$$

Figure 3.9: ROC for Noise Uncertainty: $N=1000$

Under the noise uncertainty, generalized energy detector with $p = 2$ i.e. conventional energy detector, is the best energy detector. But conventional energy detector may not be the best energy detector in the absence of noise uncertainty. Also as the noise uncertainty increases and becomes significant (generally greater than 0.5 dB), the detection performance of generalized energy detector becomes independent of p .

3.7 Summary

Detailed analysis of energy detection technique of spectrum sensing has been given in this chapter. Section 3.1 represents system model to measure the performance of energy detector over AWGN channel. Probability of Detection and Probability of False alarm are derived for AWGN channel in section 3.2. The performance of energy detector over Rayleigh fading channel is discussed in section 3.3. The performance of energy detector is further improved in case of Rayleigh fading channel by introducing EGC diversity scheme in section 3.4. Introduction and need for Improved Energy detector is given in section 3.5. Performance of Generalized Energy detector can be observed for different values of positive power constant p is obtained by doing mathematical analysis and simulations in the same. Section 3.6 discusses the effect of noise uncertainty on the performance of the generalized energy detector.

Chapter 4

Energy detection through Stochastic Resonance

4.1 Introduction of Stochastic Resonance

Stochastic resonance is a nonlinear effect wherein the noise turns out to be beneficial to the transmission or detection of an information-carrying signal. This paradoxical effect has now been reported in a large variety of nonlinear systems, including electronic circuits, optical devices, neuronal systems, material-physics phenomena, chemical reactions. Stochastic resonance can take place under various forms, according to the types considered for the noise, for the information-carrying signal, for the nonlinear system realizing the transmission or detection, and for the quantitative measure of performance receiving improvement from the noise [21], [23].

Stochastic resonance, as illustrated by Fig. 4.1, involves four essential ingredients:

- an information signal $s(t)$, which can be of many different types, deterministic, periodic or non, random;
- a noise $\eta(t)$, whose statistical properties can be of various kinds: white or colored, Gaussian or non;



Figure 4.1: Stochastic Resonance system as a black box

- a transmission or processing system, which generally is nonlinear, receiving $s(t)$ and $\eta(t)$ as inputs under the influence of which it produces the output signal $y(t)$;
- a measure of performance, which quantifies the efficacy of the processing or transmission of $s(t)$ into $y(t)$ in the presence of $\eta(t)$, and which can also be of many different types, according to the context: signal-to-noise ratio, correlation coefficient, Shannon mutual information, etc.

Stochastic resonance then takes place each time it is possible to improve the measure of performance by means of an increase in the level of the noise $\eta(t)$.

SR studies have concentrated on a periodic coherent signal $s(t)$, transmitted by nonlinear systems of a dynamic and bistable type. There are two classes of bistable system: the double-well (continuous) system and the two-state (discrete) system. Stochastic resonance has essentially been addressed with a sinusoidal $s(t)$ added to a white Gaussian $\eta(t)$ transmitted by a nonlinear dynamic system governed by a double-well potential and measured by a signal-to-noise ratio in the frequency domain.

SR in bistable dynamic systems:

This form of SR is based on the evolution equation,

$$\tau_a \dot{x}(t) = -\frac{dU(x)}{dx} + s(t) + \eta(t) \quad (4.1)$$

Such an equation represents a dynamic system whose state $x(t)$ is forced by the input $s(t)+\eta(t)$, and whose free relaxation $\tau_a \dot{x} = -dU/dx$ is governed by a potential $U(x)$ which generally is a **double-well potential**. A form of the potential is

$$U(x) = -\frac{x^2}{2} + \frac{x^4}{4X_b^2} \quad (4.2)$$

with parameter $X_b > 0$, whose shape is depicted in Fig. 4.2.

Because of its double-well potential $U(x)$, the dynamic system of eq. (4.1) has two stable stationary states. A mechanical interpretation of this system allows a concrete description of the occurrence of the SR phenomenon. In such an interpretation, eq. (4.1) describes the motion in an overdamped regime, of a particle in a potential $U(x)$ subjected to the external force $s(t) + \eta(t)$. If a periodic input $s(t) = A \cos(2\pi t/T_s)$ is applied alone and with a too weak amplitude A , then the particle cannot jump over the potential barrier between the two wells; it remains confined in one of the wells around a potential minimum, with no transitions between wells. One can introduce here a binary output signal $y(t)$, with two states say $y(t) = \pm 1$, indicating which of the two wells the particle is in at time t , for instance

$$y(t) = \text{sign} [s(t) + \eta(t) - \theta] = \pm 1 \quad (4.3)$$

When the amplitude of $s(t)$ is below the quantization threshold, no transition is

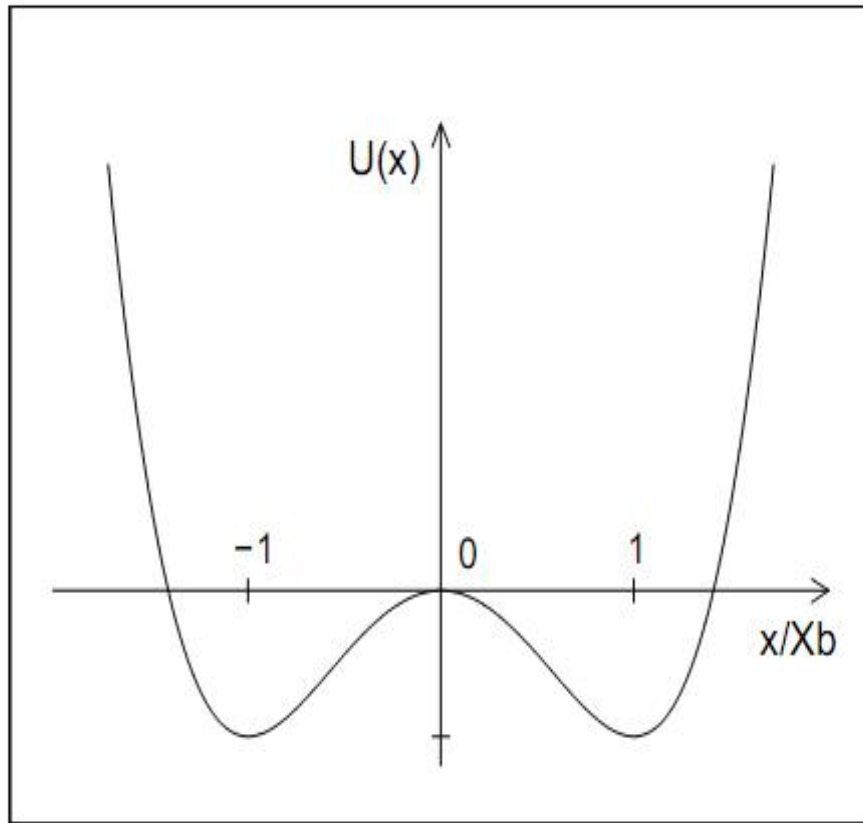


Figure 4.2: Double-well potential

induced in $y(t)$ in the absence of noise. As the input noise level is raised above zero, a cooperative effect can take place where the noise $\eta(t)$ assists the periodic input $s(t)$ in overcoming the threshold. As no transitions take place between wells, $y(t)$ remains stuck in one of its two states. Then, if a small noise $\eta(t)$ is added, a cooperative effect between the signal $s(t)$ and the noise becomes possible, enabling occasionally the particle to jump over the potential barrier. This translates into transitions between wells which are correlated with the periodic input $s(t)$ as it plays a part in their production (in conjunction with the noise). When the noise level is raised, the probability of occurrence of such coherent transitions first increases, thus reinforcing the correlation of the output $y(t)$ with the periodic input $s(t)$. For stronger noise levels, incoherent transitions induced by the noise alone will become more and more

frequent, and will gradually destroy the correlation of the output with the periodic input. The noise thus has a nonmonotonic influence, first enhancing the correlation of the output with the periodic input, up to an optimum, after which the correlation is gradually destroyed.

The output $y(t)$ is a random signal, because of the influence of the noise input $\eta(t)$, yet it bears correlation with the periodic input $s(t)$. To quantify the correlation of $y(t)$ with $s(t)$, the standard method starts with the calculation of the autocorrelation function of $y(t)$, and then through Fourier transform, to its power spectral density. In the power spectral density of $y(t)$, the influence of the periodic input $s(t)$ shows up as spectral lines at integer multiples of the coherent frequency $1/T_s$. These lines emerge out of a broadband continuous noise background stemming from incoherent transitions due to $\eta(t)$.

Fig. 4.3 shows that when a sinewave is applied at the input of SR system, the overall spectral power at the output of the system increases. Similarly, the same SR system model is verified for different types of i/p signals like BPSK signal, QPSK signal, unipolar signal, bipolar signal, FSK signal, PSK signal, etc.

4.2 Energy Detection through Stochastic Resonance system

4.2.1 Introduction

A novel spectrum-sensing approach based on the stochastic resonance (SR) technique can be employed. By introducing the received signal into a dynamic SR system, the SNR of PU signals can be increased. This enlarges the spectrum power of PU signals and improves spectrum-sensing performance in CR networks, particularly under low SNR conditions, i.e., $\text{SNR} < -10$ dB. The detection probability using this SR approach

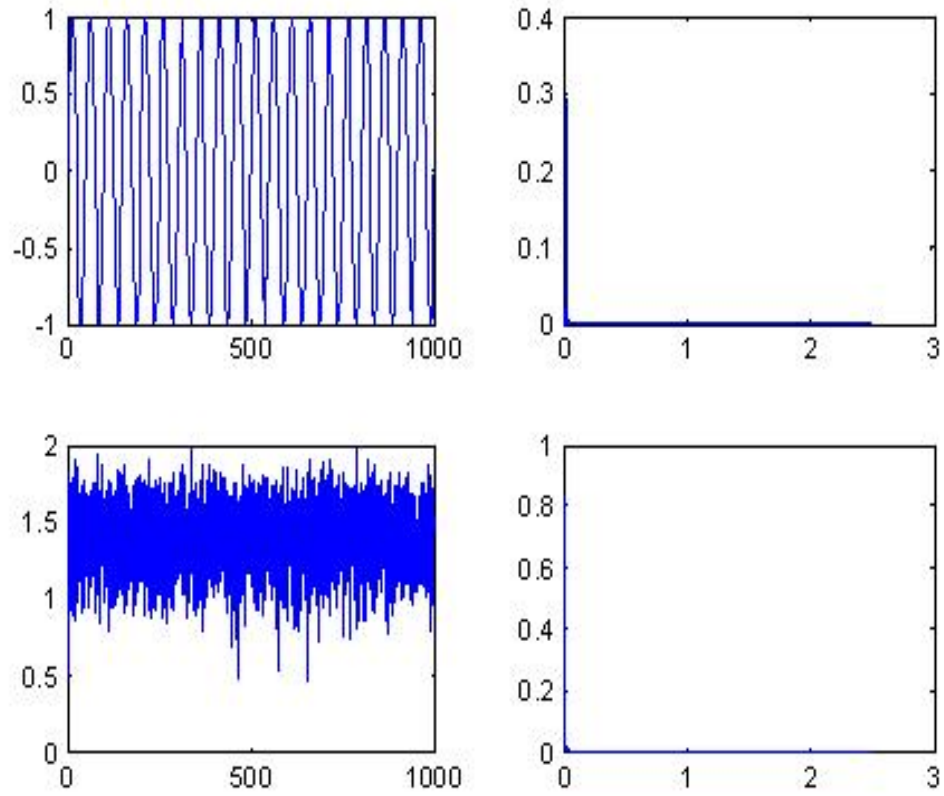


Figure 4.3: input and putput of SR system

can be improved [20].

The energy detection problem considered over here can be again started with the binary hypothesis model:

$$\begin{aligned}
 H_0 : r(t) &= n(t) \\
 H_1 : r(t) &= h.s(t) + n(t)
 \end{aligned} \tag{4.4}$$

where $t = 1, \dots, N$ indexes the samples of received signal at cognitive receiver end by cognitive radio user, where $r(t)$ is the received signal, $s(t)$ is sinewave PU signal represented by $s(t) = A_p \sin(\omega_p t + \phi_p)$ to be detected in CR end, and h is

the channel gain of the sensing channel between the PU and the SU, which can be supposed to be Rayleigh distribution with second-order moment $E[h^2] = m_h^2$ and is independent to $s(t)$. $n(t)$ is the additive white Gaussian noise (AWGN) with mean zero and variance σ_n^2 .

In conventional energy detector (CED), the received signal samples are first squared, then summed over the number of samples collected and then compared with a predetermined threshold to take decision on presence or absence of PU. The test statistics (T) can be represented as

$$T = \frac{1}{N} \sum_{t=1}^N |r(t)|^2 \quad (4.5)$$

where N is the number of samples.

The probability of detection and false alarm can be represented as

$$\begin{aligned} P_d &= P(T > \lambda \mid H_1) \\ P_f &= P(T > \lambda \mid H_0) \end{aligned} \quad (4.6)$$

λ is a predetermined threshold.

Where N is the number of samples.

The false-alarm rate $P_{f(ED)}$ and the detection probability $P_{d(ED)}$ of the energy detector can be calculated by the following expressions [20]:

$$P_{f(ED)} = P_r \{T(r) > \gamma_{ED}; H_0\}$$

$$P_{f(ED)} = P_r \left\{ \frac{T(r)}{\sigma_n^2} > \frac{\gamma_{ED}}{\sigma_n^2}; H_0 \right\}$$

$$P_{f(ED)} = Q_{\chi_N^2} \left(\frac{\gamma_{ED}}{\sigma_n^2} \right)$$

And

$$P_{d(ED)} = P_r \{T(r) > \gamma_{ED}; H_1\}$$

$$P_{d(ED)} = P_r \left\{ \frac{T(r)}{m_h^2 \sigma_s^2 + \sigma_n^2} > \frac{\gamma_{ED}}{m_h^2 \sigma_s^2 + \sigma_n^2}; H_1 \right\}$$

$$P_{d(ED)} = Q_{\chi_N^2} \left(\frac{\gamma_{ED}}{m_h^2 \sigma_s^2 + \sigma_n^2} \right)$$

where $Q_{\chi_N^2}(\cdot)$ is the right-tail probability of the central chi-squared pdf with N degrees of freedom.

The proposed SR based energy detection approach is shown in Fig. 4.4.

A unique property of SR is that it can be used to amplify the SNR of the input signal, which makes it more suitable for the weak target detection problem, particularly under low SNR circumstances such as SNR < -10 dB. In the SR system, when the input signal, the SR noise, and the system's nonlinearity can reach the given match point, noise energy can be converted into signal energy, which results in the enhancement of the output SNR toward maximization (peak point). In Fig. 4.4 a novel spectrum-sensing approach is presented based on the combination of SR and energy detection. By passing the received signal in CR receiving end through an SR system, an amplified response to the input signal can be observed at the output. The amplified signal is then passed through the energy detector to get the final decision.

In this proposed scheme based on SR [20], first, we set the normalized signal of $r(t)$ in eq. (4.4), for example, $r_0(t)$, as the input of an SR system $f[\cdot]$. Then, we have

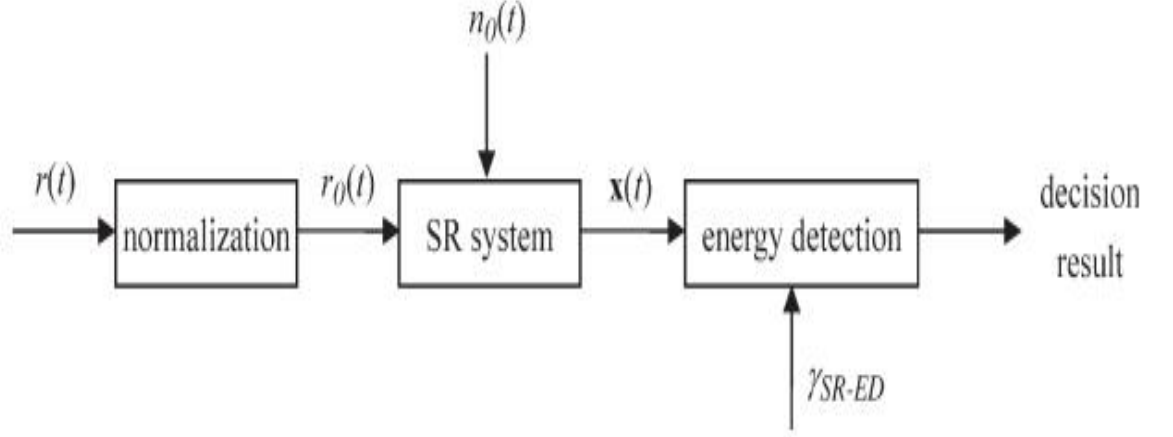


Figure 4.4: Block Diagram of Energy Detedtion through SR system

$$\begin{aligned} \dot{X}(t) &= f[X(t), r_0(t) + n_0(t)] \\ r_0(t) &= \frac{r(t)}{\sqrt{\text{var}[r(t)]}}, (t = 0, 1, \dots, N - 1) \end{aligned} \quad (4.7)$$

where $x(t)$ is the SR system status vector, and $n_0(t)$ is the introduced SR noise with mean zero and variance $\sigma_{n_0}^2$; therefore, $r_0(t) + n_0(t)$ can be regarded as the drive signal to the SR system.

A kind of discrete overdamped bistable oscillator is usually discussed and utilized, which can be expressed as

$$\frac{x(t) + x(t + \Delta t)}{\Delta t} = 2x(t) - x^3(t) + k \cdot r_0(t) + n_0(t) \quad (4.8)$$

where Δt is the sampling interval, k is a constant driving parameter ($k=0.3$), and the initial value of the status variable $x(0)$ can be randomly selected within $(-1, +1)$.

When the energy detection is used after the SR process, a new test statistic $T(x)$ is established as

$$T(X) = \sum_{t=0}^{N-1} \|X(t)\|^2 > \gamma_{SR-ED} \quad (4.9)$$

where γ_{SR-ED} is the decision threshold that is used by the SR-based energy detector. Based on the linear response theory in SR [12], the output of the SR system can also be divided into two additive parts, that is

$$X(t) = S_{SR}(t) + n_{SR}(t) \quad (4.10)$$

Where $S_{SR}(t)$ represents system response corresponding to normalized PU signal $h \cdot s(t) / \sqrt{\text{var}[r(t)]}$, and $n_{SR}(t)$ is the system response corresponding to the noise signal $n(t) / \sqrt{\text{var}[r(t)]} + n_0(t)$.

4.2.2 Optimum value of SR noise to be added

Define SNR_i as the SNR of the SR systems input signal $r(t)$ under hypotheses H_1 and SNR_0 as the output SNR of the SR system status vector $x(t)$. Then, we can get,

$$SNR_i = \frac{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=0}^{N-1} h^2 s^2(t)}{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=0}^{N-1} n^2} = \frac{m_h^2 \sigma_s^2}{\sigma_n^2} \quad (4.11)$$

$$SNR_0 = \frac{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=0}^{N-1} \|S_{SR}(t)\|^2}{\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{t=0}^{N-1} \|n_{SR}(t)\|^2} = \frac{\sigma_{s(SR)}^2}{\sigma_{n(SR)}^2} \quad (4.12)$$

where $\|\cdot\|$ is the modulus function, σ_s^2 and $\sigma_{s(SR)}^2$ are the power of $s(t)$ and $s_{SR}(t)$, respectively, and $\sigma_{n(SR)}^2$ represents the variance of $n_{SR}(t)$. For example, when the discrete overdamped bistable oscillator in eq. (4.8) is used and assuming that a

sinusoidal PU signal is introduced as

$$s(t) = A_P \cdot \sin(\omega_P t + \phi_P) \quad (4.13)$$

Where A_P , ω_P , and ϕ_P are the amplitude, angular frequency, and phase of the PU sinusoidal signal, then SNR_i and SNR_o of the bistable oscillator can be expressed by

$$SNR_i = \frac{\frac{1}{2} m_h^2 A_P^2}{\sigma_n^2} \quad (4.14)$$

$$SNR_o = \frac{4\sqrt{2} k^2 m_h^2 A_P^2}{(k^2 \sigma_n^2 + \sigma_{n0}^2)} e^{-\frac{2}{k^2 \sigma_n^2 + \sigma_{n0}^2}} \quad (4.15)$$

From the above analyses, to reach a maximal SNR_o , the optimal variance of the introduced SR noise σ_{n0}^2 can be calculated by

$$\sigma_{n0(opt)}^2 = \arg \max_{\sigma_{n0}^2} SNR_o = \arg \max_{\sigma_{n0}^2} \frac{\sigma_{s(SR)}^2}{\sigma_{n(SR)}^2} \quad (4.16)$$

It can be found in the figure that SNR_o will reach a maximal value when an optimal σ_{n0} is selected. In other words, due to the reason that the SNR of the input signal is fixed and is not adjustable, an SR noise $n_0(t)$ is then introduced here to reach the optimal performance or maximal SNR_o . The power of the introduced SR noise will transfer to that of the PU signal $s(t)$ to enlarge the SNR_o , which reveals the physical phenomenon of SR. Under hypotheses H_1 , $r_0(t)$ is composed of the PU signal and AWGN; therefore, according to the requirement that the total noise $n(t)/\sqrt{\text{var}[r(t)]} + n_0(t)$ be even symmetric in the SR system and $n(t)$ takes part in the SR system as a part of the SR noise, the introduced SR noise $n_0(t)$ should have the same pdf with $n(t)$ as an AWGN signal. Furthermore, when $\sigma_{n0(opt)}^2 = 0$, it becomes a special case that no more SR noise $n_0(t)$ needs to be introduced to the SR system, and the additive

channel noise $n(t)$ can fully play the role of the SR noise. Thus, the SNR_0 can be changed to

$$SNR_0 = \frac{4\sqrt{2}k^2m_h^2A_P^2}{\left(k^2\sigma_n^2 + \sigma_{n0(opt)}^2\right)} e^{-\frac{2}{k^2\sigma_n^2 + \sigma_{n0(opt)}^2}} \quad (4.17)$$

To get the optimal value of $\sigma_{n0(opt)}^2$, let $\frac{\partial SNR_0}{\partial \sigma_{n0}^2} = 0$ and we have,

$$\sigma_{n0(opt)}^2 = 1 - k^2\sigma_n^2 \quad (4.18)$$

Output SNR of the discrete overdamped bistable oscillator versus is shown in Figure 4.5

4.2.3 Performance Analysis

When the energy detection is used after the SR process, a new test statistic $T(x)$ is established as

$$T(X) = \sum_{t=0}^{N-1} \|X(t)\|^2 > \gamma_{SR-ED} \quad (4.19)$$

The false-alarm rate using the proposed SR-based approach, i.e., $P_{f(SRED)}$, can then be expressed as [20]

$$P_{f(SR-ED)} = P_r \{T(X) > \gamma_{SR-ED}; H_0\}$$

$$P_{f(SR-ED)} = P_r \left\{ \frac{T(X)}{\sigma_n^2(SR)} > \frac{\gamma_{SR-ED}}{\sigma_n^2(SR)}; H_0 \right\}$$

$$P_{f(SR-ED)} = Q_{\chi_N^2} \left(\frac{\gamma_{SR-ED}}{\sigma_n^2(SR)} \right)$$

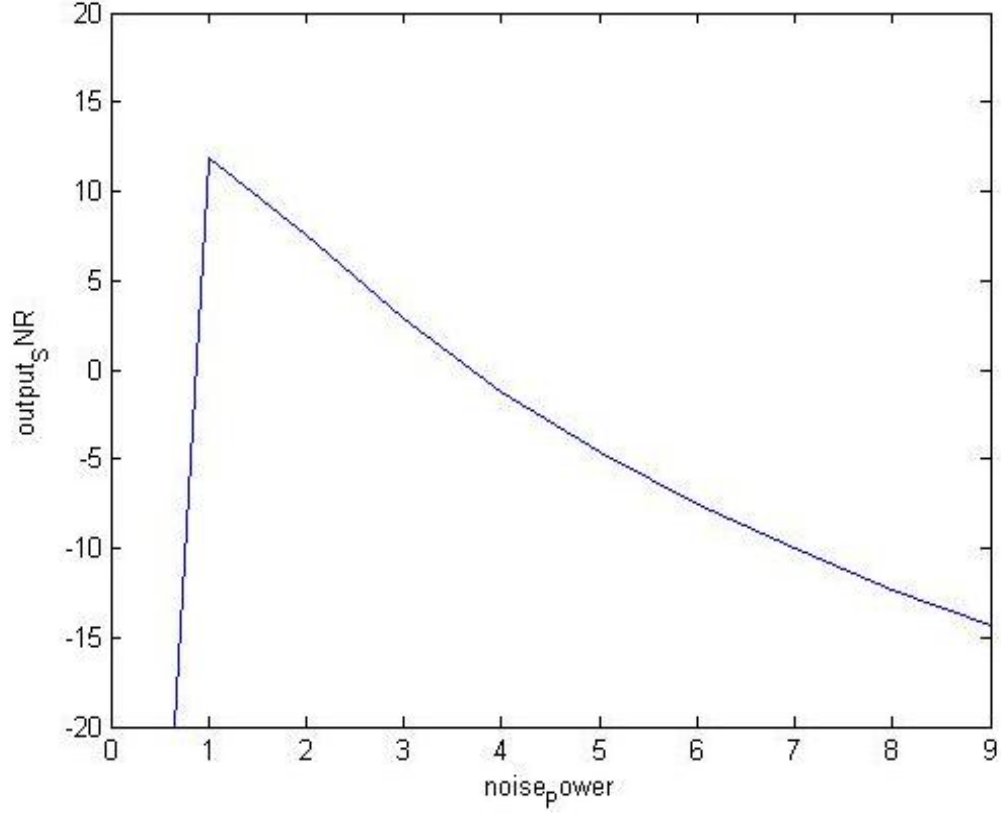


Figure 4.5: Output SNR versus noise power

The detection probability of the proposed SR-based energy detection approach, which is denoted by $P_{d(SR-ED)}$ can be calculated by

$$P_{d(SR-ED)} = P_r \{T(X) > \gamma_{SR-ED}; H_1\}$$

$$P_{d(SR-ED)} = P_r \left\{ \frac{T(X)}{\sigma_{s(SR)}^2 + \sigma_{n(SR)}^2} > \frac{\gamma_{SR-ED}}{\sigma_{s(SR)}^2 + \sigma_{n(SR)}^2}; H_1 \right\}$$

$$P_{d(SR-ED)} = Q_{\chi_N^2} \left(\frac{\gamma_{SR-ED}}{\sigma_{s(SR)}^2 + \sigma_{n(SR)}^2} \right)$$

4.2.4 Simulation Result

In the computer simulations, the discrete overdamped bistable oscillator in Eq. (8) is utilized as the SR system model, where k is the constant set as $k=0.3$, and $r_0(t)$ is the normalized received signal, which is also a part of the SR system input. Here, PU sinusoidal signal through a Rayleigh fading channel with AWGN noise are used as the input of discrete overdamped bistable oscillator, that is

$$r(t) = h. [A_P \sin(\omega_P t + \phi_P)] + n(t) \quad (4.20)$$

where h is rayleigh channel gain with mean 1, A_P , ω_P , and ϕ_P are the amplitude, angular frequency, and phase of the PU sinusoidal signal. Power of AWGN noise signal $n(t)$ is $\sigma_n^2 = 1$. The optimal variance of the introduced white Gaussian SR noise [10] with mean 0 can be calculated as $\sigma_{n_{0(opt)}}^2 = 1 - k^2$. Fig. 4.6 shows the receiver operating characteristic (ROC) curves of the proposed SR-based energy detector and the traditional energy detector under SNR = -15 dB. The sample number is chosen as $N=10^3$. Fig. 4.7 shows P_d vs. SNR curves of the proposed SR-based energy detector and the traditional energy detector.

Pf versus SNR curve for Rayleigh Channel with SR noise:

The curve obtained in Fig 4.7 shows that the performance of energy detector with SR noise in terms of Pf is as good as AWGN channel. That is detection probability is increased without sacrifice in terms of Probability of false alarm.

4.2.5 Computational Complexity Analysis

For CR applications, spectrum sensing should not take a long time to perform and should result in a reliable detection outcome. Robust techniques, relatively low computational complexity, and/or a high-performance processing architecture are re-

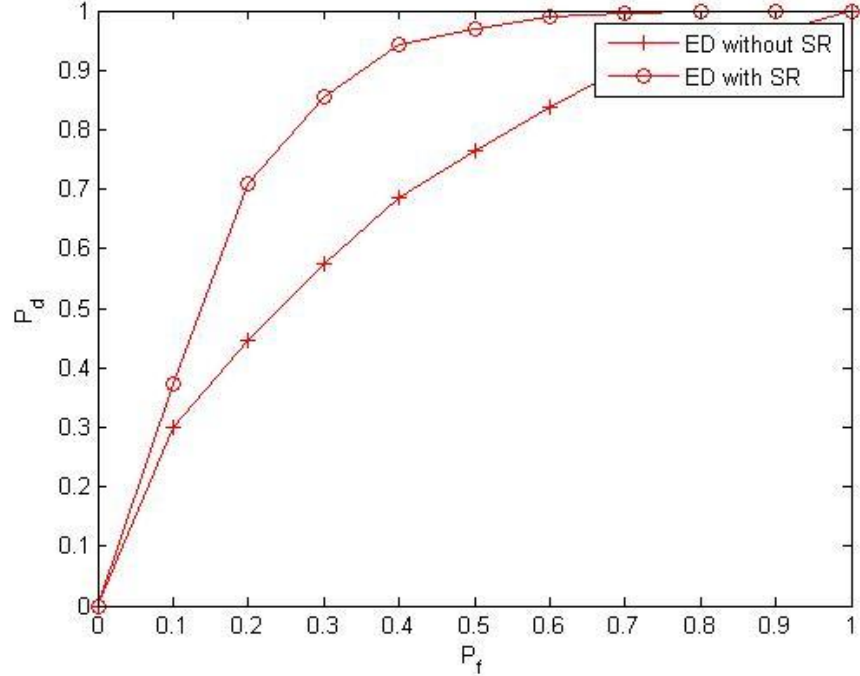


Figure 4.6: ROC for energy detection (with and without SR noise)

quired to achieve this. By using the energy detector, given the sample number N and the threshold γ_{ED} , one has to carry out N times of multiplication operations; therefore, the computational complexity can be regarded as $O(N)$. On the other hand, in the proposed SR-based approach, the normalized received signal has to be sent into the SR system first and then must carry out the energy detection; therefore, the SR process will produce extra computational costs compared with the traditional energy detector. As in most commonly used SR systems such as the bistable oscillators, the dynamic equation shown in eq. (4.1) has an ordinary explicit expression, or at least it possesses the same computational costs with the linear dynamic equation, for example, $O(N)$. For example, if the discrete overdamped bistable oscillator in eq. (4.1) is utilized, it can be observed that an output sample $x(t)$ requires just four multiplication operations. The next stage of injecting this sample into the energy detection only requires an additional multiplication operation. A total of five multiplication operations per sample are required to implement the proposed SR-based

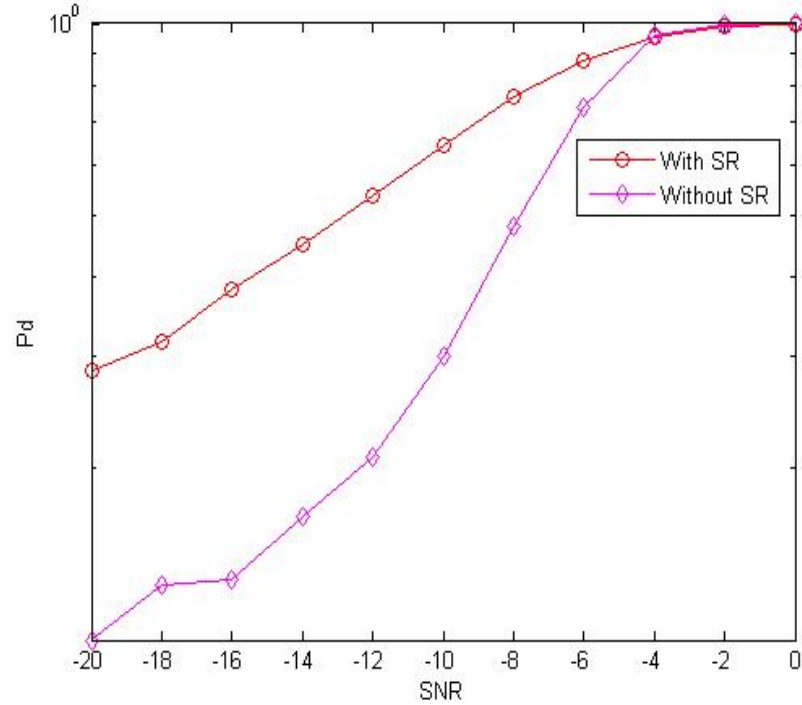


Figure 4.7: Pd Vs. SNR curve (with and without SR noise)

detection scheme. The computational complexity is $O(N)$, which is similar to that of a traditional energy detection approach. However, the key difference is that the detection performance is improved under low SNR conditions and that the SNR of the output signal using an SR-based approach is also increased. This represents a significant enhancement when compared with existing spectrum-sensing methods.

4.3 Generalized Energy Detection through Stochastic Resonance system

4.3.1 Introduction of Generalized Energy Detection

The test statistic T_{CED} for conventional energy detector is given as

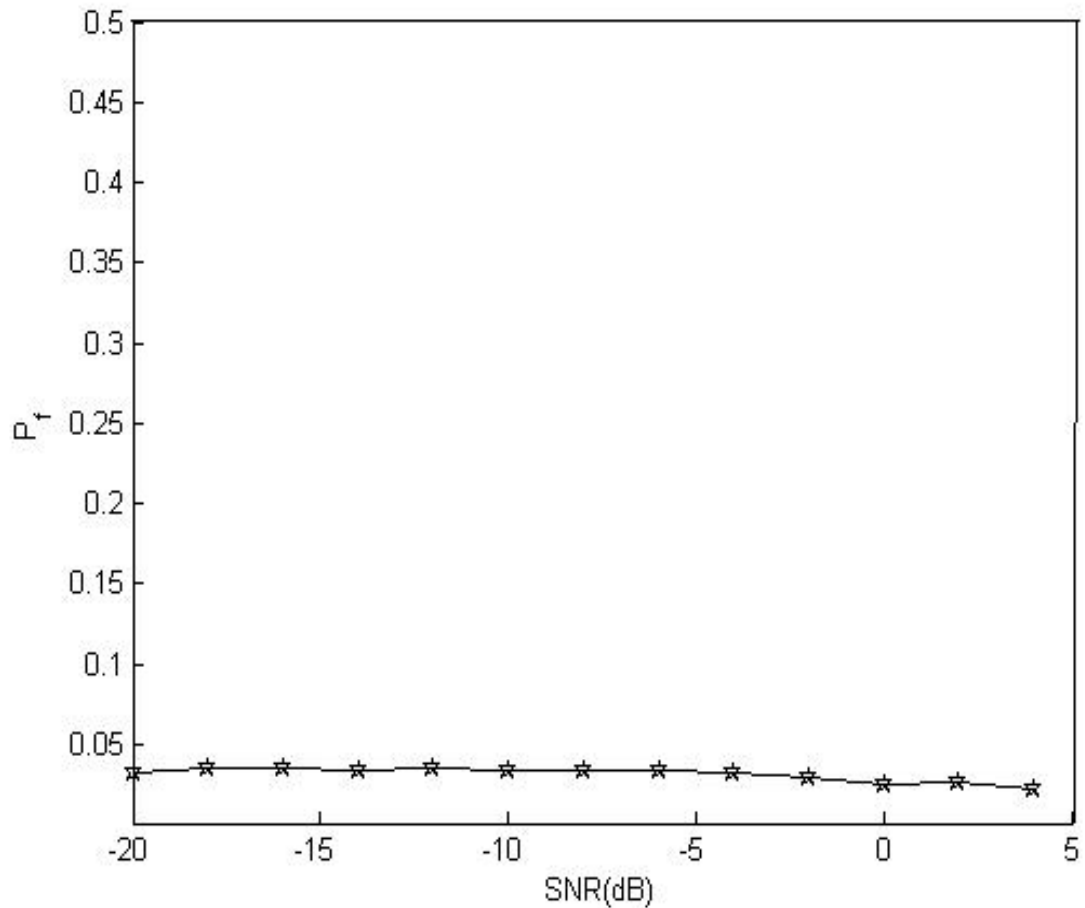


Figure 4.8: Pf versus SNR curve for Rayleigh fading Channel with SR noise

$$T_{CED} = \frac{1}{N} \sum_{t=1}^N |r(t)|^2 \quad (4.21)$$

We can transform conventional energy detector to generalized energy detector (GED) by replacing squaring operation by an arbitrary positive operation p [16]. Then the test statistic for GED is given as

$$T_{GED} = \frac{1}{N} \sum_{t=1}^N |r(t)|^p \quad (4.22)$$

where $p > 0$ is an arbitrary constant. It can be seen that CED is a special case of GED with $p = 2$. For large N and thus invoking central limit theorem (CLT), we can define probability of detection P_d and probability of false alarm P_f for GED as [17]

$$P_f = P_r(T_{GED} > T | H_0) = Q\left(\frac{T - \mu_0}{\sigma_0/\sqrt{N}}\right) \quad (4.23)$$

$$P_d = P_r(T_{GED} > T | H_1) = Q\left(\frac{T - \mu_1}{\sigma_1/\sqrt{N}}\right) \quad (4.24)$$

where

$$Q(t) = \frac{1}{2\pi} \int_t^{\infty} e^{-\frac{x^2}{2}} dx \quad (4.25)$$

and T is the predetermined threshold which can be obtained by fixing probability of false alarm, μ_1 and μ_0 are means of T_{GED} under H_1 and H_0 respectively, σ_1^2 and σ_0^2 are variances of T_{GED} under H_1 and H_0 respectively, which can be given as sec. 3.5.

$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma^p$$

$$\sigma_0^2 = \frac{2^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right]$$

$$\mu_1 = \frac{2^{p/2}(1+\gamma)^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \sigma^p$$

$$\sigma_1^2 = \frac{2^p(1+\gamma)^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \sigma^{2p}$$

with γ is received signal-to-noise ratio.

This Generalized Energy Detection can be performed on received signal after passing received signal through SR system. Probability of detection can be improved further. Fig. 4.8 shows simulation results for generalized energy detector for the same system parameters as considered in section 4.2.4 and introducing SR noise. This ROC plot is obtained at SNR=-20 dB.

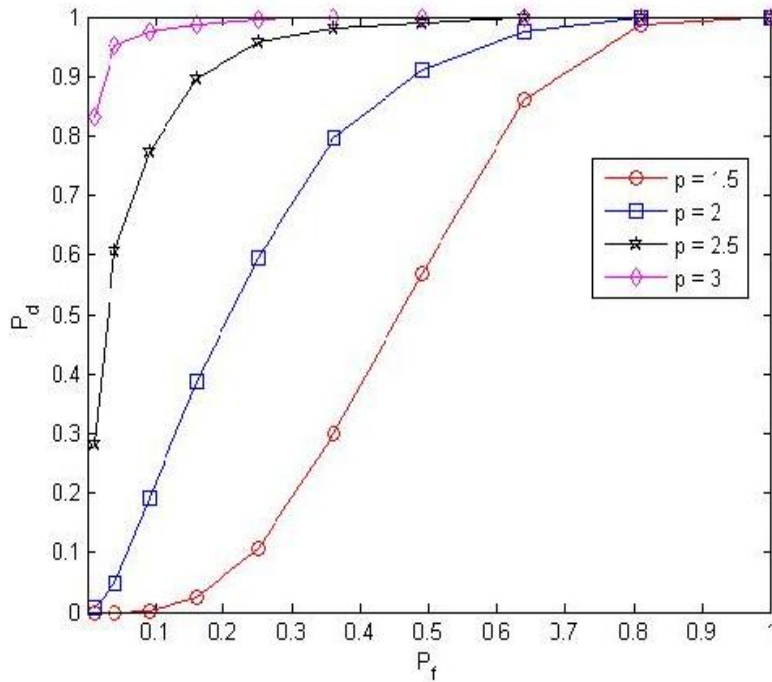


Figure 4.9: ROC for Generalized Energy Detection (with SR noise)

Pf versus SNR curve for Generalized Energy detector with SR noise:

Fig 4.9 shows that the probability of False alarm remains almost same in case of generalized energy detector with SR noise for value of $p=3$. As number of samples

and value of p changes P_f will be changed.

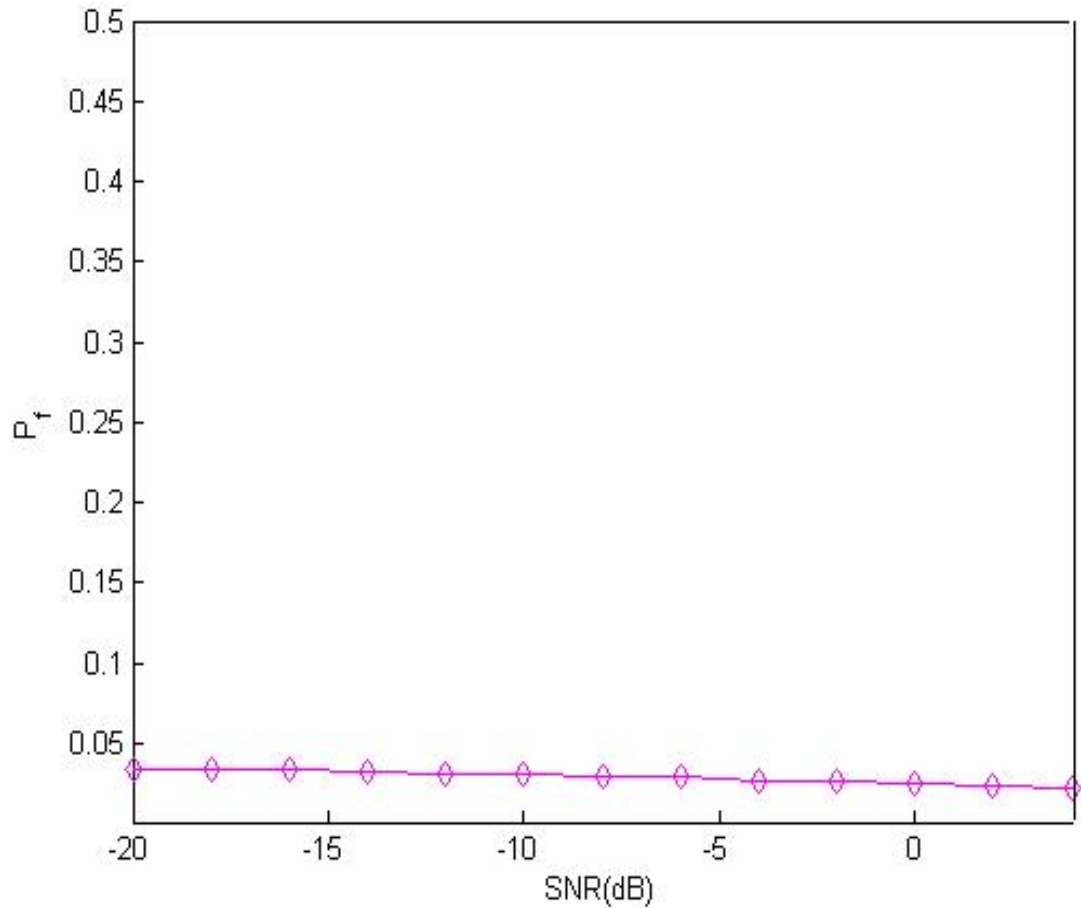


Figure 4.10: P_f versus SNR curve for GED with SR noise

4.3.2 Performance of SR based GED under noise uncertainty

Concept of SNR wall:

The sensitivity of detectors can be improved by increasing the sensing time and so the sample complexity gives us an actual way to compare different spectrum sensing methods. However, one must also consider the impact of real-world uncertainties on the performance of detectors since robustness is important. Doing this reveals that the sample complexity blows up to infinity as the detector sensitivity approaches cer-

tain critical values called SNR walls [30]. Below these SNR walls, it is completely impossible to robustly distinguish the two hypotheses. The location of the walls themselves depends on what is known about the signal being sensed as well as the size of certain critical uncertainties in the noise distribution and fading process.

Sample Complexity Analysis:

Under the noise uncertainty model given in [30], the sample complexity of detection also depends on the parameter $\rho = 10^{x/10}$. Sample complexity with noise uncertainty for a fixed value of P_d and P_f is given as

$$N = \frac{2[Q^{-1}(P_f) - Q^{-1}(1 - P_d)]^2}{[SNR_i - (\rho - \frac{1}{\rho})]^2} \quad (4.26)$$

The fundamental effect of stochastic resonance is improvement in the received signal SNR. This improvement is given by [29]

$$SNR_{gain} = SNR_0 - SNR_i \quad (4.27)$$

Where SNR_0 is the SNR of SR system output signal and SNR_i is the SNR of SR system input i.e. received signal in CR receiving end. Thus, sample complexity in case of SR based energy detector is given by

$$N_{SR} = \frac{2[Q^{-1}(P_f) - Q^{-1}(1 - P_d)]^2}{[SNR_0 - (\rho - \frac{1}{\rho})]^2} \quad (4.28)$$

As $SNR_0 > SNR_i$, sample complexity will be reduced in SR based energy detector compared to traditional energy detector.

This can be represented in terms of SNR wall also. For traditional energy detector, $SNR_{wall} = (\rho - \frac{1}{\rho})$. The improvement in SNR for SR based energy detector is given by SNR_{gain} . Thus, reduction in SNR wall for SR based energy detector is given by

$$SNR_{wall}^{SR} = \left(\rho - \frac{1}{\rho} \right) - SNR_{gain} \quad (4.29)$$

Fig. 4.8 shows the sample complexities of traditional energy detector, SR based energy detector and SR based GED under noise uncertainty ($\alpha=0.3$ dB). The fixed values of $P_f=1-P_d=0.1$. It shows the reduction in SNR wall for SR based energy detector compared to traditional energy detector, which is about 7 dB. It has been shown that sample complexity is greatly reduced for GED (for power constant $p=3$) for the same value of SNR compared to SR based conventional energy detector ($p=2$).

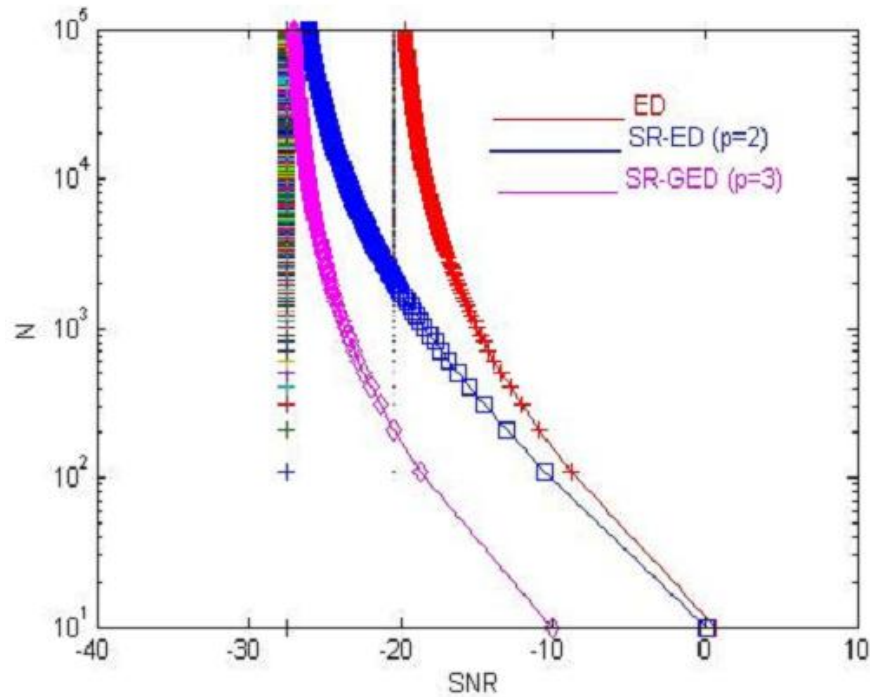


Figure 4.11: Sample complexity of traditional ED, SR based ED and SR based GED under noise uncertainty

4.4 Summary

Detailed analysis of energy detection technique of spectrum sensing has been given in this chapter. Section 3.1 represents system model to measure the performance of energy detector over AWGN channel. Probability of Detection and Probability of False alarm are derived for AWGN channel in section 3.2. The performance of energy detector over Rayleigh fading channel is discussed in section 3.3. The performance of energy detector is further improved in case of Rayleigh fading channel by introducing EGC diversity scheme in section 3.4. Introduction and need for Improved Energy detector is given in section 3.5. Performance of Generalized Energy detector can be observed for different values of positive power constant p is obtained by doing mathematical analysis and simulations in the same. Section 3.6 discusses the effect of noise uncertainty on the performance of the generalized energy detector.

Chapter 5

Conclusion and Future Work

5.1 Conclusion

Spectrum Sensing is the most important task to use the spectrum opportunistically in cognitive radio network. Various techniques have been proposed and in use for spectrum sensing. In transmitter Detection technique (Non cooperative Detection), the weak primary transmitter signal is detected based on the local observation of Cognitive Radio user. Matched filter detection is the most complex but most accurate method of spectrum sensing. Cyclostationary Feature Detection is computationally complex and may require significantly long observation time. Energy Detection is the least complex and does not require a priori information of primary user signal. The aim of this thesis is to study energy detection technique in detail because of its above-mentioned advantage and analyze the performance of energy detection over AWGN channel and Rayleigh fading channel. Performance improvement can be obtained by introducing diversity in fading channel. Also, performance of energy detector deteriorates under low SNR. Under the condition of low SNR, probability of detection can be improved by using generalized energy detector. Also, noise uncertainty exists in actual practice. Effect of noise uncertainty is studied in generalized energy detection and it has been found that conventional energy detector is best under noise uncertainty. The spectral power of primary users (PUs) can be amplified, and the signal-to-noise ratio (SNR) of a received signal can be increased using Stochastic

Resonance (SR). Detection probability has been increased significantly and drastic reduction in SNR wall has been obtained. Further, by combining Generalized Energy Detector and Stochastic Resonance, improvement in detection probability as well as significant reduction in sample complexity is achieved.

5.2 Future Work

There exist certain challenges in energy detection technique. The performance of energy detector is susceptible to noise uncertainty, which exist in practice. Stochastic Resonance based energy detector can be evaluated by adding noise uncertainty. Under the constraint of the false alarm probability, energy detection with stochastic resonance can be applied for multiple nodes i.e., cooperative sensing for maximizing the probability of detection. Further, diversity techniques like Equal gain Combining (EGC) or Selection Combining (SC) can be applied. This can achieve much better performance than traditional energy detectors [31].

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