Beamforming using FIR Eigenfilter Approach in Cognitive Radio Network

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Abstract
Beamforming technique allows the cognitive users to opportunistically access the licensed spectrum without interfering the licensed users by exploiting the spatial domain in the radio transmission for cognitive radio network. In this paper, the eigenfilter approach to solve general least-squares approximation problems is extended. Such extension unifies previous work in eigenfilters and many other filter design problems, including spectral/spatial filtering, one-dimensional or multidimensional filters etc. With this approach, various filter design problems are transformed into problems of finding an eigenvector of a positive definite matrix that is determined by filter design specifications. A number of design examples are presented to show the usefulness and flexibility of the proposed approach.

Keywords
Eigenfilter, Eigenvector, Rayleigh Principle, Positive Definite, Beamforming, Direction of Arrival, Wieght Function

I. Introduction
In the modern era, there is a common believe that spectrum scarcity is happening at frequencies which can be used economically for wireless communication. This concept arises due to the heavy occupancy of frequency spectrum below 3 GHz which is caused by the ever growing demand for wireless services by the customers [1]. This problem has placed a heavy burden to the resource allocation policy maker to accommodate between the demand and the available spectrum resources. This fact is contradicting with the mindset of spectrum scarcity, since we can see that we have spectrum abundance.

An approach which is expected to solve the problem of spectrum scarcity is the Cognitive Radio (CR) [2-3]. The CR system is developed to be able to sense the spectral environment over the available band and to use the unused spectrum as long as it doesn't interfere with the licensed user. The cognitive radio network usually consists of the primary or Licensed Users (LU) which has the priority and legality to access the communication spectrum, and the secondary or the cognitive radio users (RU) who use the spectrum only if they do not create interference to the primary users. This is where the cognitive radio technique is used by the secondary users to ensure non-interfering condition with the primary users. There are several ways to achieve the spectrum sharing with cognitive radio, such as space, time, frequency, and region. One of the strategies is to have the cognitive users to scan the spectrum and search for idleness, then access it when an unused slot is detected.

Beamforming is a well-known spatial filtering technique which can be used to direct the communication transmission or reception energy in the presence of noise and interference. By using this technology, we can enable simultaneous communication links between the primary and secondary users with minimized or even total avoidance of interference. Beamforming allows the establishment of a communication link between the secondary users by exploiting the absence of a licensed user's communication link in a certain geographical location, also known as the spatial spectrum holes. The definition of spectrum holes is the frequency bands which are assigned to primary users, which at a particular time and specific geographical location are not used by them. The basic idea of Beamforming in cognitive radio is to direct the radio signal to the direction of the destination, and to minimize the transmission energy towards the primary users [4]. This way we can suppress the interference caused by the secondary users to the primary users. In a multiple-antenna system, Beamforming exploits channel knowledge in the transmitter to maximize the Signal-to-Noise Ratio (SNR) at the receiver. Beamforming can also be used in the uplink or downlink in a multiuser system to maximize the Signal-to-Interference-Noise Ratio (SINR) to a specific user. However several challenges still exist if we want to implement Beamforming into a cognitive radio system.

The paper is organized as follows. In section II, conventional Beamforming technique with necessary formulation and beam patterns are described. In section III, Eigenfilter design for FIR is discussed with necessary literature. In section IV, implementation of Eigenfilter approach in Beamforming is discussed. In section V, simulation results using Eigenfilter weights are done and compared weights obtained with conventional method. Lastly, concluding remarks are made in section VI.

II. Conventional Beamforming
In Beamforming, we estimate the signal of interest arriving from some specific directions in the presence of noise and interfering signals with the aid of an array of sensors. These sensors are located at different spatial positions and sample the propagating waves in space. The collected spatial samples are then processed to attenuate/null out the interfering signals and spatially extract the desired signal. As a result, a specific spatial response of the array system is achieved with 'beams' pointing to the desired signals and 'nulls' towards the interfering ones [4].

Fig. 1, shows a simple Beamforming structure based on a linear array, where M sensors sample the wave field spatially and the output y(t) at time t is given by an instantaneous linear combination of these spatial samples $x_m(t)$, m = 0, 1, . . . , M − 1, as:

$$y(t) = \sum_{m=0}^{M-1} x_m(t) w_m^* \quad (1)$$

where, $*$ denotes the complex conjugate.

The beamformer associated with this structure is only useful for sinusoidal or narrowband signals, where the term 'narrowband' means that the bandwidth of the impinging signal should be narrow enough to make sure that the signals received by the opposite ends of the array are still correlated with each other, and hence it is termed a narrowband beamformer [5].

We now analyse the array's response to an impinging complex plane wave $e^{j\omega t}$ with an angular frequency ω and a DOA angle θ, where θ ∈ [−π/2 π/2] is measured with respect to the broadside of the linear array, as shown in fig. 2. For convenience, we assume the phase of the signal is zero at the first sensor. Then the signal received by the first sensor is $x_0(t) = e^{j\omega t}$ and by the mth sensor is $x_m(t) = e^{j\omega(t-\tau_m)}$, m = 1, 2, . . . , M − 1, where τ_m is the propagation delay for the signal from sensor 0 to sensor m and is a function of θ. Then the beamformer output is:

$$y(t) = e^{i\omega t}\sum_{m=0}^{M-1} e^{-i\omega t_m} w_m^* \quad (2)$$

with $\tau_0 = 0$. The response of this beamformer is given by:

$$P(\omega,\theta) = \sum_{m=0}^{M-1} e^{-i\omega t_m} w_m^* = w^H d(\omega,\theta) \quad (3)$$

where, the weight vector w holds the M complex conjugate coefficients of the sensors, given by:

$$w = [w_0\ w_1 \ldots\ldots\ldots\ w_{M-1}]^T$$

$$d(\omega,\theta) = [1\ e^{-i\omega\tau_1} \ldots\ldots\ldots\ e^{-i\omega\tau_{M-1}}]^T \quad (4)$$

We refer to d (θ,ω) as the array response vector, which is also known as the steering vector or direction vector.

For signals having the same angular frequency ω and the corresponding wavelength λ, but different DOAs θ_1 and θ_2 satisfying the condition $(\theta_1, \theta_2) \in [-\pi/2\ \pi/2]$, aliasing implies that we have $d(\theta_1, \omega) = d(\theta_2,\omega)$, namely:

$$e^{-i\omega\tau_m(\theta_1)} = e^{-i\omega\tau_m(\theta_2)}$$

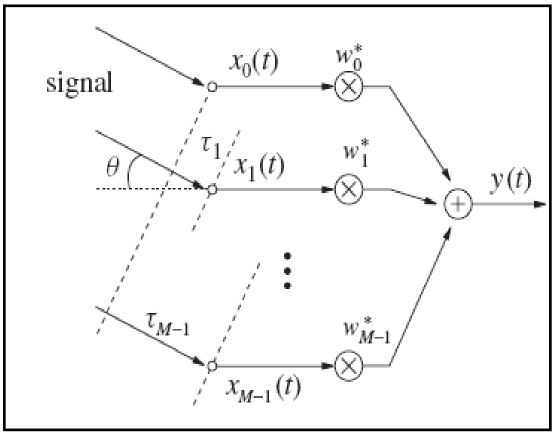

Fig. 1: Narrow Band Beamforming

For a uniformly spaced linear array with an inter-element spacing d, we have $\tau_m = m\tau_1 = m(d \sin\theta)/c$ and $\omega\tau_m = m(2\pi d \sin\theta)/\lambda$. Then Equation (1.15) changes to:

$$e^{-jm(2\pi d\sin\theta_1)/\lambda} = e^{-jm(2\pi d\sin\theta_2)/\lambda}$$

we will always set d = λ/2, unless otherwise specified, then $\omega\tau_m = m\pi\sin\theta$ and the response of the uniformly spaced narrowband beamformer is given by:

$$P(\omega,\theta) = \sum_{m=0}^{M-1} e^{-im\pi\sin\theta}\ w_m^* \quad (5)$$

Note for an FIR (finite impulse response) filter with the same set of coefficients its frequency response is given by:

$$P(\Omega) = \sum_{m=0}^{M-1} e^{-im\Omega}\ w_m^* \quad (6)$$

with $\Omega \in [-\pi\ \pi]$ being the normalized frequency. For the response of the beamformer given by Equation, when θ changes from −π/2(−90∘) to π/2(90∘), π sin θ changes from −π to π accordingly, which is in the same range as in Equation (6). With this correspondence, the design of uniformly spaced linear arrays can be achieved by the existing FIR filter design approaches directly.

Suppose we want to form a flat beam response pointing to the directions $\theta \in [-\pi/6\ \pi/6]$ ([−30∘ 30∘]), while suppressing signals from directions $\theta \in [-\pi/2\ -\pi/4]$ and [π/4 π/2], then it is equivalent to designing an FIR filter with a passband of $\Omega \in [-0.5\pi\ 0.5\pi]$ and a stopband of $\Omega \in [-\pi - 0.71\pi]$ and [0.71π π] (sin π/6 = 0.5 and sin π/4 = 0.71). We can use the MATLAB function remez to design such a filter, and then use the result directly as the coefficients of the desired beamformer.

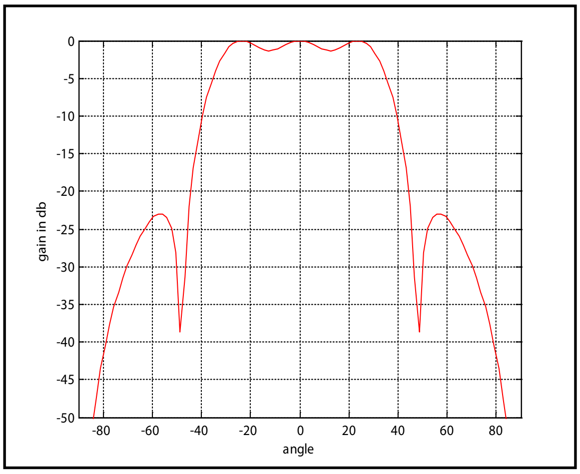

Fig. 2: Beam Pattern

The beam pattern of the resultant narrowband beamformer Substituting this result into Equation, we can draw the resultant amplitude response |P(θ,ω)| of the beamformer with respect to the DOA angle θ. |P(θ,ω)| is called the beam pattern of the beamformer to describe the sensitivity of the beamformer with respect to signals arriving from different directions and with different frequencies. Fig. 2, shows the Beam Pattern (BP) in dB, which is defined as follows:

$$BP = 20\log_{10}\frac{|P(\theta,\omega)|}{\max\ P(\theta,\omega)|} \quad (7)$$

III. Eigenfilter Design

The Eigenfilter method for digital filter design involves the computation of filter coefficients as the eigenvector of an appropriate Hermitian matrix by Rayleigh principle [6]. As opposed to the least-squares approach, which requires the computation of a matrix inverse which may be susceptible to numerical inaccuracies, the Eigenfilter method has a much lower design complexity and remains robust even when ill-conditioned matrices are present in the design problem [8].

Let, h(n) be a causal FIR filter of length N, so that $H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}$. Defining the N X 1 vectors

$$h = [h(0)\ h(1) \ldots\ldots\ldots\ h(N-1)]^T \quad (8)$$

$$e(z) = [1\ z^{-1} \ldots\ldots\ldots\ Z^{-(N-1)}]^T \quad (9)$$

we clearly have $H(z) = h^T e(z)$.

To develop an Eigenfilter based method for approximating a complex valued desired response D(ω), following is the least-squares objective function [7],

$$\xi_{WLS} \triangleq \frac{1}{2\Pi}\int W(\omega)|\ D(\omega) - H(e^{j\omega})|^2\, d\omega \quad (10)$$

in order to express it as a quadratic form in terms of the vector . The frequency region is now a subset of the interval and the desired response is allowed to be complex, subject to the unit norm constraint. Here, the matrix is a complex- valued Hermitian, positive definite matrix. Under the unit norm constraint, the optimal filter coefficients can be found using Rayleigh's principle, which applies for any Hermitian matrix, be it real or complex. After obtaining the optimal coefficients, the resulting filter must be scaled in order to satisfy the reference frequency condition. The objective is a quadratic form in terms of the vector of real and imaginary parts of h(n). It involved an N X N complex matrix or 2N X 2N real matrix.

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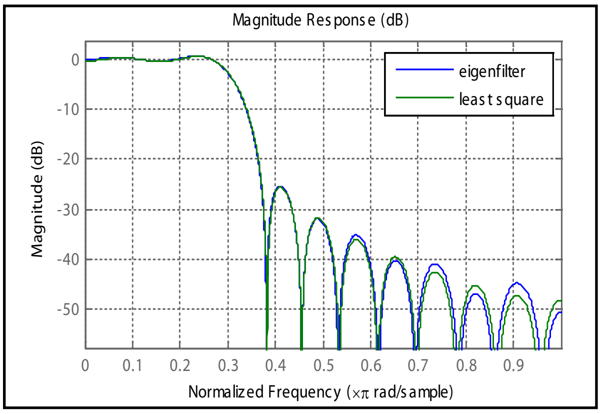

Fig. 3:

Magnitude responses obtained using the least-squares approach along with the eigenfilter approach. In fig. 3, the magnitude responses using the Eigenfilter method as well as the least-squares approach are plotted for a low-pass filter with $\omega_p = 0.3$ and $\omega_p = 0.4$. Here, the filter order was chosen to be and equal weighing was used for both the passband and stopband. The eigenfilter was rescaled to have unity gain at ω=0. From the plots, it can be seen that the Eigenfilter is very similar to the least-squares filter, although the former performs slightly worse in some parts of the stopband. As N increases, the two responses become more and more alike. The close agreement between the least-squares approach and the eigenfilter method, along with the lower complexity of the latter, show the merits of the eigenfilter method.

IV. Beamforming: Eigenfilter Approach

Consider the following approximation problem, which turns out to be general enough to cover many filter design problems. Suppose we want to approximate a given function g(z) (possibly complex function) defined for real x ϵ χ, by using the linear combination of a set of N functions $b_0(z), b_1(z), \ldots, b_{N-1}(z)$ which are also well-defined in χ. Let W_n^*, the conjugate of w_n, be the weights of such linear combination [9-10]. With w denoting the vector $[w_0\ w_1 \ldots\ldots w_{N-1}]$ T and b(x) denoting the vector $[b_0(z), b_1(z), \ldots, b_{N-1}(z)]$ T the result of such a linear combination can be expressed as

$$f(x) = \sum_{n=0}^{N-1} w_n^* b_n(x) = w^t b(x) \quad (11)$$

By properly choosing w, we want to make f(x) 'close' to g(x) for z E X. We now reformulate this problem into an eigenvector problem. First, we add the normalization constraint $\sum_{n=0}^{N-1} |w_n|^2 = 1$, $w^t w = 1$. Under this constraint, we have no control of the 'scale' of f(x) any more. Therefore, we shall make $f(x)/f(x_0)$ approximate $g(x)/g(x_0)$, where, x_0 is a reference point in X. Equivalently, we minimize the following quadratic error measure

$$E = \int \left| f(x) - f(x_0)\frac{g(x)}{g(x_0)} \right|^2 dx \quad (12)$$

Using (1). We can rewrite E as

$$E = \int \left| w^t b(x) - w^t b(x_0)\frac{g(x)}{g(x_0)} \right|^2 dx = w^t P w \quad (13)$$

Where

$$P = \int \left[b(x) - b(x_0)\frac{g(x)}{g(x_0)} \right] \left[b(x) b(x_0)\frac{g(x)}{g(x_0)} \right]^t dx \quad (14)$$

We can see that $P^t = P$, i.e., P is a Hermitian matrix. Also, the error $E = w^t P w$ is positive for any nonzero w, so P is positive-definite. (It can be positive semidefinite if there exists w such that E = 0, which implies that the approximation can be made exact.) Therefore, all the eigenvalues of P are real and positive.

According to the Rayleigh-Ritz theorem, the vector w which minimizes $E = w^t P w$ under the constraint $w^t w = 1$ is the eigenvector of P corresponding to the smallest eigenvalue. Such eigenvector can be computed using the power method. When the ratio between the two smallest eigenvalues of P is sufficiently large, this method converges very fast. Hence, the required computation complexity is relatively low, compared with other approximation schemes which usually involve matrix inversion operations.

V. Simulation Results

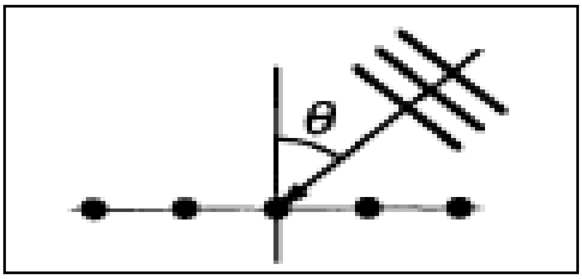

Fig. 4: A Liner Array

A. linear array

Consider a linear array having N sensors spaced at one-half wavelength as shown in fig. 4. The output of the n-th sensor is weighted by W_n^* to produce the overall output. The gain of this sensor array with respect to a uniform plane wave arriving at angle θ is

$$f(\theta) = \sum_{n=0}^{N-1} w_n^* [\exp(-jn\pi \sin\theta)] = w^t b(\theta)$$

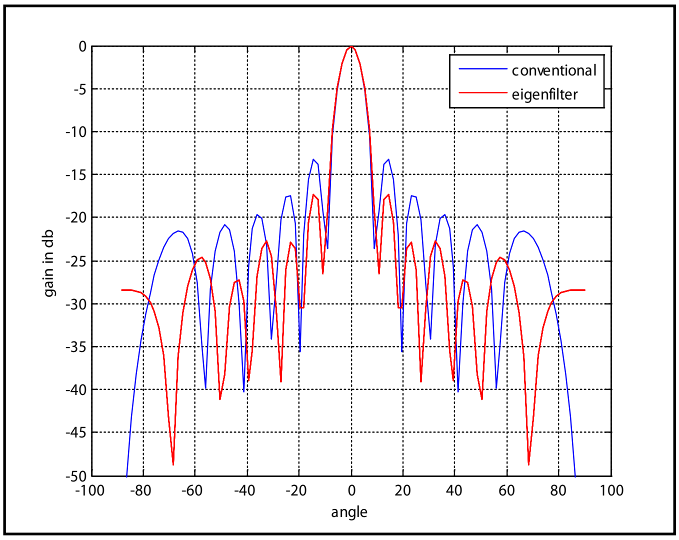

Fig. 5: Gain of a Linear Array

So, the appropriate bn(θ) functions for this problem are bn(θ) = exp(- jnπsinθ). In fact, the vector b(θ) obtained above is the so-called steering vector or direction vector. Then, we can use the eigen-approach to compute the proper weights . For the case N = 12, suppose we want to 'steer' the array to an arrival angle of 0º. We can achieve this by constraining the gain to be unity for θ = 0º and minimizing the energy elsewhere using the Eigen-approach. Fig. 5, shows the resulting gain versus θ. As we can see the eigenfilter approach gives better beam response than conventional method with same number of sensors N = 12.

VI. Conclusion

In this paper, a generalized eigenfilter approach for Beamforming is proposed, where Beamforming technique allows the cognitive users to opportunistically access the licensed spectrum without interfering the licensed users by exploiting the spatial domain in the radio transmission for cognitive radio network. With this approach, various filter design problems are transformed into problems of finding an eigenvector of a positive definite matrix that is determined by Beamforming techniques.

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