Analysis of Network Observability of PMU under Contingency Scenario

Major Project Report

Submitted in Partial Fulfillment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

IN

ELECTRICAL ENGINEERING
(Electrical Power Systems)

By

PURVI DOSHI
13**MEEE**19

DEPARTMENT OF ELECTRICAL ENGINEERING
INSTITUTE OF TECHNOLOGY
NIRMA UNIVERSITY
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Undertaking for Originality of the Work

I, **Purvi Doshi** (**Roll.No.13MEEE19**), give undertaking that the Major Project entitled "**Analysis of Network Observability of PMU under Contingency Scenario**" submitted by me, towards the partial fulfillment of the requirements for the degree of Master of Technology in Electrical Power Systems of Nirma University, Ahmedabad, is the original work carried out by me and I give assurance that no attempt of plagiarism has been made. I understand that in the event of any similarity found subsequently with any published work or any dissertation work elsewhere; it will result in severe disciplinary action.

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Certificate

This is to certify that the Major Project Report entitled "**Analysis of Network Observability of PMU under Contingency Scenario**"submitted by **Ms. Purvi Doshi** (**Roll No:13MEEE19**) towards the partial fulfillment of the requirements for the award of degree in Master of Technology (Electrical Engineering) in the field of Electrical Power Systems of Nirma University is the record of work carried out by her under our supervision and guidance. The work submitted has in our opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of our knowledge have not been submitted to any other University or Institution for award of any degree or diploma.
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Abstract

Currently, rapid growth of power demands, disproportionate growth of power generation and transmission systems, power system restructuring, and other factors have overloaded the existing electrical networks and subsequently decreased the stability margin of these networks. In such circumstances, to ensure the stable and proper operation of the system, a precise measurement and monitoring of the system states are required. This monitoring was conventionally performed by utilizing the Supervisory Control and Data Acquisition (SCADA) system, in which state estimation is derived based on measurements that are not usually synchronized. To overcome this limitation in the SCADA, the wide-area monitoring, protection and control (WAMPAC) system has been employed, in which Phasor Measurement Units are placed at different locations in power network. These units, which are time synchronized with clock signals from global positioning system (GPS) satellites, are able to provide synchronized phasor measurements. To install optimal number of PMUs in a system network is an important task. The complete observability of the system is a prerequisite to the state estimation to achieve the same with economical considerations generally certain minimum number of appropriately distributed PMUs. The complete observability of the power system, using phasor measurements, implies that each bus of the network must have one voltage phasor measurement or a voltage phasor pseudo-measurement. The observability of power system is defined in terms of numerical observability as well as topological observability of the system. A network is said to be topological observable if it contains a spanning tree of full rank. When the measurement Jacobian is of full rank, the network is said to be numerically observable. To achieve full network observability, both topological and numerical observability is needed and for this optimal PMU placement is needed. The proposed method for network observability is tested for different standard test systems considering contingencies such as loss of single PMU, single line outage and communication constraints. Thus the optimal placement problem (OPP) is formulated such that minimizing the number of PMU installations for full network observability.

Nomenclature

V_1	Phasor voltage
I_{12}	Phasor current
X	State vector
e	Error
Z	Measurement Vector
N	Number of buses
R_H	Rank of Jacobian Matrix
k1, k2	Constant complex value
H	Jacobian matrix

Abbreviations

PMU	Phasor Measurement Units
GPS	Global Positioning System
WAMS	Wide Area Monitoring Systems
SCADA	Supervisory Control and Data Aquisition
SA	Simulated Annealing
WSCC	Western System Coordinating Council
OPP	Optimal PMU Placement
ZIB	Zero Injection Bus
PSAT	Power System Analysis Toolbox
KCL	Kirchhoff's Current Law
KVL	Kirchhoff's Voltage Law
WAMPAC	Wide-area Monitoring, Protection and Control

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The complexity of the power system is increasing day by day, the need for real time monitoring of the dynamic behavior of the power system became one of the difficulties faced by power system engineers. Synchrophasor technologies can overcome these difficulties. Phasor Measurement Units gives time synchronized phasor measurement in a power system. Synchronization in PMU measurements is achieved by time stamping of current and voltage waveforms using a available common synchronizing signal from global positioning system[1]. The important step in employing phasor measurement technology, using PMUs, is their optimal location in the power system in order to fully observe the network, which is a prerequisite for an efficient and accurate state estimation and control of the power system[2]. This dissertation report mainly focuses on analysis of complete network observability i.e. topological and numerical observability of the system both in intact condition as well as considering contingencies.

1.1 Problem Identification

The traditional Supervisory Control and Data Aquisition (SCADA) system has been utilized for several decades to provide steady state monitoring of the electrical power systems, its ability to detect dynamic changes in the network and prevent severe blackouts has been limited. This has motivated the development of Synchrophasor technology. Phasor Measurement Units (PMUs) are devices that provide synchronized measurements of real-time phasors of voltages and currents at different buses in a power network. Since the voltage and current phasors are being measured, the state estimation equations become linear. It is easier to find the solution in linear state than the nonlinear system state estimation. The PMUs can enhance many applications such as measurement loss and branch outage, bad data detection and fault location studies. However, the overall cost of the metering system will limit the number and locations of PMUs. This requires of finding the minimum number of PMUs to make the system completely observable, as well as the optimal placement of these PMUs. The optimal PMU placement methodologies applied include the complete system observability during normal operating conditions and during various critical contingencies such as line outages, single PMU loss, communication channel constraint with less calculation effort and less time-consuming. Network observability determines whether a state estimator will be able to determine a unique state solution for a given set of measurements, their location, and a specified network topology. The problem is independent of the measurement errors (or even the measurement values), branch parameters as well as the operating state of the system.

1.2 Objective

The objective of this project is to analyze the observability of a power system network among with consideration of optimal placement of Phasor Measurement Units to guarantee full observability of a power system as well as maximizing the measurement redundancy under various contingencies such as single PMU loss or single line outage. Phasor measurement units (PMUs) are devices that provide synchronized measurements of real-time phasors of voltages and currents. Synchronization is achieved by same-time sampling of voltage and current waveforms using timing signals from the Global Positioning System(GPS). The most important issue for full network observability is installation of the PMUs on all of the system buses is impossible because of their high cost and the lack of communication facilities. Thus, one of the important issues is to find the optimal number and location of PMUs according to the desired objectives. The main goal is therefore to achieve full observability that is topological as well as numerical observability of the system in normal operation and also in the case of various critical contingencies such as single PMU loss or a single line outage using possible minimum number of PMUs.

1.3 Methodology

- Literature survey
- Simulation study of IEEE standard test system for placement of PMUs considering practical approach
- Comparison of network observability techniques
- Selection of suitable techniques
- Deciding rules for network observability
- Development of algorithm for selected method
- Verification of developed algorithm on different IEEE standard test systems
- Achieving ensured full network observability along with optimal PMU placement consideration under intact and contingency scenario

1.4 Scope of Work

The scope of project work is to attain full network observability of the power system with optimal PMUs placement under selected contingencies.

- Study of conventional and phasor measurement of power system parameters.
- Study of power system behaviour under faulty conditions.
- Study of power system network observability techniques.
- Comparison and selection of technique for achieving full network observability i.e. Topological and Numerical observability.

- Proposing optimal solution for location of phasor measurement units to achieve complete network observability.
- Analysis of network observability under selected contingency scenario.

1.5 Literature Survey

For the better understanding of project literature survey plays a very important role. Literature survey consists of papers and books referred which gives fundamental knowledge of network observability.

- **A.G.Phadke and J.S.Thorp** [1] In book entitled "Synchronized Phasor Measurements and Their Applications" gives information about fundamentals of phasor measurement techniques, phasor measurement units. It enlightens about synchronized phasor measurement applications in power system control and protection scheme.
- **Nikolas M.Manousakis, George N.Korres** [2] In the paper entitled "Taxanomy of PMU Placement Methodologies" discusses about the Optimal PMU Placement problem to provide the minimal PMU installations to ensure full observability of the power system.
- **IEEE Std C37.118 for Synchrophasors for Power Systems.**[3] The standard defines the measurement, provides a method of quantifying the measurements, and quality test specifications. It also defines data data transmission formats for real time data reporting.
- **Reynaldo F. Nuqui, Arun G. Phadke** [4] In the paper entitled "Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability" discusses depth of unobservability and its impact on the number of PMU placements is explained. Initially use of spanning trees of the power system graph and a tree search technique to find the optimal location of PMUs.
- **T.L. Baldwin, L. Mili M.B. Boisen, Jr. R. Adapa** [5] In the paper entitled "Power System Observability With Minimal Phasor Measurement Placement"

discussed how the complete system observability is achieved by placing minimum PMU sets. It also proposed graph theory based optimization techniques like depth first and spanning measurement subgraphs.

- **A.Y.Abdelaziz, Amr M. Ibrahim , Reham H. Salem** [6] In the paper entitled “Power system observability with minimum phasor measurement units placement” discussed the Optimal PMU Placement methodologies applied include the system observability during normal operating conditions, as well as during single branch forced outages aiming at reducing the computational burden in optimal placement problems. In order to improve the speed of convergence, an initial PMU placement is provided by graph-theoretic procedure.

- **Ranjana Sodhi, S.C. Srivastava, S.N. Singh** [7] In the paper entitled “Optimal PMU placement method for complete topological and numerical observability of power system” discussed a two-stage PMU placement method, where stage-1 finds out the minimum number of PMUs required to make the power system topologically observable and stage-2 is proposed to check if the resulted PMU placement (from stage-1) leads to a full ranked measurement Jacobian. In case PMUs placed, ensuring topological observability in stage-1, do not lead to the Jacobian of full rank, a sequential elimination algorithm (SEA) is proposed in stage-2 to find the optimal locations of additional PMUs, required to be placed to make the system numerically observable as well.

- **Ranjana Sodhi, S.C. Srivastava, S.N. Singh**[8] In the paper entitled “Optimal PMU Placement to Ensure System Observability under Contingencies” discussed a simple and effective method for optimal placement of PMUs considering critical contingencies. A voltage stability based contingency screening method has been utilized to select critical contingency cases. An Integer Linear Programming (ILP) based algorithm for the PMU placement has been modified to determine optimal PMU locations under the system intact and the critical contingency cases.

1.6 Outline of Thesis

- **Chapter 1** introduces the limitations and certain typical problem associated with conventional measurements and thereby requirement of use of phasor measurement units for network observability of the system. The literature survey and methodology adopted is also described.

- **Chapter 2** gives a a general idea of measurement units. The traditional and modern methods of measurements are then discussed briefly. Modern method i.e phasor measurement units and limitations of conventional measurements are also described.

- **Chapter 3** includes different methods of network observability such as Topological and Numerical observability. The basics rules of network observability and PMU placement rules for the same are discussed. Different PMU placement algorithms such as depth first method, Graph Theoretic Procedure, Recursive N-1 Spanning are also discussed.

- **Chapter 4** describes about power system observability under intact and contingency condition. Different ways of network observability such as topological and numerical observability are discussed. Binary integer linear programming method is discussed for topological observability of the system.

- **Chapter 5** presents simulation results of topological observability of the standard test systems with and without considering zero injection busbar. For this ILP based approach has been implemented to test system along with OPP under steady state and several selected contingencies such as single PMU loss, line outage of power system. It also incorporates impact of communication constraints on network observability. Also conventional measurements such as flow measurements are considered for topological observability. Further, jacobian matrix for numerical observability is find to make the system numerically observable for normal as well as under contingency scenario.

- **Chapter 6** includes conclusion and future scope.

Chapter 2

Power System Monitoring

Electricity sector has been in the process of continuous development, and is one of the most essential commodities today, required for the economic growth of a country. While the traditional Supervisory Control and Data Acquisition (SCADA) system has been utilized for several decades to provide steady state monitoring of the electrical power systems, its ability to detect dynamic changes in the network and prevent severe blackouts has been limited. Now there is a challenge to maintain system availability and reliability. To achieve that some of the important power system parameters like active power, reactive power, load angle, frequency, bus voltage to be measured in synchronized manner and in real time. For these measurements, Phasor Measurement Unit is employed at particular locations to monitor the voltage and current waveform. It is synchronised with a clocking signal from the Global Positioning System (GPS) such that it can transmit data to the respective grid. This has motivated the development of the synchrophasor technology, and paved its way to the emerging form of Wide Area Monitoring Systems (WAMS).

2.1 Conventional Measurements

Power system can be monitored in real time either by SCADA based Energy Management System or by Synchrophasors based WAMS system. SCADA systems are implemented for several decades, where as synchrophasor is an emerging technology which is being developed in several utility system. SCADA system gets the data from transducers, meters. The gathered data includes information of breakers and switches, real and reactive power flows, voltage magnitudes. The main characteristics of SCADA based conventional measurements are given below:-

a. All the measurements are being taken within a time window, that is typically of few seconds long.

b. SCADA measurements provides only voltage and current magnitudes. No direct measurements of phase angles are possible[1].

c. Data from different locations are time skewed as the data is not time synchronized.

Based on the collected measurements, a state estimator is run to estimate the states i.e. voltage magnitude and phase angles at all buses in the system.

2.2 Limitations of Conventional Power System Monitoring

Conventional system monitor can fail when system state is changing quickly and when critical data are missing. When the power system state is changing quickly, measurements taken in a time window of a few seconds are not consistent with each other. The inconsistencies between any two analog measurements are proportional to the time difference between the measurements and the rate at which the states are changing.

2.3 Wide Area Monitoring Systems

A Wide Area Measurement System consists of Phasor Measurement Units, which provide the phasor measurements in the power system networks. The measurement includes both the magnitude and the phase angles of the bus voltage and incident branch current signal, and are time synchronized via Global Positioning System(GPS) with an accuracy of 1 microsecond[13]. The phasors, measured at the same time instant, provide snapshot of the power system network and by comparing the snapshots of two consecutive time instants with each other, not only the steady state, but also the dynamic states of the systems can be monitored.

2.4 Phasor Measurements

Phasors are basic tools of AC circuit analysis that represents a sinusoidal signal represented by the quantity of its magnitude and phase with respect to a reference. As depicted in Fig.2.1, the distance between sinusoidal peak of the signal and the time reference (e.g. time = 0) is defined as a phase angle and it is transferred to an angular measurement in the phasor representation[1].

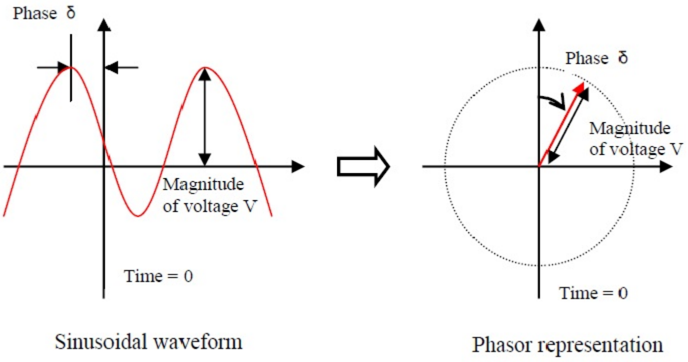

Figure 2.1: Sinusoidal waveform and its phasor representation

Phasor technology including the Phasor Measurement Unit (PMU) is a valuable measurement technology in the power system for monitoring the condition of transmission and distribution networks. As shown in Fig.2.2, the phasor of the 50 Hz component is obtained based on the digitally-sampled analog voltage waveform that is further synchronized with the clocking signal from the GPS receiver in distributed locations (say #1and #2). The time reference is titled as a *'common reference'* signal and it helps to synchronize the different waveforms at all different sites. The amplitude difference between Signal 1 and Signal 2 in Fig.2.2 is due to the signal attenuation on the overhead transmission line.

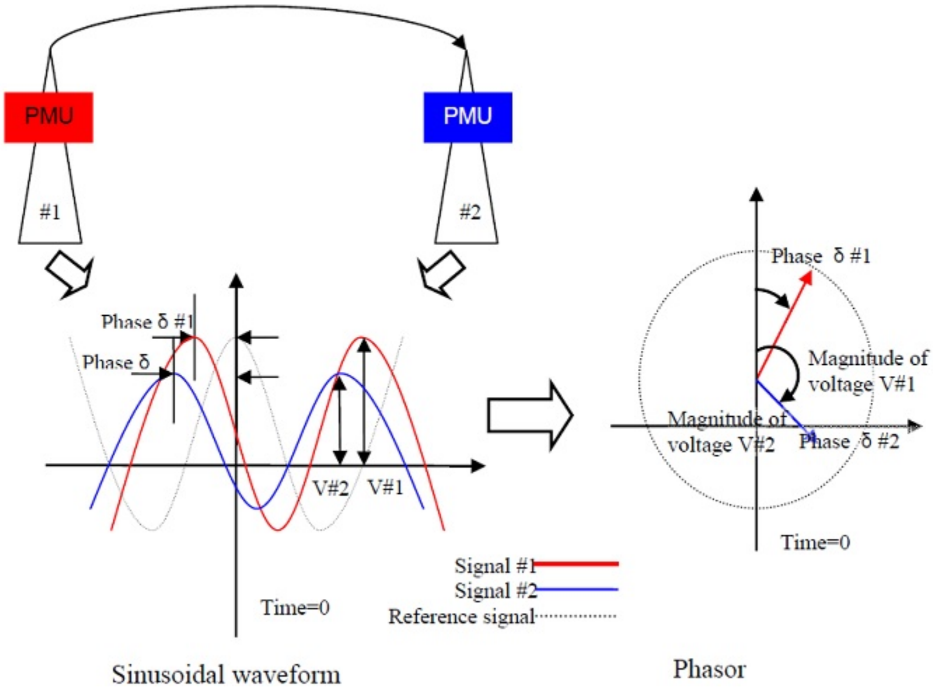

Figure 2.2: Signals received by PMUs[1]

2.5 Phasor Measurement Unit

The Phasor Measurement Unit embeds the Global Positioning System (GPS) receiver clocks to achieve the synchronising of sampled signals at nominated locations of the entire power network. The GPS receiver provides the 1 pulse-per-second(pps) signal, and a time tag, which consists of the year, day, hour, minute and second. The l-pps signal is divided by a phase-locked oscillator into the required number of pulses per second for sampling of the analog signals. In the real-life system, the PMU receives the voltage and current waveforms as inputs, which are derived from standard Current Transformer (CT) and Potential Transformer (PT). The input signals are being filtered and sampling is done at an effective rate of 48 samples per cycle of the fundamental frequency[2]. The complete unit meets the IEEE Surge Withstand Capability standard, C37.9.1. The microprocessor determines the positive sequence phasors according to the recursive algorithm described, and the timing message from the GPS, along with the sample number at the beginning of a window, is assigned to the phasor as its identifying tag. The computed string of phasors, one for each of the positive sequence measurements, is assembled in a message stream to be communicated to a remote site. The messages are transmitted over a dedicated communication line through the modems. A 4800 baud communication line can support the transmission of the phasor stream at the rate of about every 2-5 cycles of the fundamental frequency, depending upon the number of positive sequence phasors being transmitted[13].

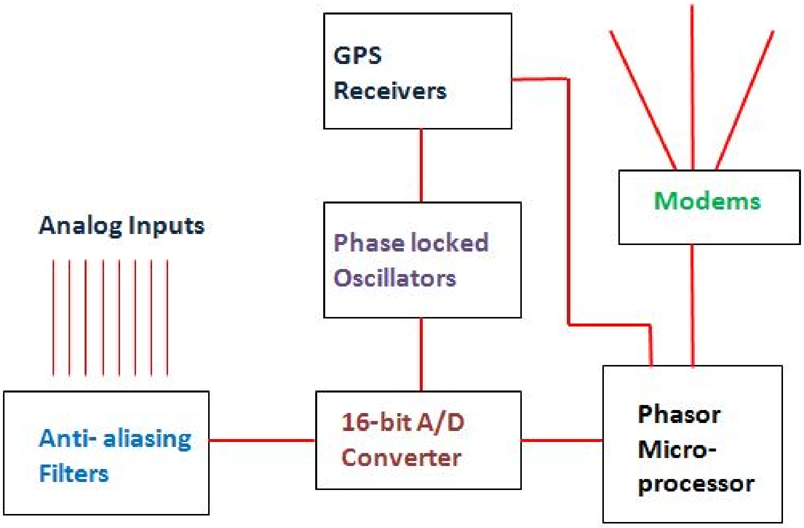

Figure 2.3: Block Digram of PMU

The phasor microprocessor, as shown in Fig.2.3, uses the recursive Discrete Fourier Transform (DFT) algorithm is being used to calculate the positive sequence, fundamental frequency, voltage and current phasors from the sampled data[1]. The resulting time-tagged phasors are immediately available for local or remote applications via the standard communications ports according to IEEEC37.118 standard. By operating interrelated software of the PMU, the users are capable of monitoring the phasors across the whole transmission network for any abnormal events.

2.6 Summary

This chapter provides a general description of measurement units. The traditional and modern methods of measurements are then discussed briefly. This, chapter together with the previous one provides general background, lays the groundwork of whole dissertation work.

Chapter 3

Network Observability

In previous chapter the conventional and synchronised measurement units are discussed. Further, limitations of conventional measurements are also discussed in brief. In this chapter, different methods of network observability i.e Topological and Numerical Observability, rules for the network observability and PMU placement rules for the same will be discussed. Also different PMU placement methods such as depth first method, Graph Theoretic Procedure, Recursive N-1 Spanning method will be discussed.

3.1 Network Observability

Network observability determines whether a state estimator will be able to determine a unique state solution for a given set of measurements. There are three basic approaches to conduct network observability analysis; namely numerical, topological, and hybrid approaches. A system is said to be observable when sufficient measurements are available, such that all the states of the system, consisting of bus voltage magnitude and angles at all buses can be estimated.

3.2 Observability Analysis

The principle of optimal placement of PMUs in a power system mostly is power system observability. After placing a new PMU, whatever method is used, the observability of the power system must be checked. If the system is observable, the placement stops, else the placement must be continued. There are several types of algorithms available for analyzing power network observability analysis i.e. topological observability and numerical methods[7].

(1) **Topological Observability**: This method uses the decoupled measurement model and graph theory. In this topological method, decision is based on logical operations. Thus, it requires only information about network connectivity, measurement types and their locations, if a full rank spanning tree can be constructed with the current measurement set; the system will be observable[7].

(2) **Numerical Observability**: This method uses either fully coupled or decoupled measurement models. These methods are based on numerical factorization of the measurement Jacobian or measurement information gain matrix. If any of these matrices is of full rank, the system is said to be observable[7].

3.3 Rules of Network Observability

Based on the fundamental laws of branch current and node voltage in circuit theory, several rules are applied to analyse the network to ensure that the network is fully observable.

Case 1: For PMU installed buses, voltage phasor and current phasor of all its incident branches are known. These are called **direct measurements**[7]. According to the function of the phasor measurement unit, a PMU located in the Bus D, as shown in Fig.3.1, indicates that the voltage in this bus can be directly measured. Meanwhile, the branch currents attached to the node are also measured by the PMU.[7]

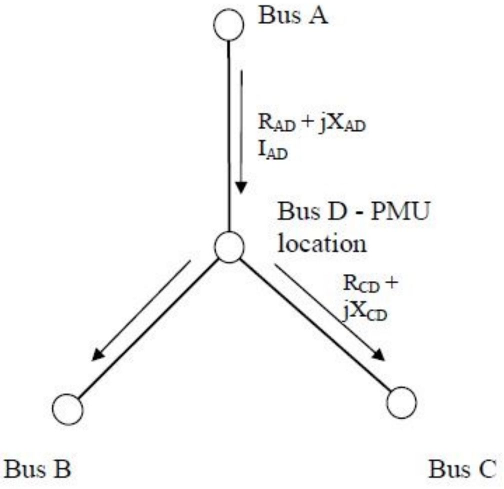

Figure 3.1: The first observability rule

Case 2: If the voltage and current phasors at one end of a branch are known, the voltage phasor at the other end of the branch can be obtained. These are called **pseudo measurements**[7].

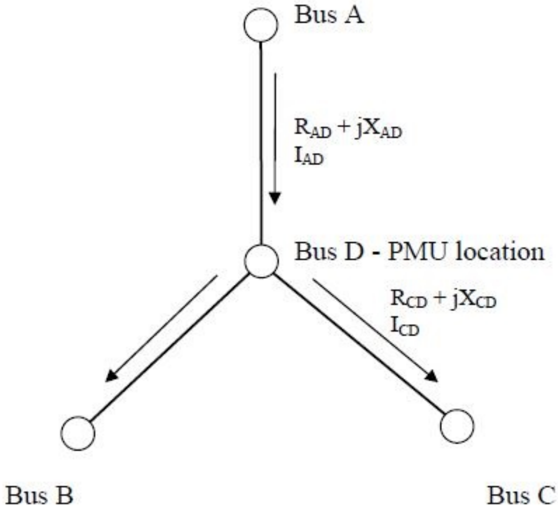

Figure 3.2: The second observability rule

Case 3: If voltage phasors of both ends of a branch are known, the current phasor of this branch can be obtained directly. These measurements are also known as **pseudo measurements**[7].

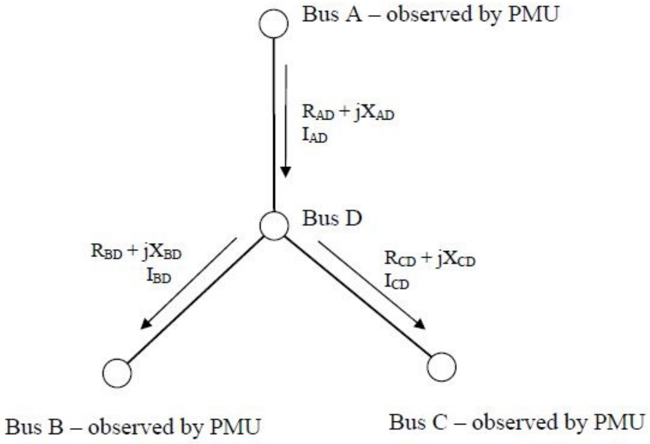

Figure 3.3: The third observability rule

Case 4: If there is zero injection bus without PMU whose incident branches current phasors are all known but one, then the current phasor of the unknown one could be obtainable by KCL equations.[7]

Case 5: If a zero-injection bus with unknown voltage phasor and voltage phasors of its adjacent buses are all known, then the voltage phasor of the zero-injection bus can be found using the nodal equations.[7]

Case 6: If a group of adjacent zero-injection buses exists, whose voltage phasors are unknown but the voltage phasors of all adjacent buses to the group are known, then the voltage phasors of zero-injection buses can be obtained using the nodal equations.[7]

Finally Fig.3.4 shows the flowchart of observability analysis based on aforementioned rules.

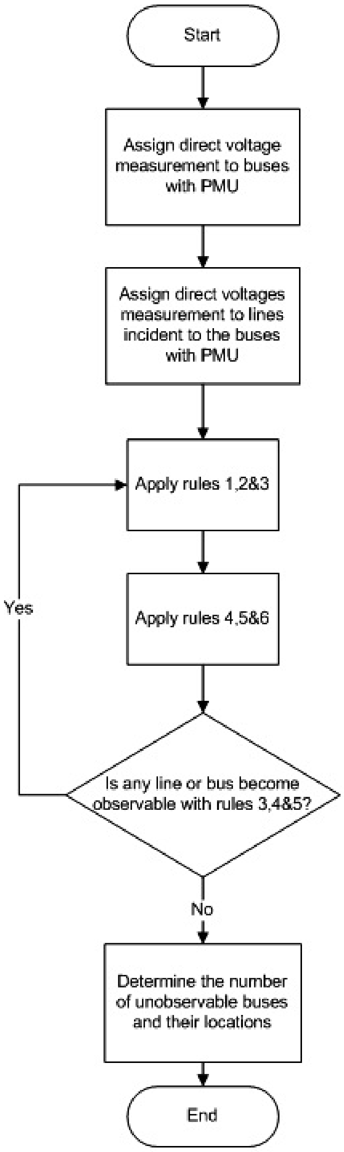

Figure 3.4: Flow chart of topological observability analysis

3.4 PMU Placement Rules

A PMU is able to measure the voltage phasor of the installed bus and the current phasors of some or all the lines connected to that bus. The following generalized rules are used for PMU Placement[4]

- Rule 1: Assign one voltage measurement to a bus where a PMU has been placed, including one current measurement to each branch connected to the bus itself[4].
- Rule 2: Assign one voltage pseudo- measurement to each node reached by an-

other equipped with a PMU[4].

- Rule 3: Assign one current pseudo- measurement to each branch connecting two buses where voltages are known. This allows interconnecting observed zones[4].
- Rule 4: Assign one current pseudo-measurement to each branch where current can be indirectly calculated by the Kirchhoff current law (KCL). This rule applies when the current balance at one node is known, i.e. if the node has no power injections (if N-1 currents insert to the node are known, the last current can be computed by difference)[4].

The pseudo-measurement proposed doesnt mean measure directly, but calculate the required measurement indirectly by the KCL, KVL. It can improve the convergence of the result by applying this rule in a variety of algorithms, so that it can reduce the number of the PMUs, which has the widespread economic practicability.

3.5 PMU Placement Algorithm

Various types of optimization techniques like Heuristic methods-depth first, graph theoretic procedure and Meta-Heuristic methods- Recursive N-1 Spanning, Simulated Annealing are discussed.[5]

3.5.1 Depth First Algorithm

This method uses only Rules from 1 to 3 (it does not consider pure transit nodes). The first PMU is placed at the bus with the largest number of connected branches. If there is more than one bus with this characteristic, one is randomly chosen. Following PMUs are placed with the same criterion, until the complete network observability is obtained, as depicted in Fig.3.5 [4].

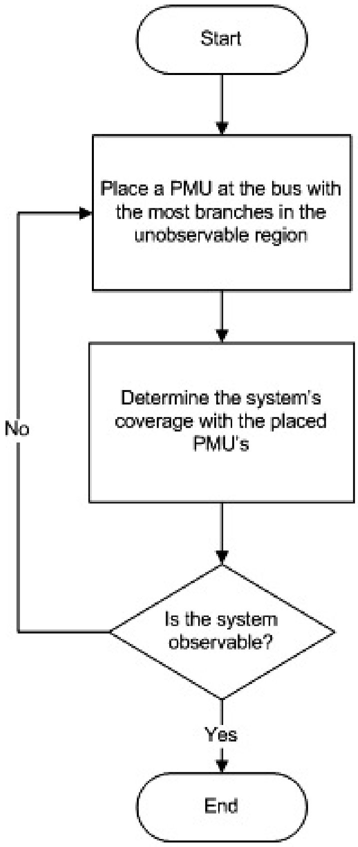

Figure 3.5: Depth First Algorithm

3.5.2 Graph Theoretic Procedure

This method was originally proposed in [Baldwin et al. 1993] and is similar to the depth first algorithm, except for taking into account of pure transit nodes (Rule 4).[4]

3.5.3 Simulated Annealing Algorithm

Simulated annealing method is put forward by Metropolis in 1953. Simulated Annealing (SA) is a technique that finds a good solution to an optimization problem, by trying random variations of the current solution. A worse variation is accepted as the new solution with a probability that decreases as the computation proceeds.[4]

3.5.4 Recursive N-1 Spanning Algorithm

The rules for minimal PMU placement assume a fixed network topology and a complete reliability of measurement devices. Simple criteria which yield a complete observability in case of line outages (N-1 security) are proposed in [3] and are based on the following definition: A bus is said to be observable if at least one of the two following conditions applies:[4]

Rule 1: a PMU is placed at the node;

Rule 2: the node is connected at least to two nodes equipped with a PMU.

Rule 2 is ignored if the bus is connected to single-end line.

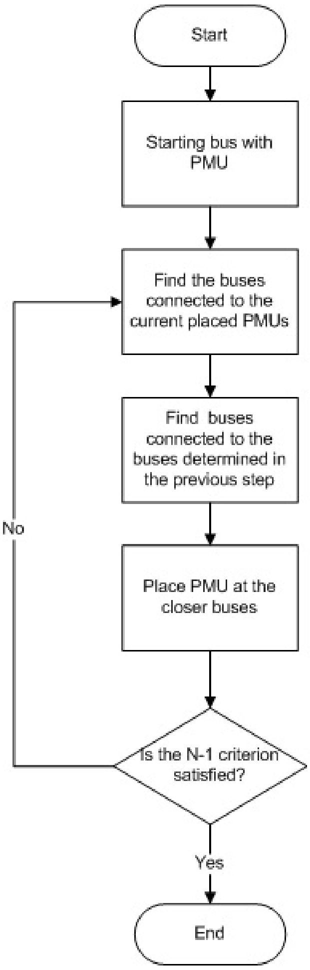

Figure 3.6: Recursive N-1 Algorithm

3.6 Summary

The chapter is indented to provide overview of different methods of network observability such as topological and numerical observability. The basic rules of network observability and PMU placement rules for the same are discussed. Different PMU placement algorithms such as depth first method, Graph Theoretic Procedure, Recursive N-1 Spanning are also discussed.

Chapter 4

Power System Observability Analysis by OPP

A power system state estimator demands complete observability of the network from the available measurements. Power system is observable by a set of measurements when the measurements can build a spanning tree of full rank of the system [10]. Power system observability analysis is usually carried out in two different ways namely numerical and topological observability analysis[10].

4.1 Power System Observability by OPP under Intact condition

4.1.1 Numerical observability analysis

The measurement model used in state estimation is

$$z = h(x) + e \tag{4.1}$$

where x is the system state vector i.e. the voltage phasor of all buses of the network, z is the measurement vector, h(x) is the non-linear function that relates the measurement vector to the state vector of the system and e represents the measurement error vector. Since PMUs provide accurate measurements (voltage and current phasors) the measurement error is small and can be neglected. Numerical observability analysis checks whether the rank of measurement matrix is full or not. For an N bus network the measurement sets (voltage and current phasors) obtained from PMUs can make the system numerically observable if[10]

$$Rank(h) = 2N - 1 \tag{4.2}$$

Optimal PMU placement for numerical observability of power system can be carried out in two different ways

Method I: Introduction of PMU in the network one by one to improve the rank of the measurement matrix. No further PMU is introduced when Rank(h)= 2N-1 is satisfied.

Method II: Considering PMU at all buses, PMUs are sequentially eliminated from different buses one by one. Elimination process is overruled at a bus which makes the rank of the measurement matrix deficient.

4.1.2 Topological observability analysis

Basic assumptions of the proposed method- The power system network is represented as an undirected graph G=(N, E), which consists of N nodes representing the number of busbars in the system, connected by edges of the set E corresponding to network branches between the busbars. To assess a system's topological observability with placement of PMUs, the following assumptions and simple rules have been applied [11].

a. If voltage phasor and current phasor at one end of a branch are known, voltage phasor at the other end of the branch can be calculated using Ohm's law.

b. If voltage phasors at both the ends of a branch are known, the branch current can be calculated.

c. If there is zero injection busbar without a PMU, whose outgoing currents are known except for one, then the unknown outgoing current can be calculated using Kirchhoff's Current Law (KCL)[7].

The measurements such as busbar voltage phasors and outgoing currents, directly obtained from PMUs, are referred as direct measurements and measurements derived by utilizing the above three rules are referred as pseudo measurements. Using this concept, many graph methods, e.g. depth first search, spanning tree based methods, integer linear programming based method, etc. have been suggested to optimally place PMUs in the system for ensuring the topological observability of the system[7]. This work has not considered the PMU outage or the communication link failure in the optimal PMU placement, which may be incorporated with the additional constraints in the formulation.

Proposed technique for PMU Placement

The optimal PMU placement problem can, therefore, be formulated as an **Integer linear programming** (ILP) problem[9, 10, 11] with an objective to minimize the total cost of PMU installations, while ensuring that each node in the system is observable. If PMUs are to be placed in an N bus system where cost of placing a PMU at busbar i is c_i, and U represents the vector consisting of binary decision variables u_i, then the objective function can be written as follows and the optimal solution of the ILP problem can be marked as U[7, 9, 10, 11].

$$Minimize \sum_{j=1}^{N} c_i u_i \tag{4.3}$$

subject to constraints

$$AU \geqslant 1 \tag{4.4}$$

$$U_j = (0/1), abinaryvariable \tag{4.5}$$

where, c_j is the cost of installing a PMU at jth busbar. In the present study, cost of PMU installation at all the busbars is assumed to be equal to 1 per unit. However, the realistic cost of adding PMU at each busbar can be easily considered in the formulation. A is the binary connectivity matrix of the system network of size($N \times N$), with entries a_{ij} as: a_{ij} =1, if i =j or if i and j are connected

$a_{ij} = 0$, otherwise

Numerical observability

Jacobian H, which relates the measurements to busbar voltage phasors (2N state variables) in a conventional weighted least square (WLS) estimator is of full rank. A voltage and current phasor measurement set (considered in this study) allows complete observability of the network if,

$$Rank(H) = 2N - 1 \tag{4.6}$$

Generally, for a topological based placement technique, Eq.(4.6) is not always true, and in that case the state estimation process cannot be carried out. For this reason, it is essential to compliment the topological observability with numerical observability analysis[7]. Eq.(4.6) needs the formulation of the measurement Jacobian, which is explained as follows.

Consider an N bus system provided with PMUs, to acquire m phasor measurements. The measurement vector Z can be linearly related to N dimensional state vector V, which contains N nodal phasor voltages. The measurement set is composed of voltage and current measurements. Using a measurement set, composed of voltage and current phasors, a linear state estimator can be formulated. The least square estimation in such formulation requires only one iteration[7].

The phasor measurements have been used in rectangular coordinates in this work. The constraint formulation is briefly discussed with a two busbar example as follows. Consider a two busbar system, as shown in Fig.4.1

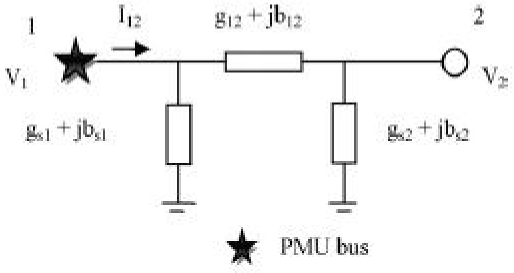

Figure 4.1: Two bus system with measurements

Assume that the busbar 1 is equipped with a PMU and measures phasor voltage V_1 and phasor current I_{12}. In rectangular coordinates, the voltage measurement can be represented as $V_1 = (E_1 + jF_1)$ and line current as $I_{12} = (C_{12} + jD_{12})$[9]. The phasor line current I_{12} can be expressed as,

$$I_{12} = [(g_{12} + jb_{12}) + (gs_1 + jbs_1)] \times V_1 - j(g_{12} + jb_{12}) \times V_2 \tag{4.7}$$

where $(g_{12} + jb_{12})$ is the series admittance of the line and $(gs_1 + jbs_1)$ is the shunt admittance of the line at busbar 1. In a simplified form, Eq.(4.7) can be re-written as,

$$I_{12} = k1 \times V_1 + k2 \times V_2 \tag{4.8}$$

where k1 and k2 are constant complex values. The measurement vector Z can, thus, be expressed as,

$$Z = \begin{pmatrix} V_1 \\ I_{12} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ k1 & k2 \end{pmatrix} * \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} + e = HV + e \tag{4.9}$$

where e is an error vector, H is (2×2) measurement Jacobian of linear equations. In general, for an N bus system with m-phasor measurements, H is $(m \times N)$measurement Jacobian of linear equations. In state estimation, usually one busbar is chosen as the reference busbar with its voltage angle taken as zero, in order to get the relative phase angles at other buses. The imaginary part of the corresponding reference voltage phasor is, thus, zero and X is, thereby, (2N-1) state vector. Z is $(2m \times 1)$ measurement vector and H is $(2m \times 2N - 1)$ measurement Jacobian. The system is said to be numerically observable, if rank of the measurement Jacobian H is (2N-1)[7, 9, 10].

4.2 Power System Observability by OPP under Contingencies

A power system state estimator demands complete observability of the network from the available measurements. Power system is observable by a set of measurements when the measurements can build a spanning tree of full rank of the system [8]. Optimal placement of Phasor Measurement Units (PMUs), to ensure complete observability of power system, should not only be decided at the base case operating conditions but also must consider the contingency cases such as single PMU loss, line outages, communication channel limit. Power system observability analysis under contingencies is usually carried out in two different ways namely numerical and topological observability analysis[8]. PMU malfunction and system line outages are probable occurrences, as are failures of other equipment used in power systems. Thus, continuous state estimation in a power system requires that the placement of these units be performed such that full observability of the system is maintained in either condition. However, the high cost of PMUs restricts the number of units that can be installed in a power system. Therefore, they should be placed optimally so as to reduce the number of units required to as few as possible[8].

4.3 Summary

This chapter provides a brief introduction about power system observability under intact and contingency condition. Different ways of Network observability such as topological and numerical observability is discussed. Binary integer linear programming method is discussed for topological observability of the system.

Chapter 5

Simulation Results and Discussion

In this simulation work, popular Western System Coordinating Council (WSCC) 3 generators, 9 bus system and IEEE 14-bus system used. This system is simulated using MATLAB.

5.1 Case 1:Topological Observability Analysis using PSAT

Analysis of topological observability is done by using PSAT toolbox of MATLAB. Standard test systems such as WSCC 9-bus system, IEEE 14-bus system are considered for network observability by considering optimised placement of phasor measurement units.

5.1.1 WSCC 9-bus system

The algorithms discussed in chapter-3 are applied to WSCC 9-Bus system Fig.5.1 represents single line diagram of the system. Power system analysis toolbox (PSAT) is used to gain the results for all the methods and described as per Table-5.1. For complete system observability simulated annealing(SA) suggests minimum two PMUs instead of three from depth first algorithm, hence it would be beneficial in cost comparison. As the speed of placement is less in SA method, Recursive N-1 Algorithm would be more preferable as it includes single outage of system component[12].

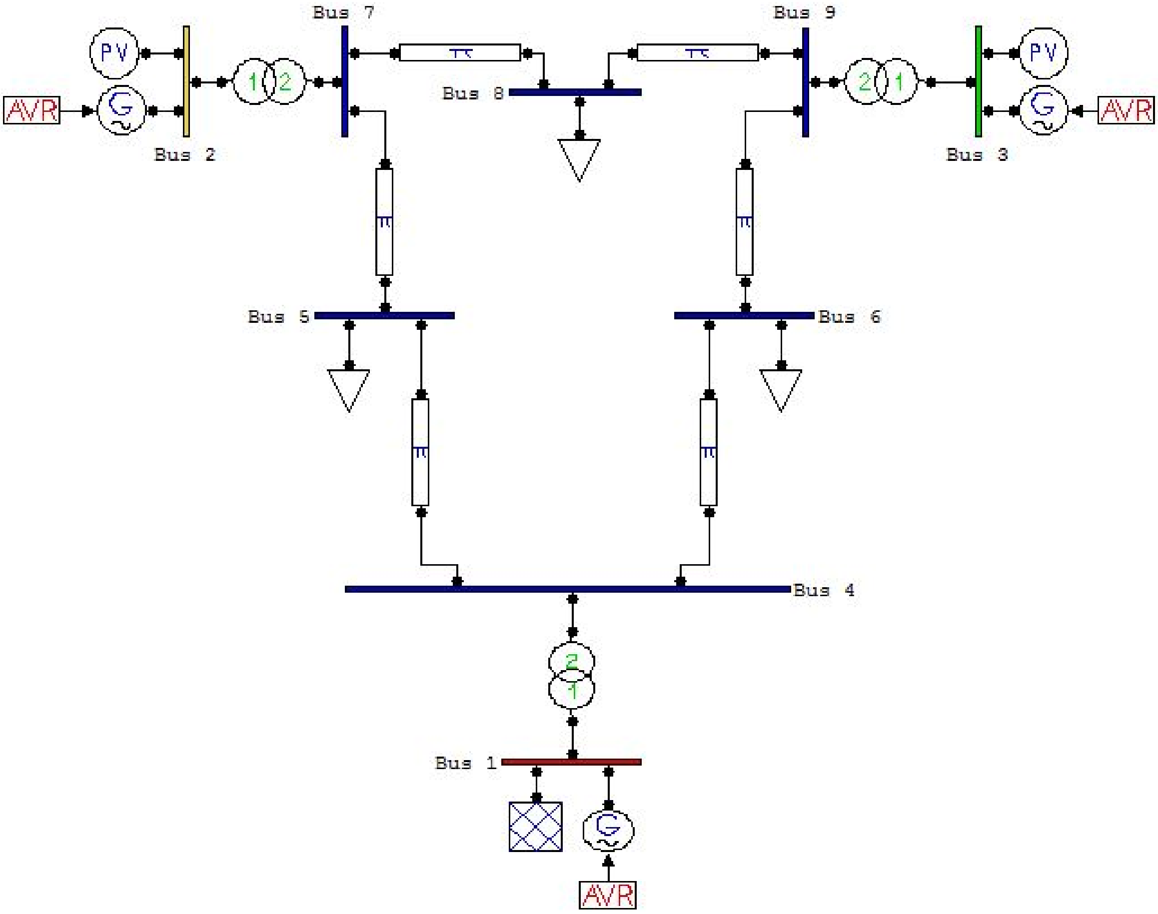

Figure 5.1: WSCC 9-Bus System

The graph model shown in Fig.5.2 depicts that the nodes-4,7,9 has maximum number of incident lines to it.

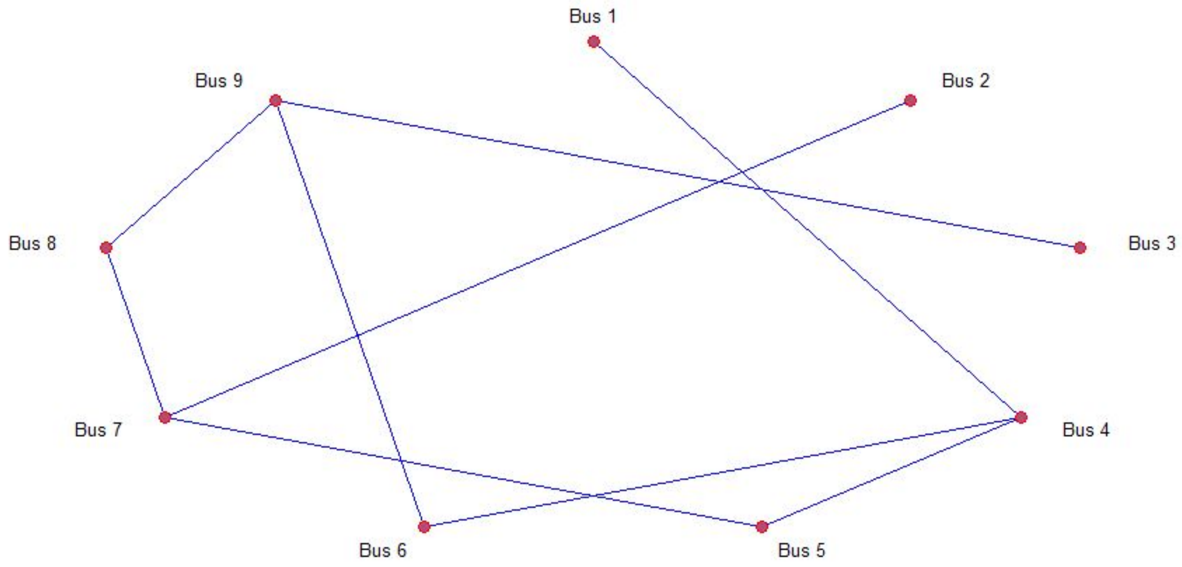

Figure 5.2: Graph Model of WSCC 9-Bus System

Table 5.1: PMU Placement Result of WSCC-9 Bus System

Method	Elapsed time	No of PMUs	Set	Bus Location
Depth First	0.047602s	03	01	4, 7, 9
Simulated Annealing	1.0783s	02	01	4, 8
Recursive N-1 Spanning	0.3301s	03	06	4,7,9

5.1.2 IEEE 14-bus system

The algorithms discussed in chapter-3 are applied to IEEE 14-bus system. It has five synchronous machines, three of which are synchronous condensers used for reactive power support and twenty branches. Single line diagram of this test system is shown in Fig.5.3. Node 7 is called a pure transit node; we also call it no load node. There are no loads to consume the power, no generators to inject the power either. The power injected by node 8 and node 4 transmits to node 9 completely that is why it is named pure transit node.

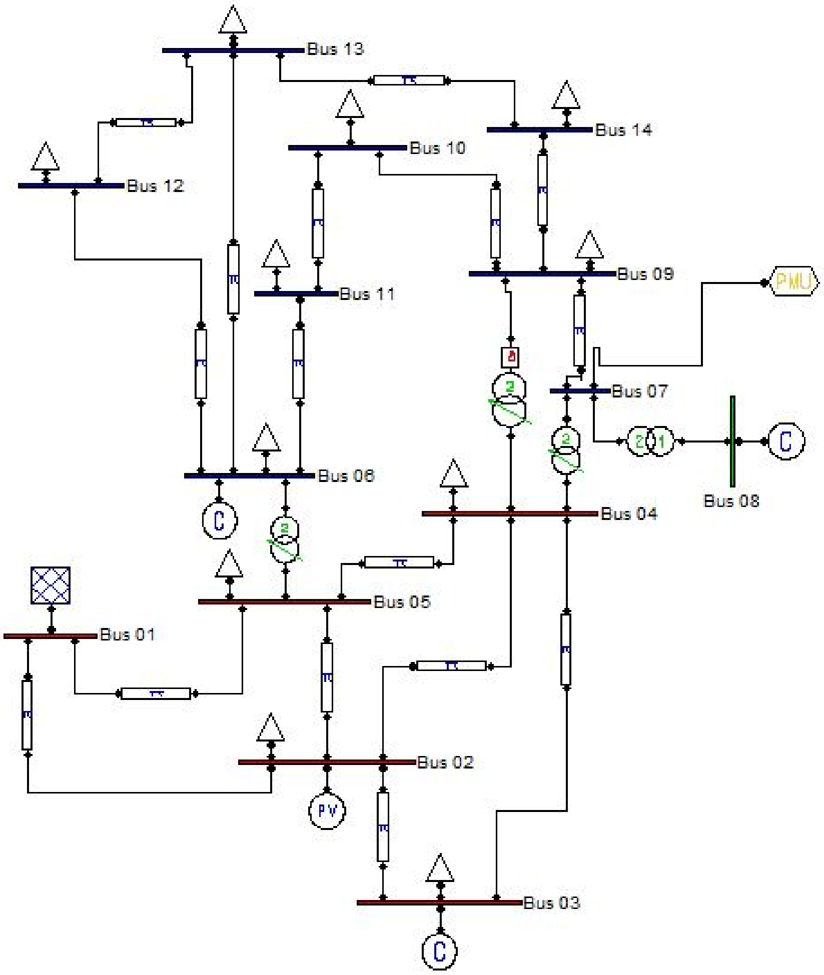

Figure 5.3: IEEE 14-Bus System

The graph model is given in Fig.5.4. The graph model shows that the nodes-2,4,5,6,7,9 has the maximum number of incident lines to it.

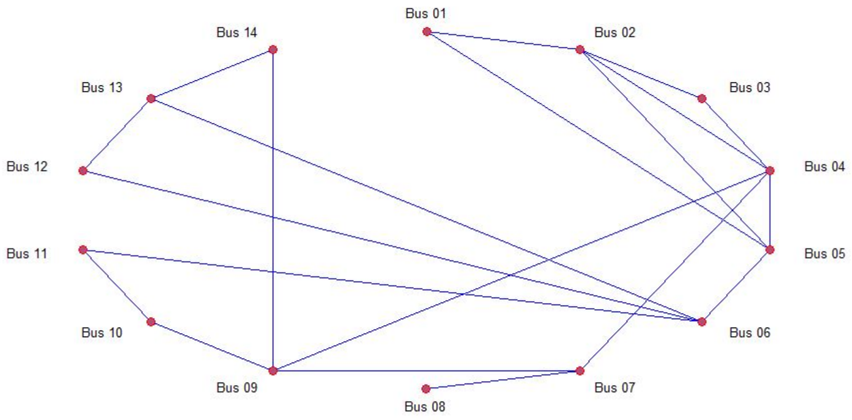

Figure 5.4: Graph Model of IEEE 14 Bus System

Table 5.2: PMU Placement Result of IEEE-14 Bus System

Method	Elapsed time	No of PMUs	Set	Bus Location
Depth First	0.04394s	06	01	1, 4, 6, 8, 10, 14
Simulated Annealing	0.86934s	04	01	1, 4, 10, 13
Recursive N-1 Spanning	0.2344s	08	08	2,5,6,7,9,10,13,14

For complete system observability simulated annealing(SA) suggests minimum four PMUs, hence it is beneficial in cost comparison. As the speed of placement is restricted in SA method, Recursive N-1 Algorithm is preferred as it includes single outage of system component.

5.2 Case 2: Complete Observability by ILP method under Intact Condition

Different test systems such as WSCC 9-bus system, IEEE 14-bus system are considered for topological and numerical observability analysis.

5.2.1 WSCC 9-bus System

The details of this system is given in Appendix-A. The basic PMU placement algorithm, as explained in chapter 4, is applied on this system for complete observability of the system. In WSCC 9-bus system, U contains 9 binary decision variables if zero injection is not considered. The constraint at each bus is formulated to ensure that each bus is topologically observable. The decision vector U contains 9 decision variables and the complete ILP can be formulated as-

$$Minimize \sum_{j=1}^{9} c_j u_j \tag{5.1}$$

subject to:

$$f1 : u_1 + u_4 \geq 1 \tag{5.2}$$

$$f2 : u_2 + u_7 \geq 1 \tag{5.3}$$

$$f3 : u_3 + u_9 \geq 1 \tag{5.4}$$

$$f4 : u_1 + u_4 + u_6 \geq 1 \tag{5.5}$$

$$f5 : u_5 + u_7 \geq 1 \tag{5.6}$$

$$f6 : u_4 + u_6 + u_9 \geq 1 \tag{5.7}$$

$$f7 : u_2 + u_5 + u_7 + u_8 \geq 1 \tag{5.8}$$

$$f8 : u_7 + u_8 + u_9 \geq 1 \tag{5.9}$$

$$f9 : u_3 + u_6 + u_8 + u_9 \geq 1 \tag{5.10}$$

The solution of the above ILP , with $c_j = 1$ p.u provides the basic optimal PMU locations at buses 4,7,9. This provides 3 phasor voltage measurements, 8 phasor current measurements and therefore, 22 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 17. It indicates that topological observability has resulted into the numerical observability of the power system.As rank of jacobian is 17.

5.2.2 IEEE 14 bus System

The details of this system is given in Appendix-A. The basic PMU placement algorithm, as explained in chapter 4, is applied on this system with and without considering zero injection busbar.

Without considering Zero injection busbar

Fig.5.5 shows the single line diagram of the IEEE 14-bus system without considering zero injection. It has five synchronous machines, three of which are synchronous condensers used for reactive power support and twenty branches. Node 7 is called a pure transit node; we also call it no load node. There are no loads to consume the power, no generators to inject the power either. In IEEE-14 bus system, U contains 14 binary decision variables if zero injection is not considered. The constraint at each bus is formulated to ensure that each bus is topologically observable. The decision vector U contains 14 decision variables and the complete ILP can be formulated as-

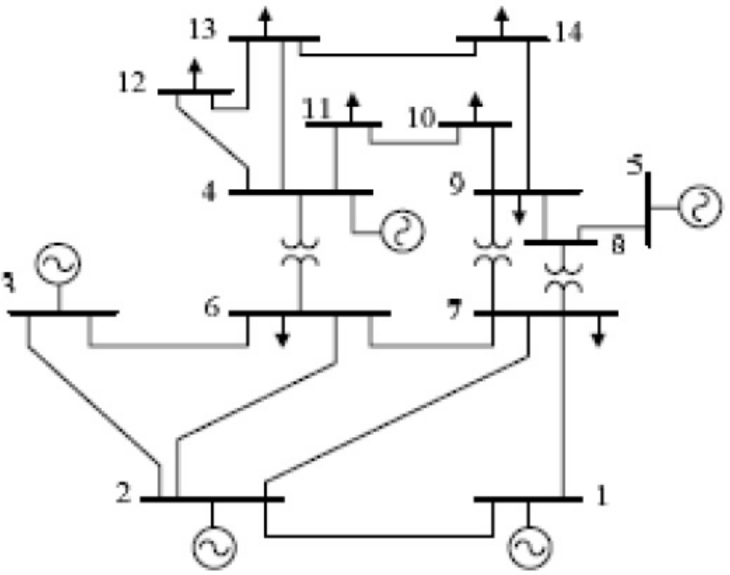

Figure 5.5: Single line diagram of IEEE-14 bus System

$$Minimize \sum_{j=1}^{14} c_j u_j \tag{5.11}$$

subject to:

$$f1 : u_1 + u_2 + u_5 \geq 1 \tag{5.12}$$

$$f2 : u_1 + u_2 + u_3 + u_4 + u_5 \geq 1 \tag{5.13}$$

$$f3 : u_2 + u_3 + u_4 \geq 1 \tag{5.14}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_7 + u_9 \geq 1 \tag{5.15}$$

$$f5 : u_1 + u_2 + u_4 + u_5 + u_6 \geq 1 \tag{5.16}$$

$$f6 : u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 1 \tag{5.17}$$

$$f7 : u_4 + u_7 + u_8 + u_9 \geq 1 \tag{5.18}$$

$$f8 : u_7 + u_8 \geq 1 \tag{5.19}$$

$$f9 : u_4 + u_7 + u_9 + u_{10} + u_{14} \geq 1 \tag{5.20}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 1 \tag{5.21}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 1 \tag{5.22}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 1 \tag{5.23}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 1 \tag{5.24}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 1 \tag{5.25}$$

The solution of the above ILP , with $c_j = 1$ p.u provides the basic optimal PMU locations at buses 2,6,7,9. This provides 4 phasor voltage measurements, 15 phasor current measurements and therefore, 38 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 27. It indicates that topological observability has resulted into the numerical observability of the power system.As rank of jacobian is 27.

With considering Zero injection busbar

Fig.5.6 shows the single line diagram of the IEEE 14-bus system considering zero injection busbar. In IEEE-14 bus system, U contains 14 binary decision variables if zero injection is not considered. However the system gets modified when a zero injection is considered at bus-7. It has been obtained by merging buses 7 and 8 and creating a new another fictious bus 7‘.

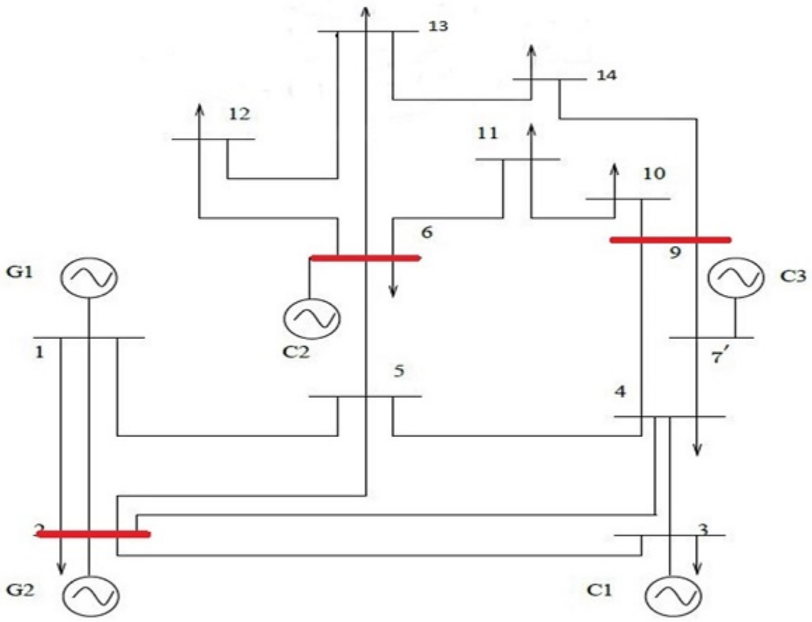

Figure 5.6: IEEE 14-Bus System considering zero injection

The constraint at each bus is formulated to ensure that each bus is topologically observable.The decision vector U contains 13 decision variables and the complete ILP can be formulated as-

$$Minimize \sum_{j=1}^{13} c_j u_j \tag{5.26}$$

subject to:

$$f1 : u_1 + u_2 + u_5 \geq 1 \tag{5.27}$$

$$f2 : u_1 + u_2 + u_3 + u_4 + u_5 \geq 1 \tag{5.28}$$

$$f3 : u_2 + u_3 + u_4 \geq 1 \tag{5.29}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_{7‘} + u_9 \geq 1 \tag{5.30}$$

$$f5 : u_1 + u_2 + u_4 + u_5 + u_6 \geq 1 \tag{5.31}$$

$$f6 : u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 1 \tag{5.32}$$

$$f7' : u_4 + u_{7'} + u_9 \geq 1 \tag{5.33}$$

$$f9 : u_4 + u_{7'} + u_9 + u_{11} + u_{14} \geq 1 \tag{5.34}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 1 \tag{5.35}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 1 \tag{5.36}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 1 \tag{5.37}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 1 \tag{5.38}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 1 \tag{5.39}$$

The solution of the above ILP, with c$_j$ = 1p.u provides the basic optimal PMU locations at buses 2,6,9 placed at these three optimal locations. This provides 3 phasor voltage measurements, 12 phasor current measurements and therefore, 30 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 25. It indicates that topological observability has resulted into the numerical observability of the power system.As rank of jacobian is 25.

Table 5.3: PMU Placement Result considering zero injection bus of IEEE 14 Bus System by ILP Method

Method	ZIB Location	No of PMUs	Set	Bus Location
Integer linear programming	7	03	01	$2, 6, 9$

Table 5.4: PMU Placement Result without considering zero injection bus of IEEE 14 Bus System by ILP Method

Method	No of PMUs	Set	Bus Location
Integer linear programming	04	01	$2, 6, 7, 9$

5.3 Case 3:Complete Observability by ILP under Contingencies

In order to illustrate the effect of a contingency on system topological and numerical observability and optimal placement of PMUs to ensure the full observability, an Integer Linear Programming (ILP) based PMU placement technique [10] has been used. A brief description of the method is given in the above section. Different contingencies such as single line outage,loss of single PMU is considered in **WSCC 9-bus system**, **IEEE 14-bus system**.

5.3.1 WSCC 9-bus system

In the event of an outage of the line 1-4 , current phasor measurement of line 1-4 provided by PMU at bus 4 gets lost and PMUs fail to observe node 1. Similarly, outage of critical line 4-6, leaves node 6 unobservable. The analysis clearly shows that the optimally placed PMUs in power system, considering only the system intact conditions, fail to ensure complete observability under contingency cases.

(a) Single line Outage

The ILP-1 solution, in normal state, is 4, 7 and 9. The outage of line 1-4 is considered to be the first critical contingency.

(1) Topological Observability: The constraints are modified as,

$$f1 : u_1 \geq 1 \tag{5.40}$$

$$f2 : u_2 + u_7 \geq 1 \tag{5.41}$$

$$f3 : u_3 + u_9 \geq 1 \tag{5.42}$$

$$f4 : u_4 + u_6 \geq 1 \tag{5.43}$$

$$f5 : u_5 + u_7 \geq 1 \tag{5.44}$$

$$f6 : u_4 + u_6 + u_9 \geq 1 \tag{5.45}$$

$$f7 : u_2 + u_5 + u_7 + u_8 \geq 1 \tag{5.46}$$

$$f8 : u_7 + u_8 + u_9 \geq 1 \tag{5.47}$$

$$f9 : u_3 + u_6 + u_8 + u_9 \geq 1 \tag{5.48}$$

Similarly for all line outages will be considered.

Table 5.5: PMU Placement under Single line Outage of WSCC 9-Bus System by ILP

Case	No of PMUs	Bus Location
Intact	03	$4, 7, 9$
Line 1-4 Outage	04	$1, 6, 7, 9$
Line 4-6 Outage	03	$1, 7, 9$
Line 3-9 Outage	04	$3, 4, 7, 9$
Line 6-9 Outage	03	$3, 4, 7$
Line 5-7 Outage	04	$4, 5, 7, 9$
Line 7-8 Outage	04	$4, 7, 8, 9$
Line 2-7 Outage	04	$2, 4, 7, 9$
Line 8-9 Outage	03	$4, 7, 9$

(2) Numerical Observability

a. When basic optimal PMU locations are 3,4,7,9 i.e for line outages of 3-9,6-9. This provides 4 phasor voltage measurements, 10 phasor current measurements and therefore, 28 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 17. It indicates that topological observability has resulted into the numerical observability of the power system. As rank of jacobian should be 17.

b. When basic optimal PMU locations are 1,6,7,9 i.e for line outages of 1-4,4-6. This provides 4 phasor voltage measurements, 9 phasor current measurements and therefore, 26 measurements in the measurement vector Z . The rank of the measurement Jacobian H is found as 17. It indicates that topological observability has resulted into the numerical observability of the power system. As rank of jacobian should be 17.

(b) Loss of Single PMU

Till now, it is considered that each bus is observable at least by one PMU and these PMUs, placed by proposed algorithm, will function properly. PMUs highly reliable but, if because of any disturbance in power system or for maintenance purpose any of these optimally placed PMU is out from system than some of the buses connected through it may not remain observable. To guard against such unexpected failure of PMUs, a strategy is developed to account for single PMU loss. This objective is accomplished if all buses are observable by at least two PMUs. To get these sets of PMU, the objective and constraint function will remain same with the only change in matrix b[13]. For this case the elements of matrix b will be equal to 2 instead of 1, as for previous case, as shown below:

(1) Topological Observability The constraints are modified as:

$$f1 : u_1 + u_4 \geq 2 \tag{5.49}$$

$$f2 : u_2 + u_7 \geq 2 \tag{5.50}$$

$$f3 : u_3 + u_9 \geq 2 \tag{5.51}$$

$$f4 : u_1 + u_4 + u_6 \geq 2 \tag{5.52}$$

$$f5 : u_5 + u_7 \geq 2 \tag{5.53}$$

$$f6 : u_4 + u_6 + u_9 \geq 2 \tag{5.54}$$

$$f7 : u_2 + u_5 + u_7 + u_8 \geq 2 \tag{5.55}$$

$$f8 : u_7 + u_8 + u_9 \geq 2 \tag{5.56}$$

$$f9 : u_3 + u_6 + u_8 + u_9 \geq 2 \tag{5.57}$$

The ILP solution considering loss of single PMU of IEEE 14-bus system is 1,2,4,6,7,8,9, 11,13.

(2) Numerical Observability

When basic optimal PMU locations are 1,2,3,4,5,6,7,8,9 i.e for loss of single PMU. This provides 9 phasor voltage measurements, 16 phasor current measurements and therefore, 50 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 17.

5.3.2 IEEE 14-bus system

Without considering zero injection busbar

In the event of an outage of the critical line 1-2, current phasor measurement of line 1-2 provided by PMU at bus 2 gets lost and PMUs fail to observe node 1. Similarly, outage of critical line 2-3, leaves node 3 unobservable. And, the optimal scheme, in the event of the outage of line 1-5, still observes all nodes of the IEEE 14-bus system. The analysis clearly shows that the optimally placed PMUs in power system, considering only the system intact conditions, fail to ensure complete observability under contingency cases.

(a) Single line Outage

The ILP-1 solution, in normal state, is 2, 6, 7, and 9. The outage of line 1-2 is considered to be the first critical contingency.

(1) Topological Observability: The constraints are modified as,

$$f1 : u_1 + u_5 \geq 1 \tag{5.58}$$

$$f2 : u_2 + u_3 + u_4 + u_5 \geq 1 \tag{5.59}$$

$$f3 : u_2 + u_3 + u_4 \geq 1 \tag{5.60}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_7 + u_9 \geq 1 \tag{5.61}$$

$$f5 : u_1 + u_2 + u_4 + u_5 + u_6 \geq 1 \tag{5.62}$$

$$f6 : u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 1 \tag{5.63}$$

$$f7 : u_4 + u_7 + u_8 + u_9 \geq 1 \tag{5.64}$$

$$f8 : u_7 + u_8 \geq 1 \tag{5.65}$$

$$f9 : u_4 + u_7 + u_9 + u_{11} + u_{14} \geq 1 \tag{5.66}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 1 \tag{5.67}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 1 \tag{5.68}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 1 \tag{5.69}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 1 \tag{5.70}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 1 \tag{5.71}$$

Similarly for all line outages will be considered as given in Table 5.6

(2) Numerical Observability

a. When basic optimal PMU locations are 4,5,6,7,9 i.e for line outages of 1-2,2-5,2-3. This provides 5 phasor voltage measurements, 19 phasor current measurements and therefore, 48 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 27. It indicates that topological observability has resulted into the numerical observability of the power system. As rank of jacobian should be 27.

b. When basic optimal PMU locations are 2,7,10,13 i.e for line outages of 6-12,14-9,9-7,7-4,13-6,6-11. This provides 4 phasor voltage measurements, 12 phasor current measurements and therefore, 32 measurements in the measurement vector Z . The rank of the measurement Jacobian H is found as 27. It indicates that topological observability has resulted into the numerical observability of the power system. As rank of jacobian should be 27.

Table 5.6: PMU Placement under Single line Outage of IEEE 14 Bus System by ILP

Case	No of PMUs	Bus Location
Intact	04	$2, 6, 7, 9$
Line 1-2 Outage	05	$4, 5, 6, 7, 9$
Line 2-5 Outage	05	$4, 5, 6, 7, 9$
Line 2-3 Outage	05	$4, 5, 6, 7, 9$
Line 1-5 Outage	04	$2, 6, 7, 9$
Line 9-10 Outage	04	$2, 7, 10, 13$
Line 5-6 Outage	04	$2, 6, 7, 9$
Line 12-13 Outage	04	$2, 6, 7, 9$
Line 13-14 Outage	04	$2, 6, 7, 9$
Line 7-8 Outage	04	$2, 6, 7, 9$
Line 4-3 Outage	04	$2, 6, 7, 9$
Line 4-2 Outage	04	$2, 6, 7, 9$
Line 4-5 Outage	04	$2, 6, 7, 9$
Line 9-4 Outage	04	$2, 6, 7, 9$
Line 10-11 Outage	04	$2, 6, 7, 9$
Line 6-12 Outage	04	$2, 7, 10, 13$
Line 14-9 Outage	04	$2, 7, 10, 13$
Line 9-7 Outage	04	$2, 7, 10, 13$
Line 7-4 Outage	04	$2, 7, 10, 13$
Line 13-6 Outage	04	$2, 7, 10, 13$
Line 6-11 Outage	04	$2, 7, 10, 13$

(b) Loss of Single PMU

Till now, it is considered that each bus is observable atleast by one PMU and these PMUs, placed by proposed algorithm, will function properly. PMUs highly reliable but, if because of any disturbance in power system or for maintenance purpose any of these optimally placed PMU is out from system than some of the buses connected through it may not remain observable. To guard against such unexpected failure of PMUs, a strategy is developed to account for single PMU loss. This objective is accomplished if all buses are observable by at least two PMUs. So, if one PMU from primary set will not function than backup set will make the system observable. To get these sets of PMU, the objective and constraint function will remain same with the only change in matrix b[13]. For this case the elements of matrix b will be equal to 2 instead of 1, as for previous case, as shown below:

(1) Topological Observability

$$f1 : u_1 + u_2 + u_5 \geq 2 \tag{5.72}$$

$$f2 : u_1 + u_2 + u_3 + u_4 + u_5 \geq 2 \tag{5.73}$$

$$f3 : u_2 + u_3 + u_4 \geq 2 \tag{5.74}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_7 + u_9 \geq 2 \tag{5.75}$$

$$f5 : u_1 + u_2 + u_4 + u_5 + u_6 \geq 2 \tag{5.76}$$

$$f6 : u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 2 \tag{5.77}$$

$$f7 : u_4 + u_7 + u_8 + u_9 \geq 2 \tag{5.78}$$

$$f8 : u_7 + u_8 \geq 2 \tag{5.79}$$

$$f9 : u_4 + u_7 + u_9 + u_{11} + u_{14} \geq 2 \tag{5.80}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 2 \tag{5.81}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 2 \tag{5.82}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 2 \tag{5.83}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 2 \tag{5.84}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 2 \tag{5.85}$$

The ILP solution considering loss of single PMU of IEEE 14-bus system is 1,2,4,6,7,8,9, 11,13.

Table 5.7: PMU Placement Result under Loss of Single PMU of IEEE 14 Bus System by ILP Method

Method	No of PMUs	Set	Bus Location
Integer linear programming	9	01	$1, 2, 4, 6, 7, 8, 9, 11, 13$

(2) Numerical Observability

When basic optimal PMU locations are 1,2,4,6,7,8,9,11,13 i.e for loss of single PMU. This provides 9 phasor voltage measurements, 27 phasor current measurements and therefore, 72 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 27.

With considering Zero injection busbar

In IEEE-14 bus system, the system gets modified when a zero injection is considered at bus-7. It has been obtained by merging buses 7,8 and creating a new bus 7'. The constraint at each bus is formulated to ensure that each bus is topologically observable. Following results are obtained for single line outage and loss of single PMU.

(a) Single line Outage

Considering zero injection busbar in IEEE 14-bus system, following are the results for topological and numerical observability.

(1) Topological Observability

The constraint at each bus is formulated as described in chapter 4 to ensure that each bus is topologically observable. Table 5.8 shows PMU Placement under single line outage considering zero injection.

(2) Numerical Observability

a. When basic optimal PMU locations are 4,5,6,9 i.e for line outages of 1-2,2-5,2-3,9-7 considering zero injection. This provides 4 phasor voltage measurements, 17 phasor current measurements and therefore, 42 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 25. It indicates that topological observability has resulted into the numerical observability of the power system as rank of jacobian should be 25.

Table 5.8: PMU Placement under Single line Outage of IEEE 14 Bus System considering zero injection

Case	No of PMUs	Bus Location
Intact	04	$2, 6, 7, 9$
Line 1-2 Outage	04	$4, 5, 6, 9$
Line 2-5 Outage	03	$2, 6, 9$
Line 2-3 Outage	04	$4, 5, 6, 9$
Line 1-5 Outage	04	$4, 5, 6, 9$
Line 9-10 Outage	04	$2, 6, 9, 10$
Line 5-6 Outage	03	$2, 6, 9$
Line 12-13 Outage	03	$2, 6, 9$
Line 13-14 Outage	03	$2, 6, 9$
Line 4-3 Outage	03	$2, 6, 9$
Line 4-2 Outage	03	$2, 6, 9$
Line 4-5 Outage	03	$2, 6, 9$
Line 9-4 Outage	03	$2, 6, 9$
Line 10-11 Outage	03	$2, 6, 9$
Line 6-12 Outage	03	$2, 6, 9$
Line 14-9 Outage	04	$2, 9, 10, 12$
Line 9-7 Outage	04	$4, 5, 6, 9$
Line 7-4 Outage	03	$2, 6, 9$
Line 13-6 Outage	04	$2, 9, 10, 11$
Line 6-11 Outage	04	$2, 9, 10, 12$

b. When basic optimal PMU locations are 2,6,9 i.e for line outages of 4-2,4-3,4-5,5-6,7-4,9-4,11-10,12-13,13-14 . This provides 3 phasor voltage measurements, 12 phasor current measurements and therefore, 30 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 25. It indicates that topological observability has resulted into the numerical observability of the power system as rank of jacobian should be 25.

c. When basic optimal PMU locations are 2,9,10,12 i.e for line outages of 13-6,14-9,6-11,6-12,9-10. This provides 4 phasor voltage measurements, 11 phasor current measurements and therefore, 30 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 25. It indicates that topological observability has resulted into the numerical observability of the power system as rank of jacobian should be 25.

(b) Loss of Single PMU

Considering zero injection busbar following are the results for loss of single PMU.

(1) Topological Observability

Table 5.9: PMU Placement Result under Loss of Single PMU of IEEE 14 Bus System considering zero injection

Method	No of PMUs	Set	Bus Location
Integer linear programming	07	01	$2, 4, 5, 6, 8, 10, 12$

(2) Numerical Observability

When basic optimal PMU locations are 2,4,5,6,8,10,12 i.e for loss of single PMU considering zero injection. This provides 7 phasor voltage measurements, 22 phasor current measurements and therefore, 58 measurements in the measurement vector Z. The rank of the measurement Jacobian H is found as 25.

5.4 Case 4: Network Observability Considering Conventional Measurements

5.4.1 Considering Flow Measurements

Power grid has a number of conventional measurement devices distributed throughout the grid. These measurements may be utilized in developing optimal PMU placement algorithm[16]. Therefore availability of such measurements may reduce the number of PMUs required for complete observability. Flow measurement is considered as conventional measurement. Flow measurement is considered at line 5-6. So in IEEE 14-bus system,the constraints for ILP can be modified as-

$$f1 : u_1 + u_2 + u_5 \geq 1 \tag{5.86}$$

$$f2 : u_1 + u_2 + u_3 + u_4 + u_5 \geq 1 \tag{5.87}$$

$$f3 : u_2 + u_3 + u_4 \geq 1 \tag{5.88}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_7 + u_9 \geq 1 \tag{5.89}$$

$$f5_{new} : u_1 + u_2 + u_4 + u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 1 \tag{5.90}$$

$$f7 : u_4 + u_7 + u_8 + u_9 \geq 1 \tag{5.91}$$

$$f8 : u_7 + u_8 \geq 1 \tag{5.92}$$

$$f9 : u_4 + u_7 + u_9 + u_{10} + u_{14} \geq 1 \tag{5.93}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 1 \tag{5.94}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 1 \tag{5.95}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 1 \tag{5.96}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 1 \tag{5.97}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 1 \tag{5.98}$$

The solution of the above ILP considering conventional measurements, with $c_j = 1$ p.u provides the basic optimal PMU locations at buses 2,6,7,9.

5.4.2 Considering both Zero injection and flow measurements

Considering zero injection as well as flow measurements in IEEE 14-bus system, the constraints for ILP can be modified as-

$$f1 : u_1 + u_2 + u_5 \geq 1 \tag{5.99}$$

$$f2 : u_1 + u_2 + u_3 + u_4 + u_5 \geq 1 \tag{5.100}$$

$$f3 : u_2 + u_3 + u_4 \geq 1 \tag{5.101}$$

$$f4 : u_2 + u_3 + u_4 + u_5 + u_7 + u_9 \geq 1 \tag{5.102}$$

$$f5_{new} : u_1 + u_2 + u_4 + u_5 + u_6 + u_{11} + u_{12} + u_{13} \geq 1 \tag{5.103}$$

$$f8 : u_7 + u_8 + u_9 \geq 1 \tag{5.104}$$

$$f9 : u_4 + u_8 + u_9 + u_{10} + u_{14} \geq 1 \tag{5.105}$$

$$f10 : u_9 + u_{10} + u_{11} \geq 1 \tag{5.106}$$

$$f11 : u_6 + u_{10} + u_{11} \geq 1 \tag{5.107}$$

$$f12 : u_6 + u_{12} + u_{13} \geq 1 \tag{5.108}$$

$$f13 : u_6 + u_{12} + u_{13} + u_{14} \geq 1 \tag{5.109}$$

$$f14 : u_9 + u_{13} + u_{14} \geq 1 \tag{5.110}$$

The solution of the above ILP considering conventional measurements and zero injection, with c$_j$ = 1 p.u provides the basic optimal PMU locations at buses 4,5,6,9.

5.5 Case 5: Network Observability Considering Limited Availability of PMU Channels

To introduce the effect of the channel limits on the number of PMUs while achieving the entire system observability, the matrix B is formed using the matrix A as discussed in Chapter 4, whose number of rows for each bus can be found as follows:

$$r_k = \begin{pmatrix} N_k \\ L \end{pmatrix} if L \leq N_k \tag{5.111}$$

$$r_k = 1 if N_K < L \tag{5.112}$$

where N$_k$ is the number of branches connected to bus k, L is the specified channel limit for the PMUs. Each row of the matrix B is then formed based on the number of L combinations of N$_k$[15]. However, when N$_k$ < L, i.e., the number of branches incident to bus k is smaller than the channel limit of the PMUs, the associated row is kept unchanged[15].

As an illustration, consider the buses connected to bus 2, which are buses 1, 3, 4, and 5; therefore, the 2- combinations of this set will result in the pairs 1-3, 1-4, 1-5, 3-4, 3-5, and 4-5. In other words, the rows associated with bus 2 in matrix B will be nothing but the decomposed 2 combination versions of the row related to bus 2 in matrix A, where each row will include a 1 corresponding to bus 2[15]. Further, the optimum number of PMUs required for any L-channel PMU is described below.

Table 5.10: PMU Placement considering limited availability of PMU Channels of IEEE 14-Bus System

Channel Limit(L)	No of PMUs	Bus Location
1	08	$1, 2, 3, 4, 5, 7, 10, 13$
2	04	2,6,7,9
3	04	2,6,7,9

5.6 Summary

This chapter includes simulation results of topological observability of the standard test systems with and without considering zero injection busbar. For this ILP based approach has been implemented to test system along with OPP under steady state and several selected contingencies such as single PMU loss, line outage of power system. It also incorporates impact of communication constraints on network observability. Also conventional measurements such as flow measurements are considered for topological observability. Further, jacobian matrix for numerical observability is calculated to make the system numerically observable for normal as well as under contingency scenario.

Chapter 6

Conclusion and Future Scope

6.1 Conclusion

For successful operation and control of large power network, it is mandatory to have complete network observability under steady state as well as contingency scenario. With this aim, detailed study of network observability has been carried out. In order to ensure complete network observability i.e. topological and numerical observability, WSCC 9-bus and IEEE 14-bus test system have been considered. ILP based approach has been implemented to test system along with OPP under steady state and several selected contingency of power system. It also incorporates impact of communication constraints on network observability. Important findings can be listed as follows-

- Out of three algorithms considered namely, Depth first, Simulated Annealing and Recursive N-1 method, SA is giving minimum 2 PMUs for complete topological obse rvability. SA depends on initial assumption of system configuration and also it does not includes effect of line outage or so further it is taking longer time for calculation and requires additional complimentary method for achieving optimal solution. Thus ILP based approach has been found to be simple and better and is implemented.
- Based on ILP algorithm implementation, system observability has been ensured, 3 PMUs for WSCC 9-bus and 4 PMUs for IEEE 14-bus system has been suggested for complete network observability. The same PMU locations also

ensures the numerical observability as well.

- Considering impact of ZIB, number of PMUs in IEEE 14-bus system can further be reduced to 3 under steady state condition.
- Under line outage consideration without ZIB, it has been observed to have 4 PMUs and with ZIB, 3 PMUs are sufficient to ensure complete observability. The same PMU locations also ensures the numerical observability as well.
- Further, it has been observed that under certain typical line outage, that can be said as critical contingency, number of PMUs required are 5 without ZIB and is further reduced to 4 or in some cases 3 as well for the same contingency cases.
- For single PMU loss case, it has been observed that without ZIB, in addition to the PMU placed under intact condition , additional 5 PMUs are required for topological observability. On the other side with ZIB, additionally 3 PMUs are required but location of PMU is affected under several typical contingency. The same PMU locations also ensures the numerical observability as well.
- The effect of PMU channel has been studied. It has been noticed that channel constraints with less channel limit requires drastic change in number of PMUs and locations. But as channel limit increases number of PMUs and locations are not affected greatly.
- PMU placement in system is gradual and time being process, hence the impact of conventional flow measurements along with phasor measurements have also been tested for both considering and without considering ZIB.

6.2 Future Scope

- ILP does not consider multiple objectives, hence multiple objective based function and method can be considered for further improvised results.
- Variable PMU cost can be considered.
- Multiple flow measurements availability can be considered.
- Based on achieved phasor measurements, linear state estimation under steady state condition can be carried out.
- The achieved phasor measurements can be utilized for further system studies such as stability studies.

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Appendix A

System Data

A.1 Data for WSCC 9-Bus System

Table A.1: Line data WSCC 9-Bus System

Line No.	From bus	To bus	Half line charging B(p.u.)	Reactance	Resistance
1	1	4	0.0	0.0576	0.0
2	4	6	0.079	0.092	0.017
3	3	9	0.0	0.0586	0.0
4	6	9	0.179	0.17	0.039
5	5	7	0.153	0.161	0.032
6	7	8	0.0745	0.072	0.0085
7	2	7	0.0	0.0625	0.0
8	8	9	0.1045	0.1008	0.0119

A.2 Data for IEEE-14 Bus System

Table A.2: Line data IEEE-14 Bus System

Line No.	From bus	To bus	Resistance	Reactance	Half line charging B(p.u.)
1	1	2	0.01938	0.05917	0.02640
2	1	5	0.05403	0.22304	0.02190
3	2	3	0.04699	0.19797	0.01870
4	2	4	0.05811	0.17632	0.02460
5	2	5	0.05695	0.17388	0.01700
6	3	4	0.06701	0.17103	0.01730
7	4	5	0.01335	0.04211	0.00640
8	4	7	0	0.20912	0
9	4	9	0	0.55618	0
10	5	6	0	0.25202	0
11	6	11	0.09498	0.1989	0
12	6	12	0.12291	0.25581	0
13	6	13	0.06615	0.13027	0
14	7	8	0	0.17615	0
15	7	9	0	0.11001	0
16	9	10	0.03181	0.0845	0
17	9	14	0.12711	0.27038	0
18	10	11	0.08205	0.19207	0
19	12	13	0.22092	0.19988	0
20	13	14	0.17093	0.34802	0