Evaluation of Moment-Curvature Relationship for Reinforced Concrete Beam Element

By

Jiten A. Patel

12MCLC20

DEPARTMENT OF CIVIL ENGINEERING

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Evaluation of Moment-Curvature Relationship for Reinforced Concrete Beam Element

Major Project Report

Submitted in partial fulfillment of the requirements

For the degree of

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(Computer Aided Structural Analysis And Design)

By

Jiten Patel

(12MCLC20)

Guided By

Prof. (Dr.)Sharad Purohit

DEPARTMENT OF CIVIL ENGINEERING

INSTITUTE OF TECHNOLOGY

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AHMEDABAD-382481

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Declaration

This is to certify that

i) The thesis comprises my original work towards the degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) at Nirma University and has not been submitted elsewhere for a degree.

ii) Due acknowledgement has been made in the text to all other material used.

Jiten A. Patel

Certificate

This is to certify that the Major Project Report entitled **"Evaluation of Moment-Curvature Relationship for RC Beam Element"** submitted by **Mr. Jiten A. Patel (Roll No: 12MCLC20)**, towards the partial fulfillment of the requirements for the degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis And Design.) of Nirma University is the record of work carried out by him under our supervision and guidance.The work submitted has in our opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other University or Institution for award of any degree or diploma.

Date:

Prof.(Dr.) Sharad Purohit
Guide and Professor,
Department of Civil Engineering,
Institute of Technology,
Nirma University,
Ahmedabad.

Dr. P. V. Patel
Professor and Head
Department of Civil Engineering,
Institute of Technology,
Nirma University,
Ahmedabad.

Dr. K. Kotecha
Director,
Institute of Technology,
Nirma University,
Ahmedabad.

_____
Examiner

_____
Date of Examination

Abstract

The relationship between Moment and Curvature of Reinforced Concrete sections is an important parameter for Nonlinear Analysis of RC framed structure. This helps in determining strength, stiffness, ductility, and energy dissipation capacity of the RC sections. The moment-curvature relationship would enable us to observe the strength reduction beyond the yield point and degradation of the flexural rigidity. Theory of beam flexure states that curvature is the second derivate of displacement. Moment-Curvature (M-∅) relation is then very important as it helps to calculation of displacement due to the forces acting in to the inelastic elements.

Analytical as well as experimental study is carried out for RC beam section. A well-established models of stress-strain for concrete and steel material are used. Models showing relationship of stress-strain for both confine as well as unconfined concrete are studied. Six deferent stress-strain models are selected. Among those IS 456 Model, Hongnestad Model, Kent and Park Model, IRC Model are pertaining to unconfined concrete and Cusson Model and Mendar Model are pertaining to confined concrete. For steel material King Model is used. A MATLAB based program is developed to derive Moment-Curvature relationship for RC beam elements using above mentioned models for concrete and steel material analytically.

An experimental program is design to evaluate Moment-Curvature relationship for RC beam elements. Both singly and doubly reinforced sections are design with under and over reinforcement for each section. Three sample specimens of dimension 120mm $\times$ 150mm $\times$ 1700mm are prepared for each case leading to total twelve numbers of test specimens. Loads and strain are measured to derive Moment-Curvature for each test specimens.

It is found that well established models of stress-strain for material to derive Moment- Curvature relationship closely match with the Moment-Curvature relationship experimentally. It is noticed that model suggested by IRC 112 shows good agrement with experimentally obtain Moment-Curvature relationship.

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12MCLC20

Abbreviations, Notations and Nomenclature

M ..... Moment of Reinforced Concrete Elements.

ϕ ..... Curvature of RC Elements.

f_{ck} ..... Characteristic Strength of Concrete.

σ_c ..... Stress at Any Point.

ε_c ..... Strain at Any Level.

f_{cc} ..... Characteristic Strength of Confined Concrete.

f_{c0} ..... Characteristic Strength of Unconfined Concrete.

f_c ..... Cylinder Strength of Concrete.

f_{cd} ..... Design Compressive Strength of Concrete.

ε_{co} ..... Peak Strain for Unconfined Concrete.

ε_{cc} ..... Peak Strain for Confined Concrete.

ε_{si} ..... Strain in Steel at Level of i^{th} Bar.

fl ..... Lateral Confinement Pressure.

M_{cr} ..... Moment at Cracking.

ϕ_{cr} ..... Curvature at Cracking.

f_r ..... Modulus of Rupture.

I_g ..... Gross Moment of Inertia.

ρ ..... Tensile Steel Ratio.

ρ' ..... Compression Steel Ratio.

m ..... Modular Ratio.

f_y ..... Yield Strength of Reinforcement.

kd ..... Neutral Axis Depth.

k ..... Neutral Axis Depth Factor.

a ..... Depth of Rectangular Stress Block.

M_y ..... Moment at Yielding.

ϕ_y ..... Curvature at Yielding.

M_u ..... Moment at Ultimate.

ϕ_u ..... Curvature at Ultimate.

C_c ..... Concrete Compressive Strength.

α ..... Mean Stress Factor.

γ ..... Centroid Factor.

T ..... Tensile Steel.

M_c ..... Moment of Compressive Force in Concrete About the Neutral Axis.

M_{cs} ..... Moment of Force in Compressive Steel About the Neutral Axis.

M_t ..... Moment of Force in Tensile Steel About the Neutral Axis.

A_{st} ..... Area of Tension Reinforcement.

A_{sc} ..... Area of Compression Reinforcement.

A_{sx} ..... Area of Transverse Reinforcement Running in X Direction.

A_{sy} ..... Area of Transverse Reinforcement Running in Y Direction.

SUR ..... Singly Under Reinforcement.

SOR ..... Singly Over Reinforcement.

DUR ..... Doubly Under Reinforcement.

DOR ..... Doubly Over Reinforcement.

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Chapter 1

Introduction

1.1 General

Reinforced concrete (RC) is the one of the most widely used structural material. Because of the non-homogeneity of the reinforced concrete, various methods for its analysis and design are used. However linear behavior is valid for a region of small response. At ultimate load condition material and geometrical non-linearity occurs which calls for non-linear analysis of structure. In present days, there is a turn in design philosophy from linear static approaches to performance based approaches. Nonlinear analysis required relationship of Moment-Curvature for section so it play important role. Moment-Curvature relationship for the section is the initial input of the Non-Linear analysis. The Moment-Curvature relationship of RC section demonstrates the strength, ductility, stiffness and energy dissipation capacity of the structural section subjected to bending. The Moment-Curvature also give as the clear view of strength reduction beyond the peak point and degradation of the flexural rigidity. In present report, a computer program is developed to evaluate Moment-Curvature relation for RC rectangular beam section analytically. An experimental program is also design to obtain Moment-Curvature for RC beam experimentally. Then experimental results are compared with the analytical results.

1.2 Introduction

Most of buildings and bridges are designed to exceed their elastic limits when attacked by the design earthquake. Exceeding elastic limits for reinforced concrete sections means cracking of concrete, yielding of reinforcement, crushing of concrete and eventual collapse of the section. The nonlinear flexural behavior of reinforced concrete sections can be assessed by a special type of section analysis called Moment-Curvature analysis. The outcome of this analysis is the relation between applied moment and related curvature in the section.

"Moment-Curvature analysis is a method to accurately determine the load-deformation behavior of the concrete section using nonlinear material stress-strain relationship."

For given axial load there exists an extreme compression fiber strain and a section curvature at which the nonlinear stress distribution is in equilibrium with the applied axial load. A unique bending moment can be calculated at this section curvature from the stress distribution. The extreme concrete compression stain and section curvature can be iterated until a range of moment-curvature values are obtain.

1.3 Significance of Moment-Curvature

The design of the structure is carried out by limit state method even though the analysis of structure is done by the conventional elastic theory, for taking in to account the material nonlinearity. This method is good for determinate as well as indeterminate structure even under factor load conditions till the Moment -Curvature relationship remain linear. For under reinforced section it is valid till the reinforcement steel is not yielded. However, once yielding has take place (at any section), the behavior of a structure enters in an inelastic phase, and the simplified conventional linear elastic structural analysis is no longer valid. There is a called for inelastic analysis of structure. One has to determine the bending moments for loading beyond the yielding stage. In simplified limit analysis the moment-curvature relation is an idealized

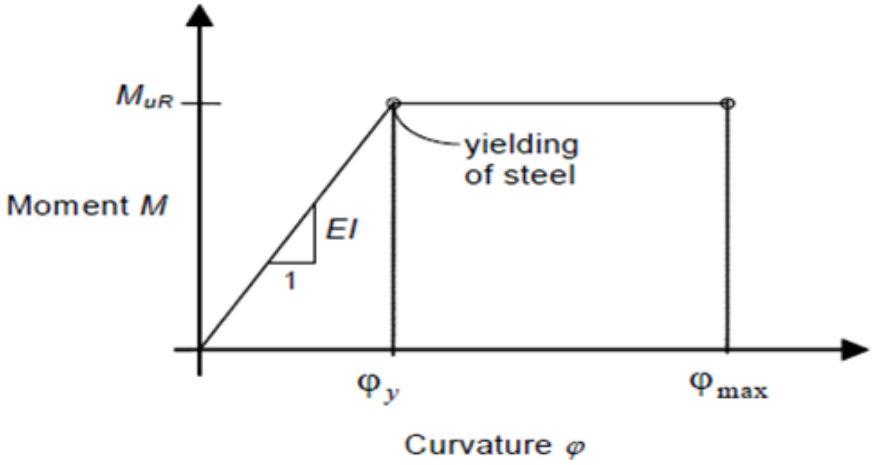

Figure 1.1: Idealized Moment-Curvature Relation [1]

bilinear elasto-plastic relation as shown in Fig. 1.1.

1.4 Objective of Study

Objectives of the present study have been identified as follows:

- To understand the various stress-strain models for confine and unconfined concrete.
- To develop computer program to calculate analytical Moment-Curvature.
- To carry out experiment to determine relation between Moment and curvature for RC beam elements.
- To explore various computational tools for Moment Curvature.

1.5 Scope of Study

In order to achieving the above objectives the scope of study are as under:

- To study need of Moment-Curvature relationship in Non-Linear analysis.
- To study various nonlinear stress-strain models for concrete and steel.
- It also include the preparation for RC specimens for deriving experimental Moment-Curvature relationship.
- To study about instrumentation for measuring the curvature in RC beam elements
- To develop a computer program to determine the moment curvature relationship for RC rectangular beam element using Matlab.
- To study the different available tools which are used for finding Moment - Curvature for RC Beam Element.
- Comparison of results obtain by experiments and analytically.

1.6 Methodology Carried Out

Methodology adopted in present study are as shown in Fig. 1.2. Here, study was divided in to two parts part-I include the experimental work and part-II include analytical work. Experiment work includes casting of specimens, preparation for instrumentation and testing of specimens and part-II include analytical evaluation of moment curvature by transform area method and strain compatibility method.

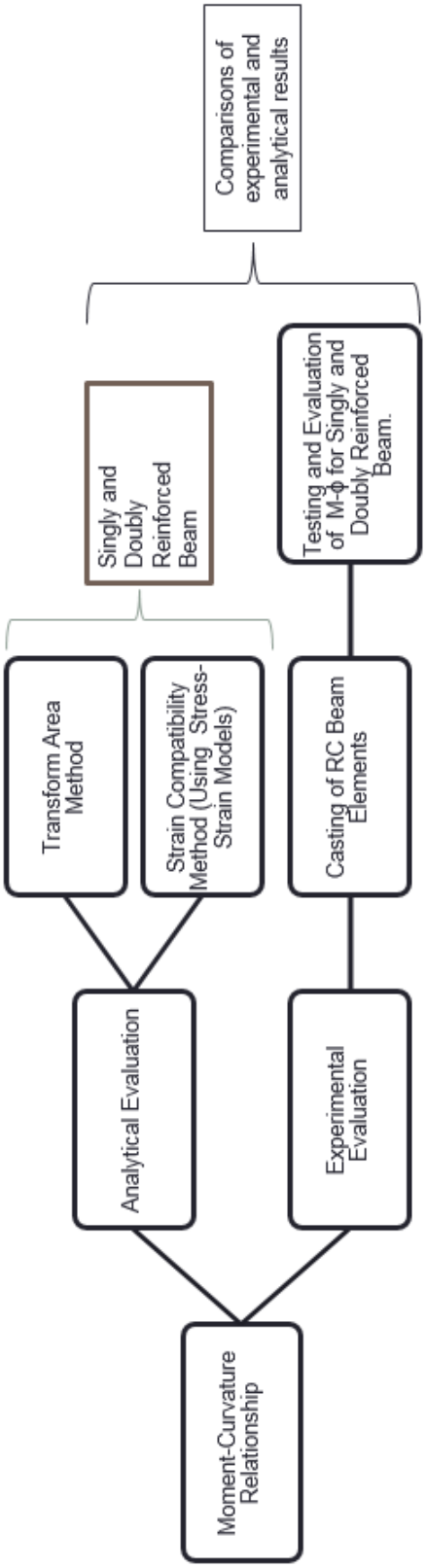

Figure 1.2: Flow of The Study

1.7 Organization of Report Work

Chapter 2 starts with a literature review which include three parts. In first part description of various stress-strain curve for concrete and steel are included. In second part the experimental evolution of moment-Curvature relationship is given. And in third part is dealing with the evolution of analytical Moment-Curvature relationship.
Chapter 3 includes the theoretical evolution of Moment Curvature relationship for unconfined concrete as well as confined concrete. these chapter also deals with the various methods for evaluation of Moment-Curvature.
Chapter 4 includes the problem statements and Moment-Curvature relationship was evaluated for the designed RC beam elements by using various mathematical stress-strain models. In this chapter, a Moment-Curvature relationship was also developed by using various available tools.
Chapter 5 discusses about the experimental setup and experimental evaluation of the moment curvature relationship for RC beam Element.
Chapter 6 includes the experimental results for concrete properties as well as experimental Moment-Curvature. It also deals with the comparison of analytical Moment-Curvature and experimental Moment-Curvature.
Chapter 7 deals with the summery of the study. It also include the conclusions and future scope of the work.

Chapter 2

Literature review

2.1 General

This chapter deals with the most relevant literature available on the study of Moment-Curvature relationship. It includes the study of different existing confined and unconfined stress-strain models for concrete. It also deals with the literature available for evaluation of moment and curvature for reinforced beam and column elements.

2.2 Unconfined Concrete

IS Code Model[7], IS456 suggests a standard constitutive model for concrete. The constitutive behavior of concrete in compression is assumed to be parabolic up to strain 0.002 and then it follows a straight horizontal line up to failure. It ignores the degradation of concrete at strains beyond 0.002. Thus the constitutive relation is expressed as follow:

For $0 < \varepsilon_c < 0.002$

$$\sigma_c = f_{ck}[(\frac{2\varepsilon_c}{\varepsilon_{co}}) - (\frac{\varepsilon_c}{\varepsilon_{co}})^2] \tag{2.1}$$

For $0.002 < \varepsilon_c < 0.004$

$$\sigma_c = f_{ck} \tag{2.2}$$

Where,σ_c = stress in concrete at any point of strain; ε_c= strain at any point; ε_{co}=strain at which parabolic part ends=0.002; f_{ck}=characteristic compressive strength of concrete.

Hognestad Model[6] The Hognestad model includes the damage parameter of concrete. The stressstrain curve before maximum stress reached is a parabola and then the falling branch behavior is adopted depending on the limit of useful strain the relations is expressed as under;

For $0 < \varepsilon_c < 0.002$

$$\sigma_c = f_{ck}[(\frac{2\varepsilon_c}{\varepsilon_{co}}) - (\frac{\varepsilon_c}{\varepsilon_{co}})^2] \tag{2.3}$$

For $0.002 < \varepsilon_c < 0.004$

$$\sigma_c = f_{ck}[1 - 100(\varepsilon_c - \varepsilon_{co}) \tag{2.4}$$

Kent and **Park** (1971) [4] proposed a stress-strain equation for both unconfined and confined concrete. In their model they used Hognestad's (1951) equation to more completely describe the post-peak stress-strain behavior. In this model the ascending branch is represented by modifying the Hognestad second degree parabola by replacing 0.85fc' by fc' and ε_{co} by 0.002.

$$\sigma_c = f_c[(\frac{2\varepsilon_c}{\varepsilon_{co}}) - (\frac{\varepsilon_c}{\varepsilon_{co}})^2] \tag{2.5}$$

The post peak branch was assumed to be straight line whose slope was defined primarily as a function of concrete strength.

$$\sigma_c = f_c[1 - Z(\varepsilon_c - \varepsilon_{co})] \tag{2.6}$$

Where,

$$Z = \frac{0.5}{\varepsilon_{50u} - \varepsilon_{co}} \tag{2.7}$$

$$\varepsilon_{50u} = \frac{3 + 0.29fc}{145fc - 1000} \tag{2.8}$$

ε_{50u}=the strains corresponding to the stress equal to 50% of the maximum concrete strength for unconfined concrete.The Kent and park model is represented in Fig. 2.1

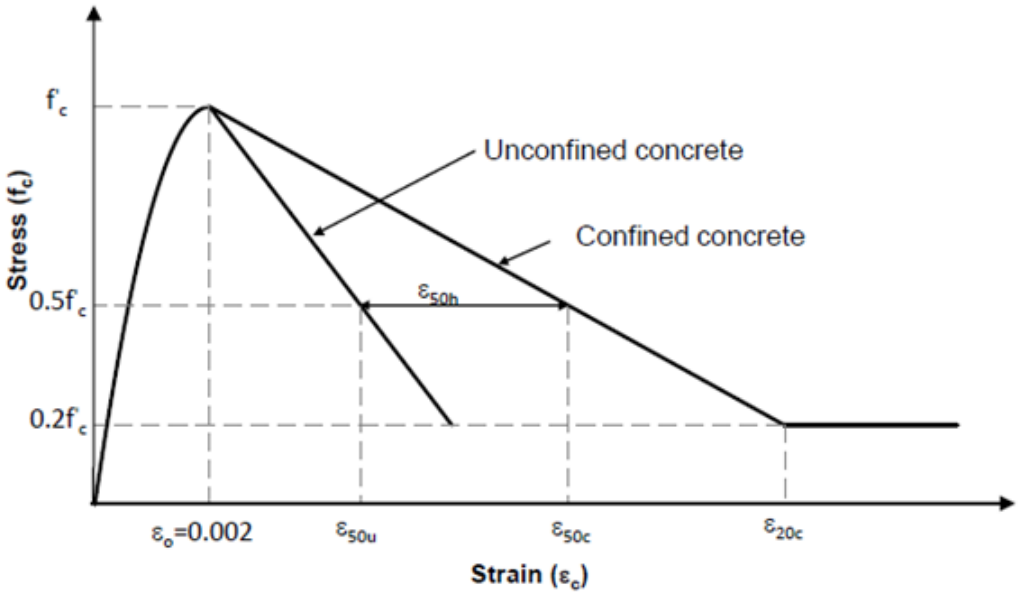

Figure 2.1: Proposed Stress-Strain Model for Confined and Unconfined Concrete by Kent and Park [4].

IRC 112 Model[8] The parabolic rectangular stress-strain block described by the IRC 112 model is valid for all situations and expression for ascending portion are as under;

$$\sigma_c = f_{cd}[1 - \left(1 - (\frac{\varepsilon_c}{\varepsilon_{c2}})^2\right)] \tag{2.9}$$

Post peak behavior of the curve describe by follow;

$$\sigma_c = f_{cd} \tag{2.10}$$

$$f_{cd} = \frac{0.67 f_{ck}}{1.5} \tag{2.11}$$

Where,f_{cd} = design compressive strength of concrete; σ_c = stress in concrete at any point of strain; ε_c = strain at any point; ε_{c2} = strain at reaching characteristic strength=0.002; ε_{cu2} = Ultimate strain=0.0035; f_{ck} = characteristic compressive strength of concrete.

Popovics (1973) [10] proposed a single equation to describe unconfined concrete stress-strain behavior. A major appeal of this model is that it only requires three parameters to control the entire pre and post peak behavior, specifically f_{ck}, ε_{co} and Ec.

$$\frac{\sigma_c}{f_{ck}} = \frac{n(\frac{\varepsilon_c}{\varepsilon_{co}})}{(n-1) + (\frac{\varepsilon_c}{\varepsilon_{co}})^n} \tag{2.12}$$

In which the power 'n' can be expressed as an approximate function of the compressive strength of normal-weight concrete as;

$$n = 0.4 \times 10^{-3} f_{ck} + 1.0 \tag{2.13}$$

Popovics equation works well for most normal strength concrete ($f_{ck} \leq 55$ MPa), but it lacks the necessary control over the slope of the post-peak branch for high strength concrete.

Thorenfeldt et al. (1987) [12] made modifications to the Popovics (1973)[10] relation to adjust the descending branch of the concrete stress-strain relation. The authors proposed the following equation for the unconfined concrete stress-strain relation.

$$\frac{\sigma_c}{f_{ck}} = \frac{n(\frac{\varepsilon_c}{\varepsilon_{co}})}{(n-1) + (\frac{\varepsilon_c}{\varepsilon_{co}})^{nk}} \tag{2.14}$$

In above eq. for $(\varepsilon_c/\varepsilon_{co}) < 1$ value of k is taken as 1 and for $(\varepsilon_c/\varepsilon_{co}) > 1$ value of k is taken as greater than 1. Thus by adjusting the value of 'k' the post-peak branch

of the stress-strain relation can be made steeper. This approach can be used for high strength concrete.

One shortcoming of those models are that they ignore the level of confinement provided by the lateral reinforcement. The useful strain in concrete depends on the confinement of concrete. So in present study confined model were also included to study the effect of the confinement.

2.3 Confined Concrete

Kent and **Park** (1971) [4] **modified** their stress-strain model for including effect of the confinement. Based on the results from tests on square columns by **Roy** and **Sozen**, it was shown that confining the concrete with rectangular or square hoops was not very effective and there was no substantial increase in the concrete compressive strength due to confinement because of this the maximum stress in the confined concrete is assumed same as that unconfined concrete so thus the ascending branch of the model is represented by the same second degree of parabola.

The slope of the post-peak branch is affected by the confinement. The empirical equation for descending branch of the stress-stain relation is given by,

$$\sigma_c = f_c[1 - Z(\varepsilon_c - \varepsilon_o)] \tag{2.15}$$

where,

$$Z = \frac{0.5}{\varepsilon_{50h} + \varepsilon_{50u} - \varepsilon_o} \tag{2.16}$$

$$\varepsilon_{50u} = \frac{3 + 0.29fc}{145fc - 1000} \tag{2.17}$$

$$\varepsilon_{50h} = \varepsilon_{50c} - \varepsilon_{50u} \tag{2.18}$$

$$\varepsilon_{50h} = \frac{3}{4} p" \sqrt{\frac{b"}{s}} \tag{2.19}$$

where ε_{50c} and ε_{50u} are the strains corresponding to the stress equal to 50% of the maximum concrete strength for confined and unconfined concrete respectively.

b" is the width of the confined core. S is the spacing of the confine reinforcement p" is the volumetric ratio of the confine hoops to the volume of the concrete core measured to the outside of the perimeter hoops and is given by

$$p" = \frac{2(b" + d")As"}{b"d"s} \tag{2.20}$$

d" is the depth of the confine core and As" is the cross sectional area of the hoop bars. It is assumed that concrete can sustain some stress at indefinitely large strains. However, the failure of the member would occur before the strains in concrete become impractically high. Hence, for this model it was assumed that the concrete can sustain a stress of 0.2fc' from a strain of ε_{20c} to infinite strain. Fig. 2.1 shows the proposed stress-strain model by Kent and Park.

Mander et al. (1988) [8] tested a full scale rectangular, square and circular column to study the effect of the transverse reinforcement on the confinement effectiveness and overall performance. From the test he observed that if the peak strain and stress co-ordinates could be found then the performance over the entire range of stress-stain curve was similar. And it was not depends on the arrangement of confinement reinforcement. Author decide the failure criteria base on **William** and **Warnker** model and **Schicert** and **Winkle** model to generate a confine model. He used **Popvics** model to describe entire stress-strain model.

$$\frac{\sigma_c}{f_{ck}} = \frac{n(\frac{\varepsilon_c}{\varepsilon_{co}})}{(n-1) + (\frac{\varepsilon_c}{\varepsilon_{co}})^n} \tag{2.21}$$

$$n = \frac{Ec}{Ec - Esec} \tag{2.22}$$

$$Ec = 5000\sqrt{f_{ck}} \tag{2.23}$$

$$Esec = \frac{fcc'}{\varepsilon_{cc}} \tag{2.24}$$

ε_{cc} is the stain at the maximum compressive strength of confined concrete fcc'.

$$\varepsilon_{cc} = \varepsilon_{co}[1 + 5(\frac{fcc'}{fc'} - 1)] \tag{2.25}$$

fcc' is the compressive strength of confine concrete. For circular section:

$$fcc' = f_{ck}(-1.254 + 2.254\sqrt{1 + \frac{7.94fl'}{f_{ck}} - \frac{2fl'}{f_{ck}}}) \tag{2.26}$$

in which fl' is the confinement pressure and is given by

$$fl' = \frac{1}{2}K_e\rho_s f_{yh} \tag{2.27}$$

ρ_s= ratio of volume of transvers confining steel to volume of confine concrete core. f_{yh}= yield strength of the transverse reinforcement. K_e= Confinement coefficient.

For rectangular section:

$$f_{cc} = f_{co} + 4.1 \times fle \tag{2.28}$$

$$fle = \sqrt{fle_x^2 + fle_y^2} \tag{2.29}$$

$$fle_x = \frac{k_e \times A_{sx} \times F_y}{s \times d_c} \tag{2.30}$$

$$fle_y = \frac{k_e \times A_{sy} \times F_y}{s \times b_c} \tag{2.31}$$

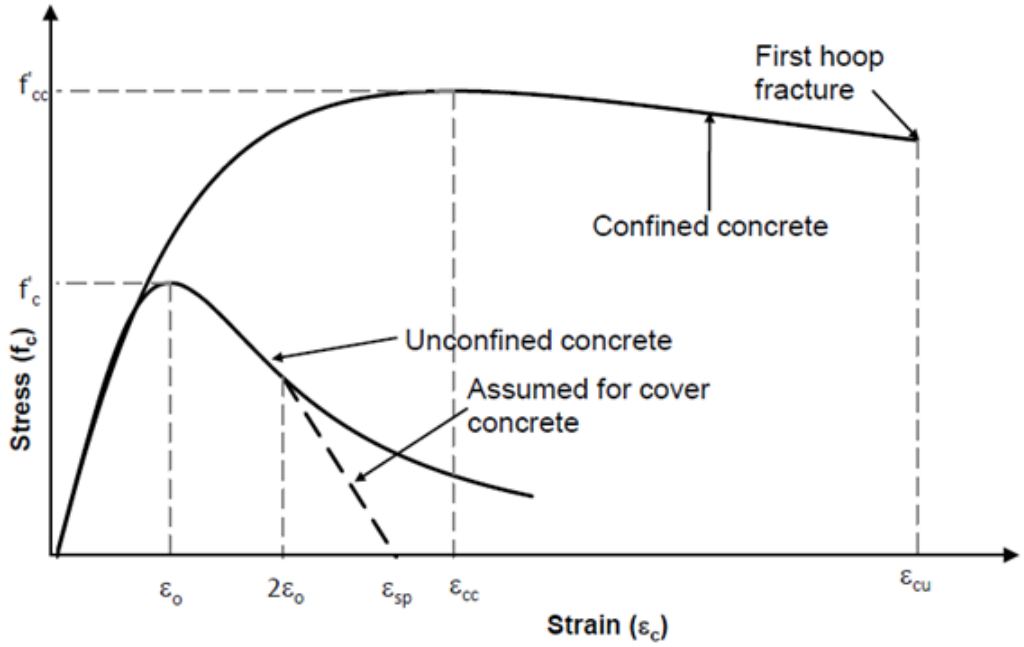

Figure 2.2: Stress-Strain Relation for Monotonic Loading of Confined and Unconfined Concrete by Mander [8]

$$k_e = \frac{(1 - \sum_{i=1}^{n} \frac{(w_i)^2}{6b_c d_c})(1 - \frac{s'}{2b_c})(1 - \frac{s'}{2d_c}))}{1 - \rho_{cc}} \tag{2.32}$$

Where, b_c and d_c are the core dimensions. ρ_{cc}= ratio of area of longitudinal reinforcement to area of core of section. S'= Spacing between hoops. A_{sx} and A_{sy} are total area of transverse steel running in the x and y directions, respectively The Mander et al. (1988) model does not handle the post-peak branch of high strength concrete particularly well and requires some modification. Fig. 2.2 shows stress-strain curve proposed by the Mander.

Cusson et al. (1995) [14] conducted an experiment on 50 large scale high strength concrete column to developed stress-strain model which includes the effect of concrete compressive strength, tie yield strength, tie configuration, transverse reinforcement ratio, tie spacing and longitudinal reinforcement ratio. Base on the experimental results he proposed following sets of equations. The ascending part(OA) Fig.

2.3 is a relationship originally proposed by **Popovics** [8] for concrete and written as:

$$\sigma_c = f_{cc}\left(\frac{K(\frac{\varepsilon_c}{\varepsilon_{cc}})}{K-1+(\frac{\varepsilon_c}{\varepsilon_{cc}})^K}\right) \tag{2.33}$$

where,

$$K = \frac{E_c}{E_c - \frac{f_{cc}}{\varepsilon_{cc}}} \tag{2.34}$$

Here, K controls the initial slope and the curvature of the ascending branch. E_c=tangent modulus of concrete. The descending part (ABC) Fig. 2.3 of the stress-strain curve is a modification of the relationship proposed by the **Fafitis and shah** [23] for confined HSC and is described as under:

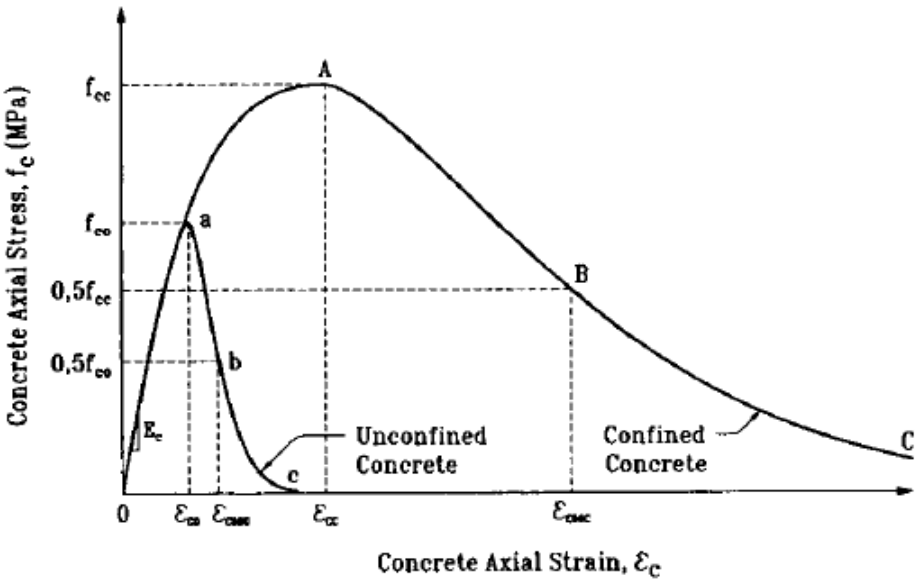

Figure 2.3: Stress-Strain Relation for Monotonic Loading of Confined and Unconfined Concrete by cusson [14].

$$\sigma_c = f_{cc}e^{k_1(\varepsilon_c-\varepsilon_{cc})^{1.5}} \tag{2.35}$$

$$k_1 = \frac{ln0.5}{(\varepsilon_{c50c}-\varepsilon_{cc})^{1.5}} \tag{2.36}$$

k_1 control the general slope of the descending branch of the curve.

Li et al. (2000) [5] conducted an experimental investigation on circular and square reinforced concrete columns to study the behavior of high-strength concrete columns confined by normal and high-yield strength transverse reinforcement and with different confinement ratio and configurations. From the tests they concluded that volumetric ratio and the yield strength of confining reinforcement significantly affect the shape of the stress-strain curve. Based on their experimental study, Li et al. (2001) proposed a three branch stress-strain model for high strength concrete confined by either normal or high-yield strength transverse reinforcement. The equations are:

$$\sigma_c = \begin{cases} E_c\varepsilon_c + (\frac{fck - E_c\varepsilon_{co}}{\varepsilon_{co}^2})\varepsilon_c^2 & 0<\varepsilon_c \leq \varepsilon_{co} \\ fcc' - \frac{(fcc' - fck)}{(\varepsilon_{cc} - \varepsilon_{co})^2}(\varepsilon_c - \varepsilon_{cc})^2 & \varepsilon_{co} < \varepsilon_c \leq \varepsilon_{cc} \\ fcc' - \frac{\beta fcc'}{\varepsilon_{cc}}(\varepsilon_c - \varepsilon_{cc})^2 \geq 0.4fcc' & \varepsilon_c > \varepsilon_{cc} \end{cases}$$

The term β controls the slope of the post-peak branch of the stress-strain model. The maximum confined concrete compressive strength is given by

$$fcc' = f_{ck}(-1.254 + 2.254\sqrt{1 + \frac{7.94fl'}{fc'}} - \frac{2\alpha_s fl'}{fc'}) \tag{2.37}$$

Where

$$\alpha_s = \begin{cases} (21.2 - 0.35fck)\frac{fl'}{fck} & \text{fck} \leq 52\text{Mpa} \\ 3.1\frac{fl'}{fck} & \text{fck} > 52\text{Mpa} \end{cases}$$

Where fl' is the effective lateral confining pressure, calculated using the equations proposed by Mander et al. (1988).

2.4 Inelastic Analysis of RC Elements

Shamim A. Sheikh [22] developed the computer program by using four stress-strain model (i.e Mander model, Fafitis model, Sheikh and Uzumeri model, and modified Kent and park model.) to predict the moment curvature relationship for the column

section under axial load. By comparing the experimental results with the analytical results he found that the model by Mander at el. overestimate the moment curvature for RC element and analytical results from the modified Kent and park model underestimate the sectional capacity.

Srikanth at el. (2007) [17] found out the analytical Moment-Curvature behavior for statically determinate RC beam elements by considering effect of the confinement reinforcement in the compression zone. Author developed Moment-Curvature relationship by using six different stress-strain model as a stress block. Analytical Moment-Curvature obtain from different model was compared with the experimental results. Author has found that the results obtain from the Cusson and Mendis model are very closer from the experimental results. Srikanth also carried out parametric study to find out the effect of various parameters on the Moment-Curvature relationship.

Noor [25] developed experimental program to find ductility for simply supported beam subjected to the two point loading. Noor considered moment-curvature relation, depth of natural axis and strain in the material as a performance criteria. Here author found out the effect of the high strength steel and high strength concrete on the ductility and moment curvature. Noor concluded that by using high strength steel ductility and stiffness were increased and in the case of high strength concrete it was decreased.

Kwak at el. (2000) [16] carried out the material nonlinear analysis of RC beam using Moment-Curvature instead of using sophisticated layered approach because it has some limitation in application to large structure with many degree of freedom. Kent and park and later extended by Scott [11] model was used as a stress block for the concrete in the computer program of Moment Curvature. Kwak found the moment- Curvature at three different point such as cracking, yielding and ultimate.

Suarez at el. [20] Developed tool known as RC-analysis to find out the Moment-Curvature for RC square, rectangular and circular column and rectangular RC beam element. RC-analysis models nonlinear behavior of confine concrete, unconfined con-

crete and reinforcing bars to obtain curvature response. The shear strength plot is obtain by modified Kowalsky and priestly model. From the moment curvature plot different response limit states can be identified. The stress-strain relation for reinforcing steel is same used by the king program.

2.5 Summary

Present study include different stress-strain blocks for confine and unconfined concrete member. Literature review shows that Mander[9] and Cusson [12] gives close solution to the experimental solution.

In literatures L/d ratio for simply supported beams were taken around 10. And b/d ratio was taken around 0.5 to 0.9.

Literature also shows that as the concrete grade is increased probability of brittle failure of RC section is also increased.

Chapter 3

Analytical Evaluation of Moment Curvature

3.1 General

This chapter deals with the analytical evaluation of Moment Curvature of RC rectangular beam element. A matlab program is developed to evaluate the Moment-Curvature relationship for rectangular RC beam element by transformed area method as well as by strain compatibility method. Here the stress-strain curves which are used in this study are also discuss. This chapter is include various computational tools available for Moment-Curvature relationship for confine as well as unconfined RC elements.

3.2 Evaluation of Moment-Curvature

Moment-Curvature can be evaluate by using Theoretical Method or Strain Compatibility method anad Transform area method. The transform area method is simple but approximate method to evaluate moment curvature while strain compatibility method is accurate but iterative method. In strain compatibility method material models used as a stress block. Following section explain in detail about the evalua-

tion of Moment-Curvature by these two methods.

3.3 Moment-Curvature by Transform Area Method

In transform area method, Moment-Curvature relation is found out for three different points say cracking, yielding and ultimate to plot whole curve. Each state is described in details in following sections.

3.3.1 Moment-Curvature at Cracking

When the stress in the concrete reached equal to the modulus of the rupture of section, the first oneset crack is occur and critical moment is calculated by using following equation;

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{3.1}$$

Where, f_r is the modulus of rupture of concrete, I_g is the moment of inertia of gross section considering the contribution from reinforcements, and y_t is the distance of extreme tension fiber from neutral axis of the section. Considering the contribution of reinforcements to moment of inertia. Corresponding curvature can be find out by using following equation;

$$\phi = \frac{f_r}{E_c y} \tag{3.2}$$

This gives the cracking point on the curve (M_{cr} , ϕ_{cr}). Where, E_c is modulus of elasticity of concrete.

3.3.2 Moment-Curvature at Yielding

As the moment is increased the cracks in the tension zone propagates through the cross-section. The moment-curvature relation is linear till the tension reinforcement yields. At yielding, strain in the tension reinforcement is ε_y and neutral axis shifts towards the compression area. The neutral axis at yielding is given as distance kd from

extreme compression fiber, where the depth factor **k** is calculated using expression in case of doubly reinforced section;

$$k = \sqrt{(\rho + \rho')^2 m^2 + 2\left(\rho + \frac{\rho' d'}{d}\right) m} - (\rho + \rho') \tag{3.3}$$

In case of singly reinforced section **k** is calculated by calculating roots of following equation;

$$k^2 + 2m\rho k - 2m\rho = 0 \tag{3.4}$$

Where ρ and ρ' are the tension and compression steel ratios, m is the modular ratio, and d and d' are the distance of compression and tension steel from extreme compression fiber. Moment is given by following equation;

$$M_y = jdf_y A_s \tag{3.5}$$

Where, f_y is the yielding of the reinforcement. A_s is the tension reinforcement and jd is the distance from centroid of compressive forces in the steel and concrete to the centroid of tension. Corresponding curvature is given by following equation;

$$\phi_y = \frac{\varepsilon_s}{(d - kd)} \tag{3.6}$$

This gives the cracking point on the curve (M_y , ϕ_y).

3.3.3 Moment-Curvature at Ultimate

After yielding of tension steel, stress remains constant but strain keeps increasing until compressive strain in extreme fiber of concrete reaches its maximum value. In order to address the nonlinearity in concrete at high strains, whitney-block is used to convert the parabolic stress distribution in concrete to an equivalent rectangular stress-block representation. The ultimate Curvature and moment of doubly reinforced section for the case where the tension steel is yielding may be found using following

equations;

$$a = \frac{A_s f_y - A'_s f_y}{0.85 f'_c b} \tag{3.7}$$

$$M = 0.85 f'_c ab \left(d - \frac{a}{2}\right) + A'_s f_y (d - d') \tag{3.8}$$

The ultimate Curvature and moment of singly reinforced section for the case where the compression steel is yielding may be found using following set of equations. Equivalent depth of the rectangular stress block can be calculated by solving following equation;

$$\left(\frac{0.85 f'_c}{0.003 E_c \rho}\right) a^2 + ad - 0.85 d^2 = 0 \tag{3.9}$$

Ultimate moment can be calculated by;

$$M_u = 0.85 f'_c ab(d - 0.5a) \tag{3.10}$$

where, d is the effective depth of the section. f'_c is the concrete compressive strength. E_c is the modulus of the elasticity of concrete. b is the width of the section. Corresponding Curvature is given by

$$\phi_u = \frac{0.85 \varepsilon_c}{a} \tag{3.11}$$

Where, ε_c is ultimate strain in concrete at maximum stress. Assumption of yielding in tension reinforcement is now checked by ensuring:

$$\varepsilon'_s = \varepsilon_c \left(\frac{c - d'}{c}\right) \leq \varepsilon_y \tag{3.12}$$

If the above condition is satisfied then assumption made is true and obtained value of defines the ultimate state on the moment-curvature curve (M_u , ϕ_u). A Matlab program was written to calculate the moment-curvature values for three states as per principles explained in above sections. Code is attached in the appendix.

3.4 Strain Compatibility Method

By strain compatibility method, Moment Curvature for the RC section with flexural and axial load can be derived on bases of the assumptions similar to those used for the determination of the flexural strength. Assumptions are as under :

- Plane section before bending remain plane after bending.
- The tensile strength of concrete is neglected.
- Stress- strain curves for steel and concrete are known.
- The steel is perfectly bonded.
- The variation of the strain across the section is taken as a linear up to failure.
- The stress-stain relation proposed in a model is taken as a stress block.

The curvature associated with the large range of bending moment can be find out by using above assumptions, from the equation of equilibrium, strain compatibility and stress-strain models for concrete and steel. fy= yielding strength of steel fc"=strength of concrete in a member . For a given concrete strain in the extreme compression fiber ε_{cm} and neutral axis depth kd, the steel strain (i.e ε_{s1} and ε_{s2}) can be determine by using similar triangular strain diagram.

$$\varepsilon_{si} = \varepsilon_{cm}\frac{kd - d_i}{kd} \tag{3.13}$$

The stress corresponding strain can be determine by using stress-strain curve for steel. Fig. 3.1 show typical stress-stain curves for steel. Then steel force can be found out by multiplying stress in to steel and area of the steel. Fig. 3.2 shows distribution of the forces. For given concrete strain ε_{cm} in the compressive fiber, the concrete compressive force is given by

$$C_c = \alpha f c" bkd \tag{3.14}$$

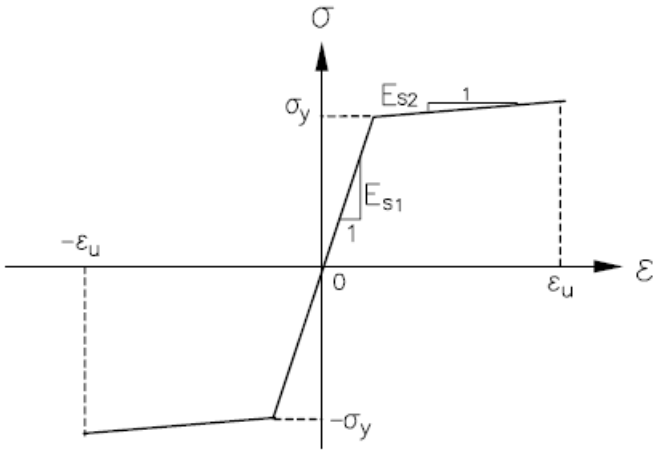

Figure 3.1: Stress-Strain Curve for Steel [3]

It is acting at γkd from the extreme compression fiber. α is the mean stress factor. γ is the centroid factor. α and γ is given by following eq.

$$\alpha = \frac{\int_0^{\varepsilon_{cm}} f_c \mathrm{d}\varepsilon_c}{f_c" \varepsilon_{cm}} \tag{3.15}$$

$$\gamma = 1 - \frac{\int_0^{\varepsilon_{cm}} \varepsilon_c f_c \mathrm{d}\varepsilon_c}{\varepsilon_{cm} \int_0^{\varepsilon_{cm}} f_c \mathrm{d}\varepsilon_c} \tag{3.16}$$

If the stress in the concrete fc can be written in terms of the strain ε_c , the concrete force and its line of action may be determine from above equation.The force equilibrium equations can be written as follow;

$$P = \alpha f_c" bkd + \sum_{i=1}^{n} f_{si} A_{st} \tag{3.17}$$

$$M = \alpha f_c" bkd \left(\frac{h}{2} - \gamma kd\right) + \sum_{i=1}^{n} f_{si} A_{st} \left(\frac{h}{2} - d_i\right) \tag{3.18}$$

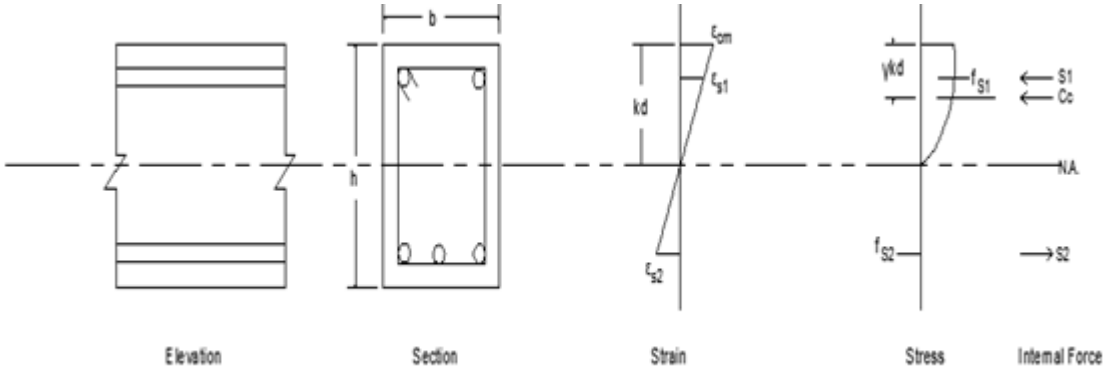

Figure 3.2: Section With Strain, Stress and Force Distribution [3]

Where, M is the Moment and P is the axial load. Here in this case it was taken as zero. Corresponding curvature is given by following equation;

$$\phi = \frac{\varepsilon_{cm}}{kd} \tag{3.19}$$

3.4.1 Procedure for Strain Compatibility Method

For deriving the complete moment-curvature relationship for RC rectangular beam element, discrete values of concrete strains (ε_c) were selected such that even distributions of points on the plot, both before and after the maximum were obtained. The procedure used in computation is given below:

- The extreme fiber concrete compressive strain was assumed (i.e. 1/10 of ultimate strain).
- For the value of assume strain N.A. depth, kd was assumed. Say 0.5d.
- For this value of N.A.,compressive forces C_c in concrete was calculated by using stress-strain models for concrete.
- The strain in tension and compression steel was calculated base on the strain compatibility.

- Based on the strain in the tension steel, corresponding stresses were calculated from the stress-strain curve of steel.
- The total tensile force (T) in tensile steel was calculated.
- Same procedure was repeated for compressive steel to calculated compressive force in compression steel.
- The total compression force C (C=C_c+C_t) acting in the compression zone was calculated.
- If C=T then assumed value of N.A. depth was corrected and M and ϕ ia calculated otherwise the N.A. depth was modified until the C=T was achieved.
- Same procedure was repeated till the assume strain is equal to the ultimate strain.

Now, the total moment about N.A. was given by:
M=M_c + M_{cs} + M_t
where,
M_c = Moment of compressive force in concrete about the N.A.
M_{cs} = Moment of force in compressive steel about the N.A.
M_t = Moment of force in tensile steel about the N.A.
Corresponding curvature is given by using Eqn. 3.19.
A Matlab code is developed for evaluation of Moment-Curvature relationship by using various stress-strain model for concrete as well as steel. Code is attached in appendix.Flow chart of the matlab program is shown in Fig 3.3.

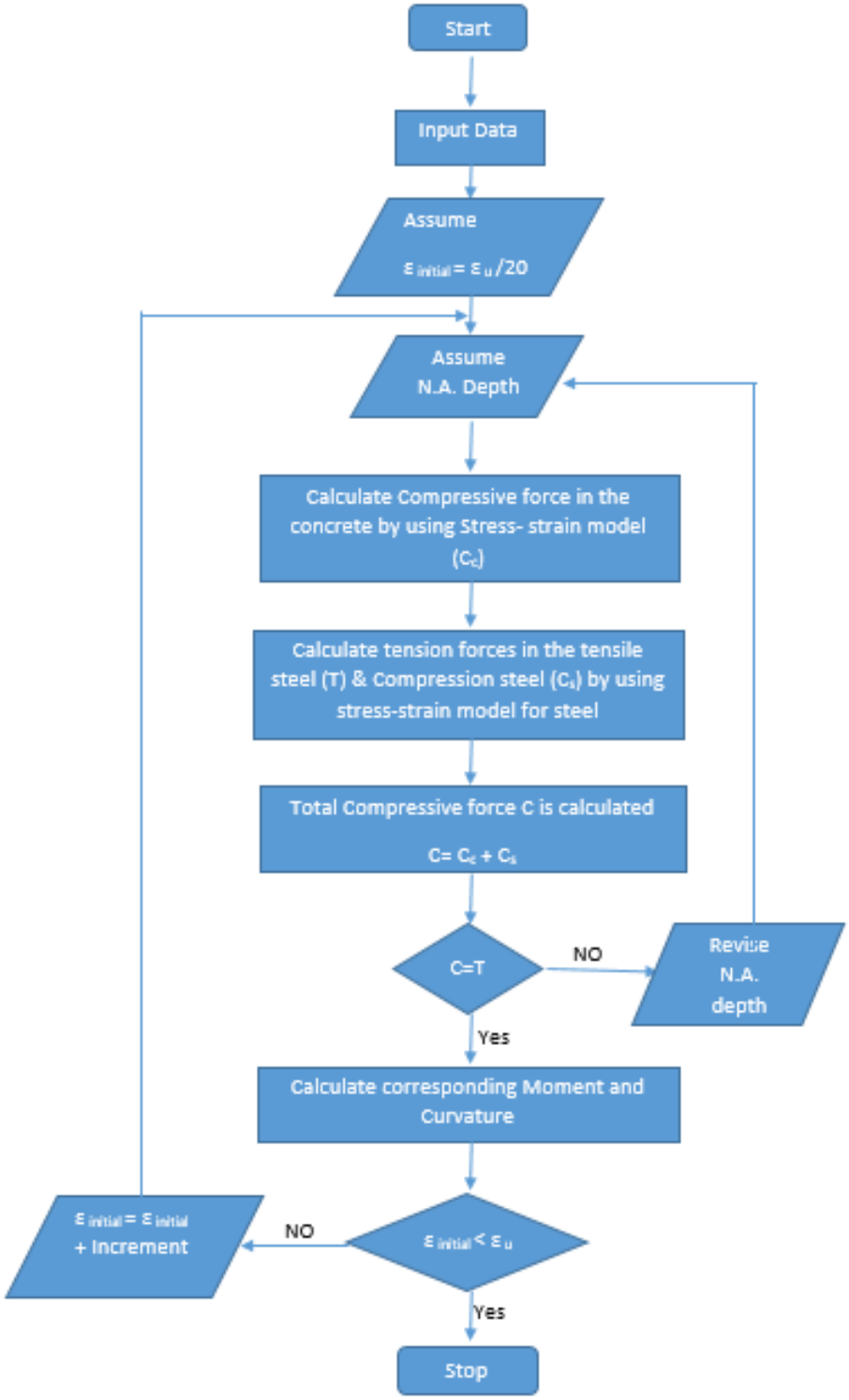

Figure 3.3: Flow Chart of Computer Program

3.5 Computational Tools for Moment Curvature

- **CUMBIA** is a software written in Matlab language by **Luis A. Montejo**, North Carolina State University. The software is able to perform monotonic moment-curvature analysis and force-displacement response of reinforced concrete column.

- **Response 2000** is a software developed by **Evan Bentz and Michael P. Collins**, University of Toronto. It is used for sectional analysis program that will calculate the strength and ductility of a reinforced concrete cross-section subjected to shear, moment, and axial load. All three loads are considered simultaneously to find the full load-deformation response using the modified compression field theory. He used popovics and thorenfeldt equation for basic stress-strain block.

- **RC-Analysis** is a software developed by **Suarez V. A. and Hurtado J. C.**. It is used for moment-curvature and shear strength analysis of reinforced concrete sections.

- **XSECTION, CONSEC, UCFyber** are also used to predict moment curvature relationship.

In the present study, RC-Analysis and Response 2000 are used in order to determine moment-curvature relationship for a beam section. Yield curvature and ultimate curvature are obtained using these tools.

Chapter 4

Analysis of Moment-Curvature for RC Beam Elements

4.1 General

Development of the software programs requires clear understanding of programming language and the domain knowledge. Matlab is used for the coding of the program. The entire program is based on the formal mathematical models which have been coded for stress-strain block. Results obtain by analytical method are compared to the experimental results. This chapter includes the problem statement. It also deals with the analytical evaluation of Moment-Curvature of designed RC rectangular beam element by using transform area method and strain compatibility method.

4.2 Problem Statement

In present study four different type of rectangular specimens (Singly Under Reinforced beam, Singly Over Reinforced, Doubly Under Reinforced and Doubly Over Reinforced) were designed to calculate Moment-Curvature. From the literature it is found that b/d and L/d ratio for the simply supported beam was taken between 0.5 to 0.9 and 10 respectively. Where, L is the span of the beam, d is the depth of the

Table 4.1: Geometric Specification for Beam

Description	**SUR**	**SOR**	**DUR**	**DOR**
b (mm)	120	120	120	120
D (mm)	150	150	150	150
d (mm)	135	135	135	135
f_{ck} **(MPa)**	20	20	20	20
f_y **(MPa)**	415	415	415	415
A_{sc} (mm^2)	2-6ϕ	2-6ϕ	2-8ϕ	2-8ϕ
A_{st} (mm^2)	3-8ϕ	3-10ϕ	2-10ϕ +1-8ϕ	3-10ϕ
Xu (mm)	63.01	112.9	45.21	71.59
M (KNm)	5.9	10.51	6.64	10.9
P (KN)	23.6	42.04	35.32	43.6

beam and b is the width of the beam. So in present study, b/d ratio of the beam was taken as 0.8 and L/d ratio was taken as 10. Dimensions of the RC specimens were 120 mm $\times$ 150 mm $\times$ 1700 mm, where span of the beam was taken as 1500mm. General specifications of beams are shown in Table 4.1. Fig. 4.1 shows the cross section and reinforcement details for singly under as well as over reinforced beam. and Fig. 4.2 shows the cross section and reinforcement detail for Doubly under reinforced as well as Doubly over reinforced RC beam element.

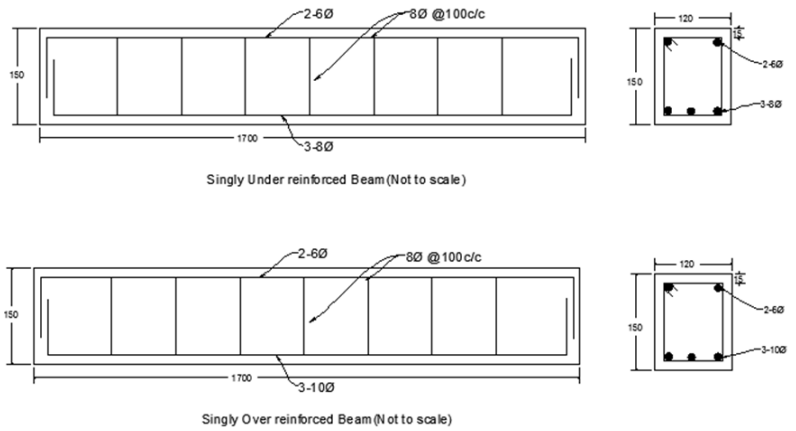

Figure 4.1: Cross Section of Beam (a) Singly Under Reinforced (b) Singly Over Reinforced

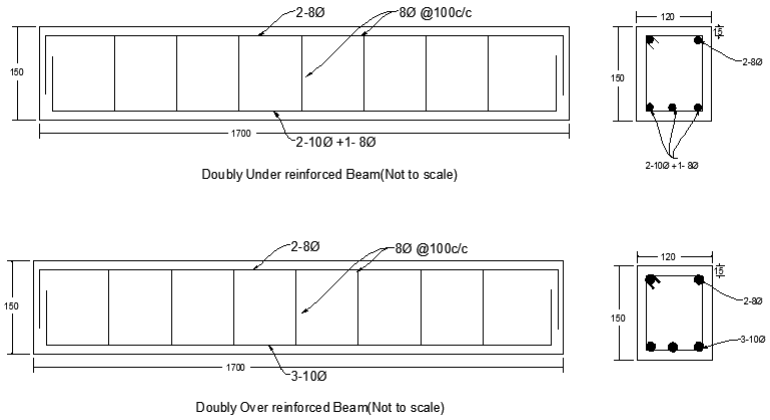

Figure 4.2: Cross Section of Beam (a) Doubly Under Reinforced (b) Doubly Over Reinforced

4.3 Analytical Moment-Curvature

In this section Moment-Curvature was evaluated for selected problem by using transform area method as well as strain compatibility method. Transform area method is the simple but approximate method where strain compatibility method is accurate method for beam under flexure and axial load. But this method is iterative method.

4.3.1 Transform Area Method

In present study, it is assumed that 100 mm spacing between ties is small for any confining effects so beam can be treated as an unconfined section for obtaining moment-curvature relationship[3]. Plots obtained from the matlab are as shown in Fig. 4.3 and Fig. 4.4.From Fig. 4.3, it is observed that in case of singly under reinforced RC beam there is a minor drop in moment carrying capacity after the yielding of the reinforcement. In this case ultimate moment carrying capacity is less compare to the singly over reinforced section but the ultimate curvature is more. In case of singly over reinforced section, degradation in strength after peak is comparatively more.

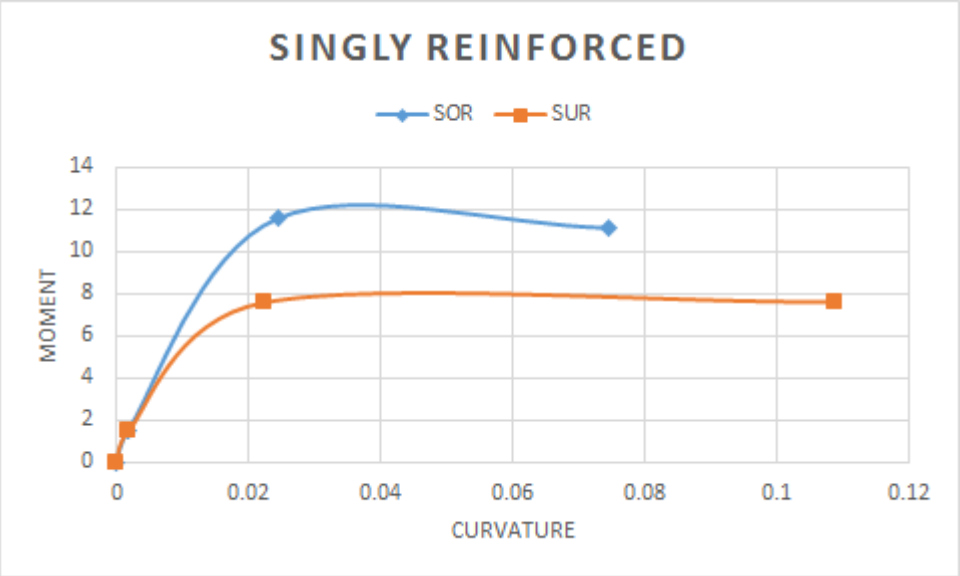

Figure 4.3: Moment-Curvature Relationship for Singly Reinforced Beam

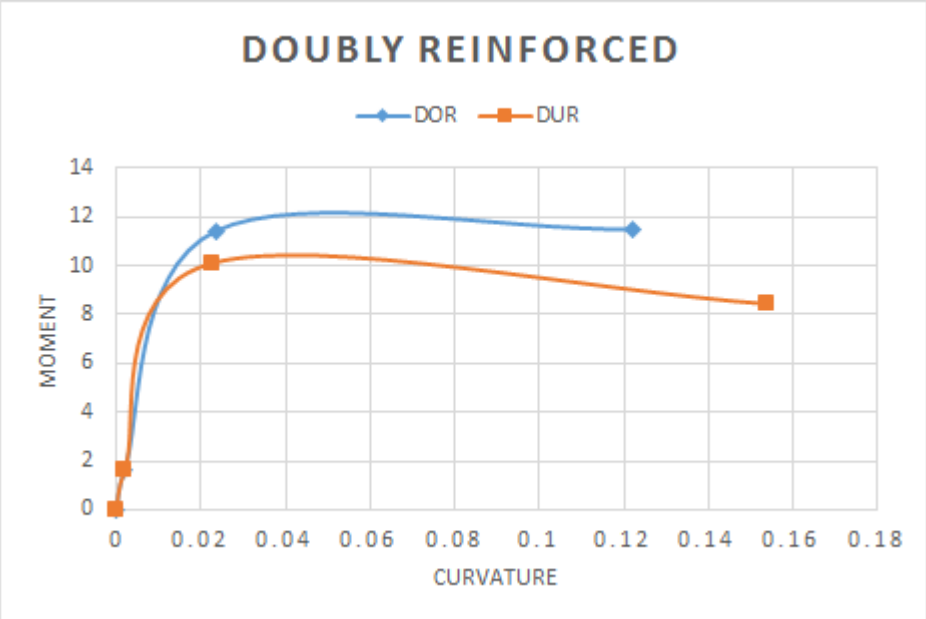

Figure 4.4: Moment-Curvature Relationship for Doubly Reinforced Beam

Curvature ductility, the ratio of ultimate curvature to the curvature at yielding of reinforcement, is 4.88 in case of singly under reinforced beam and 2.80 in case of singly over reinforced section. So even though singly over reinforced section has more moment carrying capacity ductility is less as compare to the Singly under reinforced section.

From Fig. 4.4,in both the case (Under reinforced and Over reinforced) of doubly reinforced beam, degradation in strength after peak is less compare to the singly over reinforced section. Curvature ductility for doubly under reinforced section is 6.72 and for doubly over reinforced section is 5.2.

4.3.2 Strain Compatibility Method

To evaluate Moment-Curvature relationship for RC rectangular beam elements by Strain Compatibility Method, six different existing confine as well as unconfine mathematical stress-strain models were used as stress block. Among these six models IS 456 Model, Hongnestad Model, Kent and Park Model and IRC 112 Models were unconfined concrete models. Cusson Model and Mander Model were confined concrete model. One shortcoming of unconfined models are that it ignores the level of confinement provided by the lateral reinforcement. The useful strain in concrete depends on the confinement of concrete so to study the effect of the confinement on the Moment-Curvature confine stress-strain models were also used. Fig. 4.5 shows various stress-strain models which were used in present study.

Table 4.2 shows Moment-Curvature relationship at different strain level by using all six stress-strain models as a stress block for singly under reinforced beam. Cusson Model and Mander Model are for confined concrete as the effective strain in concrete depends on the confinement of concrete so they were used to study the effect of the confinement on the Moment-Curvature relationship. Fig. 4.6 shows Moment-Curvature for singly under reinforced beam. Form Fig. 4.6, it is observed that Moment-Curvature plot for singly under reinforced (SUR) remain linear in elastic

Model	Ascending Branch	Decsending Branch
IS 456 Model	$f_c = f'_c\left(\left(\frac{2\varepsilon_c}{\varepsilon_{c0}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)^2\right)$	$f_c = f'_c$
Hongnestad Model	$f_c = f'_c\left(\left(\frac{2\varepsilon_c}{\varepsilon_{c0}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)^2\right)$	$f_c = f'_c[1 - 100(\varepsilon_c - \varepsilon_{co})]$
Kent and Park Model	$f_c = f'_c\left(\left(\frac{2\varepsilon_c}{\varepsilon_{c0}}\right) - \left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)^2\right)$	$f_c = f'_c[1 - Z(\varepsilon_c - \varepsilon_{co})]$ $\quad Z = \frac{0.5}{\varepsilon_{50u} - \varepsilon_{co}}$ $\quad \varepsilon_{50u} = \frac{3 + 0.29f_c'}{145f_c' - 1000}$
IRC 112 Model	$f_c = f_{cd}\left[1 - \left[1 - \frac{\varepsilon_c}{0.002}\right]^2\right]$ $\quad f_{cd} = \frac{0.67f_{ck}}{1.5}$	$f_c = f_{cd}$ $\quad f_{cd} = \frac{0.67f_{ck}}{1.5}$
Cusson Model	$f_c = f_{cc}\left[\frac{K\left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)}{K - 1 + \left(\frac{\varepsilon_c}{\varepsilon_{c0}}\right)^K}\right]$ $\quad K = \frac{E_c}{E_c - \left(f_{cc}/\varepsilon_{cc}\right)}$	$f_c = f'_c\, exp[k1(\varepsilon_c - \varepsilon_{co})^{1.5}]$ $\quad k_1 = \frac{ln0.5}{(\varepsilon_{c50c} - \varepsilon_{co})^{1.5}}$
Mander Model	$f_c = \frac{f'_{cc}Xr}{r - 1 + X^r}$ $\quad X = \left(\frac{\varepsilon_c}{\varepsilon_{cc}}\right)$ $\quad \varepsilon_{cc} = \varepsilon_{co}\left[1 + 5\left(\frac{f_{cc}'}{f_{co}} - 1\right)\right]$ $\quad r = \frac{E_c}{E_c - \left(f_{cc}'/\varepsilon_{cc}\right)}$	

Figure 4.5: Various Stress-Strain Models

zone and remain horizontal in inelastic zone. This is due to the increase in lever-arm in to the RC beam after the yield. Here, 0.00105 was consider as a yielding strain for all models excluding IRC 112 Models. In IRC model 0.0014 was consider as a yielding strain. Curvature ductility, ratio of ultimate curvature to curvature at yield for SUR beam was observed 4.3, 3.66, 2.54, 2.72, 4.71, and 5.22 form IS Code Model, Hongnestad Model, Kent and Park Model, IRC Model, Cusson Model and Mander Model respectively. Curvature ductility obtain for SUR from IS code Model and Mander Model are near to the Curvature ductility obtain from Transform area method.

Table 4.2: Moment-Curvature for Singly Under Reinforced RC Beam.

Strain	**IS CODE MODEL**		**Hongnestad Model**		**Kent and Park Model**		**IRC 112 Model**		**Cussion Model**		**Mander Model**	
	M (kNm)	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**
0	0	0	0	0	0	0	0	0	0	0	0	0
0.00035	4.2851	0.0112	4.2851	0.0112	4.2311	0.0112	2.7829	0.008	4.378	0.0114	4.2502	0.0112
0.0007	8.1257	0.0214	8.1257	0.0214	8.0242	0.0215	5.2642	0.0154	8.3835	0.0219	8.0437	0.0215
0.00105	8.5359	0.04	8.5359	0.04	8.4732	0.04	7.4191	0.0221	8.4854	0.0426	8.4702	0.04
0.0014	8.4112	0.0627	8.4112	0.0627	8.3485	0.0627	9.2155	0.0282	8.3664	0.067	8.3442	0.0632
0.00175	8.3576	0.0847	8.3576	0.0847	8.2912	0.0848	9.1619	0.0378	8.3066	0.0921	8.2731	0.088
0.0021	8.3485	0.1033	8.3567	0.1022	8.3037	0.1005	9.138	0.0461	8.2758	0.1163	8.2291	0.1133
0.00245	8.3485	0.1205	8.3792	0.1151	8.3705	0.1058	9.138	0.0538	8.2645	0.1391	8.2041	0.1384
0.0028	8.3485	0.1377	8.4046	0.1267	8.463	0.1077	9.138	0.0615	8.2582	0.1603	8.1866	0.1632
0.00315	8.3485	0.1549	8.4306	0.1371	8.5776	0.1063	9.138	0.0692	8.2576	0.1804	8.1768	0.1873
0.0035	8.3485	0.1722	8.4616	0.1463	8.7268	0.1017	9.138	0.0769	8.2569	0.2006	8.1698	0.2107

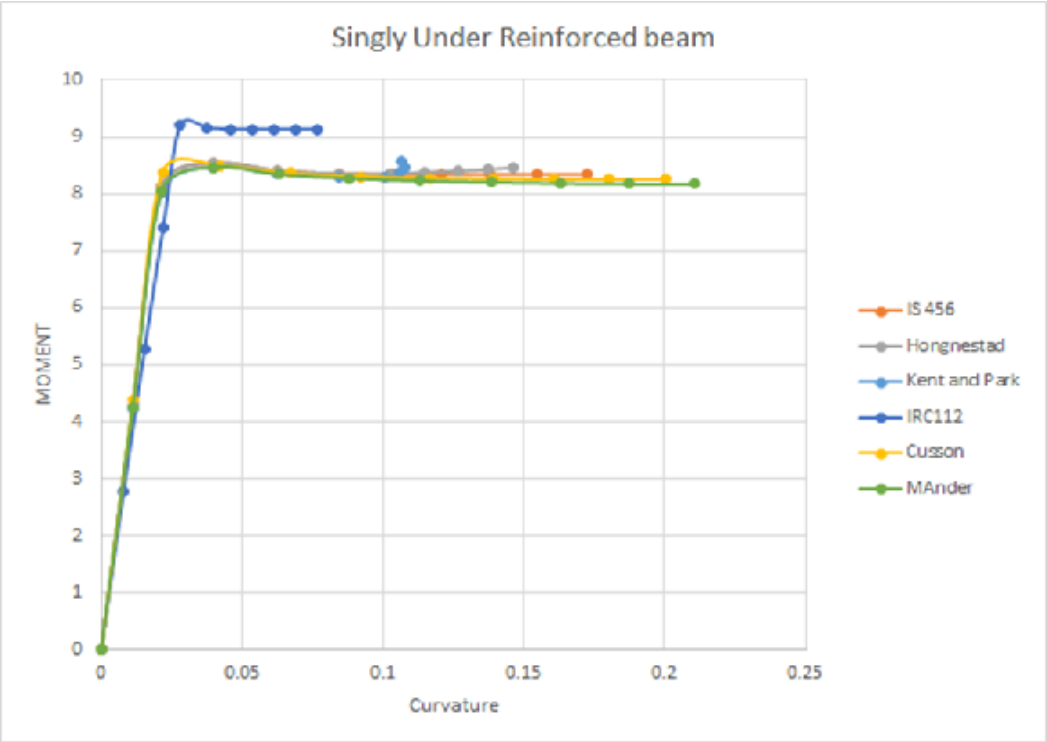

Figure 4.6: Moment-Curvature for Singly Under Reinforced Beam

From Fig. 4.6, it is observed IRC 112 model shows more ultimate moment but less curvature than other models. Neutral axis depth is inversely proposed to the curvature. Here neutral axis depth was observed more than other models because of this curvature is less compared to the other models. After the yielding of reinforcement slight increased in moment than drop in moment carrying capacity was observed.

Table 4.3shows the Moment and corresponding curvature at different strain level for singly over reinforced beam(SOR) for all six concrete models. Fig. 4.7 shows Moment-Curvature plot for SOR. Here Moment-curvature is liner in elastic zone but minor drop was observed in strength after the peak. Curvature ductility calculated for SOR was 4.3, 3.66, 2.63, 1.72, 4.7 and 5.25 form IS Code Model, Hongnestad Model, Kent and Park Model, IRC Model, Cusson Model and Mander Model respectively. Curvature ductility obtain for SUR from Hongnestad Model are near to the Curvature ductility obtain from Transform area method.

Table 4.3: Moment-Curvature for Singly Over Reinforced RC Beam.

Strain	**IS CODE MODEL**		**Hongnestad Model**		**Kent and Park Model**		**IRC 112 Model**		**Cussion Model**		**Mander Model**	
	M (kNm)	**ϕ (rad/m)**	**M (kNm)**	**ϕ (rad/m)**	**M (kNm)**	**ϕ (rad/m)**	**M (kNm)**	**ϕ (rad/m)**	**M (kNm)**	**ϕ (rad/m)**	**M (kNm)**	**ϕ (rad/m)**
0	0	0	0	0	0	0	0	0	0	0	0	0
0.00035	5.2846	0.0093	5.2846	0.00925	4.7809	0.0087	3.3886	0.0068	5.4021	0.0094	5.2375	0.0093
0.0007	10.0097	0.0178	10.0097	0.01777	9.0509	0.0167	6.3967	0.013	10.3337	0.0182	9.9009	0.0179
0.00105	14.0606	0.0256	14.0606	0.02559	12.7669	0.024	8.9926	0.0188	13.9359	0.0273	13.9354	0.0257
0.0014	13.7596	0.0401	13.7596	0.0401	13.9602	0.0341	11.1353	0.024	13.6481	0.0429	13.6204	0.0405
0.00175	13.6296	0.0542	13.6296	0.05423	13.809	0.0461	12.7728	0.0285	13.5059	0.0589	13.4515	0.0564
0.0021	13.6065	0.0661	13.6219	0.06544	13.8221	0.055	13.857	0.0324	13.4316	0.0744	13.3485	0.0726
0.00245	13.6065	0.0771	13.6809	0.07364	13.9819	0.0592	14.6977	0.036	13.3962	0.0891	13.2824	0.0887
0.0028	13.6065	0.0881	13.7409	0.08109	14.1675	0.0619	15.4428	0.0396	13.3864	0.1026	13.2414	0.1046
0.00315	13.6065	0.0992	13.8092	0.08774	14.3889	0.0633	15.5336	0.0443	13.3877	0.1154	13.2169	0.12
0.0035	13.6065	0.1102	13.8821	0.09363	14.6638	0.0632	15.5336	0.0492	13.3845	0.1283	13.1991	0.1351

Figure 4.7: Moment-Curvature for Singly Over Reinforced Beam

From Fig. 4.7, it is observed that the slope of the Moment-Curvature plot in elastic zone is more compare to the case of singly under reinforced beam. Here also IRC 112 shows more ultimate moment and less curvature. Mander and Cusson models shows more degradation in strength compare to other models.

Table 4.4shows the Moment-Curvature at different strain level for Doubly Under Reinforced beam(DUR) for all six concrete models. Fig. 4.8 shows Moment-Curvature plot for DUR. Here Moment-curvature is liner up to the yielding of reinforcement but after this sudden drop was observed in strength. In case of Moment-Curvature obtain by using Cusson Model and Mander Model drop in strength was not drastically as it was in other models. Curvature ductility calculated for DUR was 4.9, 4.4, 3.644, 3.72, 5.105 and 5.525 form IS Code Model, Hongnestad Model, Kent and Park Model, IRC Model, Cusson Model and Mander Model respectively.

Table 4.4: Moment-Curvature for Doubly Under Reinforced RC Beam.

Strain	**IS CODE MODEL**		**Hongnestad Model**		**Kent and Park Model**		**IRC 112 Model**		**Cussion Model**		**Mander Model**	
	M (kNm)	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(dar/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**
0	0	0	0	0	0	0	0	0	0	0	0	0
0.00035	4.8385	0.0103	4.9306	0.0102	4.8729	0.0103	3.276	0.0077	4.9711	0.0104	4.8905	0.0103
0.0007	9.1793	0.0198	9.3597	0.0197	9.2525	0.0198	6.233	0.0149	9.5218	0.0202	9.2642	0.0198
0.00105	10.7518	0.0335	10.9149	0.0335	10.8605	0.0334	8.8506	0.0216	10.8172	0.0352	10.8403	0.0335
0.0014	10.337	0.0532	10.5077	0.0532	10.4489	0.0532	11.1035	0.0277	10.4564	0.056	10.4335	0.0536
0.00175	10.0775	0.073	10.2556	0.073	10.1985	0.0729	10.9308	0.038	10.2522	0.0775	10.2076	0.0751
0.0021	9.8422	0.091	10.0228	0.0904	9.9488	0.0891	10.3956	0.0493	10.1065	0.0988	10.0627	0.0972
0.00245	9.6218	0.1087	9.764	0.1054	9.6332	0.0993	9.9136	0.0611	9.9769	0.1195	9.9587	0.1194
0.0028	9.4195	0.127	9.5091	0.1204	9.2904	0.1082	9.4548	0.074	9.8476	0.1396	9.8714	0.1415
0.00315	9.2319	0.1459	9.2502	0.1355	8.9357	0.1157	9.0263	0.088	9.7129	0.1595	9.7925	0.1634
0.0035	9.0634	0.1652	8.9913	0.1505	8.5797	0.1217	8.6294	0.103	9.5948	0.1797	9.7153	0.1851

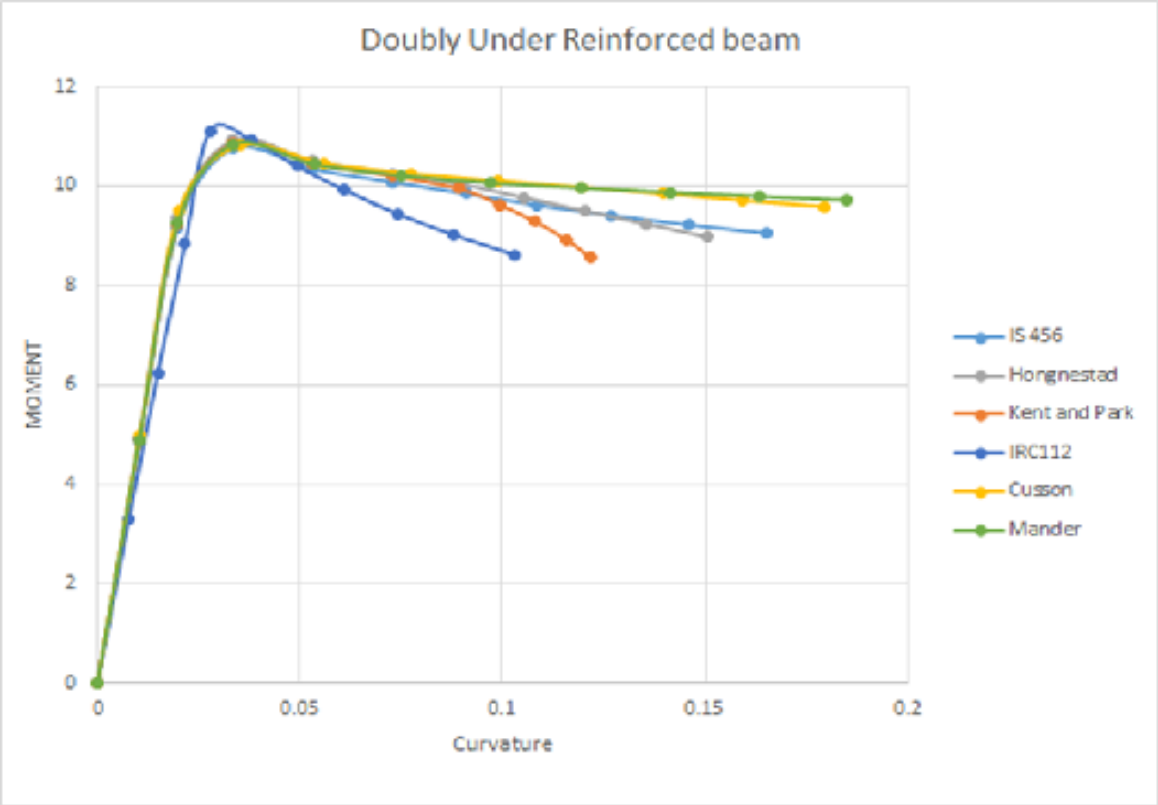

Figure 4.8: Moment-Curvature for Doubly Under Reinforced Beam

From Fig. 4.8, In case of doubly under reinforced section IRC 112 shows maximum degradation in strength after the peak. Here also IRC shows more ultimate moment and less curvature.

Table 4.5shows the Moment-Curvature for Doubly Over Reinforced beam(DOR) for confine as well as unconfine concrete models. Fig. 4.9 shows Moment-Curvature for DUR. Here also Moment-curvature is liner up to the yielding of reinforcement but after this sudden drop was observed in strength. Curvature ductility calculated for DOR was 5.05, 4.6, 3.64, 2.62, 5.3 and 5.8 form IS Code Model, Hongnestad Model, Kent and Park Model, IRC Model, Cusson Model and Mander Model respectively.

Table 4.5: Moment-Curvature for Doubly Over Reinforced RC Beam.

Strain	**IS CODE MODEL**		**Hongnestad Model**		**Kent and Park Model**		**IRC 112 Model**		**Cussion Model**		**Mander Model**	
	M (kNm)	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rda/m)**
0	0	0	0	0	0	0	0	0	0	0	0	0
0.00035	5.1603	0.0097	5.2578	0.0097	5.1931	0.0097	3.4877	0.0073	5.2964	0.0099	5.2154	0.0098
0.0007	9.7897	0.0187	9.9815	0.0187	9.857	0.0187	6.6342	0.0142	10.1448	0.0191	9.8788	0.0188
0.00105	12.589	0.0293	12.773	0.0293	12.6873	0.0293	9.4183	0.0205	12.6042	0.0309	12.6851	0.0293
0.0014	12.0118	0.0468	12.208	0.0468	12.1202	0.0468	11.8114	0.0263	12.0891	0.0494	12.1165	0.0472
0.00175	11.6408	0.0646	11.8488	0.0646	11.7665	0.0646	13.2151	0.0327	11.77	0.0688	11.7637	0.0666
0.0021	11.3318	0.0808	11.551	0.0802	11.468	0.0791	12.6149	0.0422	11.5379	0.0883	11.5202	0.0867
0.00245	11.0435	0.097	11.2573	0.0937	11.1517	0.0881	12.0739	0.0523	11.3344	0.1074	11.3311	0.1071
0.0028	10.7797	0.1137	10.96	0.1073	10.8252	0.0958	11.5456	0.0634	11.1436	0.126	11.1729	0.1276
0.00315	10.5331	0.131	10.6673	0.1209	10.5078	0.102	11.0319	0.0756	10.9596	0.1444	11.031	0.1481
0.0035	10.3089	0.1489	10.3751	0.1346	10.2055	0.1067	10.8489	0.0857	10.7863	0.1634	10.8983	0.1685

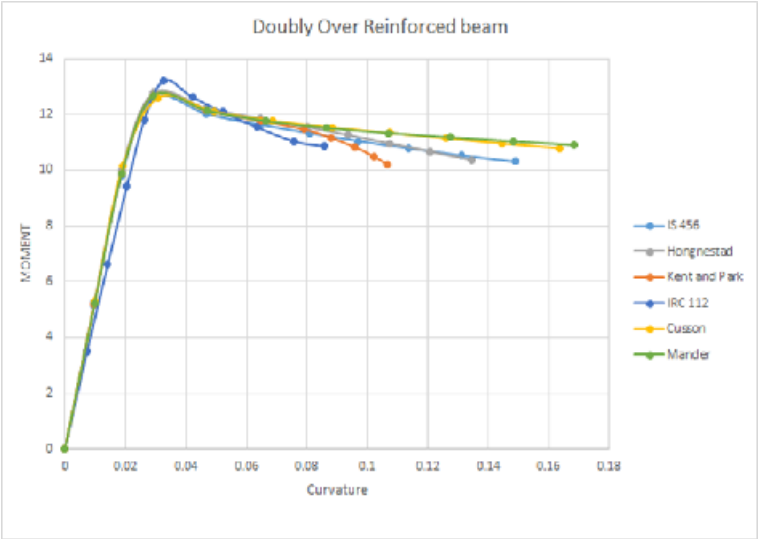

Figure 4.9: Moment-Curvature for Doubly Over Reinforced Beam

From Fig. 4.9, In case of doubly over reinforced section IRC 112 shows maximum degradation in strength after the peak. Here also IRC shows more ultimate moment and less curvature. As Mander and Cusson models includes the effect of the confinement it shows less degradation in strength after the peak.

4.4 Moment-Curvature by Computational Tools

In present study, among various available computational tools RESPONSE 2000 and RC-analysis were used to evaluate Moment-Curvature for design problem.

4.4.1 Moment-Curvature Relationship by RC-Analysis

Table4.6 shows the Moment and corresponding Curvature for different strain level. These values were obtain by using RC-ANALYSIS which used Mander model for concrete and king model for steel. The curvature ductility obtain by this tools were 5.73, 4.17, 5.72 and 5.62 for SUR, SOR, DUR and DOR. Fig 4.10 shows moment-curvature relationship for beam elements. This tools consider the effect of the shear

also so it under estimate the moment carrying capacity of the section.

Table 4.6: Moment-Curvature Relationship by Using RC-ANALYSIS.

Strain	SUR		SOR		DUR		DOR	
	M (kNm)	ϕ (rad/m)	M (kNm)	ϕ (rad/m)	M (kNm)	ϕ (rad/m)	M (kNm)	ϕ (rad/m)
0	0	0	0	0	0	0	0	0
0.00112	6.73885	0.02808	8.02358	0.02257	7.96076	0.02427	8.29511	0.02304
0.00223	6.9637	0.08239	10.33318	0.05464	9.20419	0.06543	10.34796	0.05811
0.00335	7.4642	0.12518	10.33489	0.09218	9.42523	0.10619	10.4271	0.0974
0.00446	7.71671	0.16127	10.57575	0.12142	9.70586	0.14023	10.7437	0.12946

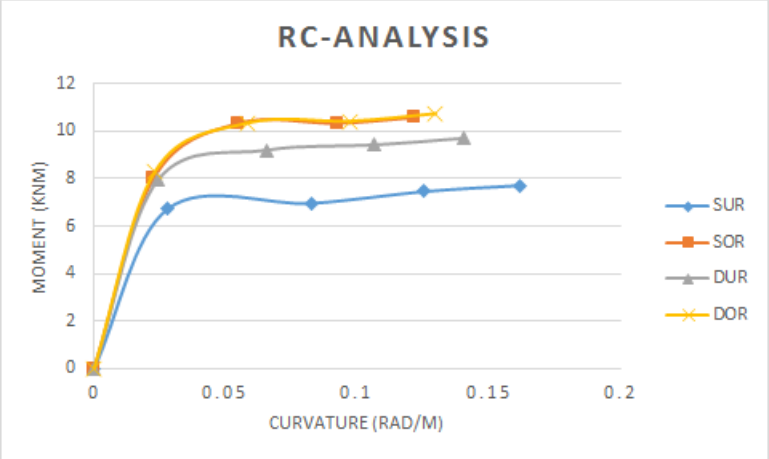

Figure 4.10: Moment-Curvature Relationship from RC-ANALYSIS

4.4.2 Moment-Curvature Relationship by RESPONSE 2000

Fig. 4.11 shows moment-curvature relationship for all four type of RC beam elements which were derived by using **RESPONSE 2000**. Table4.7 shows Moment and corresponding curvature obtain from RESPONSE 2000 tool. RESPONSE 2000 was used Popovics model for the concrete. Comprssion field theorem was used to derive Moment-Curvature relationship.

Table 4.7: Moment-Curvature Relationship by Using RESPONSE 2000

SUR		**SOR**		**DUR**		**DOR**	
M (kNm)	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**	**M (kNm)**	ϕ **(rad/m)**
0	0	0	0.00000	0	0.00000	0	0.00000
0.76	0.00088	0.787	0.00088	0.796	0.00088	0.805	0.00088
1.396	0.00221	1.529	0.00221	1.511	0.00221	1.554	0.00221
1.782	0.00354	2.046	0.00354	1.992	0.00354	2.078	0.00354
2.131	0.00488	2.532	0.00488	2.44	0.00488	2.57	0.00488
2.472	0.00621	3.009	0.00621	2.88	0.00621	3.053	0.00621
2.81	0.00754	3.484	0.00754	3.317	0.00754	3.534	0.00754
3.147	0.00888	3.956	0.00888	3.752	0.00888	4.012	0.00888
3.484	0.01021	4.426	0.01021	4.186	0.01021	4.489	0.01021
3.819	0.01154	4.894	0.01154	4.618	0.01154	4.962	0.01154
4.154	0.01288	5.357	0.01288	5.048	0.01288	5.433	0.01288
4.488	0.01421	5.817	0.01421	5.476	0.01421	5.901	0.01421
4.843	0.01563	6.302	0.01563	5.928	0.01563	6.394	0.01563
5.23	0.01720	6.829	0.01720	6.421	0.01720	6.931	0.01720
5.654	0.01891	7.4	0.01891	6.957	0.01891	7.514	0.01891
6.114	0.02081	8.015	0.02081	7.538	0.02081	8.144	0.02081
6.449	0.02289	8.674	0.02289	8.165	0.02289	8.821	0.02289
6.794	0.02518	9.377	0.02518	8.74	0.02518	9.547	0.02518
6.751	0.02769	9.972	0.02769	9.032	0.02769	10.13	0.02769
6.78	0.03046	10.156	0.03046	9.071	0.03046	10.181	0.03046
6.807	0.03351	10.199	0.03351	9.106	0.03351	10.222	0.03351
6.833	0.03686	10.236	0.03686	9.14	0.03686	10.259	0.03686
6.865	0.04055	10.269	0.04055	9.168	0.04055	10.29	0.04055
6.95	0.04460	10.295	0.04460	9.236	0.04460	10.332	0.04460
7.035	0.04906	10.402	0.04906	9.342	0.04906	10.446	0.04906
7.124	0.05397	10.505	0.05397	9.448	0.05397	10.558	0.05397
7.214	0.05936	10.604	0.05936	9.554	0.05936	10.667	0.05936
7.297	0.06530	10.667	0.06530	9.629	0.06530	10.737	0.06530
7.341	0.07183	10.686	0.07183	9.662	0.07183	10.765	0.07183
7.378	0.07901	10.681	0.07901	9.675	0.07901	10.771	0.07901
7.43	0.08691	10.667	0.08691	9.715	0.08691	10.788	0.08691
7.477	0.09560	10.685	0.09560	9.758	0.09560	10.828	0.09560
7.519	0.10516	10.697	0.10516	9.798	0.10516	10.866	0.10516
7.556	0.11568	-	-	9.835	0.11568	10.904	0.11568
7.587	0.12725	-	-	9.871	0.12725	-	-

5.7, 3.8, 5.03 and 4.59 were evaluated curvature ductility by RESPONSE 2000 for SUR, SOR, DUR and DOR RC elements respectively. These values are much closer to the values obtain by Cusson and Hongnestand model.

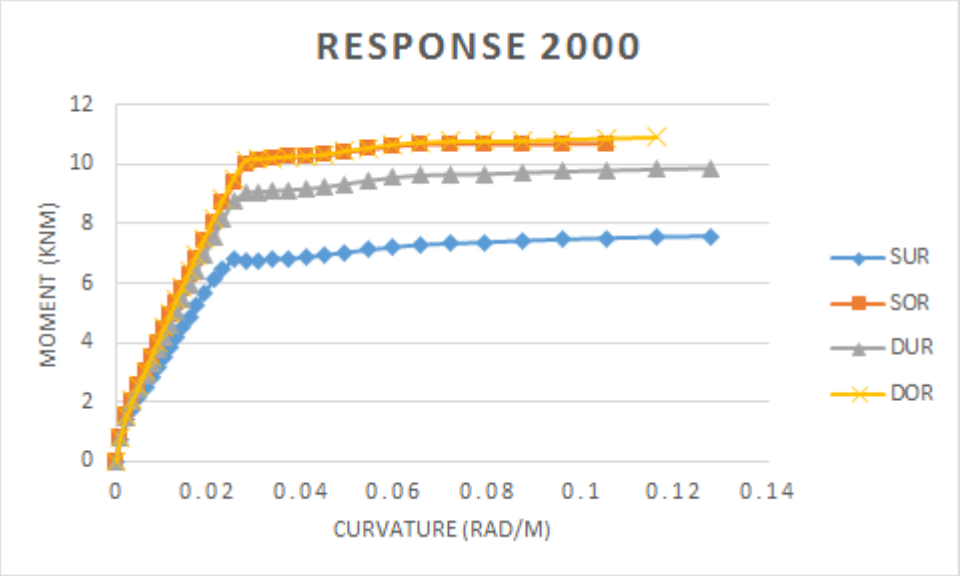

Figure 4.11: Moment-Curvature Relationship from RESPONSE 2000

4.5 Summery

This chapter discussed the various methods for analytically evaluation of Moment-Curvature relationship. The transform area method is the approximate method while the strain compatibility method is accurate but iterative method. Response 2000, which is based on the Popovics model is gives the Moment-curvature near to the Kent and park model. and RC-Analysis is also include the shear effect on the RC beam so it under estimate the moment carrying capacity of the beam. By calculating moment curvature for any beam one can estimate the curvature ductility also estimate the degradation in strength after the peak.

Chapter 5

Experimental Program

5.1 General

This chapter deals with the experimental evaluation of Moment-Curvature. The experimental program was design to compare the experimental values to the values obtain by analytical Methods. The experimental program consist of casting and testing of twelve RC beams of M20 grade concrete and Fe 415 grade steel. Two point loading was applied in middle one third portion. The beams were testes as a simply supported beams.

5.2 Concepts for Evaluation of Moment-Curvature

By using Moment-Curvature relationship we can estimate stiffness, ductility, energy dissipation and strength beyond the post peak of the stress-stain curve for any RC elements. Here Fig 5.1 shows an initially straight element of RC member having equal end moments and axial force. Let R is the radius of the curvature. It is measured from the neutral axis of the section. Neutral axis is located at depth kd from the top fiber of RC element. ε_c and ε_s is the concrete strain in the extreme compression fiber and strain in the steel respectively. Then considering only small length dx of the member, the rotation between two ends is given by following equation;

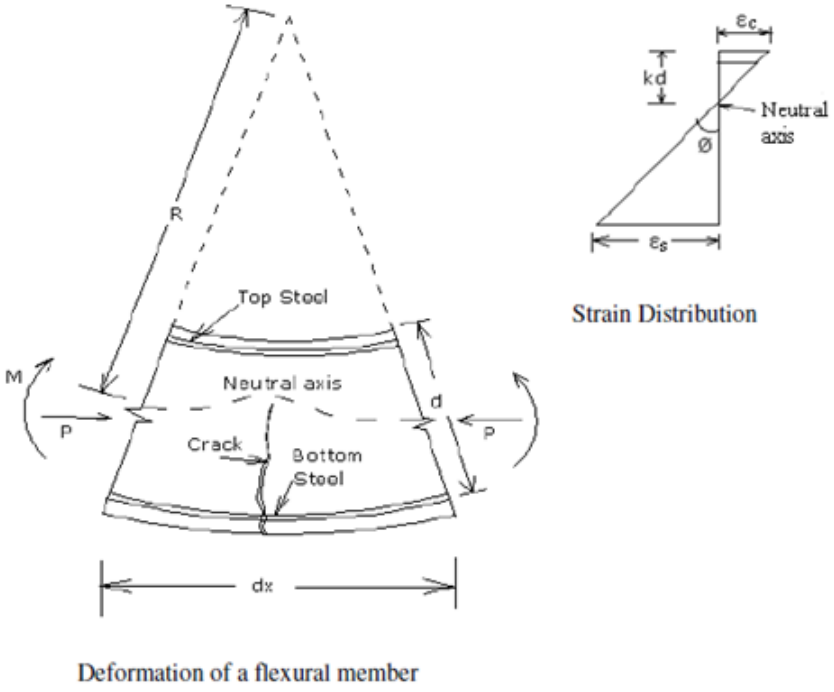

Figure 5.1: Curvature of Member [3]

$$\frac{dx}{R} = \frac{\varepsilon_c dx}{kd} = \frac{\varepsilon_s dx}{d(1-k)} \tag{5.1}$$

$$\frac{1}{R} = \frac{\varepsilon_c}{kd} = \frac{\varepsilon_s}{d(1-k)} \tag{5.2}$$

$$\phi = \frac{1}{R} = \frac{\varepsilon_c}{kd} = \frac{\varepsilon_s}{d(1-k)} = \frac{\varepsilon_c + \varepsilon_s}{d} \tag{5.3}$$

Here, $\frac{1}{R}$ is the curvature of the elements and is denoted by ϕ. It is also called as rotation per unit length of member.

Curvature is also called as a gradient of the stain profile. It can be vary along the length of the member because of the fluctuation of the neutral axis depth and strains between the cracks. If the strain at the critical section of RC beam are measured as the bending moment is increased to failure, the curvature can be calculated by equation 5.3. Fig 5.2 shows Moment Curvature for RC beam failing in tension and

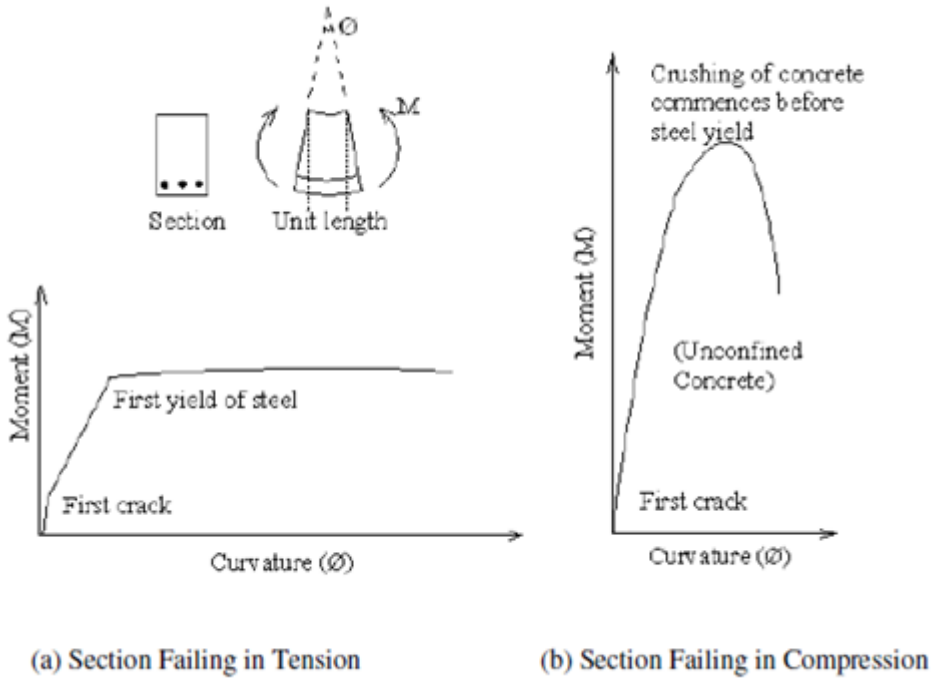

Figure 5.2: Moment-Curvature Relationship for Singly Reinforced Section

compression. Both the curves are linear in the initial stage. Relation between moment and curvature is given by classical elastic equation.

$$EI = MR = \frac{M}{\phi} \tag{5.4}$$

Where, EI is the flexural rigidity of the section. With increase in moment, cracking of the concrete reduces the flexural rigidity of the sections, the reduction in rigidity being greater for the lightly reinforced section than for the more heavily reinforced section. The behavior of section after cracking is dependent mainly on the steel content. Lightly reinforced sections shows Fig. 5.2 (a) results in practically linear M-ϕ curve up to point of steel yielding. When steel yields, a large increase in curvature occurs at nearly constant bending moment, the moment rising slowly to a maximum due to an increase in the internal lever arm, then decreasing. In heavily reinforced

sections shows in Fig. 5.2 (b), on the other hand, the M-ϕ becomes nonlinear when the concrete enters in inelastic part of stress-strain relationship, and failure can be quite brittle unless the concrete is confined by closed stirrups at close centers. If the concrete is not confined, the concrete crushes at a relatively small curvature before the steel yields, causing an immediate decrease in the moment-carrying capacity. To ensure ductile behavior in practice, steel contents less than the balanced design value are always used for beams [3].

5.3 Experimental Problem

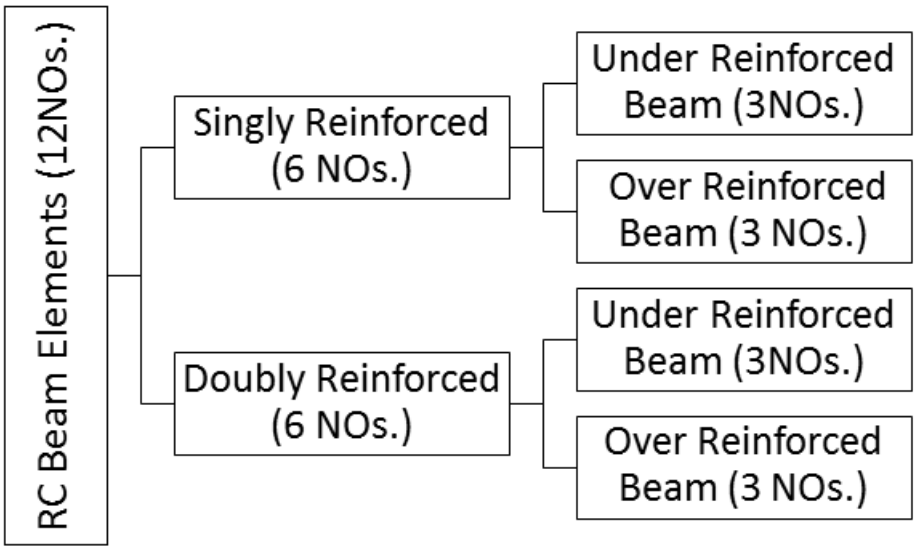

Figure 5.3: Experimental Program

The experimental program includes the casting of twelve RC rectangular beam of M20 grade concrete and Fe 415 grade steel. Dimensions of the sections were decided on the base of available literatures. In the literature, it was found that the L/d ratio of the simply supported beams was taken around 10 and b/d ratio was taken around 0.5 to 0.8. Where, L is the span of beam, d is the depth of the section and b is the width of the section. So in present study first of all L/d ratio was assumed. it was taken as

10 and b/d ratio was taken as 0.8. Hence overall dimensions of the beams 120mm X 150mm X 1700mm were selected. Here clear span of the beam was taken as a 1500mm.

In present study, all the beams were designed by using limit state method. Here, four different types of beams were design like Singly Under Reinforced, Singly Over Reinforced, Doubly Under Reinforced and Doubly Over Reinforced. Fig 5.3 shows the layout of experimental program.

Clear cover of the beams were taken as 15mm. In present study in the case of singly reinforced beams 2-6mm dia bar used as a hanger bar at the top. Two legged 8mm dia stirrups were used at 100mm c/c in all twelve beams. The details of the beams are as shown in Table 4.1. The cross sectional and reinforcment details of the beams are as in Fig. 5.4) and Fig. 5.5.

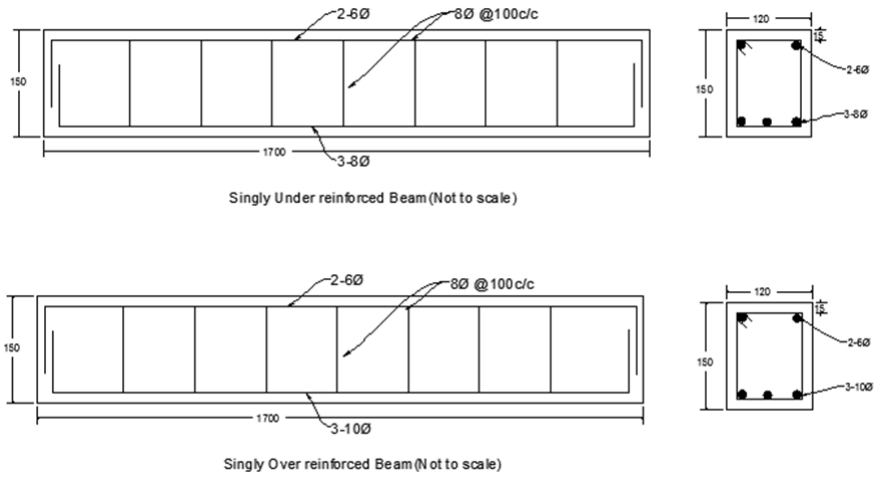

Figure 5.4: Cross Section of Beam (a) Singly Under Reinforced (b) Singly Over Reinforced

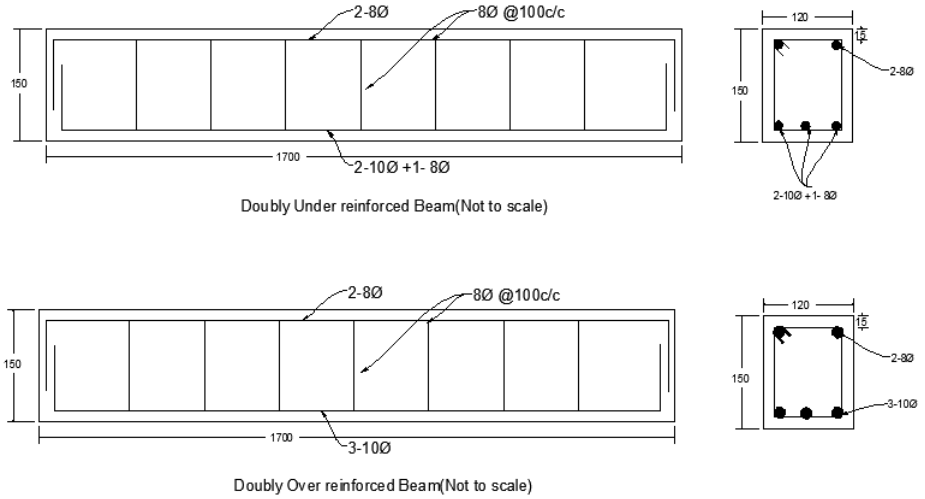

Figure 5.5: Cross Section of Beam (a) Doubly Under Reinforced (b) Doubly Over Reinforced

Table 5.1: Mix Design for M20 Grade Concrete

SR. No.	**Descri.**	**Wt (kg/m^3)**	**Wt (kg/beam)**
1	**Cement**	371	12
2	**Water**	214.14	7
3	**20 mm**	710.16	22
4	**10 mm**	473.44	15
5	**F.A.**	725.43	22.2

5.4 Casting of RC Beam

Fig. 5.6 shows 3D view and cross section of the formwork. Formwork was prepared from the wooden sheet. Wooden clamps were also prepared for the ensuring the dimensional stability during the casting of the specimens. Two sets of the formworkwas prepared so that two specimens could be casted at a time.

Mix design for M20 grade concrete was prepared. Mix design are shown in Table 5.1. During the casting of the specimens for measuring the cube strength at 7 days and 28 days, six cubes(150mm $\times$ 150 mm $\times$ 150 mm) were casted. Three cylinder(150

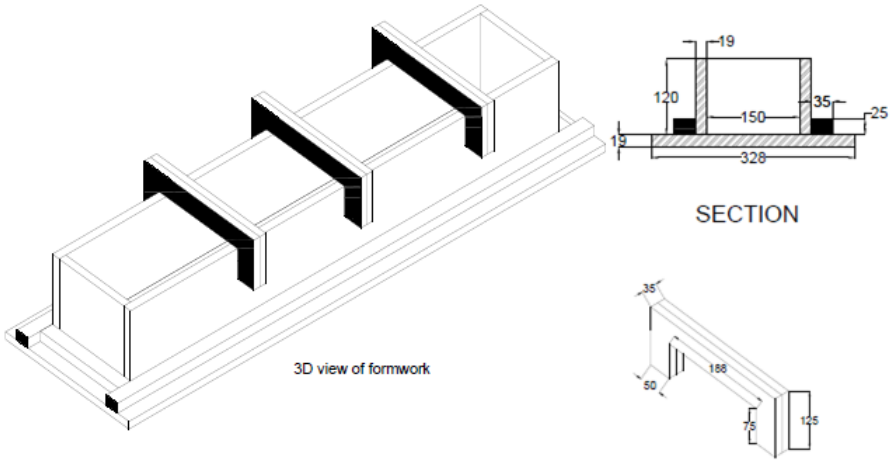

Figure 5.6: 3D View of Formwork for Beam

mm × 300 mm) and three beams(100 mm × 100 mm × 500 mm) were also casted for measuring the modulus of elasticity of concrete and flexure strength of concrete. Cube strength, Modulus of elasticity of concrete and flexure strength of concrete will be used in the analytical solution for the moment curvature of the RC beam. Fig. 5.7(a) shows drum mixer. And Fig. 5.7(b) shows casted specimens for measuring concrete properties.

Figure 5.7: Casting of Specimens for Deriving Concrete Properties

Figure 5.8: Formwork and Reinforcement Cage of RC Specimens.

Fig. 5.8(a) shows reinforcement cage and Fig. 5.8(b) shows casted reinforced concrete beam specimens.

5.5 Instrumentation and Test Setup for RC Beam

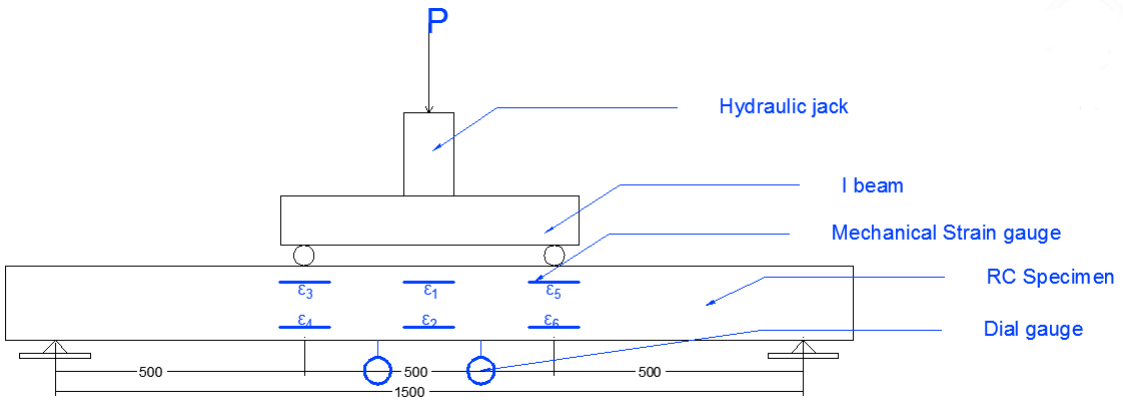

Figure 5.9: Instrumentation and Test Setup.

Fig. 5.9 shows the test set up and instrumentation diagram for all four type of RC beams. Test setup was design in such way so that in middle one third of the

beam was in pure bending zone and flexure failure was occurred in this portion. Two point load applied in the middle one third portion. As the curvature is the strain gradient, in present experiment the main focus was kept on measuring the strain in the RC rectangular beam elements. For measuring strain at the level of core, strain was measure at the top and bottom reinforcement level. Total six mechanical strain gauges were used. Among these three were fixed at top and remining were fixed at bottom.

Figure 5.10: Test Setup and Instrumentation Used for Experiments

Strain was measured by using mechanical strain gauge having 10 cm gauge length. Fig. 5.10(a) shows test setup of the experiments and also shows position of strain points at where strain was going to measure. Fig. 5.10(b) shows mechanical strain gauge. d is the distance between top and bottom strain gauge. In present experiment it was kept as 90mm.

Experimental curvature is measure by using equation 5.3. Where, ε_c is the concrete strain which was taken as a average concrete strain from the top three strain gauge and ε_s is the bottom strain or tension steel strain and was taken as a average strain from the bottom three strain gauge.

Figure 5.11: Testing of Specimens

5.6 Testing of RC Specimens

Figure 5.12: Failure of RCC Beams

Fig. 5.11 shows the testing of RC beam elements. Readings were taken at each 4 kN increment of load till the beam was failed. Corresponding Curvature were calculated from the top and bottom average strain. Corresponding moment was calculated from the load value. Then experimental Moment-Curvature relationship

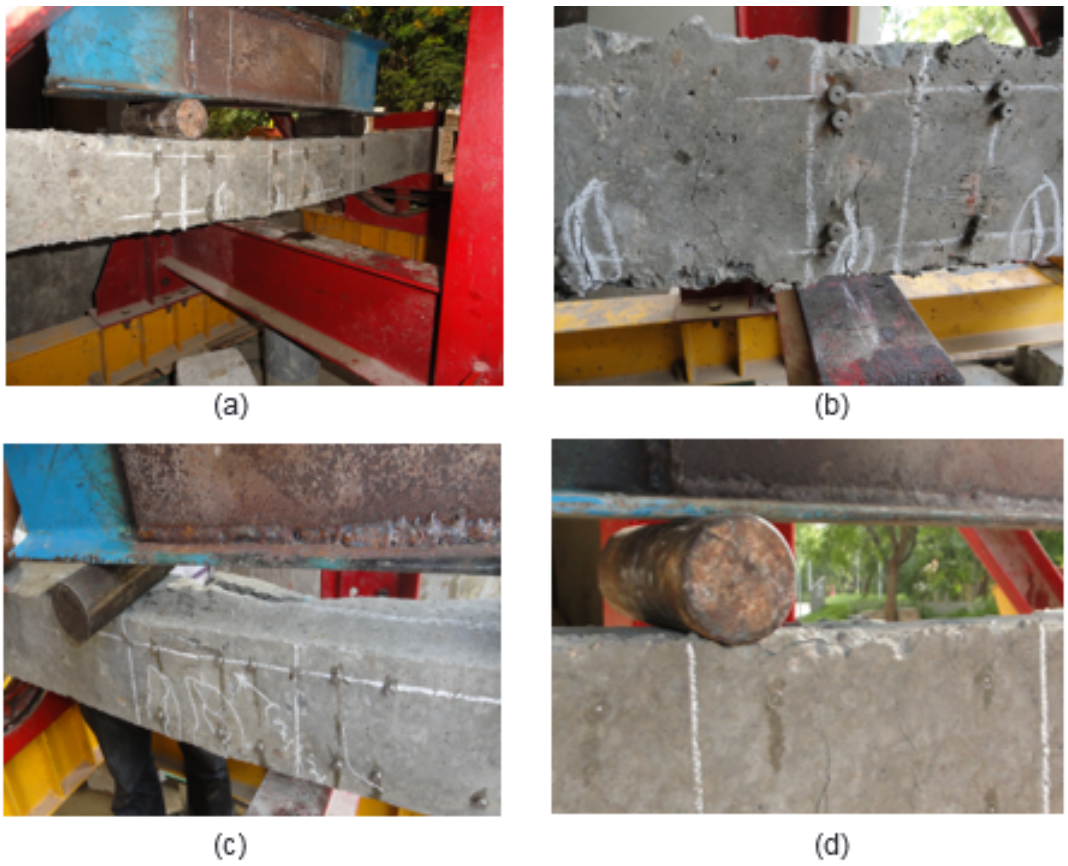

Figure 5.13: Failure Patterns of RCC Beams

was plotted. To measure the defection for the RC beam deflection gauge was used near the center of the beam. It was observed that all the beams were failed in the flexure. And cracks were observed in middle one third portion of the beams. Shear failure was not observed. Fig. 5.12 shows failure pattern of the beam elements. Fig. 5.12(a) shows the cracks in the tension zone and Fig. 5.12(b) shows deflected shape of beam after testing. Fig. 5.13(a) and 5.13(b) shows failure of specimens. Fig. 5.13(c) and 5.13(d) shows the compression failure of the RC beams.

5.7 Test Results

This section discussed the test results for concrete properties as well as the Moment-Curvature.

5.7.1 Test Results for Concrete Properties

In present experiment cubes (150mm × 150mm × 150mm), cylinders(150mm × 300mm) and beams (100mm × 100mm × 500mm) were also tested to find out the concrete properties. These properties were used in the analytical solution for Moment-Curvature. Table 5.2 shows 7 days and 28 days cube results. Table 5.5 and Table 5.4 shows the flexure strength of concrete and Modulus of elasticity of concrete which were used in the computer program to evaluate Moment-Curvature relationship analyticaly.

Table 5.2: Cube Strength

7 Days Cube strength			
Sr No	**Wt (kg)**	**Strength (Mpa)**	**Average Strength (MPa)**
1	8.54	22.2	21.8
2	8.62	21.4	
3	8.79	21.9	
28 bays Cube strength			
1	8.68	25	25.7
2	8.74	26.4	
3	8.69	25.7	

Table 5.3: Flexure Strength of Concrete

	Load (kN)	Flexure Strength	Average
M20	15	3.39	3.09
	12	2.71	
	14	3.17	

Table 5.4: Modulus of Elasticity of Concrete

L=212mm			
P (kN)	dl_1**(mm)**	dl_2 **(mm)**	dl_3**(mm)**
50	1	1	2
100	3	2	3
150	4	5	4
200	6	6	7
250	7	8	8
300	10	9	10
350	13	12	12
400	17	16	15
450	20	22	22
$E_c(\text{kN}/mm^2)$	24.69	23.436	24.34
$E_c(\text{kN}/mm^2)$	24.15		

After the testing of the cube specimens, average 7 day cube strength was observed 21.8 MPa. And 28 days cube strength was obseved 25.7MPa. Cube strength was used in transform area method as well as strain compatibility method. Flexure strength of beam was found 3.03 and Modulus of elasticity of concrete was found 24.15 kN/mm^2. These two were used only in transform area method.

5.7.2 Experimental Moment-Curvature

Twelve specimens were tested and experimental Moment-Curvature relationship was obtain for all the RC specimens.

Table 5.5: Moment-Curvature for SUR-1

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.0200	0.0533	**0.0772**
8	**1**	-0.0800	0.1133	**0.2035**
12	**1.5**	-0.1667	0.1600	**0.3439**
16	**2**	-0.1867	0.2467	**0.4561**
20	**2.5**	-0.2533	0.4067	**0.6947**
24	**3**	-0.2733	0.4867	**0.8000**
28	**3.5**	-0.3667	0.6200	**1.0386**
32	**4**	-0.4400	0.7400	**1.2421**
36	**4.5**	-0.5333	0.8067	**1.4105**
40	**5**	-0.6067	0.8333	**1.5158**
44	**5.5**	-0.7133	0.9400	**1.7404**
48	**6**	-0.7533	1.0667	**1.9158**
52	**6.5**	-0.8933	1.2333	**2.2386**
56	**7**	-0.9800	1.4600	**2.5684**
60	**7.5**	-1.1133	1.8733	**3.1439**
64	**8**	-1.3533	3.5667	**5.1789**
68	**8.5**	-1.4867	6.6267	**8.5404**
70	**8.75**	-1.7133	8.9933	**11.2702**
70	**8.75**	-2.0333	11.1600	**13.8877**
70	**8.75**	-2.1067	16.9333	**20.0421**

Table 5.5 shows Moment and corresponding Curvature for singly under reinforced beam 1. Here the Ultimate Moment was observed 8.75 kNm and Ultimate curvature was observed 0.2 rad/m. First crack was observed at 40kN load. Table 5.6 shows Moment and corresponding Curvature for singly under reinforced beam 2. In this case ultimate Moment was observed 8.625 kNm and Ultimate curvature was observed

0.191 rad/m. At 44kN load first crack was observed and similarly Table 5.7 shows the Moment-Curvature relationship for Singly under reinforced beam 3. Ultimate Moment was observed 8.625 kNm and Ultimate curvature was observed 0.166 rad/m. At 42kN load first crack was observed. Table also shows the compression and tension strain of the concrete at each increment of load and moment.

Table 5.6: Moment-Curvature for SUR-2

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0.0000	0.0000	**0.0000**
4	**0.5**	-0.0133	0.0333	**0.0519**
8	**1**	-0.1000	0.2067	**0.3407**
12	**1.5**	-0.1933	0.3933	**0.6519**
16	**2**	-0.2933	0.5067	**0.8889**
20	**2.5**	-0.3267	0.5267	**0.9481**
24	**3**	-0.2333	0.4867	**0.8000**
28	**3.5**	-0.3800	0.7067	**1.2074**
32	**4**	-0.5267	0.6467	**1.3037**
36	**4.5**	-0.6400	0.7600	**1.5556**
40	**5**	-0.7267	0.8800	**1.7852**
44	**5.5**	-0.8067	1.0333	**2.0444**
48	**6**	-0.9067	1.1400	**2.2741**
52	**6.5**	-1.0200	1.2867	**2.5630**
56	**7**	-1.1200	1.4467	**2.8519**
60	**7.5**	-1.2267	1.5800	**3.1185**
64	**8**	-1.3467	2.1533	**3.8889**
68	**8.5**	-1.3867	2.9267	**4.7926**
69	**8.625**	-1.7267	6.8667	**9.5481**
69	**8.625**	-2.0467	9.9200	**13.2963**
69	**8.625**	-2.1133	15.0867	**19.1111**

Table 5.8 shows the average Moment and corresponding curvature for singly under reinforced beam. Ultiment moment and corresponding curvture was observed 8.67 kNm and 0.186 rad/m. Fig 5.14 shows Moment-Curvature relationship for all three

Singly Under Reinforced beam. Fig 5.14 also represent the average M-ϕ relationship. The experimental plot of the M-ϕ diagram starts with the elastic region which is signified by its large slope, followed by plastic region which occurs after yielding and is signified by its parabolic shape and proceeded by ultimate and failures. In case of average Moment-Curvature, it is almost linear in elastic zone but after yielding it follows parabolic path and than horizonatal path which represent more increment in curvature for nearly constant moment.

Table 5.7: Moment-Curvature for SUR-3

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	**0.0000**	**0.0000**	**0.0000**
4	**0.5**	**-0.0200**	**0.0333**	**0.0056**
8	**1**	**-0.0800**	**0.1133**	**0.2035**
12	**1.5**	**-0.2067**	**0.1867**	**0.4140**
16	**2**	**-0.2333**	**0.3533**	**0.6175**
20	**2.5**	**-0.2133**	**0.4800**	**0.7298**
24	**3**	**-0.3333**	**0.7533**	**1.1439**
28	**3.5**	**-0.5000**	**0.6400**	**1.2000**
32	**4**	**-0.5200**	**0.8933**	**1.4877**
36	**4.5**	**-0.6067**	**1.0533**	**1.7474**
40	**5**	**-0.6733**	**1.2000**	**1.9719**
44	**5.5**	**-0.7333**	**1.2067**	**2.0421**
48	**6**	**-0.8400**	**1.3067**	**2.2596**
52	**6.5**	**-0.9600**	**1.4267**	**2.5123**
56	**7**	**-1.0333**	**1.5933**	**2.7649**
60	**7.5**	**-1.1400**	**2.4200**	**3.7474**
64	**8**	**-1.2200**	**4.3533**	**5.8667**
68	**8.5**	**-1.4200**	**6.4600**	**8.2947**
69	**8.625**	**-1.5667**	**8.6467**	**10.7509**
69	**8.625**	**-2.0733**	**9.4533**	**12.1333**
69	**8.625**	**-2.1800**	**13.5800**	**16.5895**

Fig. 5.15 shows the strain in the compression zone and tension zone for all three

Table 5.8: Average Moment-Curvature for Singly Under Reinforced RC Beam.

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0.0000	0.0000	**0.0000**
4	**0.5**	-0.0178	0.0400	**0.0617**
8	**1**	-0.0867	0.1444	**0.2493**
12	**1.5**	-0.1889	0.2467	**0.4699**
16	**2**	-0.2378	0.3689	**0.6542**
20	**2.5**	-0.2644	0.4711	**0.7909**
24	**3**	-0.2800	0.5756	**0.9146**
28	**3.5**	-0.4156	0.6556	**1.1487**
32	**4**	-0.4956	0.7600	**1.3445**
36	**4.5**	-0.5933	0.8733	**1.5712**
40	**5**	-0.6689	0.9711	**1.7576**
44	**5.5**	-0.7511	1.0600	**1.9423**
48	**6**	-0.8333	1.1711	**2.1498**
52	**6.5**	-0.9578	1.3156	**2.4379**
56	**7**	-1.0444	1.5000	**2.7284**
60	**7.5**	-1.1600	1.9578	**3.3366**
64	**8**	-1.3067	3.3578	**4.9782**
68	**8.5**	-1.4311	5.3378	**7.2092**
69.3	**8.67**	-1.6689	8.1689	**10.5231**
69.3	**8.67**	-2.0511	10.1778	**13.1058**
69.3	**8.67**	-2.1333	15.2000	**18.5809**

Singly Under Reinforced beam. Negative value indicate the concrete strain while the positive values represent steel strain. Concrete strain is closer the steel strain till the yeilding that shows good bonding between steel and concrete. After the yeilding there was drasticaly increment was observed in the tensile strain.

As the yielding has occured there was sudden increment in deflection was observed. As the deflection was incresed the rigidity of the RC beam was decreased.

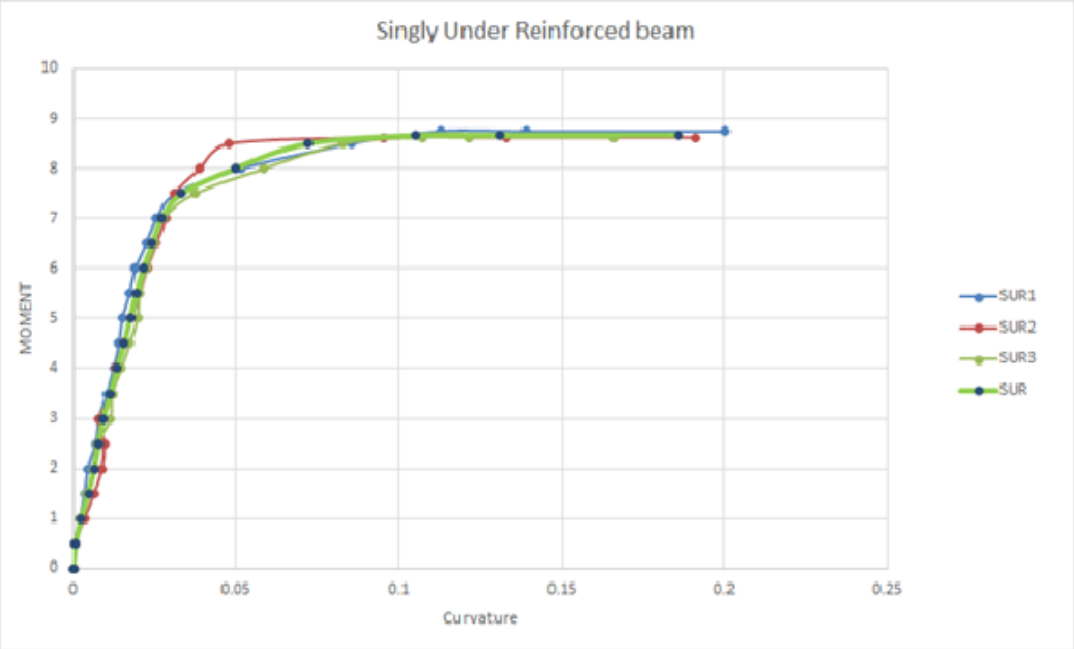

Figure 5.14: Moment-Curvature For Singly Under Reinforced Beam

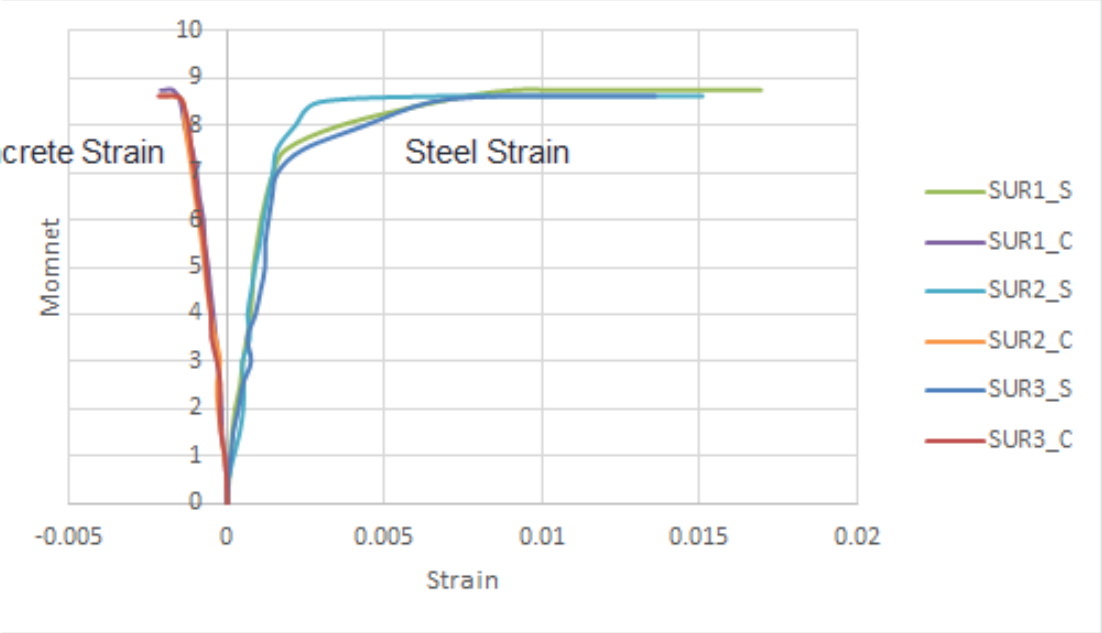

Figure 5.15: Stain at Tension and Compression Zone

Table 5.9: Moment-Curvature for Singly Over Reinforced Beam-1

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.013	0.033	**0.052**
8	**1**	-0.080	0.127	**0.230**
12	**1.5**	-0.080	0.200	**0.311**
16	**2**	-0.113	0.307	**0.467**
20	**2.5**	-0.160	0.293	**0.504**
24	**3**	-0.147	0.400	**0.607**
28	**3.5**	-0.220	0.493	**0.793**
32	**4**	-0.280	0.533	**0.904**
36	**4.5**	-0.473	0.540	**1.126**
40	**5**	-0.487	0.467	**1.059**
44	**5.5**	-0.373	0.587	**1.067**
48	**6**	-0.427	0.680	**1.230**
52	**6.5**	-0.513	0.773	**1.430**
56	**7**	-0.520	0.840	**1.511**
60	**7.5**	-0.560	0.887	**1.607**
64	**8**	-0.600	0.960	**1.733**
68	**8.5**	-0.593	1.033	**1.807**
72	**9**	-0.553	1.093	**1.830**
76	**9.5**	-0.667	1.107	**1.970**
80	**10**	-0.727	1.253	**2.200**
84	**10.5**	-0.787	1.340	**2.363**
88	**11**	-0.800	1.440	**2.489**
92	**11.5**	-0.933	1.527	**2.733**
96	**12**	-1.040	1.467	**2.785**
100	**12.5**	-1.033	1.407	**2.711**
104	**13**	-1.007	1.753	**3.067**
108	**13.5**	-1.153	2.107	**3.622**
112	**14**	-1.233	3.233	**4.963**
113	**14.125**	-1.660	7.287	**9.941**
113	**14.125**	-2.060	9.593	**12.948**

Table 5.10: Moment-Curvature for Singly Over Reinforced beam-2

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.027	0.053	**0.089**
8	**1**	-0.053	0.093	**0.163**
12	**1.5**	-0.053	0.120	**0.193**
16	**2**	-0.100	0.287	**0.430**
20	**2.5**	-0.187	0.333	**0.578**
24	**3**	-0.240	0.413	**0.726**
28	**3.5**	-0.213	0.447	**0.733**
32	**4**	-0.293	0.693	**1.096**
36	**4.5**	-0.247	0.680	**1.030**
40	**5**	-0.333	0.753	**1.207**
44	**5.5**	-0.400	0.813	**1.348**
48	**6**	-0.427	0.907	**1.481**
52	**6.5**	-0.493	0.980	**1.637**
56	**7**	-0.507	1.000	**1.674**
60	**7.5**	-0.573	1.113	**1.874**
64	**8**	-0.640	1.200	**2.044**
68	**8.5**	-0.747	1.233	**2.200**
72	**9**	-0.847	1.353	**2.444**
76	**9.5**	-0.953	1.507	**2.733**
80	**10**	-1.033	1.573	**2.896**
84	**10.5**	-1.113	1.727	**3.156**
88	**11**	-1.187	1.847	**3.370**
92	**11.5**	-1.273	2.027	**3.667**
96	**12**	-1.327	2.173	**3.889**
100	**12.5**	-1.447	2.420	**4.296**
104	**13**	-1.553	3.040	**5.104**
108	**13.5**	-1.653	4.400	**6.726**
110	**13.75**	-1.680	5.793	**8.304**
109	**13.625**	-1.993	7.887	**10.978**
110	**13.75**	-2.087	9.527	**12.904**

Table 5.11: Moment-Curvature for Singly Over Reinforced beam-3

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.033	**0.059**
8	**1**	-0.067	0.140	**0.230**
12	**1.5**	-0.100	0.180	**0.311**
16	**2**	-0.140	0.273	**0.459**
20	**2.5**	-0.173	0.313	**0.541**
24	**3**	-0.187	0.427	**0.681**
28	**3.5**	-0.233	0.500	**0.815**
32	**4**	-0.340	0.440	**0.867**
36	**4.5**	-0.413	0.520	**1.037**
40	**5**	-0.453	0.460	**1.015**
44	**5.5**	-0.433	0.607	**1.156**
48	**6**	-0.500	0.633	**1.259**
52	**6.5**	-0.507	0.680	**1.319**
56	**7**	-0.547	0.727	**1.415**
60	**7.5**	-0.553	0.807	**1.511**
64	**8**	-0.600	0.873	**1.637**
68	**8.5**	-0.620	0.920	**1.711**
72	**9**	-0.707	1.040	**1.941**
76	**9.5**	-0.713	1.127	**2.044**
80	**10**	-0.793	1.200	**2.215**
84	**10.5**	-0.880	1.313	**2.437**
88	**11**	-0.920	1.300	**2.467**
92	**11.5**	-0.993	1.467	**2.733**
96	**12**	-1.133	1.673	**3.119**
100	**12.5**	-1.207	1.860	**3.407**
104	**13**	-1.313	2.507	**4.244**
108	**13.5**	-1.440	3.367	**5.341**
112	**14**	-1.707	5.113	**7.578**
113	**14.125**	-1.887	5.947	**8.704**
113	**14.125**	-2.147	8.920	**12.296**

Table 5.12: Average Moment-Curvature for Singly Over Reinforced RC Beam.

Load (kN)	**M (kNm)**	**Top Strain** *times* 10^{-3}	**Bottom Strain** *times* 10^{-3}	ϕ *times* 10^{-2} **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.040	**0.067**
8	**1**	-0.067	0.120	**0.207**
12	**1.5**	-0.078	0.167	**0.272**
16	**2**	-0.118	0.289	**0.452**
20	**2.5**	-0.173	0.313	**0.541**
24	**3**	-0.191	0.413	**0.672**
28	**3.5**	-0.222	0.480	**0.780**
32	**4**	-0.304	0.556	**0.956**
36	**4.5**	-0.378	0.580	**1.064**
40	**5**	-0.424	0.560	**1.094**
44	**5.5**	-0.402	0.669	**1.190**
48	**6**	-0.451	0.740	**1.323**
52	**6.5**	-0.504	0.811	**1.462**
56	**7**	-0.524	0.856	**1.533**
60	**7.5**	-0.562	0.936	**1.664**
64	**8**	-0.613	1.011	**1.805**
68	**8.5**	-0.653	1.062	**1.906**
72	**9**	-0.702	1.162	**2.072**
76	**9.5**	-0.778	1.247	**2.249**
80	**10**	-0.851	1.342	**2.437**
84	**10.5**	-0.927	1.460	**2.652**
88	**11**	-0.969	1.529	**2.775**
92	**11.5**	-1.067	1.673	**3.044**
96	**12**	-1.167	1.771	**3.264**
100	**12.5**	-1.229	1.896	**3.472**
104	**13**	-1.291	2.433	**4.138**
108	**13.5**	-1.416	3.291	**5.230**
111.33	**13.92**	-1.540	4.713	**6.948**
111.67	**13.96**	-1.847	7.040	**9.874**
112	**14**	-2.098	9.347	**12.716**

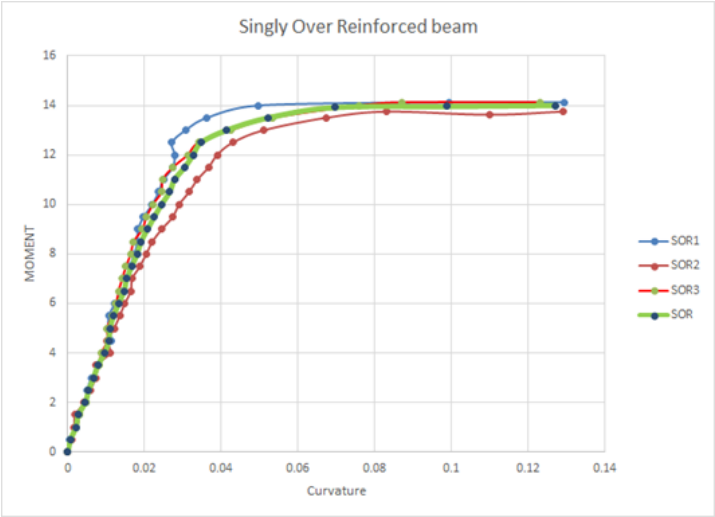

Figure 5.16: Moment-Curvature relationship for Singly Over Reinforced beam

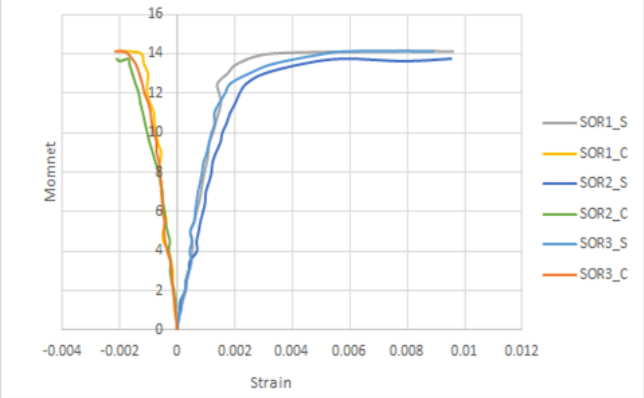

Figure 5.17: Concrete and Steel Strain of Doubly Under Reinforced Beam

Table 5.9 shows Moment and Curvature for singly over reinforced beam-1. Ultimate Moment and curavture was observed 14.125 kNm and 0.129 rad/m. Table 5.10 shows Moment and corresponding Curvature for singly over reinforced beam-2. Here, ultimate moment and corresponding Curvature was observed 13.75 kNm and 0.129 rad/m and Table 5.11 shows the Moment and corresponding curvature for Singly Over Reinforced beam-3. 14.125 kNm and 0.123 rad/m was observed ulti-

mate moment and curvature for singly over reinforced beam-3. Those tables are also shows concrete strain and steel strain for each increment of load. Fig. 5.16 shows the Moment-Curvature relationship for all three Singly Over Reinforced beam. It also shown average Moment-Curvature for rc beam. Fig. 5.17 shows the strain in RC element at the top and bottom reinforcement level.

Table 5.13: Moment-Curvature for Doubly Under Reinforced beam-1

Load (kN)	**M (kNm)**	**Top Strain** $\times$ 10^{-3}	**Bottom Strain** $\times$ 10^{-3}	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.013	0.033	**0.052**
8	**1**	-0.073	0.087	**0.178**
12	**1.5**	-0.167	0.120	**0.319**
16	**2**	-0.193	0.147	**0.378**
20	**2.5**	-0.280	0.233	**0.570**
24	**3**	-0.593	0.253	**0.941**
28	**3.5**	-0.593	0.307	**1.000**
32	**4**	-0.600	0.373	**1.081**
36	**4.5**	-0.533	0.427	**1.067**
40	**5**	-0.660	0.540	**1.333**
44	**5.5**	-0.740	0.487	**1.363**
48	**6**	-0.720	0.647	**1.519**
52	**6.5**	-0.847	0.720	**1.741**
56	**7**	-0.893	0.827	**1.911**
60	**7.5**	-1.087	0.880	**2.185**
64	**8**	-1.167	0.913	**2.311**
68	**8.5**	-1.240	1.020	**2.511**
72	**9**	-1.293	1.167	**2.733**
76	**9.5**	-1.313	1.580	**3.215**
80	**10**	-1.387	1.927	**3.681**
84	**10.5**	-1.507	2.180	**4.096**
87	**10.875**	-1.660	4.780	**7.156**
83	**10.375**	-1.873	6.660	**9.481**
80	**10**	-2.160	12.287	**16.052**

Table 5.14: Moment-Curvature for Doubly Under Reinforced beam-2

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.013	**0.037**
8	**1**	-0.047	0.060	**0.119**
12	**1.5**	-0.160	0.133	**0.326**
16	**2**	-0.180	0.300	**0.533**
20	**2.5**	-0.153	0.360	**0.570**
24	**3**	-0.227	0.373	**0.667**
28	**3.5**	-0.273	0.500	**0.859**
32	**4**	-0.400	0.647	**1.163**
36	**4.5**	-0.480	0.707	**1.319**
40	**5**	-0.560	0.767	**1.474**
44	**5.5**	-0.633	0.853	**1.652**
48	**6**	-0.640	0.920	**1.733**
52	**6.5**	-0.727	0.907	**1.815**
56	**7**	-0.847	0.993	**2.044**
60	**7.5**	-0.953	1.080	**2.259**
64	**8**	-1.040	1.147	**2.430**
68	**8.5**	-1.147	0.987	**2.370**
72	**9**	-1.073	1.187	**2.511**
76	**9.5**	-1.173	1.280	**2.726**
80	**10**	-1.340	1.733	**3.415**
84	**10.5**	-1.547	3.813	**5.956**
88	**11**	-1.940	5.833	**8.637**
82	**10.25**	-2.053	8.373	**11.585**
78	**9.75**	-2.240	11.867	**15.674**

Table 5.12 shows the average Moment and corresponding Curvature for all three singly over reinforced beam. Here, average ultimate moment and corresponding Curvature was observed 14 kNm and 0.127 rad/m. Table 5.13 represent Moment and corresponding Curvature for doubly under reinforced beam-1. From Table 5.13 ultimate Moment and Curvature was 10.875 kNm and 0.715 rad/m. Here first crack was observed at 60 kN load. Table 5.14 shows Moment-Curvature for doubly under reinforced beam-2.

Table 5.15: Moment-Curvature for Doubly Under Reinforced beam-3

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.000	**0.022**
8	**1**	-0.073	0.053	**0.141**
12	**1.5**	-0.140	0.127	**0.296**
16	**2**	-0.213	0.193	**0.452**
20	**2.5**	-0.327	0.260	**0.652**
24	**3**	-0.393	0.320	**0.793**
28	**3.5**	-0.513	0.440	**1.059**
32	**4**	-0.613	0.520	**1.259**
36	**4.5**	-0.667	0.573	**1.378**
40	**5**	-0.740	0.653	**1.548**
44	**5.5**	-0.820	0.707	**1.696**
48	**6**	-0.727	0.867	**1.770**
52	**6.5**	-0.793	0.947	**1.933**
56	**7**	-0.907	1.047	**2.170**
60	**7.5**	-0.980	1.153	**2.370**
64	**8**	-1.047	1.187	**2.481**
68	**8.5**	-1.227	1.307	**2.815**
72	**9**	-1.267	1.420	**2.985**
76	**9.5**	-1.360	1.687	**3.385**
80	**10**	-1.380	2.147	**3.919**
84	**10.5**	-1.647	3.887	**6.148**
87	**10.875**	-1.747	7.560	**10.341**
83	**10.375**	-1.980	9.867	**13.163**
78	**9.75**	-2.213	13.660	**17.637**

Table 5.16: Average Moment-Curvature for Doubly Under Reinforced RC Beam.

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.018	0.016	**0.037**
8	**1**	-0.064	0.067	**0.146**
12	**1.5**	-0.156	0.127	**0.314**
16	**2**	-0.196	0.213	**0.454**
20	**2.5**	-0.253	0.284	**0.598**
24	**3**	-0.404	0.316	**0.800**
28	**3.5**	-0.460	0.416	**0.973**
32	**4**	-0.538	0.513	**1.168**
36	**4.5**	-0.560	0.569	**1.254**
40	**5**	-0.653	0.653	**1.452**
44	**5.5**	-0.731	0.682	**1.570**
48	**6**	-0.696	0.811	**1.674**
52	**6.5**	-0.789	0.858	**1.830**
56	**7**	-0.882	0.956	**2.042**
60	**7.5**	-1.007	1.038	**2.272**
64	**8**	-1.084	1.082	**2.407**
68	**8.5**	-1.204	1.104	**2.565**
72	**9**	-1.211	1.258	**2.743**
76	**9.5**	-1.282	1.516	**3.109**
80	**10**	-1.369	1.936	**3.672**
84	**10.5**	-1.567	3.293	**5.400**
87.33	**10.92**	-1.782	6.058	**8.711**
82.67	**10.33**	-1.969	8.300	**11.410**
78.67	**9.83**	-2.204	12.604	**16.454**

In case of Beam-2 11 kNm and 0.864 rad/m were the observed ultimate moment and curvature. At the load of 56 kN first crack was observed. Table 5.15 shows the Moment Curvature for doubly under reinforced beam-3. Here, ultimate Moment and Curvature was observed 10.875 kN and 0.103 rad/m. Those tables also include strain in concrete as well as strain in steel for Doubly Under Reinforced beam. Table 5.16 shows average moment and curvature for doubly under reinforce RC beam. Average ultimate Moment and curvature of doubly under reinforced beam was observed 10.92 kNm and 0.871 rad/sec. Fig. 5.18 shows the Moment-Curvature relationship for all three doubly under reinforced beam as well as avarege Moment-Curvature. Here, Moment-Curvature is linear up to yielding of reinforcement after this it follows parabolic path. In the case of doubly under reinforced beam degardation in strength was observed after the peak strength. Fig. 5.19 shows the strain in RC element at the top and bottom reinforcement level. Strain were liner up to the yeild then it follow parabolic path. Here also drop in stregth was observed.

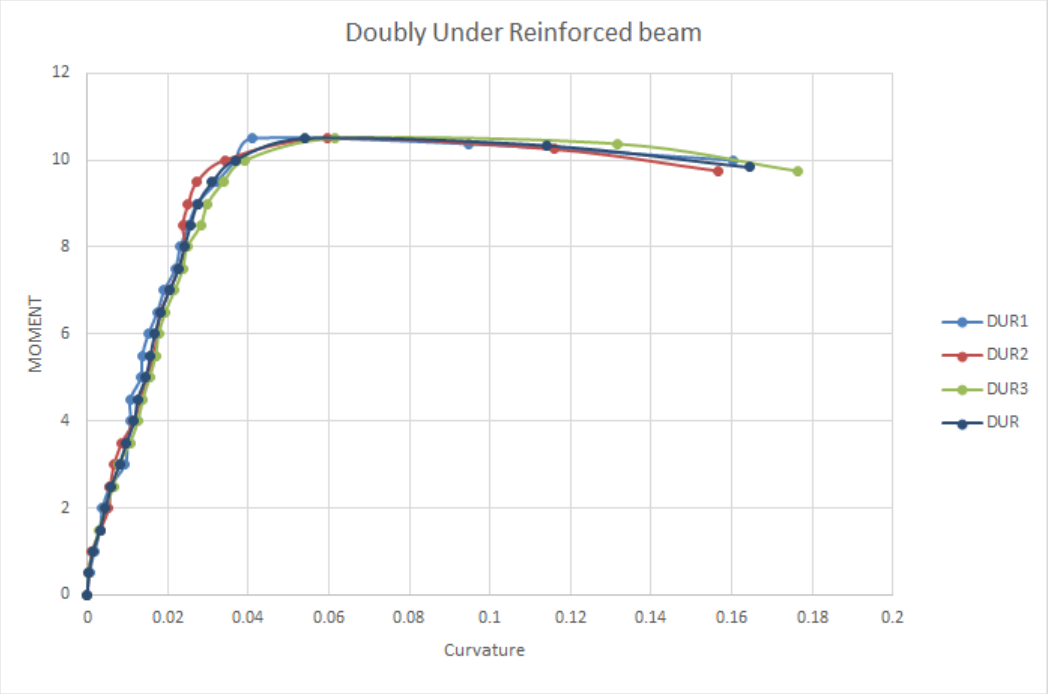

Figure 5.18: Moment-Curvature Doubly Under Reinforced

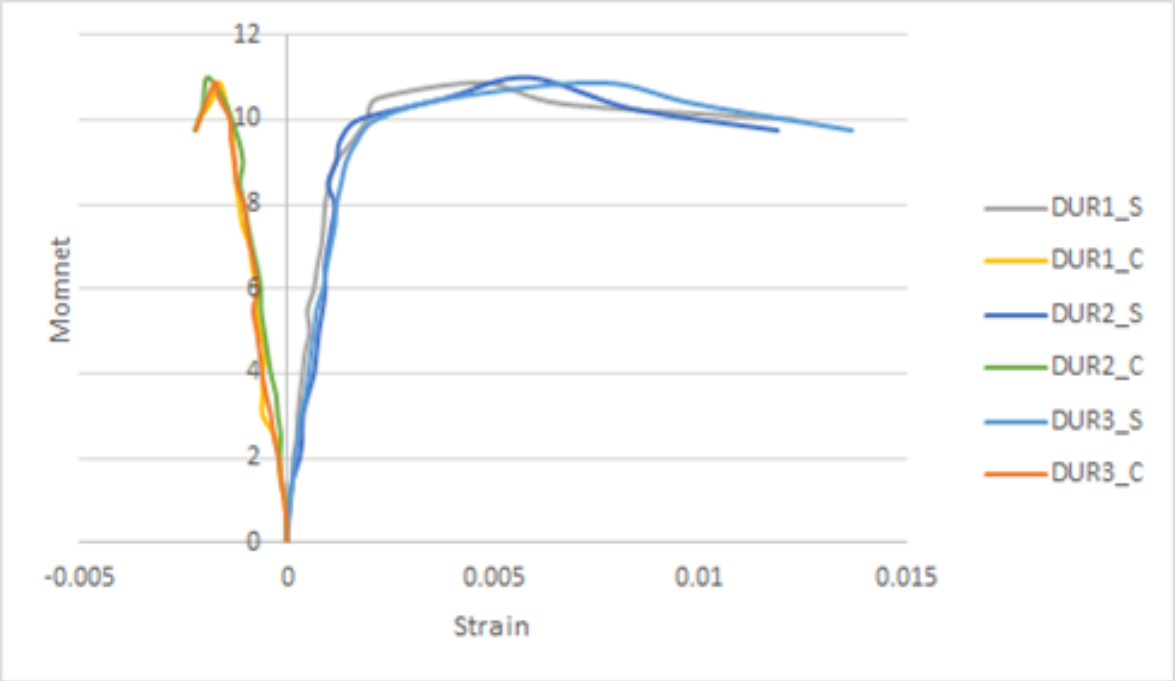

Figure 5.19: Compression Strain and Tensile Strain in Doubly Under Reinforced Beam

Table 5.17 shows Moment and Curvature for doubly over reinforced beam-1. In this case Ultimate Moment and Curvature was observed 13.5 kNm and 0.745 rad/m respectivly. At the 68 kN load first crack was observed. Table 5.18 shows Moment-Curvature for Over Reinforced beam-2. For beam-2 ultimate Moment and Curvature was observed 13.5 kNm and 0.749 rad/m respectivly. Here, first crack was occured at 72 kN load. And Table 5.19 shows the Moment Curvature for Doubly Over Reinforced beam-3. 13.5 kNm and 0.693 rad/m were the ultimate moment and curvature for beam-3. Fig. 5.20 and Fig. 5.21 shows the Moment-Curvature relationship for all three Doubly Over Reinforced beams and it also shows the strain in RC element at the top and bottom reinforcement level. Table 5.20 shows average Moment and Curvature for doubly over reinforced section. Average ultimate moment was observed 13.5 kNm and ultimate curvature was observed 0.728 rad/m.

Table 5.17: Moment-Curvature for Doubly Over Reinforced Beam-1

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.027	0.013	**0.044**
8	**1**	-0.067	0.033	**0.111**
12	**1.5**	-0.100	0.053	**0.170**
16	**2**	-0.087	0.147	**0.259**
20	**2.5**	-0.160	0.227	**0.430**
24	**3**	-0.160	0.280	**0.489**
28	**3.5**	-0.227	0.367	**0.659**
32	**4**	-0.233	0.527	**0.844**
36	**4.5**	-0.273	0.580	**0.948**
40	**5**	-0.300	0.667	**1.074**
44	**5.5**	-0.280	0.673	**1.059**
48	**6**	-0.347	0.733	**1.200**
52	**6.5**	-0.407	0.787	**1.326**
56	**7**	-0.473	0.880	**1.504**
60	**7.5**	-0.487	0.827	**1.459**
64	**8**	-0.607	0.880	**1.652**
68	**8.5**	-0.693	0.993	**1.874**
72	**9**	-0.713	1.000	**1.904**
76	**9.5**	-0.760	1.047	**2.007**
80	**10**	-0.807	1.047	**2.059**
84	**10.5**	-0.820	1.107	**2.141**
88	**11**	-0.940	1.207	**2.385**
92	**11.5**	-1.007	1.273	**2.533**
96	**12**	-1.173	1.620	**3.104**
100	**12.5**	-1.353	1.627	**3.311**
104	**13**	-1.433	2.873	**4.785**
108	**13.5**	-1.660	5.013	**7.415**
106	**13.25**	-2.007	8.507	**11.681**
98	**12.25**	-2.107	11.567	**15.193**

Table 5.18: Moment-Curvature for Doubly Over Reinforced Beam-2

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.040	**0.067**
8	**1**	-0.053	0.080	**0.148**
12	**1.5**	-0.100	0.127	**0.252**
16	**2**	-0.087	0.213	**0.333**
20	**2.5**	-0.073	0.193	**0.296**
24	**3**	-0.140	0.267	**0.452**
28	**3.5**	-0.233	0.353	**0.652**
32	**4**	-0.333	0.427	**0.844**
36	**4.5**	-0.240	0.533	**0.859**
40	**5**	-0.320	0.440	**0.844**
44	**5.5**	-0.280	0.533	**0.904**
48	**6**	-0.460	0.593	**1.170**
52	**6.5**	-0.613	0.613	**1.363**
56	**7**	-0.713	0.747	**1.622**
60	**7.5**	-0.767	0.747	**1.681**
64	**8**	-0.767	0.800	**1.741**
68	**8.5**	-0.867	0.873	**1.933**
72	**9**	-0.927	0.853	**1.978**
76	**9.5**	-0.973	0.940	**2.126**
80	**10**	-1.007	1.013	**2.244**
84	**10.5**	-1.013	1.073	**2.319**
88	**11**	-1.087	1.127	**2.459**
92	**11.5**	-1.253	1.207	**2.733**
96	**12**	-1.373	1.613	**3.319**
100	**12.5**	-1.453	1.727	**3.533**
104	**13**	-1.480	3.553	**5.593**
108	**13.5**	-1.713	5.027	**7.489**
107	**13.375**	-2.053	8.793	**12.052**
100	**12.5**	-2.120	13.000	**16.800**

Table 5.19: Moment-Curvature for Doubly Over Reinforced Beam-3

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	$\phi \times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.013	0.033	**0.052**
8	**1**	-0.060	0.060	**0.133**
12	**1.5**	-0.100	0.147	**0.274**
16	**2**	-0.127	0.100	**0.252**
20	**2.5**	-0.087	0.253	**0.378**
24	**3**	-0.180	0.280	**0.511**
28	**3.5**	-0.233	0.387	**0.689**
32	**4**	-0.340	0.473	**0.904**
36	**4.5**	-0.313	0.567	**0.978**
40	**5**	-0.373	0.633	**1.119**
44	**5.5**	-0.427	0.720	**1.274**
48	**6**	-0.447	0.780	**1.363**
52	**6.5**	-0.507	0.820	**1.474**
56	**7**	-0.567	0.880	**1.607**
60	**7.5**	-0.627	0.933	**1.733**
64	**8**	-0.607	0.987	**1.770**
68	**8.5**	-0.647	0.967	**1.793**
72	**9**	-0.667	1.013	**1.867**
76	**9.5**	-0.807	1.047	**2.059**
80	**10**	-0.907	1.173	**2.311**
84	**10.5**	-0.967	1.227	**2.437**
88	**11**	-1.167	1.573	**3.044**
92	**11.5**	-1.240	1.747	**3.319**
96	**12**	-1.307	1.827	**3.481**
100	**12.5**	-1.360	1.980	**3.711**
104	**13**	-1.400	2.833	**4.704**
108	**13.5**	-1.620	4.647	**6.963**
106	**13.25**	-2.013	8.747	**11.956**
97	**12.125**	-2.160	11.720	**15.422**

Table 5.20: Average Moment-Curvature for Doubly Over Reinforced RC Beam.

Load (kN)	**M (kNm)**	**Top Strain** $\times 10^{-3}$	**Bottom Strain** $\times 10^{-3}$	**C** $\times 10^{-2}$ **(rad/m)**
0	**0**	0	0	**0**
4	**0.5**	-0.020	0.029	**0.054**
8	**1**	-0.060	0.058	**0.131**
12	**1.5**	-0.100	0.109	**0.232**
16	**2**	-0.100	0.153	**0.281**
20	**2.5**	-0.107	0.224	**0.368**
24	**3**	-0.160	0.276	**0.484**
28	**3.5**	-0.231	0.369	**0.667**
32	**4**	-0.302	0.476	**0.864**
36	**4.5**	-0.276	0.560	**0.928**
40	**5**	-0.331	0.580	**1.012**
44	**5.5**	-0.329	0.642	**1.079**
48	**6**	-0.418	0.702	**1.244**
52	**6.5**	-0.509	0.740	**1.388**
56	**7**	-0.584	0.836	**1.578**
60	**7.5**	-0.627	0.836	**1.625**
64	**8**	-0.660	0.889	**1.721**
68	**8.5**	-0.736	0.944	**1.867**
72	**9**	-0.769	0.956	**1.916**
76	**9.5**	-0.847	1.011	**2.064**
80	**10**	-0.907	1.078	**2.205**
84	**10.5**	-0.933	1.136	**2.299**
88	**11**	-1.064	1.302	**2.630**
92	**11.5**	-1.167	1.409	**2.862**
96	**12**	-1.284	1.687	**3.301**
100	**12.5**	-1.389	1.778	**3.519**
104	**13**	-1.438	3.087	**5.027**
108	**13.5**	-1.664	4.896	**7.289**
106.33	**13.25**	-2.024	8.682	**11.896**
98.33	**12.125**	-2.129	12.096	**15.805**

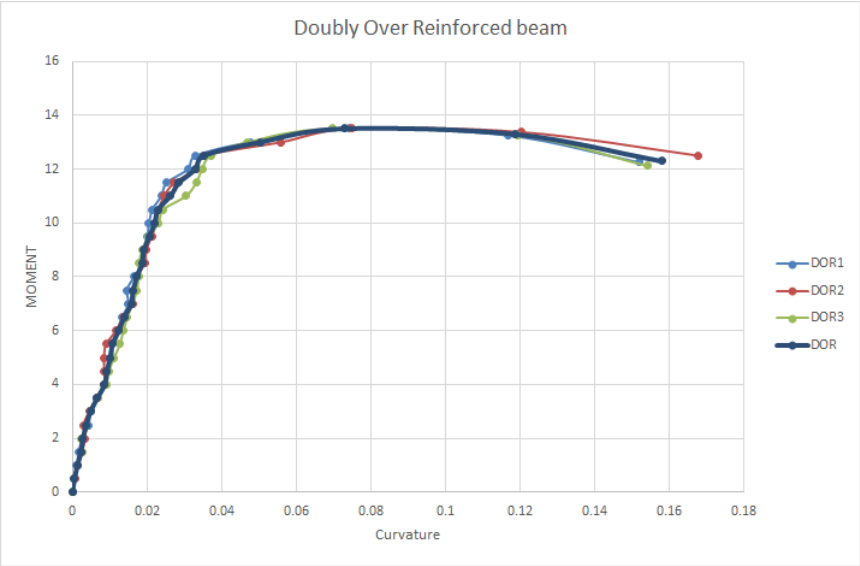

Figure 5.20: Moment-Curvature Doubly Over Reinforced

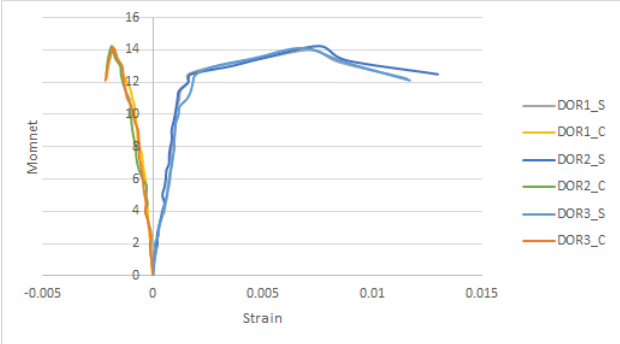

Figure 5.21: Compression Strain and Tensile Strain in Doubly Over Reinforced Beam

5.8 Summary

This chapter includes the concepts of the experimetal evalauation of Moment-Curvature relationship. It also deals with the experimental program. The experimetal program was designed in such way so that the experimental results can be compaired with

the analytical results. Here, test setup, instrumentation for measuring curvature, testing of the concrete cubes, cylider and beams to find out the concrete properties and testing of RC specimens were discusssed in detail. The experimetal curvature obtain from the testing of the RC specimens were also included in this chapter. From the Moment-Curvature one can estimate the deflection, stiffness, and ductility of the beam. Post peak behaviour of the beams can also be predicted.

Chapter 6

Results and Discussion

6.1 General

This chapter deals with the comparison of the analytical results for all four type (Singly Under Reinforced, Singly Over Reinforced, Doubly Under Reinforced and Doubly Over Reinforced) of beam elements with the experimental moment-curvature. This also include the stress-strain diagrams for selected model.

6.2 Stress-Strain Curves

Various existing stress-strain models were used to evaluate Moment-Curvature by using strain compatibility method. IS 456 model, Hongnestad model, Kent and Park Model and IRC model were unconfined models. Cusson Model and Mander Model for confined concrete was used. Fig. 6.1 shows stress-strain curves obtain from these models.

IS code model follows the parabolic path up to the peak strain than it follow straight horizontal line up to the failure. Degradation in strength after the peak was not considered in to the model. But Hongnestad include damage parameter in his model. So hongnestad model shows degradation after the peak. In Kent and Park model ascending part of the stress-strain curve is represented by a second-degree

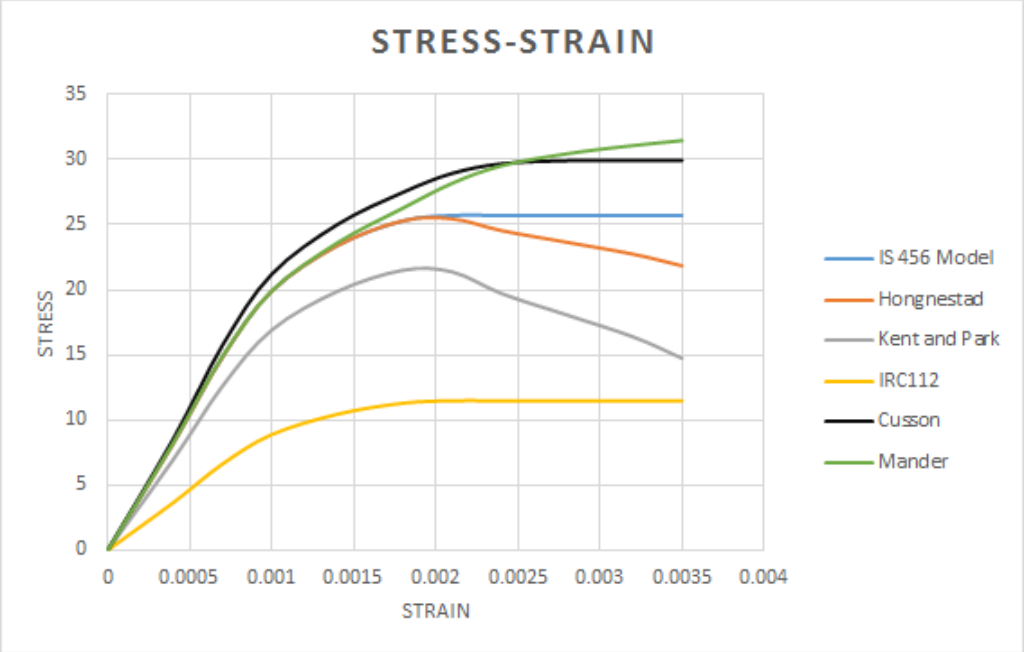

Figure 6.1: Stress-Strain Models

of parabola and here the confining steel has no effect on the shape of this portion. Descending branch depends on the concrete cylinder strength. Stress-strain curve is parabolic in the ascending portion then it follows horizontal line up to failure in the case of material model proposed in IRC112. Fig 6.1 also shows the stress-strain curve obtain by using Cusson Model and Mander Model.

6.3 Experimental and Analytical Comparison of Moment-Curvature

Fig 6.2 shows analytical as well as experimental Moment-Curvature for singly under reinforced beam. Experimental curvature are closer to the analytical values obtain by using IRC model up to the yield, after yielding it follows the value obtains by Hongnestad model. Experimental curvature was parabolic up to the peak than it follows almost straight horizontal line.

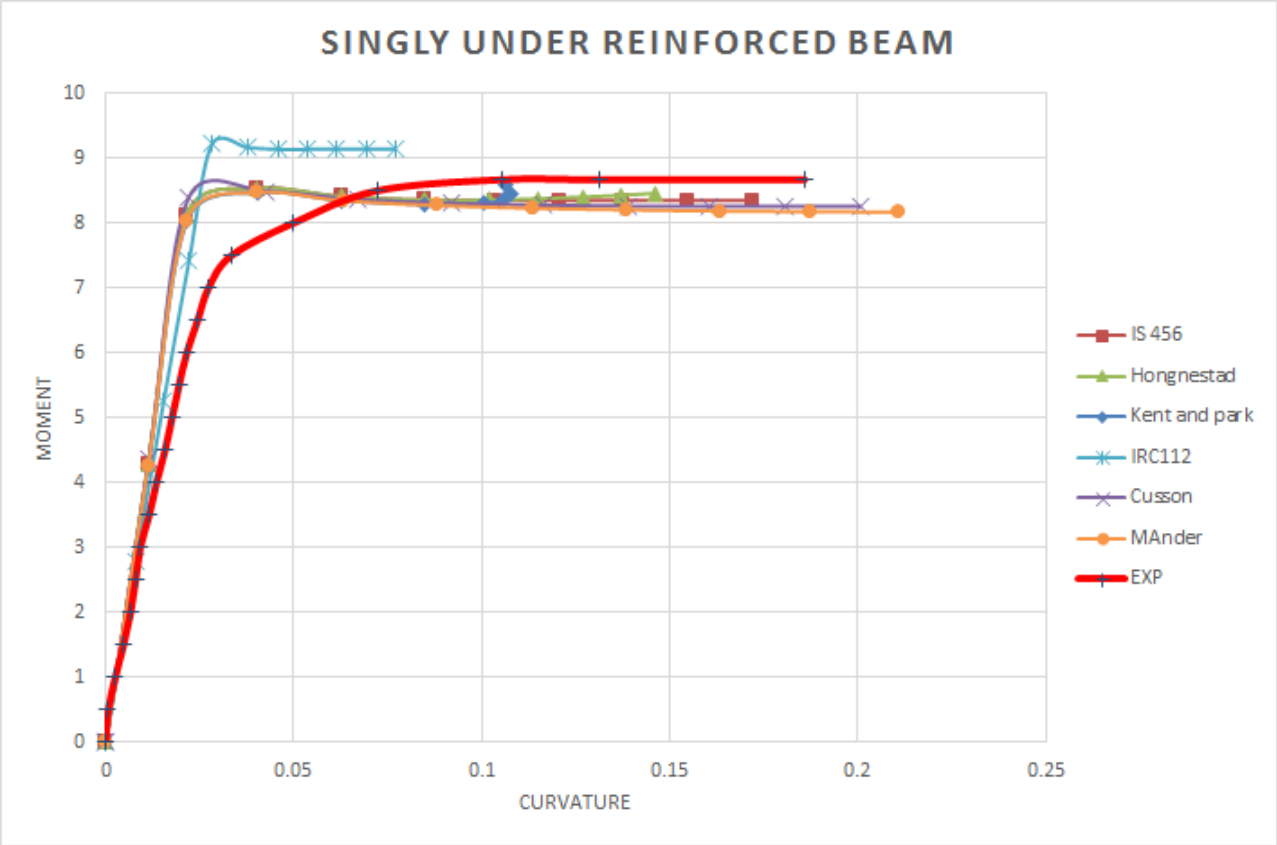

Figure 6.2: Moment-Curvature for Singly Under Reinforced Beam

Till the yielding only hairline cracks was observed but after the yielding of the reinforcement, as the moment was increased cracks were propagates. Cracking of concrete reduced the rigidity of the section due to these deflection of the beam was increased rapidly. Here, it is observed that the experimental Moment-Curvature was almost linear till the yielding than there was a large increment in curvature occurs at nearly constant bending moment. The slowly rise of the bending moment is due to the increment in to the lever arm, That represent tension failure.

Experimental values are observed very closer to the analytical value so only flexure failure was there. In case of shear failure drop in moment carrying capacity of the RC beam was observed.

If the ratio of analytical moment obtain by using IRC model to the experimental

moment was found out. It is observed around 1.05 and ratio of analytical curvature obtain by using IRC model to the experimental curvature was observed 0.8. These values are closer than 1.

If the ratio of ultimate moment evaluated by using IS code model to the experimental ultimate moment was found around 0.96. and ratio of ultimate analytical curvature to the experimental was found 0.97.

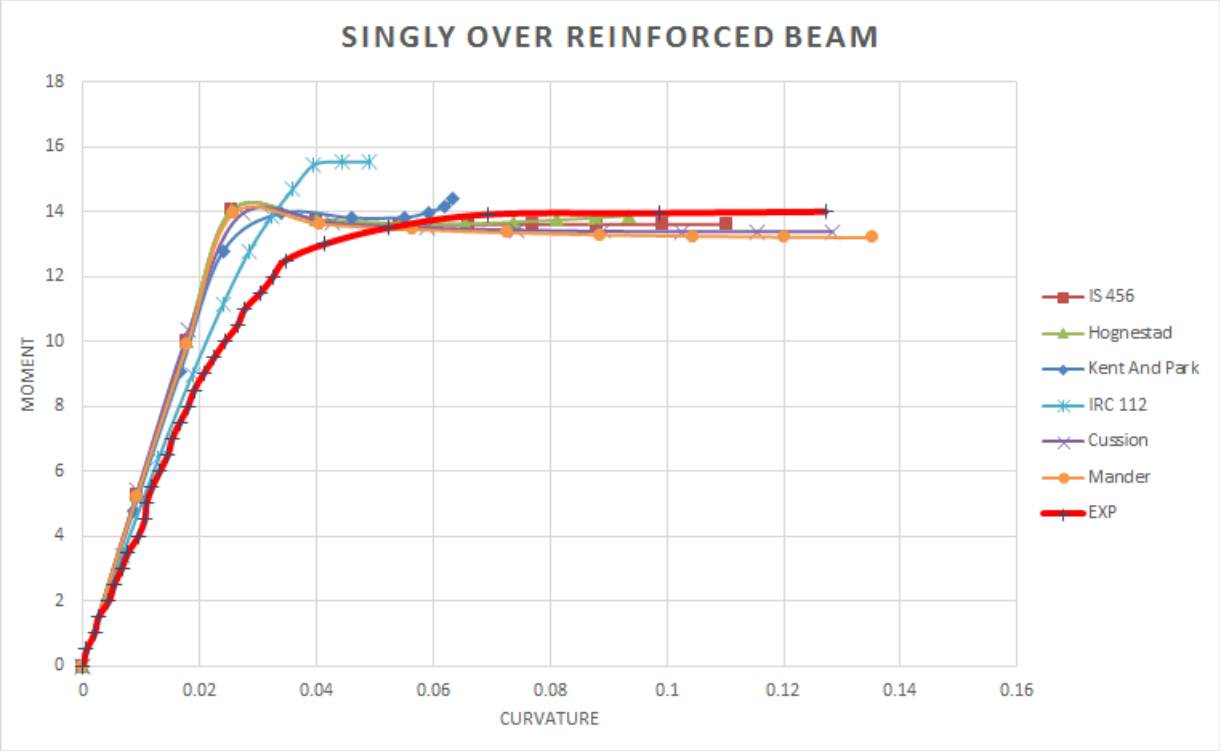

Figure 6.3: Moment-Curvature for Singly Over Reinforced Beam

Fig. 6.3 shows comparison of analytical as well as experimental Moment-Curvature for Singly Over Reinforced Beam. Here,also the analytical Moment-Curvature values obtain by using IRC model are closer to the experimental values up to the yield than it follows the value obtains by IS code model. Experimental curvature was parabolic up to the peak then it follows horizontal line. In the case of Singly Over Reinforced slope of the Moment-Curvature curve in the elastic region is less compare to the under

reinforced beam. Flexural rigidity of the over reinforced section is less compare to the under reinforced beam.

In analytical Moment-Curvature, after the peak there was minor drop in the strength was observed. Here also experimental values are observed very closer to the analytical value so only flexure failure is there. Crushing of the concrete was observed here.

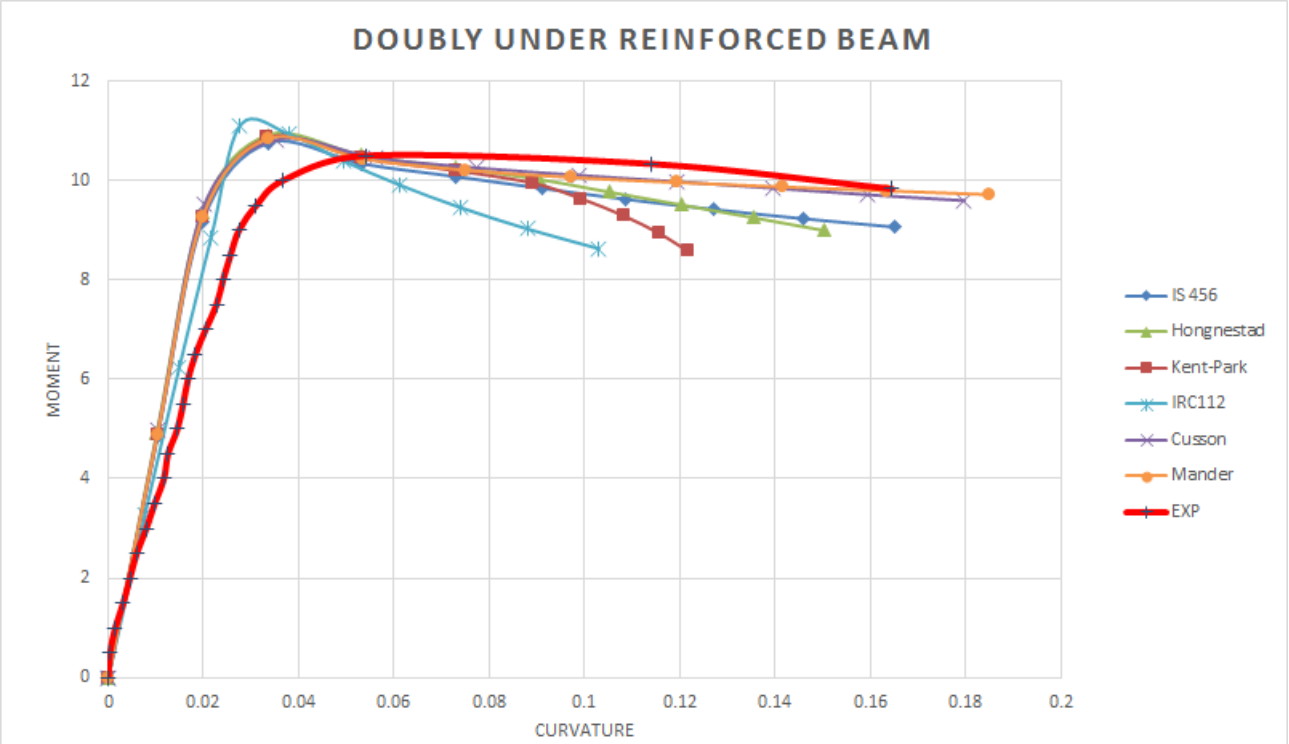

Figure 6.4: Moment-Curvature for Doubly Under Reinforced Beam

If the ratio of analytical moment obtain by using IRC model to the experimental moment was found out. It is observed around 1.022 and ratio of analytical curvature obtain by using IRC model to the experimental curvature was observed 0.814. These values are closer than 1.

If the ratio of ultimate moment evaluated by using IS code model to the experimental ultimate moment was found around 0.978 and ratio of ultimate analytical

curvature to the experimental was found 0.95.

Fig. 6.4 shows Moment-Curvature for Doubly Under Reinforced Beam. Analytical Moment-Curvature values obtain by using IRC model are closer to the experimental values up to the yield than it follows the value obtains by Mander and Cusson model. Experimental curvature was parabolic up to the peak then drop was observed in to strength. Here, Moment-Curvature become nonlinear when the concrete enters the inelastic part of the stress-strain curve. Here cursing of the concrete was observed and after the cursing sudden drop in the strength was observed.

0.96 is the ratio of analytical yielding moment which was obtain by using IRC 112 to the experimental method. and 1.185 is the ratio of analytical yielding curvature which was obtain by using IRC 112 to the experimental method. 0.967 and 1.05 is the ratio of the analytical moment to ultimate experimental moment and analytical curvature to ultimate experimental curvature respectively for doubly under reinforced beam.

Fig 6.5 shows Analytical and experimental Moment-Curvature for Doubly Over Reinforced Beam. Here also Experimental Moment-Curvature follow the analytical value obtain from the IRC model till the yielding was reached. After yielding it follows the values obtain by using Mander model and Cusson model. Experimental curvature was almost linear up to the peak then degradation in strength was observed in to Moment-Caring capacity. Here also cursing of the concrete was observed and after the cursing sudden drop in the strength as well as rapid increase in to the deflection was observed. Here flexural rigidity were almost same as in case of Doubly Under Reinforced section.

1.11 is the ratio of analytical yielding moment which was obtain by using IRC 112 to the experimental method. and 1.16 is the ratio of analytical yielding curvature which was obtain by using IRC 112 to the experimental method. 0.85 and 1.073 is ratio of the analytical moment to ultimate experimental moment and analytical curvature to ultimate experimental curvature respectively for doubly over reinforced beam.

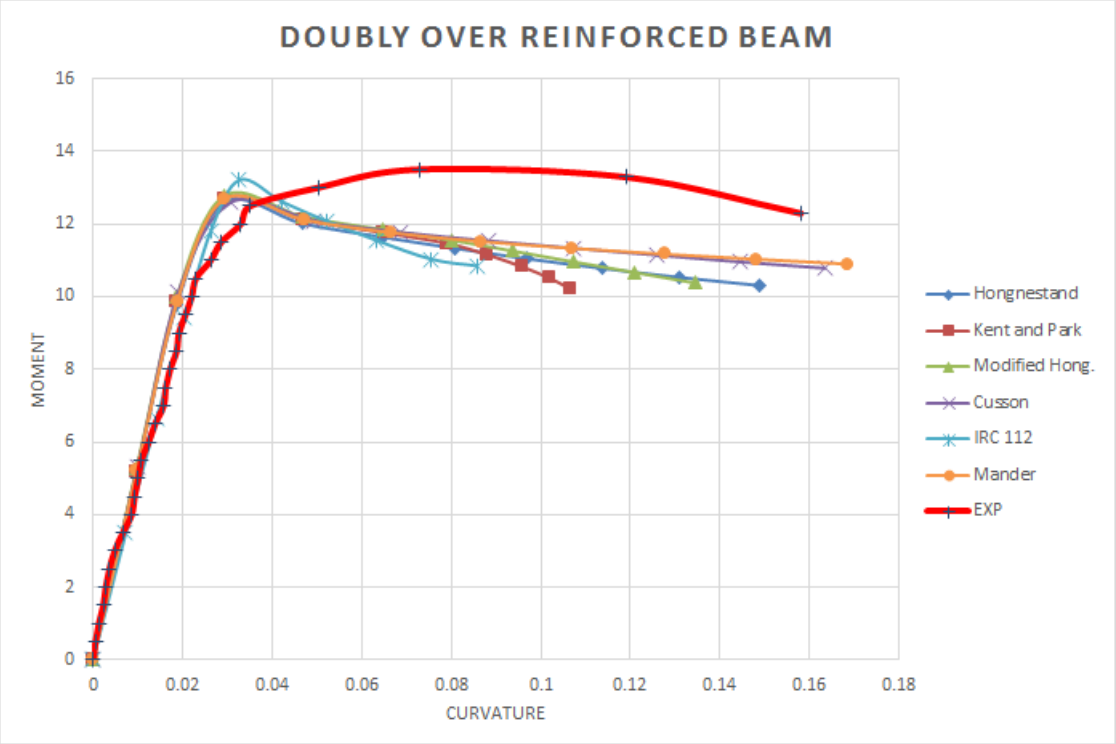

Figure 6.5: Moment-Curvature for Doubly Over Reinforced Beam

The tensile and compressive strains of reinforcement and concrete were measured at every load increment. The strain measurements against the moment for Singly Under Reinforced (SUR),Singly Over Reinforced (SOR), Doubly Under Reinforced (DUR) and Doubly Over Reinforced (DOR) are shown in the Fig 6.6. The negative values represent the compressive strain in the concrete, while the tensile strain in the reinforcement are shown in positive values.

The higher strains in beam may be due to higher deflection due to low modulus of elasticity. The strain were liner in beams until yielding od steel and then rapidly increased before failure. The higher strains in concrete beams also show that good bond between steel and concrete existed till the yielding of steel. the strain, before final failure failure may have been having higher than the strains mentioned here.

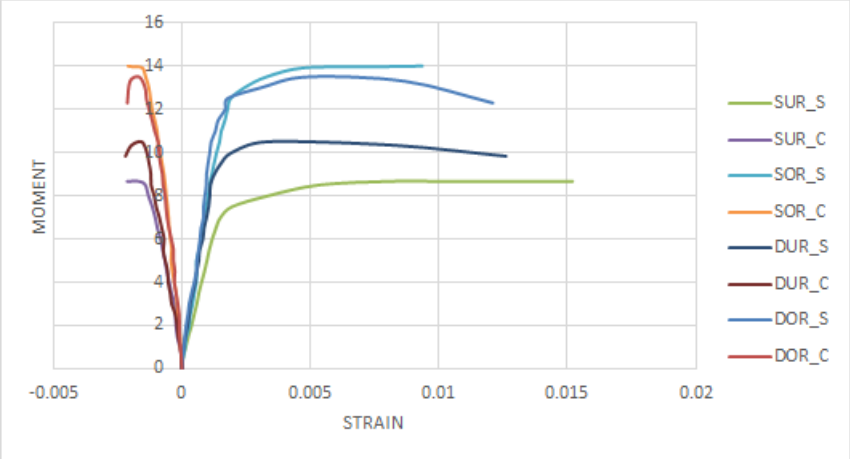

Figure 6.6: Compressive and tensile strain in concrete and steel respectively

6.4 Summary

This chapter include the comparison of experimental and analytical Moment-Curvature for RC beam elements. The strain profile was also discussed in this chapter. Analytical results by IRC model was found closer to the experimental results till the yielding and after yielding experimental results are closer to the analytical results obtain by using IS 456 model for singly reinforced beam and in case of Doubly Reinforced after the yielding, experimental results were closer to the analytical solution obtain by using Mander and Cusson Model.

Chapter 7

Summary and Conclusions

7.1 Summary

In the present study a Matlab program was developed to determine the Moment-Curvature relationship for rectangular RC beam elements by using transform area method as well as strain compatibility method. Transform area method is the approximate method and strain compatibility method is the accurate but iterative method. For evaluation of moment-curvature relationship by strain compatibility method existing material models were used as a stress block.

Experimental program was design to evaluate Moment Curvature relationship for RC rectangular beams. In experimental program casting of twelve reinforced concrete beam element were carried out. In present study singly reinforced as well as doubly reinforced RC specimens were design by limit state method. For measuring curvature, strain was measure at the top and bottom reinforcement level. Moment-Curvature obtain from the experiments was compared with the analytical results obtained from the computer program.

Attempt was also made to evaluate Moment-Curvature relationship for by using various available computational tools. In present study Response 2000 and RC-Analysis were used.

7.2 Conclusions

Following are the important conclusions made from the present study:

- By comparing the analytical results and experimental results it is observed that in the case of Singly Reinforced section the experimental Moment-Curvature follows the analytical results obtain by using IRC 112 model till the yielding of reinforcement. After this the experimental results were found closer to the analytical results obtain by IS 456.
- In the case of Doubly Reinforced section the experimental Moment-Curvature were found closer to the analytical results obtain by using IRC 112 model till the yielding of reinforcement. After this the experimental results follow the analytical results obtain by Mander and Cusson model.
- As the strain was increased, flexure rigidity of beam was decreased and deflection of the beam was increased.
- For both singly reinforced and doubly reinforced section crushing failure of the concrete was observed with the tension failure of RC beam. But in case of under reinforced section tensile cracks were observed first and then crushing failure was occurred. But in case of over reinforced crushing failure was observed first then tensile failure was observed.
- By increasing the amount of tension reinforcement there is a increase in the moment capacity of the section; however the ductility of the structure reduces drastically which is not at all desirable.
- With the increase in the grade of concrete there is a marginal increase in the moment capacity of the section but a substantial increase in the ductility of the section.

7.3 Future Scope of Work

- Moment-Curvature will be evaluated by considering the flexure as well as shear failure.
- Various parametric study will be to estimate the effects of parameters (grade of concrete, cover, grade of steel) on the Moment-Curvature relationship.
- Moment-Curvature relationship can be evaluated for Reinforced Concrete rectangular, square and circular column by using layer approach.
- Experimental evaluation of Moment-Curvature for RC Column elements.
- Moment-Curvature relationship cab be developed by considering various support condition for RC elements.
- Moment-Curvature will be evaluated for 2D Frame.

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Appendix A

Matlab Program by Transform Area Method

Moment-Curvature Relationship for Doubly Reinforced by Transform Area Method

```
Ast=input('Enter the bottom Reinforcement area(mm2):')
Asc=input('Enter the top reinforcement area(mm2):')
ecu=input('Enter the Extreme compression strain:')
f=input('MOdulus of rupture:')
Fy=input('yield strength of steel:')
b=input('width of the beam:')
D=input('depth of the beam:')
Fc=input('Cylinder strength of concrete:')
Ec=input('Modulus of the elasticity of concrete:')
Es=input('Modulus of the elasticity of concrete:')
di=input('Clear Cover:')
diab=input('diameter of top bar:')
diat=input('diameter of botom bar:')
M1=0
```

```
phi1=0
d=D-di
As=Ast+Asc

%step 1:
%First Crack

n=Es/Ec
M=[]
phi=[]
M=[M;M1]
phi=[phi;phi1]
%Using transformed area
A=b*D+(n-1)*As
Y=((b*D*D/2)+(Ast*(D-(di+diab/2)))+(Asc*(di+diat/2)))/(A)
I=((b*D^3)/12)+(b*D*((D/2-Y)^2))+
    (Ast*(((D-(di+diab/2))-Y)^2))+(Asc*(((di+diat/2)-Y)^2))
M2=(f*I)*10^-6/(Y)
phi2=(f/(Ec*Y))*10^3
M=[M;M2]
phi=[phi;phi2]

%step 2
%At first Yield

del_t=Ast/(b*D)
del_c=Asc/(b*D)
K=(((del_t+del_c)^2*n^2)+2*((del_t+(del_c*di/D))*n))^0.5-(del_t+del_c)*n
kd=K*D
```

```
es=Fy/Es
ec=es*(kd)/(d-kd)
fc=ec*Ec
esc=ec*(kd-di)/(kd)
fs=esc*Es
Cc=(fc*b*kd)/2
Cs=Asc*fs
C=Cc+Cs
j=(Cs*(di+diat)+(Cc*kd/3))/(C)
jd=d-j
M3=Fy*Ast*jd*10^-6
phi3=(10^3*es)/(d*(1-K))
M=[M;M3]
phi=[phi;phi3]

%At Ultimate

a=Fy*(Ast-Asc)/(0.85*Fc*b)
c=a/0.85
ess=ecu*(c-di)/(c)
fss=ess*Es
M4=(0.85*Fc*a*b*(d-a/2)+Asc*fss*(d-di))*10^-6
phi4=ecu*10^3/c
M=[M;M4]
phi=[phi;phi4]
plot(phi,M),grid
xlabel('Curvature of the section (phi)');
ylabel('Moment of resistance of the section (M) in kN-m');
title('Moment Curvature Relation For Over Reinforced section');
```

Moment-Curvature Relationship for Singly Reinforced by Transform Area Method

```
Ast=input('Enter the bottom Reinforcement area(mm2):')
ecu=input('Enter the Extreme compression strain:')
f=input('Modulus of rupture:')
Fy=input('yield strength of steel:')
b=input('width of the beam:')
D=input('depth of the beam:')
Fc=input('Cylinder strength of concrete:')
Ec=input('Modulus of the elasticity of concrete:')
Es=input('Modulus of the elasticity of concrete:')
di=input('Clear Cover:')
diab=input('diameter of top bar:')
diat=input('diameter of botom bar:')
M1=0
phi1=0
d=D-di
Ast=As
%step 1:
%First Crack
n=Es/Ec
M=[]
phi=[]
M=[M;M1]
phi=[phi;phi1]
%Using transformed area
A=b*D+(n-1)*Ast
Y=((b*D*D/2)+(As*(D-(di+diab/2))))/(A)
I=((b*D^3)/12)+(b*D*((D/2-Y)^2))+(As*((((D-di+diab/2))-Y)^2))
```

```
M2=(f*I)*10^-6/(Y)
phi2=(f/(Ec*Y))*10^3
M=[M;M2]
phi=[phi;phi2]

%step 2
%At first Yield

del_t=As/(b*D)
J=solve(K^2+2*n*del_t*K-2*n*del_t == 0)
K=input('Value of K from eq. K^2+2*n*del*K - 2*n*del = 0')
kd=K*d
es=Fy/Es
ec=es*(kd)/(d-kd)
fc=ec*Ec
Cc=(fc*b*kd)/2
C=Cc
jd=d-(kd/3)
M3=Fy*As*jd*10^-6
phi3=(10^3*es)/(d*(1-K))
M=[M;M3]
phi=[phi;phi3]

%At Ultimate

a=Fy*(As)/(0.85*Fc*b)
c=a/0.85
M4=As*Fy*(d-0.5*a)*10^-6
phi4=ecu*10^3/c
```

```
M=[M;M4]
phi=[phi;phi4]

plot(phi,M),grid
xlabel('Curvature of the section (phi)');
ylabel('Moment of resistance of the section (M) in kN-m');
title('Moment Curvature Relation For Over Reinforced section');
```

Appendix B

Matlab Program by Strain Compatibility Method

Moment-Curvature Relationship for Singly Reinforced Beam

```
clc
syms epsi_int z alpha gamma fc c_c X_u epsi_sh epsi_su fsu r m x
b=input('width of the beam:')
D=input('depth of the beam:')
Asc=input('Enter the top Reinforcement area(mm2):')
Ast=input('Enter the bottom Reinforcement area(mm2):')
co=input('Clear Cover:')
fcc=input('Cube strength of concrete:')
ecu=input('Enter the Extreme compression strain:')
fy=input('yield strength of steel:')
fc=input('Cylinder strength of concrete:')
Es=input('Modulus of the elasticity of concrete:')
sh=input('Spacing of Stirrups:')
fsu=input('Ultimate strength of steel:')
s_d=input('diameter of stirrups:')
```

```
b_s=input('diameter of bottom bar:')
dprime=h-co
d=h-2*co
b=B-2*co
deff=h-co-s_d-b_s/2
b_b=b-s_d
d_d=d-s_d
epsi_y=(fy/E_s)
epsi_sh=16*epsi_y
M=[]
phi=[]
for (i=1:10)
    epsi_int=(epsi_ult/10)*i
    X_u=0.5*dprime
    if (epsi_int<0.002)
       fc=((2*epsi_int/0.002)-(epsi_int/0.002)^2)*fcc
    else (0.002 <= epsi_int <= 0.004)
       fc=fcc
    end

    alpha=fc/fcc
    gamma=0.5
    for(j=1:5000)

        c_c=alpha*fcc*B*X_u*10^-3
        epsi_s=epsi_int*((deff-X_u)/(X_u))
            if (epsi_s<epsi_y)
                f_s=epsi_s*E_s
            else (epsi_y <= epsi_s < epsi_sh)
```

```
                f_s=fy
            end
        T=Ast*f_s*10^-3
            p=abs(c_c-T)

            if (p<=0.05)
               Mu=c_c*(deff-gamma*X_u)*10^-3+T*X_u*10^-3
               M(i+1)=Mu
                phiu=(epsi_int*10^3)/(X_u)
                phi(i+1)=phiu
                break;
             else
                X_u=X_u-0.01

             end
    j=j+1
    end
    i=i+1
end
plot(phi,M),grid
xlabel('Curvature of the section (phi)');
ylabel('Moment of resistance of the section (M) in kN-m');
title('Moment Curvature Relation For Over Reinforced section');
```

Moment-Curvature Relationship for Doubly Reinforced Beam

```
clc
syms epsi_int z alpha gamma fc c_c X_u epsi_sh epsi_su fsu r m x
b=input('width of the beam:')
D=input('depth of the beam:')
Asc=input('Enter the top Reinforcement area(mm2):')
Ast=input('Enter the bottom Reinforcement area(mm2):')
co=input('Clear Cover:')
fcc=input('Cube strength of concrete:')
ecu=input('Enter the Extreme compression strain:')
fy=input('yield strength of steel:')
fc=input('Cylinder strength of concrete:')
Es=input('Modulus of the elasticity of concrete:')
sh=input('Spacing of Stirrups:')
fsu=input('Ultimate strength of steel:')
s_d=input('diameter of stirrups:')
b_s=input('diameter of bottom bar:')
dprime=h-co
d=h-2*co
b=B-2*co
deff=h-co-s_d-b_s/2
b_b=b-s_d
d_d=d-s_d
epsi_y=(fy/E_s)
epsi_sh=1.5*epsi_y
M=[]
phi=[]
for (i=1:10)
    epsi_int=(epsi_ult/10)*i
```

```
X_u=0.5*dprime
if (epsi_int<0.002)
   fc=((2*epsi_int/0.002)-(epsi_int/0.002)^2)*fcc
else (0.002 <= epsi_int <= 0.004)
   fc=fcc
end
    alpha=fc/fcc
    gamma=0.5
 for(j=1:5000)

    c_c=alpha*fcc*B*X_u*10^-3
    epsi_s=epsi_int*((deff-X_u)/(X_u))
        if (epsi_s<epsi_y)
            f_s=epsi_s*E_s
        else (epsi_y <= epsi_s < epsi_sh)
            f_s=fy
        end
        epsi_sc=epsi_int*((X_u-co)/(X_u))
        if (epsi_sc<epsi_y)
            f_sc=epsi_sc*E_s
        else (epsi_y <= epsi_sc < epsi_sh)
            f_sc=fy
        end
    c_t= Asc*f_sc*10^-3
    c=c_c+c_t
    T=Ast*f_s*10^-3
        p=abs(c-T)

        if (p<=0.05)
```

```
                Mu=c_c*(deff-gamma*X_u)*10^-3+c_t*(X_u-co-s_d)*10^-3+T*X_u*10^-3
                M(i+1)=Mu
                phiu=(epsi_int*10^3)/(X_u)
                phi(i+1)=phiu
                break;
              else
                 X_u=X_u-0.01


              end
    j=j+1
    end
    i=i+1
end
plot(phi,M),grid
xlabel('Curvature of the section (phi)');
ylabel('Moment of resistance of the section (M) in kN-m');
title('Moment Curvature Relation For Over Reinforced section');
```

Appendix C

Experimental Results

Strain was measured by mechanical strain gauges. Average strain from top three strain gauge was consider as the compressive strain and average strain from bottom three strain gauge, tensile strain was evaluated. Here ε_1, ε_3, ε_5 were represent top strain. And ε_2, ε_4, ε_6 shows bottom strain. ε_c and ε_t repentant average compressive strain and average tensile strain. Fig. 5.9 shows the positions of the strain gauges.

Table C.1: Strain in Singly Under Reinforced Beam-1

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.06	-0.02	0.04	-0.02	0.06	-0.020	0.053
8	-0.08	0.12	-0.1	0.1	-0.06	0.12	-0.080	0.113
12	-0.2	0.14	-0.16	0.18	-0.14	0.16	-0.167	0.160
16	-0.18	0.2	-0.18	0.28	-0.2	0.26	-0.187	0.247
20	-0.3	0.6	-0.2	0.36	-0.26	0.26	-0.253	0.407
24	-0.24	0.7	-0.3	0.48	-0.28	0.28	-0.273	0.487
28	-0.32	0.9	-0.4	0.6	-0.38	0.36	-0.367	0.620
32	-0.4	1.04	-0.46	0.8	-0.46	0.38	-0.440	0.740
36	-0.48	1.12	-0.54	0.86	-0.58	0.44	-0.533	0.807
40	-0.56	1.2	-0.62	0.84	-0.64	0.46	-0.607	0.833
44	-0.68	1.42	-0.7	0.86	-0.76	0.54	-0.713	0.940
48	-0.64	1.58	-0.76	0.94	-0.86	0.68	-0.753	1.067
52	-0.84	1.82	-0.86	1.06	-0.98	0.82	-0.893	1.233
56	-0.94	1.8	-0.92	1.18	-1.08	1.4	-0.980	1.460
60	-1.2	2.74	-1.02	1.48	-1.12	1.4	-1.113	1.873
64	-1.48	4.36	-1.3	3.6	-1.28	2.74	-1.353	3.567
68	-1.7	9.88	-1.44	5.58	-1.32	4.42	-1.487	6.627
70	-1.64	11.76	-1.8	8.82	-1.7	6.4	-1.713	8.993
70	-2.22	14.26	-1.9	11.6	-1.98	7.62	-2.033	11.160
70	-2	19.76	-2.24	16.3	-2.08	14.74	-2.107	16.933

Table C.2: Strain in Singly Under Reinforced Beam-2

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.02	0	0.04	-0.02	0.04	-0.013	0.033
8	-0.12	0.26	-0.08	0.2	-0.1	0.16	-0.100	0.207
12	-0.16	0.58	-0.22	0.36	-0.2	0.24	-0.193	0.393
16	-0.26	0.78	-0.4	0.38	-0.22	0.36	-0.293	0.507
20	-0.1	0.94	-0.38	0.48	-0.5	0.16	-0.327	0.527
24	-0.2	0.92	-0.16	0.34	-0.34	0.2	-0.233	0.487
28	-0.24	1.36	-0.4	0.44	-0.5	0.32	-0.380	0.707
32	-0.36	0.96	-0.56	0.66	-0.66	0.32	-0.527	0.647
36	-0.5	1.1	-0.68	0.78	-0.74	0.4	-0.640	0.760
40	-0.62	1.18	-0.78	0.92	-0.78	0.54	-0.727	0.880
44	-0.74	1.34	-0.94	1.04	-0.74	0.72	-0.807	1.033
48	-0.86	1.4	-1.04	1.2	-0.82	0.82	-0.907	1.140
52	-0.98	1.5	-1.14	1.2	-0.94	1.16	-1.020	1.287
56	-1.04	1.6	-1.18	1.38	-1.14	1.36	-1.120	1.447
60	-1.14	1.66	-1.38	1.62	-1.16	1.46	-1.227	1.580
64	-1.22	2.34	-1.4	1.76	-1.42	2.36	-1.347	2.153
68	-1.38	2.9	-1.44	3.12	-1.34	2.76	-1.387	2.927
69	-1.52	10.12	-2.08	5.8	-1.58	4.68	-1.727	6.867
69	-1.7	11.12	-1.98	10.32	-2.46	8.32	-2.047	9.920
69	-1.84	11.62	-2.44	18.1	-2.06	15.54	-2.113	15.087

Table C.3: Strain in Singly Reinforced Beam 3

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.02	-0.02	0.04	-0.02	0.04	-0.02	0.033
8	-0.08	0.1	-0.08	0.1	-0.08	0.14	-0.080	0.113
12	-0.16	0.12	-0.26	0.18	-0.2	0.26	-0.207	0.187
16	-0.18	0.18	-0.22	0.42	-0.3	0.46	-0.233	0.353
20	-0.3	0.62	-0.24	0.28	-0.1	0.54	-0.213	0.480
24	-0.38	0.84	-0.38	0.4	-0.24	1.02	-0.333	0.753
28	-0.6	0.72	-0.5	0.6	-0.4	0.6	-0.500	0.640
32	-0.7	1.2	-0.66	0.62	-0.2	0.86	-0.520	0.893
36	-0.8	1.76	-0.8	0.76	-0.22	0.64	-0.607	1.053
40	-0.86	1.88	-0.88	0.88	-0.28	0.84	-0.673	1.200
44	-0.96	1.96	-0.92	0.96	-0.32	0.7	-0.733	1.207
48	-1.02	2.04	-0.98	1.06	-0.52	0.82	-0.840	1.307
52	-1.16	2.18	-1.1	1.18	-0.62	0.92	-0.960	1.427
56	-1.18	2.32	-1.2	1.32	-0.72	1.14	-1.033	1.593
60	-1.3	2.8	-1.26	1.4	-0.86	3.06	-1.140	2.420
64	-1.36	5.74	-1.3	1.62	-1	5.7	-1.220	4.353
68	-1.58	10.8	-1.78	2	-0.9	6.58	-1.420	6.460
69	-1.92	10.72	-1.7	4.4	-1.08	10.82	-1.567	8.647
69	-2	12.14	-2	4.62	-2.22	11.6	-2.073	9.453
69	-2.24	17.7	-2.3	6.74	-2	16.3	-2.180	13.580

Table C.4: Strain in Singly Over Reinforced Beam-1

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.04	0	0.04	-0.02	0.02	-0.013	0.033
8	-0.04	0.16	-0.1	0.12	-0.1	0.1	-0.080	0.127
12	-0.06	0.22	-0.06	0.2	-0.12	0.18	-0.080	0.200
16	-0.14	0.34	-0.1	0.26	-0.1	0.32	-0.113	0.307
20	-0.18	0.26	-0.16	0.3	-0.14	0.32	-0.160	0.293
24	-0.16	0.46	-0.12	0.38	-0.16	0.36	-0.147	0.400
28	-0.18	0.54	-0.2	0.44	-0.28	0.5	-0.220	0.493
32	-0.22	0.42	-0.24	0.5	-0.38	0.68	-0.280	0.533
36	-0.46	0.5	-0.48	0.52	-0.48	0.6	-0.473	0.540
40	-0.5	0.46	-0.5	0.46	-0.46	0.48	-0.487	0.467
44	-0.32	0.62	-0.42	0.56	-0.38	0.58	-0.373	0.587
48	-0.42	0.66	-0.46	0.68	-0.4	0.7	-0.427	0.680
52	-0.46	0.74	-0.62	0.78	-0.46	0.8	-0.513	0.773
56	-0.54	0.82	-0.5	0.84	-0.52	0.86	-0.520	0.840
60	-0.6	0.88	-0.52	0.88	-0.56	0.9	-0.560	0.887
64	-0.56	0.96	-0.6	0.94	-0.64	0.98	-0.600	0.960
68	-0.62	1.02	-0.62	1.02	-0.54	1.06	-0.593	1.033
72	-0.5	1.06	-0.5	1.06	-0.66	1.16	-0.553	1.093
76	-0.64	1.16	-0.66	1.12	-0.7	1.04	-0.667	1.107
80	-0.7	1.28	-0.74	1.26	-0.74	1.22	-0.727	1.253
84	-0.8	1.36	-0.76	1.36	-0.8	1.3	-0.787	1.340
88	-0.74	1.44	-0.82	1.46	-0.84	1.42	-0.800	1.440
92	-0.96	1.54	-0.9	1.56	-0.94	1.48	-0.933	1.527
96	-1.06	1.52	-1	1.62	-1.06	1.26	-1.040	1.467
100	-1.02	1.4	-1.06	1.46	-1.02	1.36	-1.033	1.407
104	-0.98	1.56	-1	1.54	-1.04	2.16	-1.007	1.753
108	-1.06	2.38	-1.18	1.46	-1.22	2.48	-1.153	2.107
112	-1.22	2.82	-1.22	2.26	-1.26	4.62	-1.233	3.233
113	-1.64	6.56	-1.64	4.64	-1.7	10.66	-1.660	7.287
113	-2.06	8.76	-2.06	7	-2.06	13.02	-2.060	9.593

Table C.5: Strain in Singly Over reinforced Beam-2

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_6	ε_c	ε_t	ε_1
0	0	0	0	0	0	0	0	0
4	-0.02	0.04	-0.02	0.1	-0.04	0.02	-0.027	0.053
8	0	0.12	-0.04	0.12	-0.12	0.04	-0.053	0.093
12	-0.02	0.08	-0.06	0.14	-0.08	0.14	-0.053	0.120
16	-0.08	0.2	-0.06	0.26	-0.16	0.4	-0.100	0.287
20	-0.1	0.28	-0.14	0.38	-0.32	0.34	-0.187	0.333
24	-0.14	0.34	-0.22	0.5	-0.36	0.4	-0.240	0.413
28	-0.1	0.46	-0.26	0.42	-0.28	0.46	-0.213	0.447
32	-0.22	0.72	-0.3	0.68	-0.36	0.68	-0.293	0.693
36	-0.18	0.56	-0.26	0.76	-0.3	0.72	-0.247	0.680
40	-0.3	0.62	-0.34	0.9	-0.36	0.74	-0.333	0.753
44	-0.34	0.68	-0.44	0.96	-0.42	0.8	-0.400	0.813
48	-0.38	0.8	-0.42	1.04	-0.48	0.88	-0.427	0.907
52	-0.48	0.9	-0.46	1.1	-0.54	0.94	-0.493	0.980
56	-0.5	1	-0.5	1	-0.52	1	-0.507	1.000
60	-0.56	0.98	-0.54	1.28	-0.62	1.08	-0.573	1.113
64	-0.66	1.1	-0.6	1.34	-0.66	1.16	-0.640	1.200
68	-0.72	1.14	-0.7	1.42	-0.82	1.14	-0.747	1.233
72	-0.78	1.22	-0.8	1.6	-0.96	1.24	-0.847	1.353
76	-0.84	1.3	-0.94	1.84	-1.08	1.38	-0.953	1.507
80	-0.82	1.38	-1.06	1.9	-1.22	1.44	-1.033	1.573
84	-0.84	1.48	-1.14	2.1	-1.36	1.6	-1.113	1.727
88	-0.9	1.6	-1.2	2.18	-1.46	1.76	-1.187	1.847
92	-0.96	1.74	-1.34	2.28	-1.52	2.06	-1.273	2.027
96	-1.12	1.88	-1.42	2.38	-1.44	2.26	-1.327	2.173
100	-1.3	1.98	-1.48	2.48	-1.56	2.8	-1.447	2.420
104	-1.48	3.6	-1.56	2.6	-1.62	2.92	-1.553	3.040
108	-1.7	5	-1.62	5.06	-1.64	3.14	-1.653	4.400
110	-1.7	7.44	-1.66	6.2	-1.68	3.74	-1.680	5.793
109	-1.8	11.18	-2.08	8.24	-2.1	4.24	-1.993	7.887
110	-2.04	13.38	-2.1	9.6	-2.12	5.6	-2.087	9.527

Table C.6: Strain in Singly Over reinforced Beam-3

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.02	-0.02	0.04	-0.02	0.04	-0.020	0.033
8	-0.06	0.14	-0.04	0.14	-0.1	0.14	-0.067	0.140
12	-0.1	0.2	-0.08	0.16	-0.12	0.18	-0.100	0.180
16	-0.2	0.32	-0.12	0.28	-0.1	0.22	-0.140	0.273
20	-0.22	0.24	-0.16	0.36	-0.14	0.34	-0.173	0.313
24	-0.28	0.44	-0.12	0.4	-0.16	0.44	-0.187	0.427
28	-0.32	0.52	-0.1	0.48	-0.28	0.5	-0.233	0.500
32	-0.4	0.4	-0.24	0.36	-0.38	0.56	-0.340	0.440
36	-0.32	0.48	-0.44	0.6	-0.48	0.48	-0.413	0.520
40	-0.36	0.44	-0.54	0.46	-0.46	0.48	-0.453	0.460
44	-0.42	0.6	-0.5	0.7	-0.38	0.52	-0.433	0.607
48	-0.46	0.64	-0.58	0.68	-0.46	0.58	-0.500	0.633
52	-0.5	0.7	-0.54	0.74	-0.48	0.6	-0.507	0.680
56	-0.54	0.74	-0.56	0.78	-0.54	0.66	-0.547	0.727
60	-0.56	0.82	-0.66	0.86	-0.44	0.74	-0.553	0.807
64	-0.58	0.9	-0.62	0.9	-0.6	0.82	-0.600	0.873
68	-0.62	0.9	-0.7	0.96	-0.54	0.9	-0.620	0.920
72	-0.68	1.04	-0.78	1.06	-0.66	1.02	-0.707	1.040
76	-0.78	1.14	-0.66	1.16	-0.7	1.08	-0.713	1.127
80	-0.86	1.2	-0.74	1.22	-0.78	1.18	-0.793	1.200
84	-0.92	1.28	-0.86	1.38	-0.86	1.28	-0.880	1.313
88	-0.96	1.36	-0.96	1.26	-0.84	1.28	-0.920	1.300
92	-1.04	1.5	-1.14	1.44	-0.8	1.46	-0.993	1.467
96	-1.12	1.96	-1.26	1.52	-1.02	1.54	-1.133	1.673
100	-1.18	2.38	-1.4	1.56	-1.04	1.64	-1.207	1.860
104	-1.28	4.24	-1.52	1.62	-1.14	1.66	-1.313	2.507
108	-1.48	6.36	-1.62	1.86	-1.22	1.88	-1.440	3.367
112	-1.58	6.8	-1.82	3.26	-1.72	5.28	-1.707	5.113
113	-1.76	7.54	-1.96	4.64	-1.94	5.66	-1.887	5.947
113	-2.04	9.74	-2.14	8	-2.26	9.02	-2.147	8.920

Table C.7: Strain in Doubly Under Reinforced Beam-1

Load(kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.02	0	0.06	-0.02	0.02	-0.013	0.033
8	-0.06	0.12	-0.04	0.1	-0.12	0.04	-0.073	0.087
12	-0.14	0.14	-0.18	0.14	-0.18	0.08	-0.167	0.120
16	-0.2	0.18	-0.14	0.16	-0.24	0.1	-0.193	0.147
20	-0.3	0.22	-0.28	0.32	-0.26	0.16	-0.280	0.233
24	-0.32	0.26	-0.32	0.36	-1.14	0.14	-0.593	0.253
28	-0.38	0.32	-0.44	0.4	-0.96	0.2	-0.593	0.307
32	-0.46	0.38	-0.52	0.5	-0.82	0.24	-0.600	0.373
36	-0.52	0.42	-0.6	0.58	-0.48	0.28	-0.533	0.427
40	-0.6	0.62	-0.68	0.68	-0.7	0.32	-0.660	0.540
44	-0.7	0.7	-0.76	0.4	-0.76	0.36	-0.740	0.487
48	-0.48	0.98	-0.8	0.5	-0.88	0.46	-0.720	0.647
52	-0.74	1.06	-0.88	0.62	-0.92	0.48	-0.847	0.720
56	-0.76	1.16	-1.02	0.7	-0.9	0.62	-0.893	0.827
60	-1.06	1.12	-1.08	0.84	-1.12	0.68	-1.087	0.880
64	-1.12	1.22	-1.16	0.76	-1.22	0.76	-1.167	0.913
68	-1.16	1.3	-1.24	0.88	-1.32	0.88	-1.240	1.020
72	-1.22	1.46	-1.28	1	-1.38	1.04	-1.293	1.167
76	-1.24	1.4	-1.44	2.16	-1.26	1.18	-1.313	1.580
80	-1.4	2.22	-1.34	2.06	-1.42	1.5	-1.387	1.927
84	-1.42	2.42	-1.54	2.48	-1.56	1.64	-1.507	2.180
87	-1.66	5.76	-1.64	6.72	-1.68	1.86	-1.660	4.780
83	-1.92	6.84	-1.88	8.88	-1.82	4.26	-1.873	6.660
80	-2.18	11	-2.22	18.02	-2.08	7.84	-2.160	12.287

Table C.8: Strain in Doubly Under Reinforced Beam-2

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.02	-0.02	0	-0.02	0.02	-0.020	0.013
8	-0.06	0.06	-0.04	0.04	-0.04	0.08	-0.047	0.060
12	-0.2	0.12	-0.14	0.1	-0.14	0.18	-0.160	0.133
16	-0.14	0.24	-0.1	0.26	-0.3	0.4	-0.180	0.300
20	-0.16	0.32	-0.16	0.32	-0.14	0.44	-0.153	0.360
24	-0.2	0.38	-0.26	0.36	-0.22	0.38	-0.227	0.373
28	-0.26	0.52	-0.38	0.5	-0.18	0.48	-0.273	0.500
32	-0.32	0.58	-0.46	0.64	-0.42	0.72	-0.400	0.647
36	-0.46	0.66	-0.52	0.7	-0.46	0.76	-0.480	0.707
40	-0.56	0.72	-0.56	0.76	-0.56	0.82	-0.560	0.767
44	-0.62	0.84	-0.66	0.84	-0.62	0.88	-0.633	0.853
48	-0.54	0.94	-0.68	0.9	-0.7	0.92	-0.640	0.920
52	-0.64	0.88	-0.74	0.86	-0.8	0.98	-0.727	0.907
56	-0.72	0.96	-0.92	0.96	-0.9	1.06	-0.847	0.993
60	-0.86	1.06	-1	1.04	-1	1.14	-0.953	1.080
64	-0.96	1.1	-1.06	1.16	-1.1	1.18	-1.040	1.147
68	-1.06	1.12	-1.14	0.98	-1.24	0.86	-1.147	0.987
72	-0.98	1.36	-0.96	1.18	-1.28	1.02	-1.073	1.187
76	-1.02	1.46	-1.12	1.32	-1.38	1.06	-1.173	1.280
80	-1.1	1.62	-1.42	2.44	-1.5	1.14	-1.340	1.733
84	-1.18	3.04	-1.7	2.82	-1.76	5.58	-1.547	3.813
88	-1.9	4.38	-2	4.36	-1.92	8.76	-1.940	5.833
82	-2	6.46	-2.06	6.44	-2.1	12.22	-2.053	8.373
78	-2.18	8.62	-2.22	8.6	-2.32	18.38	-2.240	11.867

Table C.9: Strain in Doubly Under Reinforced Beam-3

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	0	-0.02	-0.02	0	-0.04	0.02	-0.020	0.000
8	-0.06	0.04	-0.04	0.08	-0.12	0.04	-0.073	0.053
12	-0.1	0.14	-0.12	0.14	-0.2	0.1	-0.140	0.127
16	-0.2	0.18	-0.18	0.24	-0.26	0.16	-0.213	0.193
20	-0.24	0.24	-0.3	0.34	-0.44	0.2	-0.327	0.260
24	-0.34	0.32	-0.4	0.42	-0.44	0.22	-0.393	0.320
28	-0.48	0.52	-0.54	0.54	-0.52	0.26	-0.513	0.440
32	-0.56	0.68	-0.68	0.52	-0.6	0.36	-0.613	0.520
36	-0.62	0.72	-0.74	0.6	-0.64	0.4	-0.667	0.573
40	-0.64	0.78	-0.88	0.7	-0.7	0.48	-0.740	0.653
44	-0.72	0.8	-0.98	0.86	-0.76	0.46	-0.820	0.707
48	-0.68	0.86	-0.66	1.14	-0.84	0.6	-0.727	0.867
52	-0.68	0.96	-0.76	1.2	-0.94	0.68	-0.793	0.947
56	-0.84	1.02	-0.86	1.26	-1.02	0.86	-0.907	1.047
60	-0.92	1.12	-0.92	1.38	-1.1	0.96	-0.980	1.153
64	-0.96	1.24	-1	1.32	-1.18	1	-1.047	1.187
68	-1.08	1.3	-1.26	1.54	-1.34	1.08	-1.227	1.307
72	-1.14	1.4	-1.36	1.64	-1.3	1.22	-1.267	1.420
76	-1.24	1.38	-1.44	2.34	-1.4	1.34	-1.360	1.687
80	-1.34	1.52	-1.46	2.42	-1.34	2.5	-1.380	2.147
84	-1.66	2.32	-1.68	6.7	-1.6	2.64	-1.647	3.887
87	-1.76	5.6	-1.82	12.22	-1.66	4.86	-1.747	7.560
83	-2.02	6.9	-2.06	15.44	-1.86	7.26	-1.980	9.867
78	-2.28	12.06	-2.34	19.08	-2.02	9.84	-2.213	13.660

Table C.10: Strain in Doubly Over Reinforced Beam-1

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0	-0.02	0.02	-0.04	0.02	-0.027	0.013
8	-0.04	0.02	-0.04	0.04	-0.12	0.04	-0.067	0.033
12	-0.16	0.04	-0.06	0.06	-0.08	0.06	-0.100	0.053
16	-0.1	0.12	-0.1	0.18	-0.06	0.14	-0.087	0.147
20	-0.26	0.08	-0.14	0.2	-0.08	0.4	-0.160	0.227
24	-0.14	0.2	-0.18	0.3	-0.16	0.34	-0.160	0.280
28	-0.16	0.28	-0.2	0.42	-0.32	0.4	-0.227	0.367
32	-0.04	0.34	-0.3	0.78	-0.36	0.46	-0.233	0.527
36	-0.28	0.46	-0.26	0.6	-0.28	0.68	-0.273	0.580
40	-0.24	0.6	-0.3	0.68	-0.36	0.72	-0.300	0.667
44	-0.26	0.56	-0.28	0.72	-0.3	0.74	-0.280	0.673
48	-0.36	0.58	-0.34	0.8	-0.34	0.82	-0.347	0.733
52	-0.42	0.62	-0.42	0.88	-0.38	0.86	-0.407	0.787
56	-0.52	0.66	-0.46	1	-0.44	0.98	-0.473	0.880
60	-0.56	0.74	-0.5	0.72	-0.4	1.02	-0.487	0.827
64	-0.62	0.8	-0.72	0.78	-0.48	1.06	-0.607	0.880
68	-0.72	0.86	-0.74	0.86	-0.62	1.26	-0.693	0.993
72	-0.76	0.9	-0.72	0.96	-0.66	1.14	-0.713	1.000
76	-0.78	0.96	-0.76	0.94	-0.74	1.24	-0.760	1.047
80	-0.78	1.08	-0.84	0.9	-0.8	1.16	-0.807	1.047
84	-0.84	1.08	-0.8	0.94	-0.82	1.3	-0.820	1.107
88	-0.92	1.22	-0.94	1.02	-0.96	1.38	-0.940	1.207
92	-0.88	1.34	-1.06	1.18	-1.08	1.3	-1.007	1.273
96	-1.16	1.42	-1.14	1.38	-1.22	2.06	-1.173	1.620
100	-1.28	1.58	-1.3	1.52	-1.48	1.78	-1.353	1.627
104	-1.4	1.8	-1.44	3.76	-1.46	3.06	-1.433	2.873
108	-1.64	4.18	-1.66	6.12	-1.68	4.74	-1.660	5.013
112	-1.8	8.38	-1.88	8.16	-1.78	5.24	-1.820	7.260
106	-1.84	9.4	-2.08	9.52	-2.1	6.6	-2.007	8.507
98	-2.1	15.58	-2.1	10.52	-2.12	8.6	-2.107	11.567

Table C.11: Strain in Doubly Over Reinforced Beam-2

Load(kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	0	0.04	-0.04	0.02	-0.02	0.06	-0.020	0.040
8	-0.02	0.06	-0.08	0.08	-0.06	0.1	-0.053	0.080
12	-0.02	0	-0.26	0.18	-0.02	0.2	-0.100	0.127
16	-0.04	0.1	-0.18	0.3	-0.04	0.24	-0.087	0.213
20	-0.1	0.14	-0.1	0.14	-0.02	0.3	-0.073	0.193
24	-0.14	0.16	-0.18	0.24	-0.1	0.4	-0.140	0.267
28	-0.2	0.26	-0.24	0.34	-0.26	0.46	-0.233	0.353
32	-0.4	0.32	-0.3	0.4	-0.3	0.56	-0.333	0.427
36	-0.18	0.42	-0.32	0.52	-0.22	0.66	-0.240	0.533
40	-0.4	0.46	-0.26	0.4	-0.3	0.46	-0.320	0.440
44	-0.32	0.56	-0.28	0.44	-0.24	0.6	-0.280	0.533
48	-0.64	0.6	-0.4	0.44	-0.34	0.74	-0.460	0.593
52	-0.7	0.52	-0.72	0.5	-0.42	0.82	-0.613	0.613
56	-0.76	0.66	-0.82	0.72	-0.56	0.86	-0.713	0.747
60	-0.84	0.72	-0.86	0.74	-0.6	0.78	-0.767	0.747
64	-0.86	0.8	-0.8	0.78	-0.64	0.82	-0.767	0.800
68	-0.94	0.88	-0.96	0.86	-0.7	0.88	-0.867	0.873
72	-0.98	0.82	-1.04	0.78	-0.76	0.96	-0.927	0.853
76	-1.06	0.92	-1	0.9	-0.86	1	-0.973	0.940
80	-1.02	1	-1.04	1	-0.96	1.04	-1.007	1.013
84	-1.06	1.04	-1.08	1.12	-0.9	1.06	-1.013	1.073
88	-1.08	1.06	-1.16	1.18	-1.02	1.14	-1.087	1.127
92	-1.36	1.14	-1.24	1.3	-1.16	1.18	-1.253	1.207
96	-1.4	1.3	-1.3	2.24	-1.42	1.3	-1.373	1.613
100	-1.5	1.38	-1.34	2.38	-1.52	1.42	-1.453	1.727
104	-1.6	1.52	-1.44	6.48	-1.4	2.66	-1.480	3.553
108	-1.86	3.9	-1.66	7.84	-1.62	3.34	-1.713	5.027
114	-2.02	7.1	-1.88	10.88	-1.72	4.84	-1.873	7.607
107	-2.04	8.12	-2.08	12.24	-2.04	6.02	-2.053	8.793
100	-2.2	14.3	-2.1	16.24	-2.06	8.46	-2.120	13.000

Table C.12: Strain in Doubly Over Reinforced Beam-3

Load (kN)	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_c	ε_t
0	0	0	0	0	0	0	0	0
4	-0.02	0.06	0	0.04	-0.02	0	-0.013	0.033
8	-0.04	0.08	-0.04	0.08	-0.1	0.02	-0.060	0.060
12	-0.14	0.18	-0.1	0.2	-0.06	0.06	-0.100	0.147
16	-0.2	0.08	-0.14	0.12	-0.04	0.1	-0.127	0.100
20	-0.04	0.24	-0.16	0.32	-0.06	0.2	-0.087	0.253
24	-0.22	0.26	-0.18	0.24	-0.14	0.34	-0.180	0.280
28	-0.16	0.36	-0.24	0.44	-0.3	0.36	-0.233	0.387
32	-0.38	0.46	-0.3	0.54	-0.34	0.42	-0.340	0.473
36	-0.42	0.54	-0.26	0.6	-0.26	0.56	-0.313	0.567
40	-0.5	0.56	-0.28	0.68	-0.34	0.66	-0.373	0.633
44	-0.62	0.64	-0.38	0.8	-0.28	0.72	-0.427	0.720
48	-0.5	0.72	-0.46	0.8	-0.38	0.82	-0.447	0.780
52	-0.58	0.76	-0.52	0.84	-0.42	0.86	-0.507	0.820
56	-0.62	0.82	-0.6	0.88	-0.48	0.94	-0.567	0.880
60	-0.7	0.88	-0.66	0.9	-0.52	1.02	-0.627	0.933
64	-0.56	0.82	-0.72	1	-0.54	1.14	-0.607	0.987
68	-0.64	0.88	-0.7	0.92	-0.6	1.1	-0.647	0.967
72	-0.68	0.98	-0.68	0.84	-0.64	1.22	-0.667	1.013
76	-0.78	0.96	-0.84	0.98	-0.8	1.2	-0.807	1.047
80	-0.84	1.14	-0.94	1.04	-0.94	1.34	-0.907	1.173
84	-0.82	1.3	-1.02	1.12	-1.06	1.26	-0.967	1.227
88	-1.14	1.38	-1.16	1.32	-1.2	2.02	-1.167	1.573
92	-1.22	1.5	-1.24	1.54	-1.26	2.2	-1.240	1.747
96	-1.3	1.58	-1.28	1.6	-1.34	2.3	-1.307	1.827
100	-1.34	1.7	-1.32	1.74	-1.42	2.5	-1.360	1.980
104	-1.36	1.78	-1.4	3.7	-1.44	3.02	-1.400	2.833
108	-1.66	4.18	-1.54	5.06	-1.66	4.7	-1.620	4.647
113	-1.68	8.26	-1.94	7.1	-1.76	5.2	-1.793	6.853
106	-1.9	10.36	-2.06	9.32	-2.08	6.56	-2.013	8.747
97	-2.1	14.54	-2.28	12.06	-2.1	8.56	-2.160	11.720