Abstract

Solitons and solitary waves manifest in dynamical systems, when dispersion and non-linearity balance each other. These solutions of non-linear equations, which arise in diverse physical systems, can be localized or continuous. Bose-Einstein condensate (BEC) is one of the very well studied non-linear systems, where atom-atom interactions lead to rich structures. The formation dynamics and control of these waves in BEC are subjects of intense current research. The order parameter equation governing the dynamics of mean field BEC is the Gross-Pitaevskii equation, a non-linear generalization of the familiar Schrödinger equation. Other than BEC, non-linear Schrödinger equation (NLSE) manifests in non-linear fibre optics [1, 2]. Wave propagation in resonant and non-resonant atomic media is another area governed by non-linear equations.

Developing analytical tools for finding the solutions of aforementioned systems and analyzing their properties have been the goal of this thesis. In this context, we first study the strongly coupled cigar shaped Bose-Einstein condensate. Finding exact soliton solutions and a procedure for their coherent control have been achieved. Control and manipulation of BEC have been achieved through the distributed non-linearity, gain/loss and other parameters of the system. Power law-type complex soliton is then found for a quasi-one-dimensional Bose-Einstein condensate with both two and three-body interactions. We then study this system in a harmonic trap and find that the solution retains its self- similar character. Subsequently, we explore various longitudinal excitations in BEC, resulting from the time dependence of the trap frequency. Two-component BEC is then analyzed, where we have identified new sinusoidal solutions. Their behavior in a trap and periodic lattice are investigated. In particular, we find dynamical super-fluid to insulator phase transition here. Apart from studying the coherent localized solutions of non-linear equations, we have also analyzed the time evolution of a lo-

calized quantum wave packet due to a Hamiltonian whose eigen values are quadratic in the quantum number n. Specifically, we have studied the evolution of the coherent state of the Pösch-Teller potential. The phase-space study reveals sub-Planck structure arising due to fractional revival phenomenon of wave packet. We then focus our attention on another nonlinear system in non-resonant atomic media, the Maxwell-Duffing model. It is found that our procedure enables one to identify a wide class of solutions. These include, sinusoidal and cnoidal wave solutions.

(i) Exact solitons for strongly coupled cigar shaped Bose-Einstein condensates:

In weakly coupled cigar shaped BEC, localized soliton, as well as soliton trains are exact solutions, which have found experimental confirmation, although the formation mechanism of bright solitons, still remains unclear. Unlike the weakly coupled scenario, lack of exact solutions for the strongly coupled case has led to numerical investigations and variational approaches to the Thomas-Fermi limit of the Gross-Pitaevskii (GP) equation [3]. Here, we present exact soliton and soliton train solutions of the strongly coupled cigar shaped BEC. The presence of a harmonic trap, as well as the temporal variation of scattering length, loss/gain and oscillator confinement are treated analytically. This opens up the possibility of coherent control of atom laser in the strongly coupled regime.

(ii) Power-law soliton for cigar shaped BEC both with two- and three-body interactions:

The three-body interaction can be generally treated as a perturbation over the two-body case; it becomes significant for short range and larger scattering length. A number of theoretical studies have been carried out considering three body interaction in both three- and quasi-one-dimensions. We demonstrate the existence of power-law type complex solitons in the presence of repulsive two- and attractive three-body interactions. The dark solitons have a constant velocity determined by the interaction strengths, which is quite different from the Lieb mode case [4]. Their profiles can change as a function of the parameters of the theory.

(iii) Faraday waves in cigar shaped Bose-Einstein conden-

sates:

Temporal variation of scattering length or transverse frequency can lead to Faraday excitations in a cigar shaped BEC [3]. We first study these excitation in strongly coupled BEC and BEC with both two- and three-body interactions. The effect of sudden change in the oscillator frequency is then analytically modelled.

(iv) Sinusoidal Excitations in Two Component Bose-Einstein Condensates

The non-linear coupled Gross-Pitaevskii equation governing the dynamics of the two component Bose-Einstein condensate (TBEC) is shown to admit pure sinusoidal, propagating wave solutions in quasi one dimensional geometry. These solutions, which exist for a wide parameter range, are then investigated in the presence of a harmonic oscillator trap with time dependent scattering length. This illustrates the procedure for coherent control of these modes through temporal modulation of the parameters, like scattering length and oscillator frequency. We subsequently analyzed this system in an optical lattice, where the occurrence of an irreversible phase transition from superfluid to insulator phase is seen.

(v) Coherent states (CSs) of symmetric Pöschl-Teller potential (SPT) and their time evolution:

We make use of novel exponential forms of the solutions of differential equations [5], for identifying the symmetry generators underlying the hypergeometric equation. This is used to construct and study the displacement operator coherent state (DOCS) of the Pöschl-Teller potential [6]. The primary motivation for considering the Pöschl-Teller potential is that, it has a quadratic energy spectrum leading to a rich revival structure for its CS, which can lead to the formation of cat-like states. The temporal evolution, auto-correlation and quantum carpet structures of the CSs are carefully analyzed for delineating their structure and various time scales present in this problem.

(vii) Sinusoidal excitation in a non-resonant atomic medium:

One of the well studied approaches to dense non-resonant atomic medium is to consider the response of the medium as weakly nonlinear. Such situation leads to the Duffing oscillator model, where the nonlinear response of the medium is assumed to be cubic. On the other hand, the unidirectional wave propagation approximation reduces the Maxwell wave equation from second order to a first order equation. These two, together can well-describe the wave propagation in a non-resonant atomic medium and are called the reduced Maxwell-Duffing model (RMD). We present here mono frequency, sinusoidal wave excitations for RMD system. This excitation exists only in the presence of a polarizing background. General cnoidal wave solutions are found both with and without background.