Vibration Analysis of

Vertical Turbine Pump

By

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Vertical Turbine Pump

Major Project

Submitted in partial fulfillment of the requirements

For the Degree of

Master of Technology in Mechanical Engineering

(Design Engineering)

By

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Declaration

This is to certify that

- The thesis comprises my original work towards the degree of Master of Technology in Mechanical Engineering (Design Engineering) at Nirma University and has not been submitted elsewhere for a degree.
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Abstract

Turbo machines consists of many rotating parts constitutes complex dynamic system. While Designing rotors of such turbo machineries, it is very important to consider vibration characteristics into account. Rotating systems running at speeds close to the natural frequency of the system results in excessive deformation and large stresses can occur which leads to catastrophic failure of the system. This dissertation work focuses on performing vibration analysis to identify critical operating speeds for three stage vertical turbine pump.

The vibration characteristic for the above said pump is to be studied with the help of CAE tools by converting the physical model of the vertical turbine pump into a simplified lumped parameter system. The same is to be verified by analytical approach. Modification in the pump design is required in case of operating speed falling in the range of critical speed in order to make the system performance more robust and reliable.

In the present work, the bench mark study of single rotor system is performed to identify the methodology for carrying out vibration analysis (torsional and lateral) for the rotating system. The result of the bench mark study is used to produce Campbell diagram which is an essential tool to predict vibration characteristics of rotating system. As the physical system is complex, the lumped mass model considering various connections, power transmitting elements and inertias of rotating parts have been developed. The concept developed through bench mark study is applied to lumped parameter VTP model and critical speeds are evaluated. The analytical computation for determining system's natural frequencies using Holzer method shows close confirmation with FEA results.

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Nomenclature

Variables	Physical Quantity	Unit
T(t)	Time varying torque	N m
θ(t)	Angular Time Oscillation	rad
L	Length of the shaft	m
J_p	Polar moment of inertia	m^4
G	Shear modulus of material	N/m^2
K	Stiffness of shaft	Nm/rad
k	Radius of Gyration	m^2
K_e	Equvalent stiffness	Nm/rad
AF	Amplification factor	-
θ	Angle of twist	Degrees
ω	Torsional natural frequency	rad/sec.
f	Natural frequency	Hz
E	Modulus of elasticity	N/m^2
ρ	Density	kg/m^3
μ	Poission ratio	-
I_d	Polar moment of inertia of disc	$kg.m^2$
I_s	Polar moment of inertia of shaft	$kg.m^2$
D	Diameter of disc	m
R	Radius of disc	m
d	Diameter of shaft	m
r	Radius of shaft	m
VTP	Vertical turbine pump	-

Chapter 1

Introduction

1.1 Vibration of mechanical systems

Any kind of motion that repeats itself after an interval of time is called Vibration or Oscillation. Transfer of kinetic energy into potential energy and potential energy into kinetic energy occur alternatively in system subjected to vibration. The vibration theory for mechanical systems deals with the study of oscillatory motions of bodies and the forces associated with them. In case of damped system, some energy is dissipated in each cycle of vibration. If steady state vibration is to be maintained, the dissipated energy must be replaced by external source. Vibratory system includes spring or elasticity, mass or inertia and damper.[13]

1.1.1 Vibration analysis[13]

Vibration is the dynamic behavior of a system that is system oscillation about equilibrium position. The parameters consists of physical properties or characteristics of the system. By replacing distributed characteristics of continuous system into discrete ones, the vibration analysis can be simplified.[13]

Hence, in vibration analysis mathematical model is devided into two types :

1. Lumped parameter or discrete parameter system (Finite number of degrees of freedom).
2. Continuous system or distributed parameter system (infinite numbers of degrees of freedom).

Vibrating systems can be broadly characterized as linear system or nonlinear system.

- **Linear systems**:
  In case of linear vibration system, superposition principle holds and mathematical techniques are developed very well. The basic components of system such as spring, mass and damper behave linearly.

- **Nonlinear systems:**
  In nonlinear vibration system, any of the basic components behave nonlinearly. superposition principle is not valid for nonlinear vibration. Mathematical techniques for analysis are less well known.

Vibrations are also classified into mainly two types:-

1. **Free Vibration**
   If no external force acts on the system, the vibration of the system on its own after an initiall disturbance is known as free vibration.
   Under free vibration, the system vibrates with one or more its natural frequencies. These are the dynamic properties of the system established by its mass and stiffness distribution.[13]

2. **Forced Vibration**
   Forced vibration is the vibration of the system under the excitation of the external applied force.
   In case of harmonic excitation, the system is forced to vibrate with excitation frequency. A condition of resonance occure if the excitation frquency coinsides with one of the system's natural frequencies which result to large amplitude of vibration may take place . It leads to catastrophic failure of the structures like buildings, bridges or airplane wings etc.

Thus in vibration study, calculation of natural frquencies is more importanat.The vibration systems are subjected to damping to some degree due to friction and other resistances by means of dissipation of energy. Damping has very little effect on system's natural frequency so generally calculation of natural frequencies are made on the basis of no damping. In resonance condition, damping is more important in limitting the amplitude of vibration.[13]

The study of evaluating system's natural frequency along with the associated mode shapes is called **Modal Analysis**.

1.2 Torsional vibration of mechanical system[13]

Torsional vibration is basically angular oscillation of rotating components or rotors of the system. A rigid body oscillate about specific reference axis. Each rotor in system will oscillate about its rotational(spin) axis and follows a torsional disturbance to the system. Thus shaft subjected to twisting. Torsional disturbance due to twisting can produce reversals of stress which cause fatigue failure. Hence analysis of torsional vibration is vigorous in rotating machinery system.

Torsional vibration may result in rotating component from following facts:

- Start up of synchronous electric motor.
- Inertia forces in reciprocating mechanism(e.g. piston cylinder mechanism).
- During normal working cycle, torsional vibration occure due to acting of impulsive load(e.g. punch press).
- Random torsional vibration due to gear inaccuracy and ball bearing defects.

In system consists of massless/flexible shafts and rigid/massive rotors, natural frequency of torsional vibration may be very close to the source frquency range during normal operating condition. Torsional vibration creates problem in potential design. So designers should ensure about accurate prediction of system's natural frequency and frequency of source should not coinside with torsional natural frquency of the system. Thus for a dynamic system, determination of torsional natural frequency is more important.[13]

1.3 Vertical turbine pump[15, 16, 17]

Turbine pump is one type of centrifugal pump which is used to pump water from deep well or other underground to water distribution systems. There are two types of turbine pump; submersible and deep well turbine pumps. Vertical turbine pump is vertical axis centrifugal or mixed flow type pump consists of stages which accomodate rotating impellers and stationary bowls with guide vanes. The VTP also has a water intake point and a water discharge point. The driver may be electric motor or rotary engine. Figure1.2 shows the schematic diagram of vertical turbine pump.

Any vertical turbine pump consists of three parts:

1. Bowl assembly/pump element:
   There may be one or more bowl. Each bowl consists of an impeller and diffuser.

2. Discharge column.
   It conducts water from bowl assembly to head of the pump. It enclosed the rotating shaft.

3. Discharge head.
   It consists of base from which discharge column, bowl assembly, shaft are suspended.

In deep well/vertical turbine pump, generally the motor is placed above water level. The total head is the total energy taken by the pump to move the water from suction(supply tank) to the discharge point.

Working: The shaft of the pump transfer the power from electric motor to the impeller. Through the suction bell, water enters at eye of the impeller(centre of rotating impeller). Then water is accelerated and pushed out along the vanes of one impeller to the next impeller in case of multi stage and finally to exit the pump. Elecrical energy is converted into mechanical energy which leads to flow the water under a specific pressure. So more water enters the pump from water source.

Generally, turbine pumps have a constant head, and water flows uniformly at high pressure. By adding more bowl assembly or stages, the head capacity increased.

Deep well or vertical turbine pumps are cheaper in cost than submersible pumps with small diameters. VTP is fabricated specially for pumping water from wells. In VTP, open or semi-open impellers are used which must be periodically adjusted for proper functioning of the pump.

Vertical pumps are mainly used in those wells where the water surface fluctuates regularly. They are difficult to install and repair. VTP are more expensive than other centrifugal pumps, but there are two advantages of VTP; high flow rate and high efficiency.

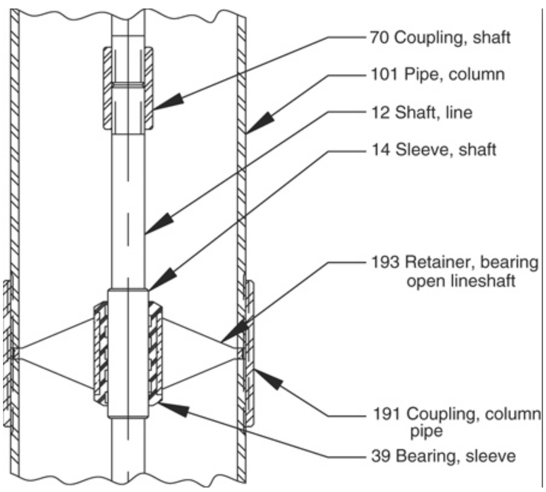

Figure 1.1: Line shaft of VTP

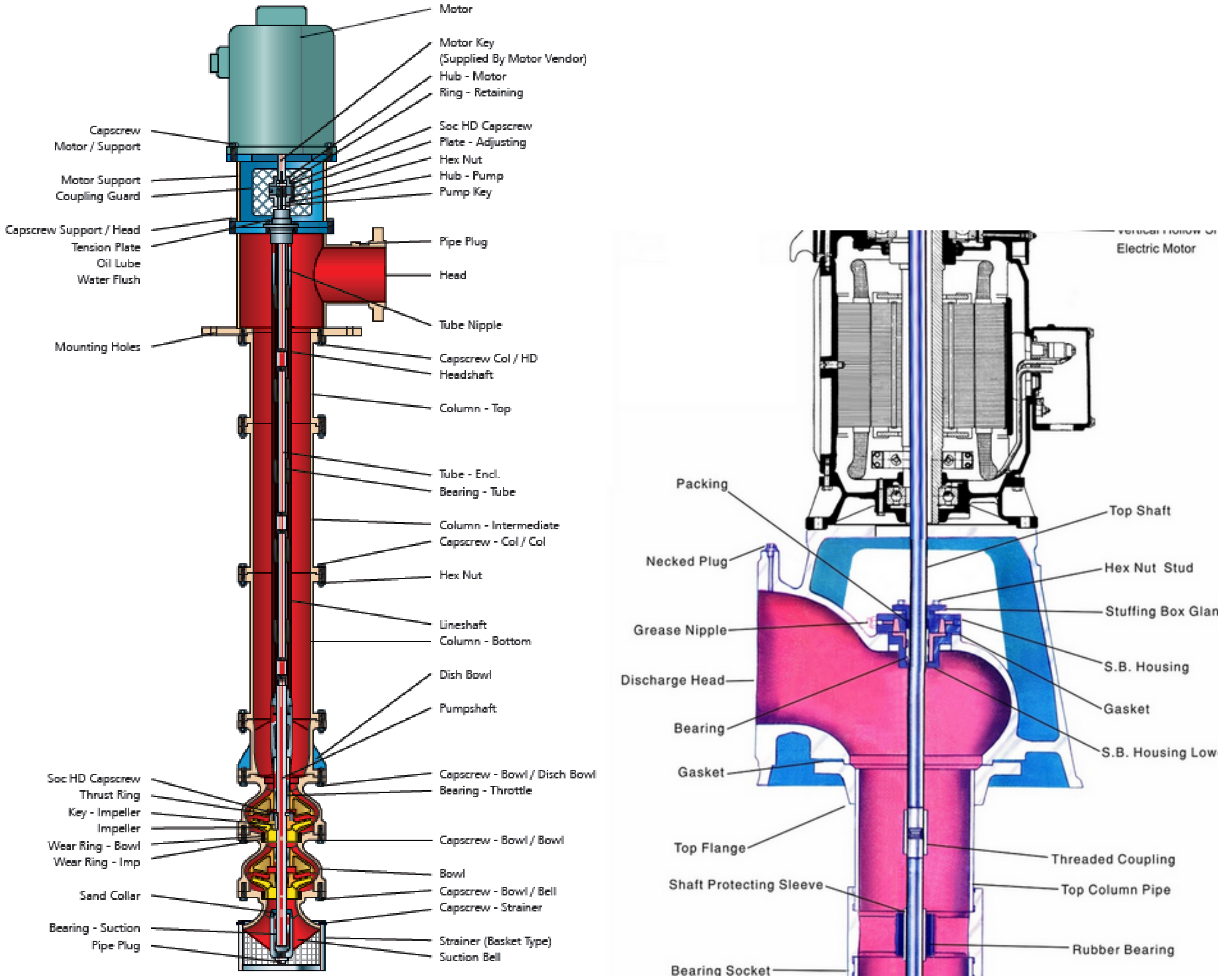

Figure 1.2: Cross section of Vertical Turbine Pump[16]

Rotor system in VTP is vertically oriented and have shaft which transmitt axial and torsional loads. Axial loads are produced by hydraullic forces from impellers at botom of system and balanced by thrust bearings at the top of the system. Between these two points of the tensile load of the rotor system, radial bearings provide position control and stability to rotating system at various intervals along the length of the system. These bearings are water lubricated rubber bearings in vertical turbine pump[17]. As shown in figure1.2, rolling element bearings are located at uppar side of the pump near the electric motor. Figure1.1 shows line shaft of vertical turbine pump and location of water lubricated rubber bearing that is line shaft bearing.

Turbine water pumps are used for various applications, such as pumping water for irrigation, fire fighting, hydroelectricity, and treatment of waste water. Turbine water pump is used for continous pumping of water from deep wells because of its advantages like low rpm , high and constant head.[15]

1.4 Objective of the study

Main objective of this dissertation work is to carryout vibration analysis of a 3 stage vertical turbine pump in order to identify torsional and lateral critical speeds of operation. The design modifications is suggested if required to make the performance reliable and robust.

1.5 Scope of work

The bench mark study using simple vertical shaft rotor model for evaluating its critical speeds is performed using FEA. The complex VTP assembly is converted in lumped parameter model which is to be analyzed through FEA. The Campbell diagram obtained as a result of vibration analysis is used to determine critical speeds. Analytical approach using Holzer method is used to evaluate natural frequencies of lumped parameter system to validate FEA results.

Chapter 2

Literature Review

Mark A. Corbo & Stanley B. Malanoski[1]shows overall methodology for torsional vibration analysis. Guidelines regarding lumped parameter modelling is given. Explanation of generation of Campbell diagram is given. Formulas for determining order number in Campbell diagram for different excitation source and excitation torque are given. They explain consideration of damping for various damping source. Description regarding modal analysis is given. Also they explain some analytical procedure for determining natural frequencies.

Martin Gula, Peter Hudk[2]The aim of this paper is to show the method for solution of rotor dynamics problem numerically using ANSYS workbench software. Description of critical speed is given and Campbell diagram is evaluated.

Ramana podugu, B.V. Ramana murthy[3]carried out modal analysis of centrifugal pump using FEM. They built mathematical as well as FEA model for obtaining natural frequencies. As per HIS guidelines, first natural frequency was very close to the operating speed. So, the model was modified by adding two stiffener plates on either side of each pedestal and again FEA analysis performed. After this modification, the natural frequency was obtained which was in acceptable region as per HIS standard. Harmonic analysis was performed to find maxiumum amplitude. Experimental test was also carried out. From the FEA analysis and experimental test, they conclude that Increase in stiffness of centrifugal pump will increase in pump natural frequency.

Naveena M & Dr. Suresh P M [4] In this paper lateral critical speed of centrifugal pump was evaluated by ANSYS software. Campbell diagram was generated for 1xAPI and 2xAPI conditions. The result was compared with theoretical calculation and RBTS (Rotor bearing testing software). Analytical solution was done using Rayleigh's method. As per API 610 standard, for 1xAPI all natural frequencies were within acceptable region but for 2x API, the first reverse mode was unacceptable. At critical points, amplitude of vibration was obtained by performing unbalance response analysis. The result of harmonic analysis is satisfied as per API 610 standards.

M.Chouksey, J.k. Dutt [5] The aim of this paper is to shows the effect of tangential force introduced by internal material damping and journal bearing on modal behavior of flexible rotor shaft system. Finite element model of rotor shft system was created and Campbell diagram for first four modes was evaluated. Curve veering phenomenon was explained. Stability diagram was generated without considering internal material damping and introduce the stability limit speed. The effect on shaft material damping on stability diagram was shown also. Oil whirl and Oil whip phenomena (in case of journal bearing) was introduced.

Erik Swanson, Chris D. Powell and Sorin Weissman[6] outlines the basic knowledge regarding how a shaft vibrates and issues affecting vibration. Conical modes and cylindrical modes are explained very well. Explanation of terminologies like forward whirling and backward whirling, gyroscopic and mass effect, critical speed was given. One case study is given and critical speed is obtained by generating Campbell diagram. They conclude that conical modes caused to split in to forward and backward whirl that increase and decrease respectively in frequency with increase in speed of rotor.

William D. Marscher[7] Tutorial discussed basics of pump rotor dynamics, various terminologies in rotor dynamics , pump rotor dynamic problems and how they can be avoided by applying right kind of vibration analysis. Consideration of fluid added mass is explained. Points regarding vertical turbine pump vibration are discussed.

M.H. Sadeghi, S. Jafari & B. Nasseroleslami[8] Carried out numerical modal analysis of turbo pump shaft. The results from finite element solution are compared with experimental modal analysis results. Hammer test and shaker tests are carried out. Comparison shows that some mode shapes are not excited in experiment due to common limitations of experimental tests. The effect of shaft bearing and fluid in contact with it during operation, on the response of the system is not considered in this analysis.

Kenneth E. Atkins, James D. Tison & J.C. Wachel[9] performed critical speed analysis and unbalance response analysis of eight stage centrifugal pump. Lumped parameter model of pump was generated and then lateral critical speed map for no seal effect was generated. Several methods for calculating dynamic properties of seals are reviewed. Unbalance response plots are generated to illustrate the effect of calculating the seal stiffness and damping coefficients by various methods. They conclude that depending on the method used to calculate the seal properties, substantial differences in the location of calculated pump critical speeds can occur.

Nagaraju Tenali and Srinivas Kadivendi[10] Rotor dynamic analysis of steam turbine rotor is carried out using ANSYS software. BEAM188 and COMBI214 elements are used to model the shaft and bearings. Critical speed is evaluated from Campbell diagram.

Unbalance response at all bearing location are predicted by applying unbalance force at center position. All results are compared with shop test results. Transient analysis was also carried out and transient response at bearing locations are predicted. As result of transient analysis, the amplitude of response is decrease with increase in time this means the system is stable. The results obtained from ANSYS and Test are in good agreement.

Ion NILA, Radu BOGATEANU, Marcel STERE, Daniela BARAN[11] Modal analysis of small vertical axis wind turbine is performed using ANSYS software. Block Lanczos method is used to analyze the natural frequency. PIPE16 (for tower), BEAM4 (for blade mass and arms), LINK 10 (for guy cables) and MASS21 for lumped mass elements has been used. Some ANSYS commands are introduced for modal analysis.

Rotor dynamic Analysis Guide[12] outlines the complete procedure of rotor dynamic analysis in ANSYS software. Terminologies in rotor dynamics are explained. ANSYS commands are introduced for rotor dynamic analysis. Some examples of Campbell diagram analysis and harmonic response analysis are given.

Shibing Liu, Bingen Yang[17] presents a experimental device to simulate rotor systems with water lubricated rubber bearing(WLRB). In experiments, unbalance mass response of a rotor system is used to determine the experimental dynamic stiffness of WLRB. Unbalance mass response of system is modelled by distributed transfer function method(DTFM). The WLRB dynamic stiffness coefficients are investigated for three cases; rotating system with one disk, two disk and three disk. The eperimental result shows that as the rotating speed increases, the dynamic stiffness coefficients increase parabolically. They also discussed about load acting on vertical shaft rotor system and advantages of WLRB used in vertical pump.

Prof. Rajiv Tiwari[19, 22] discussed identification of radial stiffness of rollling element bearing theoretically. Types of bearing, types of loads acting on the bearing with applications, bearing geometry are discussed very well. Hert's theory is used to determine rolling element bearing stifness theoretically. Theoretical explaination of calculation of stiffness coefficients of hydrodynamic beariing, seals and dampers are also given.

J.S.Rao, Uugar Yucel, Marco Tulio C. Falia[18, 21, 24] outlines the complete procedure of identification of hydrodynamic bearing stiffness and damping coefficients theoretically. From the hydrodynamic bearing theory and Raynold's equation, the expression for stiffness and damping coefficients as a function of eccentricity ratio. So, after obtaining dimensionless sommerfeld number and then eccentricity ratio the stiffness and damping coefficients of hydrodynamic bearings can be calculated easily.

2.1 Torsional Vibrations

The torsional oscillation of cylindrical shaft is defined by following formula;

$$\theta(t) = T(t) = \frac{L}{J_p G} \tag{2.1}$$

Where,

$\theta(t)$ =Angular time oscillation.

T(t)= Time varying Torque

L= Length of the Shaft

J_p =Polar moment inertia

G= Shear modulus

The above equation is for simple system which is fixed at one end and the other end free alongwith the predicted time varying torque. The first step to determine the torsional response analytically is to calculate the torsional natural frequency of the system. The stiffness and the mass moment of inertia of the shaft elements must be determined. The equation of θ(t) is related to the elastic properties of the shaft to the imposed dynamic torques. The Shaft stiffness can be determined from the elastic properties. The equation of for the uniform section shaft stiffness is;

$$K = \frac{GJ_p}{L} \tag{2.2}$$

The mass moment of inertia is calculated as;

$$J = \frac{1}{2}MR^2 \tag{2.3}$$

2.2 General Equation of Motion[12]

General equation of motion for vibration problems is given by

$$[M]\left\{\ddot{U}\right\} + [C]\left\{\dot{U}\right\} + [K]\left\{U\right\} = \left\{F\right\} \tag{2.4}$$

Where,

$[M]$= symmetric mass matrix

$[C]$= symmetric damping matrix.

$\{F\}$= external force vector

$[K]$= symmetric stiffness matrix

$\{U\}$= generalized coordinate vector.

When the gyroscopic effect and internal material damping effect considered, the equation of motion will be

$$[M]\left\{\ddot{U}\right\} + ([G] + [C])\left\{\dot{U}\right\} + ([D] + [K])\{U\} = \{F\} \tag{2.5}$$

$[G]$ and $[D]$are skew symmetric gyroscopic and circulatory or internal material damping matrices respectively. Both matrices depends on rotational velocity. $[G]$ contains inertial terms and that are derived from the kinetic energy due to gyroscopic moments acting on the rotating parts of the system.$[D]$ is contributed mainly from internal material damping of rotating elements. It modifies the stiffness the structure and can produce unstable motion.

2.3 Modeling of System[1]

In vibration analysis, for simple system natural frquency can be calculated by using simple formulas but in case of composite system, the additional details are required for each component. For intricate system, the mathematical module is synthesized into lumped parameter model or equvalent parameter model for simplicity. It responds in same way as actual system. All the damping and elastic-mass properties are necessary to assemble lunped parameter model having rigid rotor and massless shaft. The calculation of inertias of rotors and stiffness of shafts are straightforward. In case of stepped shaft, calculation becomes more complex. An error can be occur in calculation of polar mass moment of inertia of composite shapes such as impeller, turbine wheel, motor armature etc. The effective stiffness of section of shaft depends on actual coupling to shaft shrink fit. the accuracy of analysis depends on how well the assumptions matches with the actual assembly of the components. [1]

2.3.1 Torsional inertias of rotors[14]

In analysis of vibration, it is important to calculate each significant polar mass moment of inertia. Typical inertias include such components like generator and motor rotors, coupling hubs, impellers, compressor, gears, turbine wheel etc. The polar mass moment of inertia is sometimes called Wk^2. k is radius of gyration and it is not equal to shaft radius. For cylinder, k is equal to 0.707R.[14]

2.3.2 Torsional stiffness of shaft[1][13]

The stiffness of shaft having uniform diameter is;

$$K = \frac{GJ_p}{L} \tag{2.6}$$

In case of stepped shaft i.e. shaft with varying diameter, the equvalent stiffness is calculated. The equvalent stiffness(K_e) is determined by;

$$\frac{1}{K_e} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + ... \tag{2.7}$$

Where, K_1, K_2, K_3 are stiffness of each section. The torsional stiffness provided by manufecturer is often from center of shaft to end of shaft and may not be the value required for torsional model.[1]

In another way, The Equivalent torsional stiffness[13]:

$$K_e = \frac{GJ_e}{L_e} \tag{2.8}$$

where, J_e=Equivalent moment of inertia of shaft

$$J_e = \frac{\pi}{32} d_e^4 \tag{2.9}$$

d_e= Equivalent diameter of shaft

We can take smallest diameter of stepped shaft as an equivalent diameter

L_e=Equivalent length of shaft

$$L_e = J_e \left\{ \frac{L_1}{J_1} + \frac{L_2}{J_2} + \frac{L_3}{J_3} + ... + \frac{L_n}{J_n} \right\} \tag{2.10}$$

2.3.3 Couplings[14]

Coupling should be modelled as shaft having stiffness equal to coupling stiffness and rotors at each end of shaft whose inertia is equal to one half of coupling's total inertia.[1] For calculation of stiffness of coupling, vandor assumes one third penetration of shaft into hub i.e. two third of shaft length in hub has no relative motion. Couplings are modelled with torsional inertia at each hub of driving and driven equipement connected by stiffness of coupling. The stiffness of adjecent shaft sections are thus replicated up to coupling hub edge.[14]

Shaft twist freely for L/3; No slippage for 2L/3.

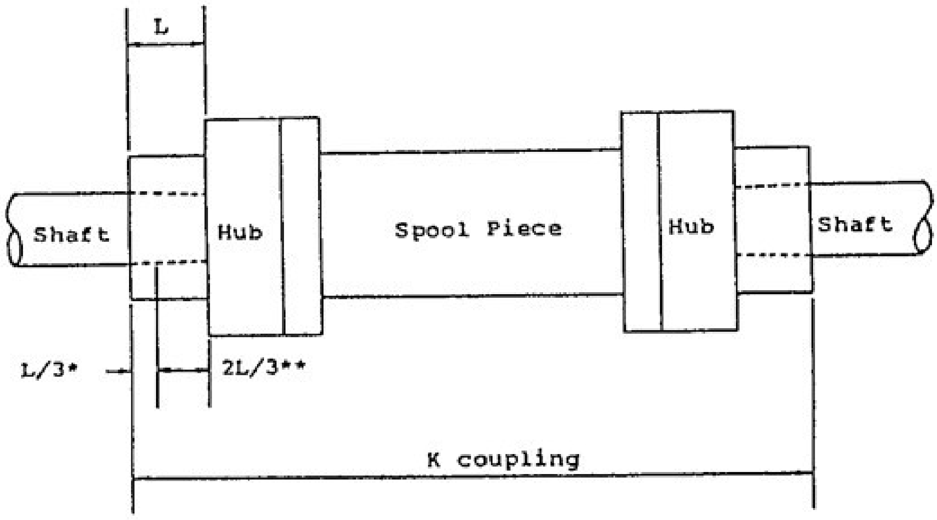

Figure 2.1: Coupling shaft penetration assumption[14]

2.3.4 Damping[1]

In turbomachinery there are two types of damping, internal damping and external damping. External damping is modelled as a dashpot between the conscious inertia and ground. damping at impeller and motor are external damping. External damping is associated with a perticular disk element. External damping is dependent on the absolute velocity of a perticular inertia. Internal damping is dependent on the difference in angular velocities between two adjecent inertias. Internal damping is associated with a specific shaft element. Internal damping is modelled as a dashpot in parallel with the torsional spring on behalf of the appropriate shaft element. Damping in couplings and shaft material hysteresis are modelled as internal dampers.[1]

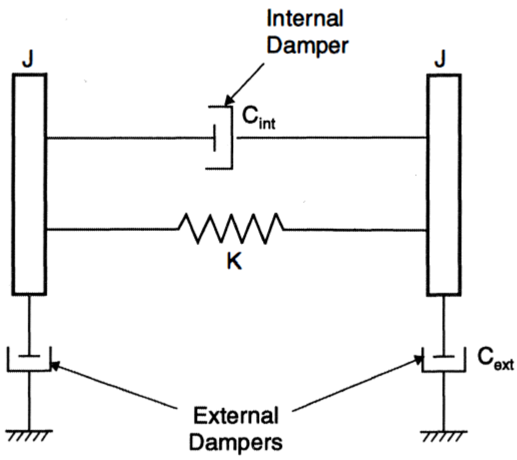

Figure 2.2: Modelling of damping[1]

2.4 Calculation of Natural Frequencies[1, 2, 4, 13, 14]

After generating lumped parameter model, natural frequency of the system is to be determined mathematically. Various methods to find natural frequencies are as follows:

- **Dunkerley's method**
  It is based on fact that in most systems, higher natural frquencies are larger compared to fundamental natural frequency. Dunkerley's formula gives approximate value of fundamental natural frquency of complex system in terms of natural frquency of its component parts. It gives approximate value always less than exact value.[13]

- **Rayleigh's method**
  This method is used only for estimating fundamental natural frequency. The method is based on rayleigh's principle that is frequency of conservative system vibrating at an equallibrium position has stationary value in neighborhood of natural mode. This stationary value, in fact is a minimum value in neighborhood of fundamental natural mode. It gives upper bound approximation of the fundamental; natural frequency of the system.[4]

- **Holzer's method**
  This method is used for multi degree of lumped mass system. It is trial and error scheme to find natural frequencies. It is iteration method. The method also gives mode shapes. It is used for damped, undamped, fixed or branched vibrating system involving linear and angular displacement, semi definate system. Computer programs are available for this method.[14]

- **Transfer matrix method**
  It is implementation of Holzer's method by using concept of state vector and transfer matrices for descrete and continous system. This method is also used for branched system.[14]

- **Finite element method**
  This method is general case of Rayleigh-Ritz method. It includes eigenvalue problem formulation by obtaining stiffness and mass matrix for each element(using direct method or Lagrange's equation) and then globle stiffness and mass metrix respectively.[2]

- **Matrix-Eigenvalue method**
  This method involves solution of differential equation of motion. Matrices are utilized to simplify the mathematical equation of motion is writting in matrix form. Then eigenvalue equation is to be obtained. Natural frequencies are squre root of the eigenvalues and this also gives mode shape that is eigen vector associated with

each eigenvalue. Computer programs are available for this method also.[1]
In this project, Holzer's method is to be used due to straight system for calculation of natural frequencies.

2.4.1 Holzer Method

Holzer developed the method for calculating torsional natural frequencies of the system in early 1900s. This method is based on iteration and it is readily adaptable to computer analysis. It can be used for branched or unbranched and damped or undamped system. In this method, first trial frequency is to be assume, when assumed frequency satisfies the constraints, the solution is reached. Generally it requires several trials. Let us consider undamped semidefinate system as shown in below figure, the Holzer's method is to be apply by Newton's 2^{nd} law,

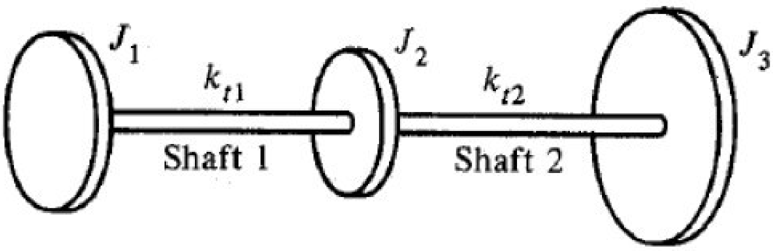

Figure 2.3: Three rotor straight torsional system for Holzer methhod

$$J_1\ddot{\theta} = -K_{t1}(\theta_1 - \theta_2) \tag{2.11}$$

$$J_2\ddot{\theta} = -K_{t2}(\theta_2 - \theta_3) - K_{t1}(\theta_2 - \theta_1) \tag{2.12}$$

$$J_3\ddot{\theta} = -K_{t3}(\theta_3 - \theta_2) \tag{2.13}$$

Now, since the motions are harmonic at principal mode of vibration,

$$\theta_i = \Theta_i sin\omega t \tag{2.14}$$

Putting (3.9) in (3.6,3.7,3.8);

$$\omega^2 J_1\Theta_1 = -K_{t1}(\theta_1 - \theta_2) \tag{2.15}$$

$$\omega^2 J_2\Theta_2 = -K_{t2}(\theta_2 - \theta_3) - K_{t1}(\theta_2 - \theta_1) \tag{2.16}$$

$$\omega^2 J_3\Theta_3 = -K_{t3}(\theta_3 - \theta_2) \tag{2.17}$$

Adding the above equations we get,

$$\sum_{i=1}^{3} J_i\Theta_i\omega^2 = 0 \tag{2.18}$$

This equation says that sum of inertia torques of semidefinate system must be zero. Now, trial frequency must satisfy this requirement for solution.

And for n rotor system,

$$\sum_{i=1}^{n} J_i\Theta_i\omega^2 = 0 \tag{2.19}$$

In this method, trial natural frquency is assumed and Θ is arbitrary chosen as unity. For n rotor system to find angular displacement,

$$\Theta_j = \Theta_{j-1} - \tfrac{\omega^2}{K_{t(j-1)}} \sum_{i=1}^{j-1} J_i\Theta_i \tag{2.20}$$

This values are substituted in equation (2.16) to verify whether constraint is satisfied or not. If equation (2.16) is not satisfied, a new trial value of ω is assumed and repeat the process. The resultant torque in equation (2.16) represents torque applied at last disk. The amplitude Θ corrosponding to ω is mode shape.

2.5 Terminologies

2.5.1 Whirling[6, 7]

Because of Centrifugal force acting on rotor during rotation, the rotor tends to bend and follow orbital or elliptical motion. This is called whirling. When the whirl direction is same as shaft spin direction it is called forward whirling. When direction of whirl is in opposite to shaft spin direction, it is called backward whirling.[7]

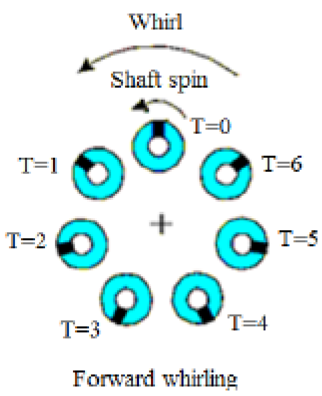

Figure 2.4: Forward whirling[6]

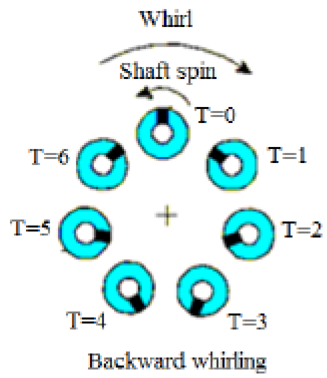

Figure 2.5: Backward Whirling[6]

2.5.2 Gyroscopic effect[6]

If a structure rotating about an axis x, if rotation is applied about an axis perpendicular to axis x (precession axis) then reaction moment appears, this is called gyroscopic moment. Its axis is perpendicular to both spin axis(x) and precession axis. Because of gyroscopic effect, each natural frequency of whirl is split in to two frequencies when rotor speed is not zero. As the rotor speed increases, gyroscopic moment stiffens the rotor stiffness and damping decreases for forward whirl and weakens the rotor stiffness and increase in damping for backward whirl. The former is called Gyroscopic stiffening and latter is called Gyroscopic softening.Gyroscopic moment shift up the forward whirl frequencies and shift down backward whirl frequencies.[6]

2.5.3 Damping[7]

Damping is defined as the ability of system to reduce its dynamic response through energy dissipation. Damping prevents the system for reaching higher amplitude of vibration due to forced resonance. For rotor system, damping classified as internal damping and external damping. Internal damping includes material damping that is provided by rotating part of structure. External damping is provided by fixed part of structure and through bearings. In some case, internal damping may decrees the stability of rotor and hence it is undesirable. External damping stabilized the system by limiting the response amplitude and somewhat increasing the critical speed.[7]

2.5.4 Whirl orbit[6, 12]

When the rotor is rotating, the discrete points or nodes on the spin axis of the rotor moves in curved path as shown in figure. The curved path is called whirl orbit. When the bearing has same stiffness value in vertical and horizontal direction, the whirl orbit is of circular form. If the supporting structure or bearing have different stiffness value in horizontal and vertical direction, whirl orbit is of elliptical form.[6]

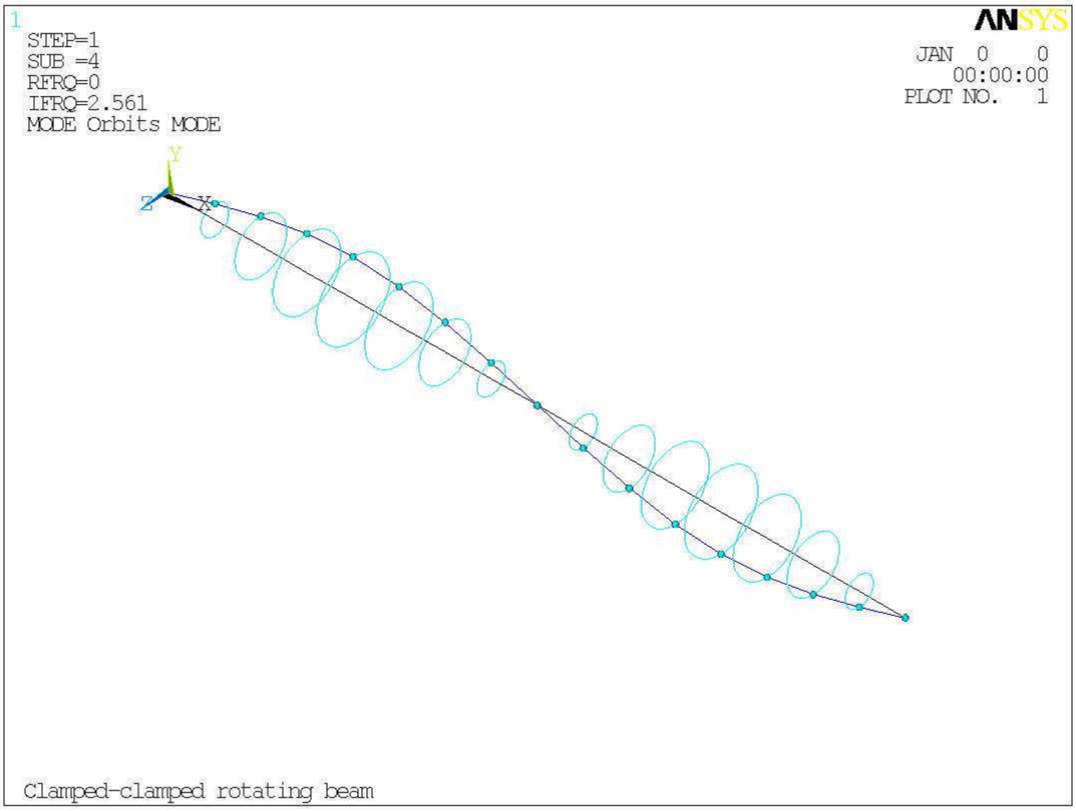

Figure 2.6: Sample of Whirl Orbit[12]

2.5.5 Critical Speed[12]

Critical speed is defined as the operating speed at which the excitation frequency is equal to the natural frequency. The excitation is Synchronous or asynchronous. The excitation due to unbalance is synchronous with rotational velocity and it is called synchronous excitation. The critical speed can be determined by Campbell diagram. Due to critical speed, vibration of system may increase drastically.[12]

2.5.6 Campbell Diagram[12][2]

Campbell diagram is graphical representation of natural frequencies verses excitation frequencies as a function of rotational speed. The rotational speed is plotted along X axis and system natural frequency plotted along Y axis. The upward slopping lines are harmonic of speed that represents the system's potential excitation. Critical speeds are calculated at intersection of modal frequency lines and excitation line. The Campbell diagram analysis allows:

- Visualize the evolution of frequencies with rotational velocity.
- Determine the critical speeds.
- Check the stability and whirl of each mode.[12]

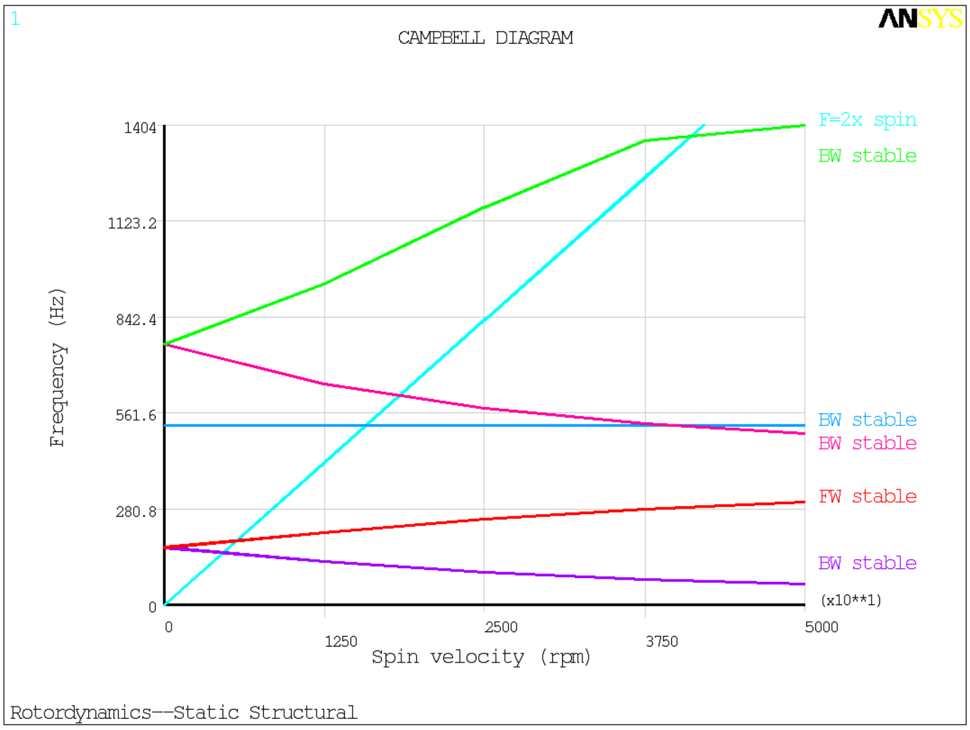

Figure 2.7: Sample of Campbell Diagram[2]

Chapter 3

Bench mark study for rotor vibration analysis

In order to carry out vibration analysis of vertical turbine pump using CAE tool, it is required to identify and establish methodology. To do so a simple vertical single rotor system will be taken into consideration. The present chapter deals with the study of such system and constructing campbell diagram for predicting torsional and lateral vibration characteristics using ANSYS.

3.1 General Procedure Of Vibration Analysis In ANSYS

Following are general steps to be considered for performing FE based critical speed analysis for rotor systems.

- Generate the model of shaft rotor system.
- Define element types and key options.
- Define real constants.
- Assign the material properties.
- Mesh the system.
- Apply appropriate boundry conditions.
- Define force and rotational speed.
- Consideration of gyroscopic effect.

- Select the required solver.
- Solve.
- Post processing for results.

3.2 Critical speed identification of simple vertical rotor model- a Bench mark study

For performing bench mark study to evaluate methodology of analysis, a simple vertical rotor model consist of massless shaft having a disk at the centre of the shaft with rigid bearings at both ends of the shaft is considered.

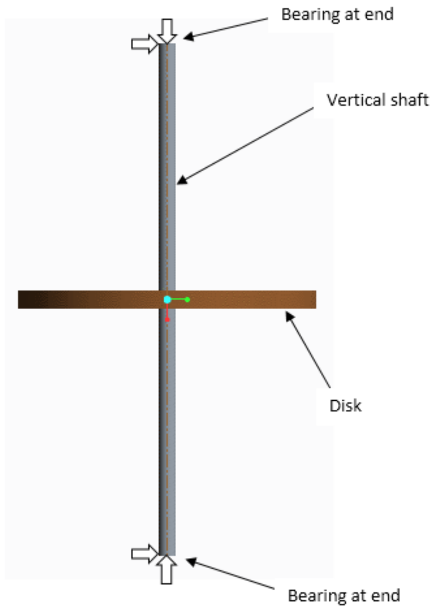

Figure 3.1: Simple Vertical Rotor Model-Test Case

The various mechanical and geometrical properties are considered for analysis as follows:

Shaft Properties:

Length of the shaft, L= 1.2 m

Diameter of the shaft, d= 0.04 m

Young's modulus, E= 2.1×10^{11} N/m^2

Poisson ratio = 0.3

Density = 7800 kg/m^3

Disk Properties:

Mass, m = 120.072 kg

Radius R= 0.35m

Thickness of the disk = 0.04 m

For evaluating critical speeds through FEA, the first step is to perform modal analysis in order to identify modal frequencies and associated mode shapes. Critical speeds for shaft rotor system are identified for campbell diagram for which the modal frequencies are required as input.

3.3 FE model of system

The physical system of simple rotor system can be simulated as continuos system or lumped mass system. Since continuos system modelling require the use of high and computing tools, a conservating approach of lumped parameter modelling is used for vibration study. Figure3.2 shows the finite element model of the simple vertical rotor system. BEAM 188 element is used for build the shaft with Key option(3)=2 which means quadratic behaviour. Rotor disk is added as lumped mass at center of the shaft. MASS21 element used for the rotor. Bearings are modeled with COMBI214 element at both ends of shaft.

The stiffness of the bearing is provided as one of the real constants of COMBI214 element to simulate the problem more precisely.

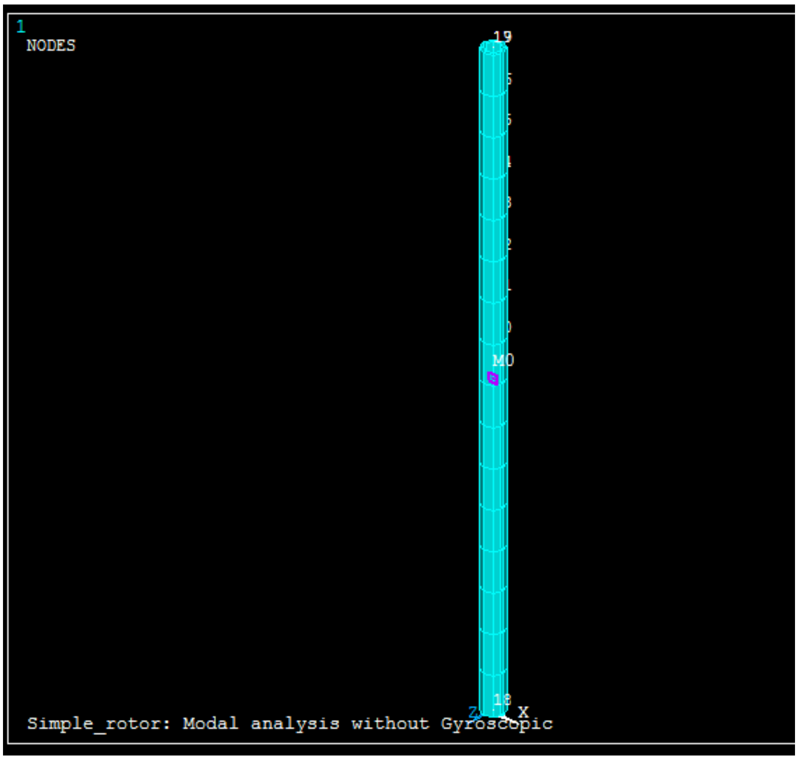

Figure 3.2: FE model of simple vertical rotor system

3.4 Loading/Boundary conditions

Proper boundary conditions are most essential input to obtain the precise results using finite element analysis. For performing modal analysis, which is used to evaluate modal frequencies, no additional external forces are required. The effect of gravity may cause prestressing within the component which leads to shifting the natural frequencies of vibration. Hence the effect of gravity is to b checked for vertical configuration. Figure3.3 shows the model with suitable boundary conditions. Shaft nodes are constrained in axial and torsional direction and bearing nodes at the base are fixed in all direction.

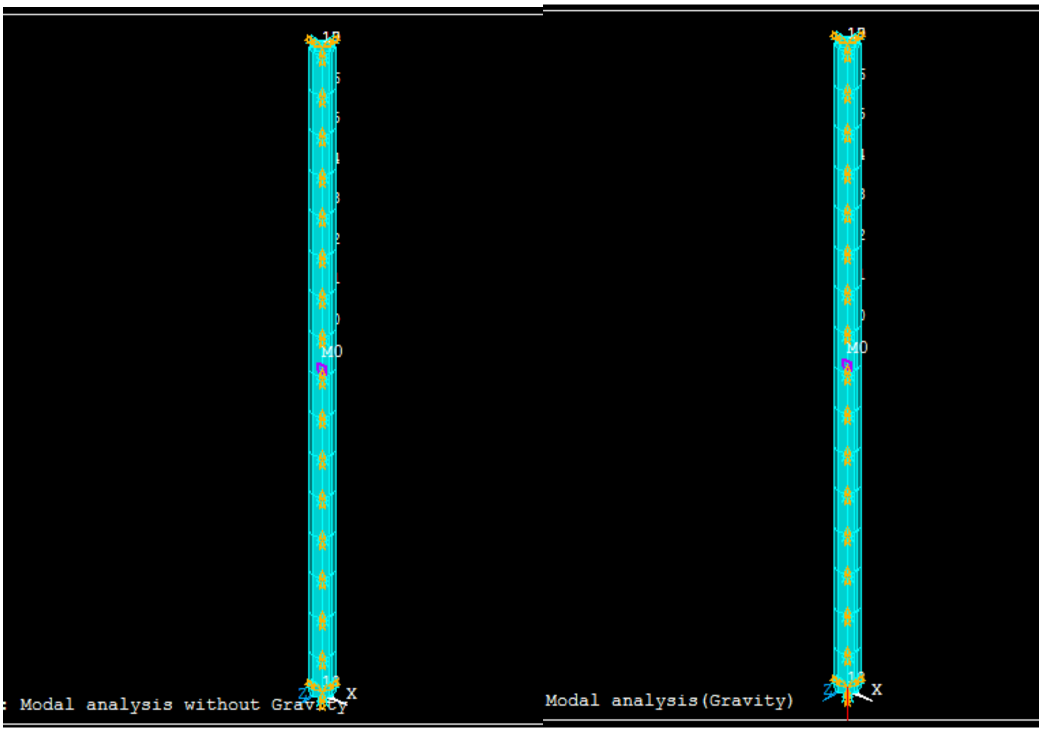

Figure 3.3: Boundary conditions for simple vertical rotor model(with/without gravity)

3.5 FE simulation results- Modal analysis

The lumped parameter model of vertical shaft with simple rotor has been undertaken for modal analysis.

For predicting the critical speed which is analogous to natural frequencies of stationary mechanical systems, it is required to generate campbell diagram for the specified inputs. For creation of campbell diagram, the range of rotational speed of system is also required. For present case, vertical shaft rotor system is considered to be rotated fom speed range of 0 to 2000 rpm. The results for modal frquencies with mode shapes without spin and with spin are shown in the following section.

The frequency values at zero speed obtained from the ANSYS are given in table3.1.

Mode No.	Frequency (Hz)
1	12.1218
2	41.9845
3	352.303

Table 3.1: Eigen frequencies at zero speed

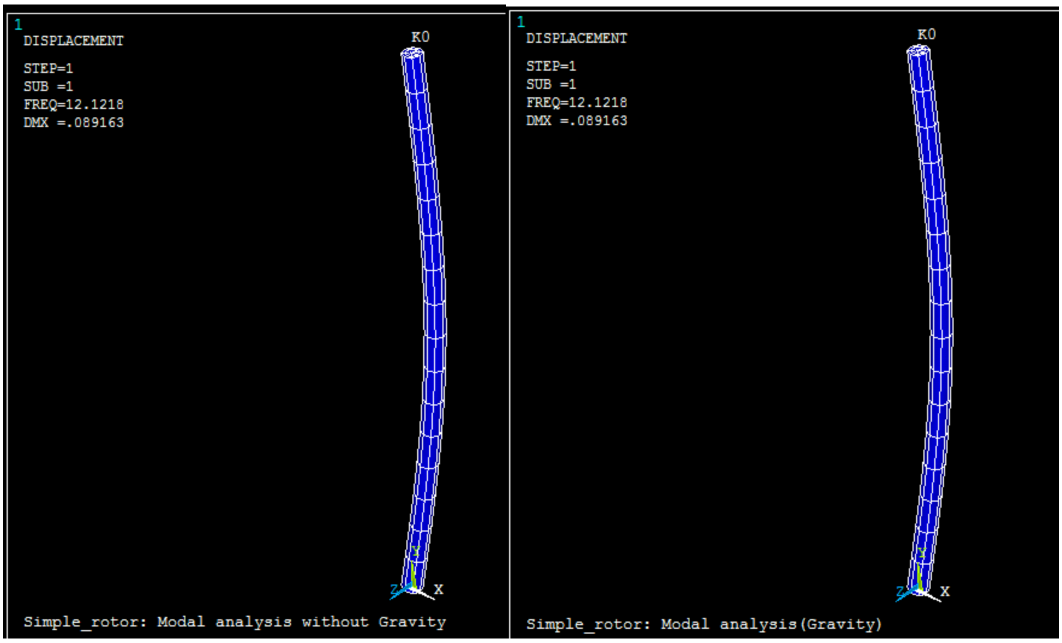

Figure 3.4: Mode 1(without spin- with/without gravity)

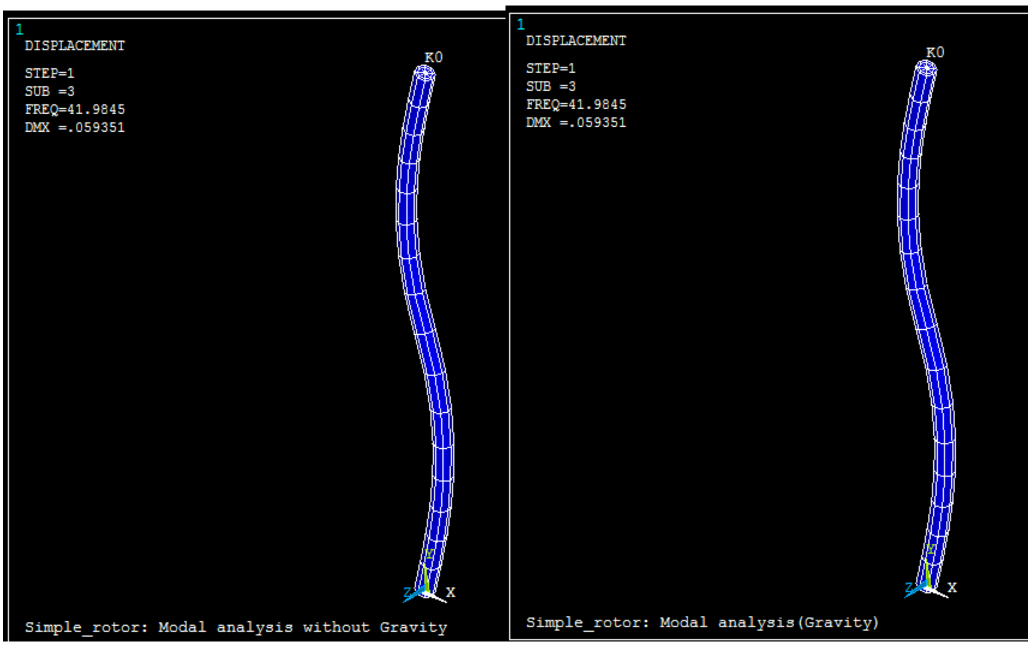

Figure 3.5: Mode 2(without spin- with/without gravity)

Figure 3.6: Mode 3(without spin- with/without gravity)

The main motive behind this modal analysis is to find out fundamental natural frequency. In order to validate the critical speed analysis through FEA, the first critical speed is to confirm to the fundamental natural frequency with considerable accuracy.

As mentioned earlier, it is required to perform modal analysis with rotational speed for the system in order to generate campbell diagram. The results derived in terms of frequency are plotted against the operating speed in the campbell diagram formulation.

The frequency values at 2000 rpm spin speed obtained from the ANSYS are given in table3.2.

Mode no.	Frequency (Hz)
1	12.1208
2	12.1229
3	20.6046
4	85.4884

Table 3.2: Frequencies at spin 2000 rpm

The associated mode shapes are also shown in figure3.7 to 3.10.

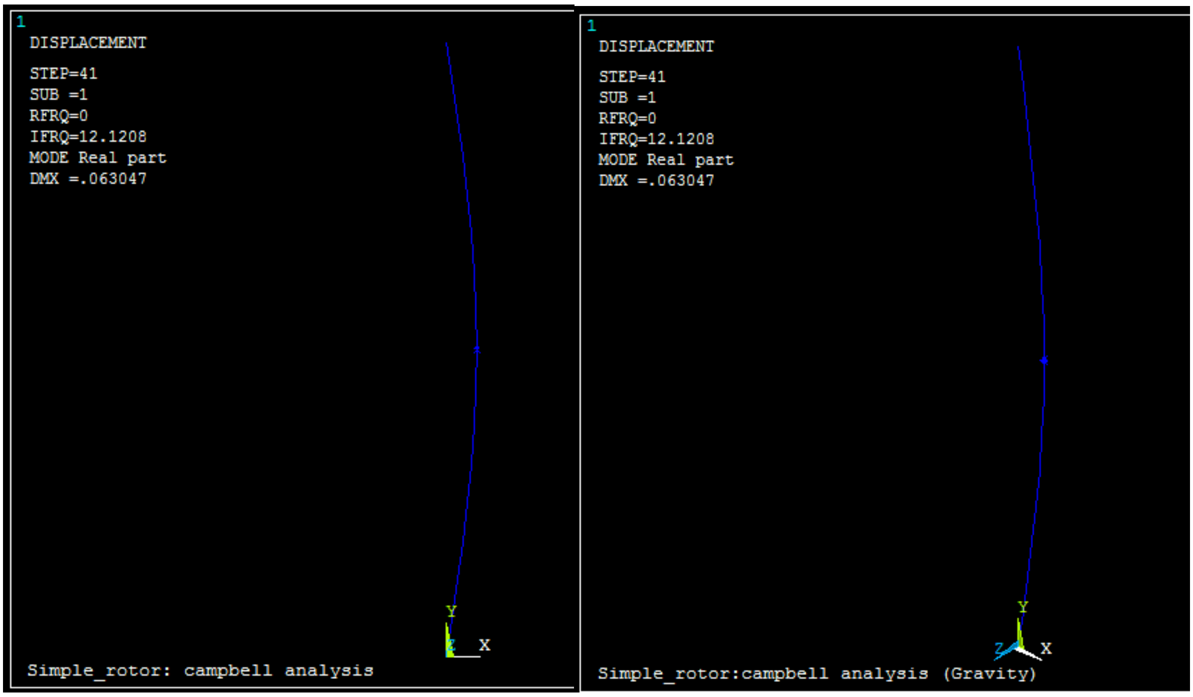

Figure 3.7: Mode 1(at spin -with/without gravity)

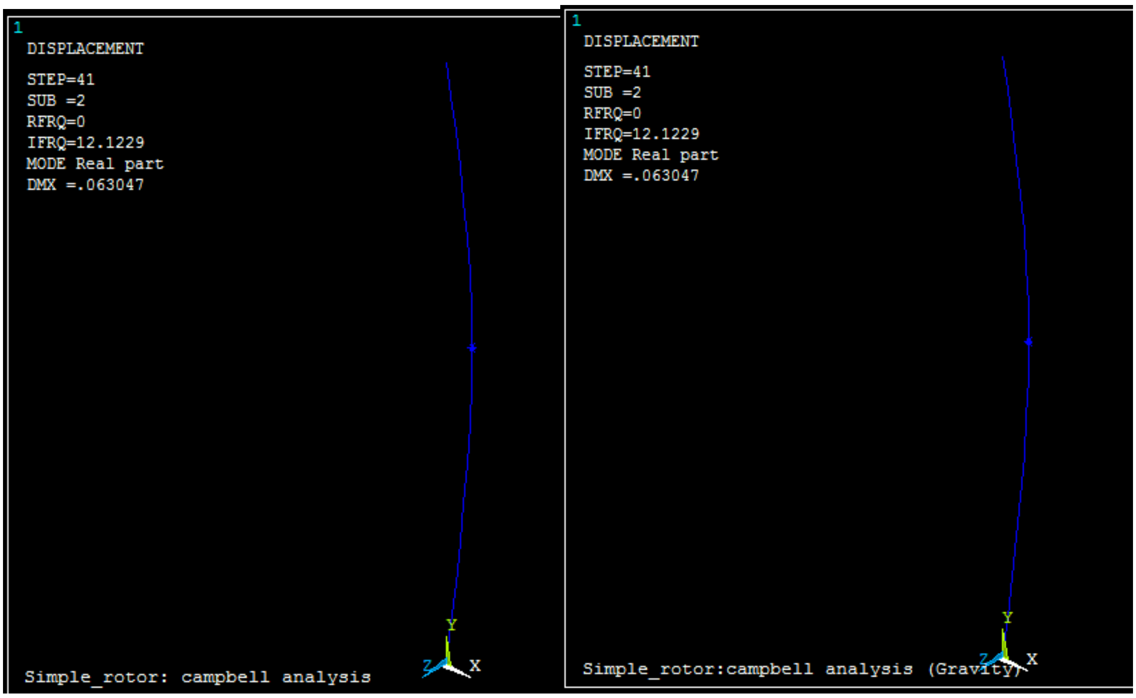

Figure 3.8: Mode 2(at spin -with/without gravity)

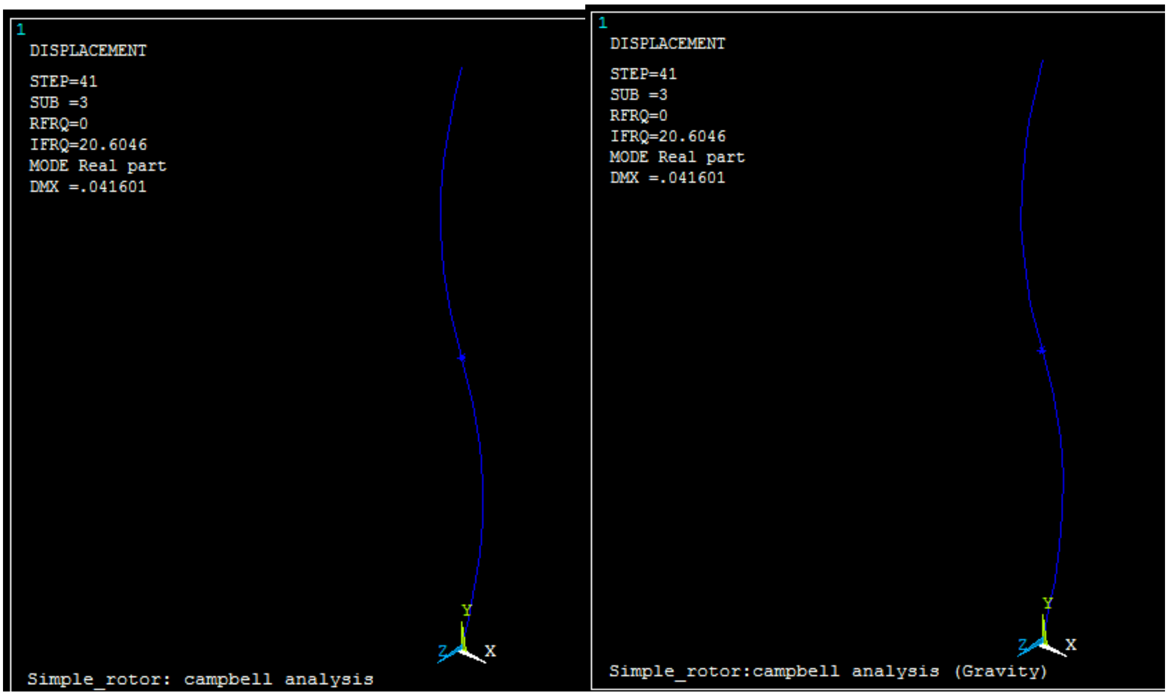

Figure 3.9: Mode 3(at spin -with/without gravity)

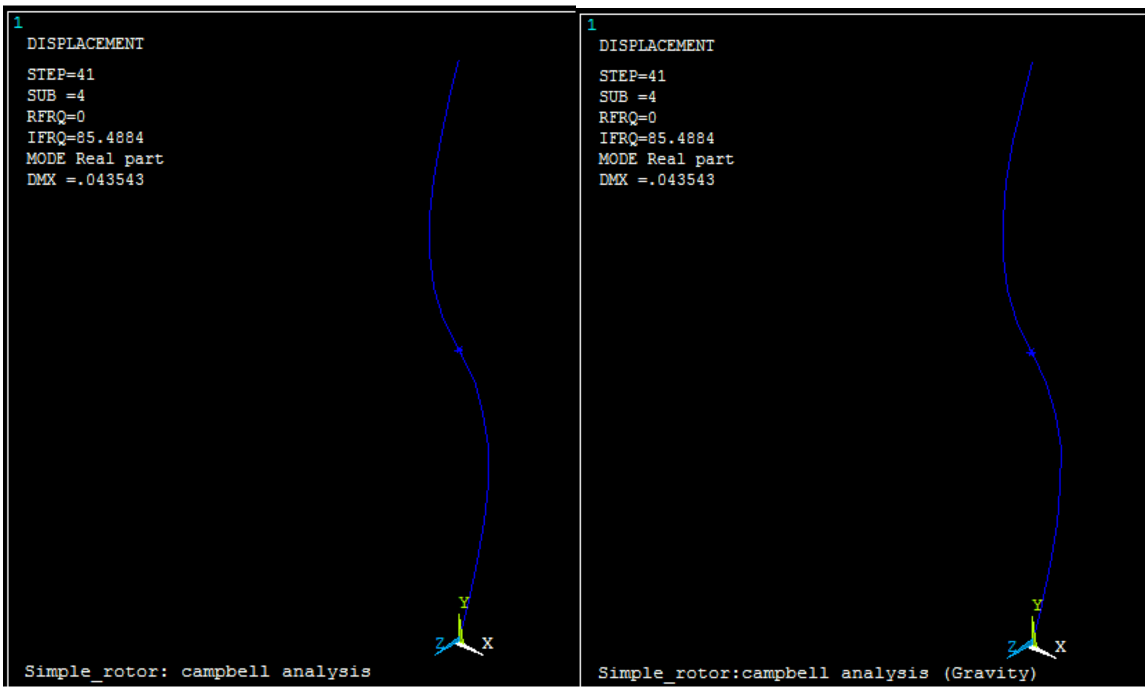

Figure 3.10: Mode 4(at spin -with/without gravity)

3.6 Orbital motion of shaft

When the structure is rotating about an axis and subjected to vibrating motion, the trajectory of a node executed around the axis is generally an ellipse designated as whirl orbit. Thus the discreate points or nodes om the spin axis of the rotor moves into curved path. This is beacause of the gyroscopic effect. When the bearing has same stiffness in vertical and horizontal direction, the whirl orbit is of circular form. If the bearing have different stiffness in both direction, whirl orbit is of elliptical form.

Whirl orbit for frequancies shown in table3.2 are shown in figure3.11to3.14.

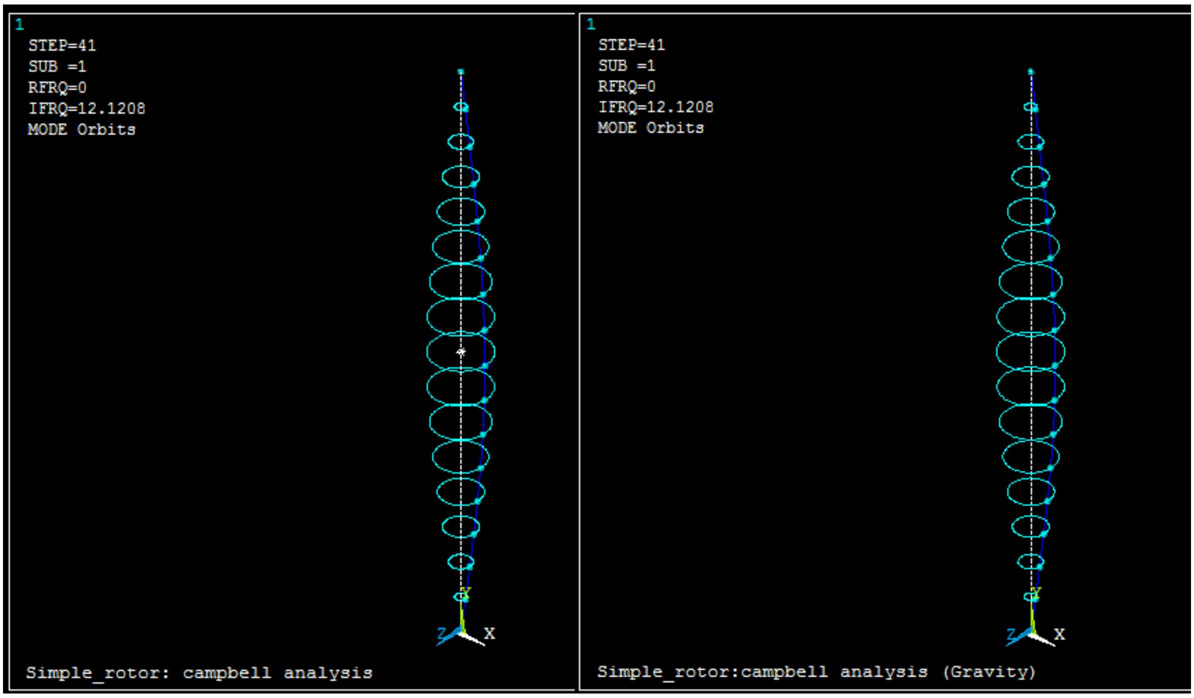

Figure 3.11: whirl orbit for 12.1208Hz

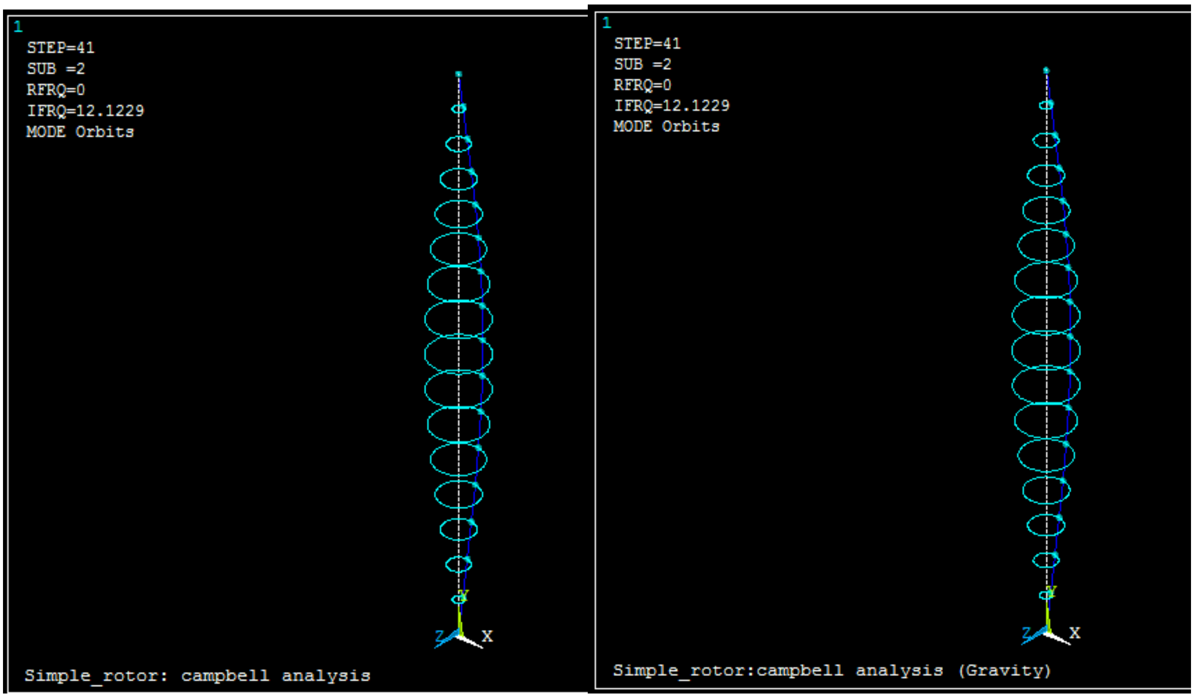

Figure 3.12: whirl orbit for 12.1229Hz

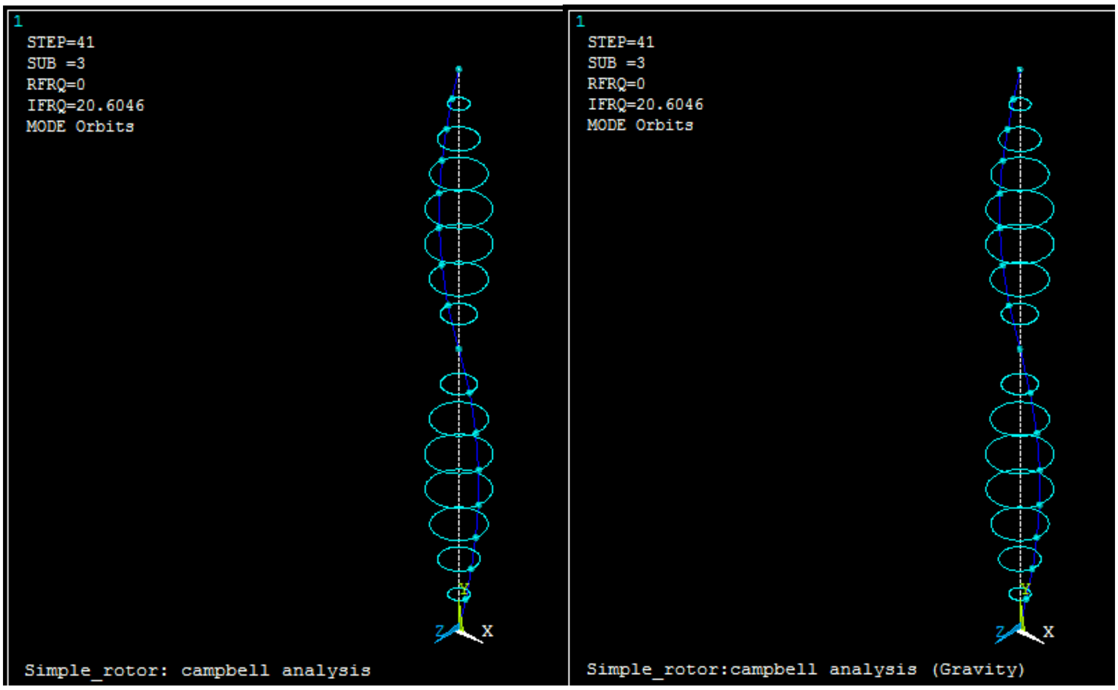

Figure 3.13: whirl orbit for 20.6046

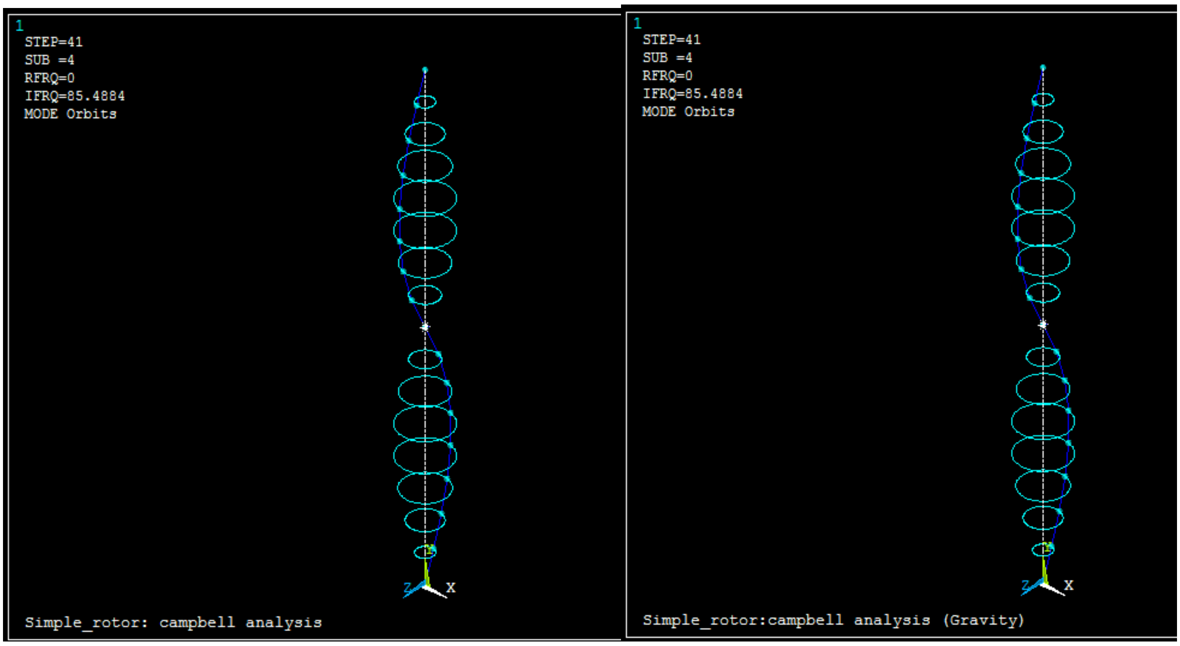

Figure 3.14: Whirl orbit for 85.4884

To display the orbit of each rotating node as well as deformed shape at time t=0(real part of solution), PLORB command is used. The PRORB command prints out the orbit characteristics semi major axis(A), semi minor axis(B), PSI(angle between local Y axis and major axis), PHI(angle between initial position{t=0} and major axis), YMAX and ZMAX(maximum displacement along Y and Z axis respectively). This commands PLORB and PRORB gives the node movemennt and orbit characteristics that are necessory for various analysis. For example, Harmonic response analysis we can visualize the maximum amplitude location by plotting and printing orbits. Also in case of multipool system, we can visualize orbital motion and orbit characteristics of each pool separately.[6, 29]

3.7 Critical speed analysis- Campbell diagram

As mentioned in literature, the campbell diagram is graphical representation of natural frequency versus excitation frequencies as a function of rotational speed. The campbell diagram can be plotted in ANSYS through PLCAMP command. The results of Campbell diagram can be obtained from PRCAMP command.

The variation of eigen frequencies between 0 rpm to 2000 rpm corrosponding to different rotational velocity are plotted in campbell diagram as shown in figure3.15. Critical speeds are determined for the excitation slope 1.

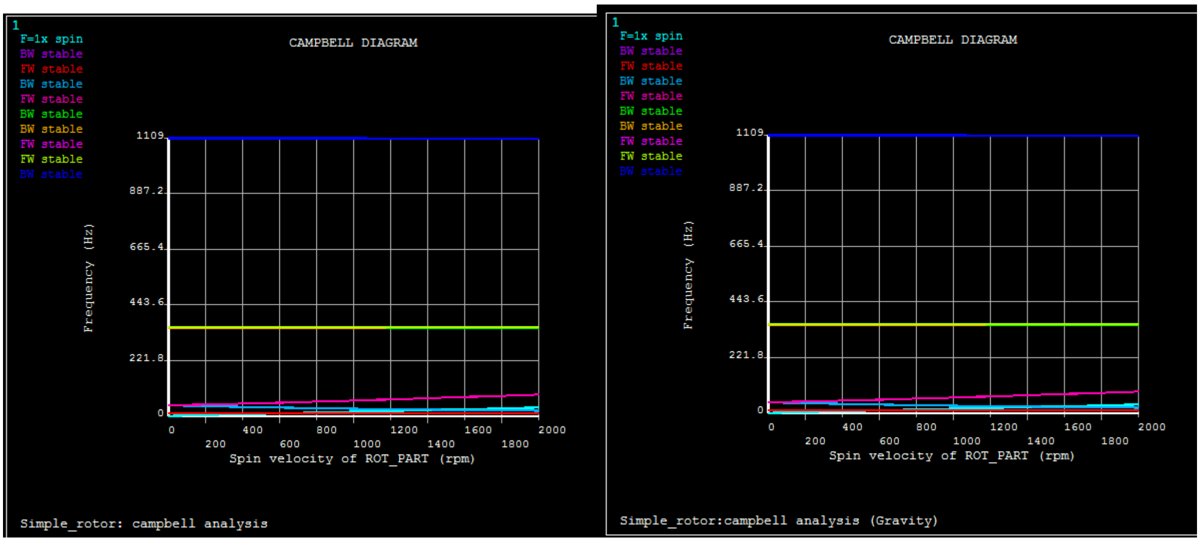

Figure 3.15: Campbell diagram for simple vertical rotor model(at spin -with/without gravity)

In the Campbell diagram, Whirl frequencies(Hz) are plotted along Y axis and rotational speeds(rpm) are plotted along the X axis. Upward slopping lines are excitation line. In present case, the slope of line is 1. Intersection of frequency line and upward slopping line gives critical speed. Campbell diagram check the stability and whirl of each mode.

The critical speeds between 0 rpm to 2000 rpm for excitation slope 1 obtained from ANSYS are given in table3.3.

SR No.	Critical speed(rpm)	Type of whirl
1	727.29	BW
2	727.33	FW
3	1467.5	BW

Table 3.3: Critical speeds for simple vertical rotor model(at spin -with/without gravity)

Because of centrifugal force acting on rotor during rotatation, the rotor is tends to bend and follow orbital motion. This is called whirling. Each natural frequency of whirl is split into two frequencies when the rotor is not stationary due to gyroscopic effect. When the whirl direction is same as shaft spin direction it is called forward whirling(FW). When direction of whirl is in opposite to shaft spin direction, it is called backward whirling(BW). As the rotor speed increses, gyroscopic moment stiffens the rotor stiffness in case of forward whirl and weakens the rotor stiffness in case of backward whirl. Hence as shown in table3.3, first and third critical speeds are associated with backward whirling and the second critical speed is associated with forward whirling.

3.8 Calculation of theoretical natural frequency

In order to validate the FE analysis for simple vertical shaft rotor system, the theoretical fundamental natural frequency has been calculated which is to be compared with the first natural frequency without spin obtained from ANSYS.

For the shaft having disc at the centre with bearings at the ends, the **fundamental natural frequency** in Hz is given by[13],

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{48EI}{mL^3}} \tag{3.1}$$

where, I = second area moment of inertia of the shaft, m^4

m = mass of the disc, kg

d=diameter of shaft, m

$$I = \frac{\pi}{64} \times d^4 \tag{3.2}$$

$$\omega_n = \frac{1}{2\pi}\sqrt{\frac{48\times 2.1x10^{11}\times\frac{\pi}{64}\times 0.04^4}{120.072\times 1.2^3}} \tag{3.3}$$

$$\omega_n = 12.43Hz \tag{3.4}$$

3.9 Results and comparison

Hence the methodology of construction of Campbell diagram to obtain critical speed of shaft rotor system has been identified by performing bench mark study on simple vertical shaft rotor system. The results obtained by FE analysis for shaft stationary condition, rotating condition and also critical speeds which are obtained from Campbell diagram are shown in table3.4. Also fundamental natural frequency obtained theoretically is shown in table3.4.

The result is same for the analysis without considering gravity and with considering gravity.

Sr.No.	Theoretical (Hz)	FEA (no spin, Hz)	FEA (with spin, Hz)	Critical speed (rpm)
1	12.43	12.1218	12.1208	727.29 (BW)
2	-	41.9845	12.1229	727.33 (FW)
3	-	352.303	20.6046	1467.23 (BW)
4	-	353.251	85.4884	-
5	-	1108.3	-	-

Table 3.4: Results for simple vertical rotor model(with/without gravity)

The result of vibration analysis of simple vertical shaft rotor system in ANSYS shows very close corelation with theoretical fundamental natural frequency. As per API 610 standard, the first frequency associated with first critical speed is close to the fundamental natural frequency. Also percentage of margin between natural frequency and operating speed should not less than $\pm 10\%$[4]. In present case as shown in table3.4 frequency associated with first critical speed(12.122Hz) is very close to the fundamental natural frequency(12.1218Hz). Also percentage of margin between natural frequency and operating speed(2000 rpm) is not less than $\pm 10\%$. So, all frequencies are within the acceptable region as per API 610 standards.

Chapter 4

Lumped Parameter Model of VTP Shaft Rotor System

4.1 CAD Model of Vertical Turbine Pump

This chapter deals with the developement of lumped parameter model of vertical turbine pump which is used to perform vibration analysis in order to identify critical speeds. The vertical turbine pump undertaken for analysis consists of three stages for operation. Each stage consists of impeller and bowl assembly.

The figure4.1 shows CAD model of single stage of vertical turbine pump. The actual turbine pump consists of three simillar stages. Hence for the formulation the parameters will remains same. The notation of various operating components are displayed also in figure4.1.

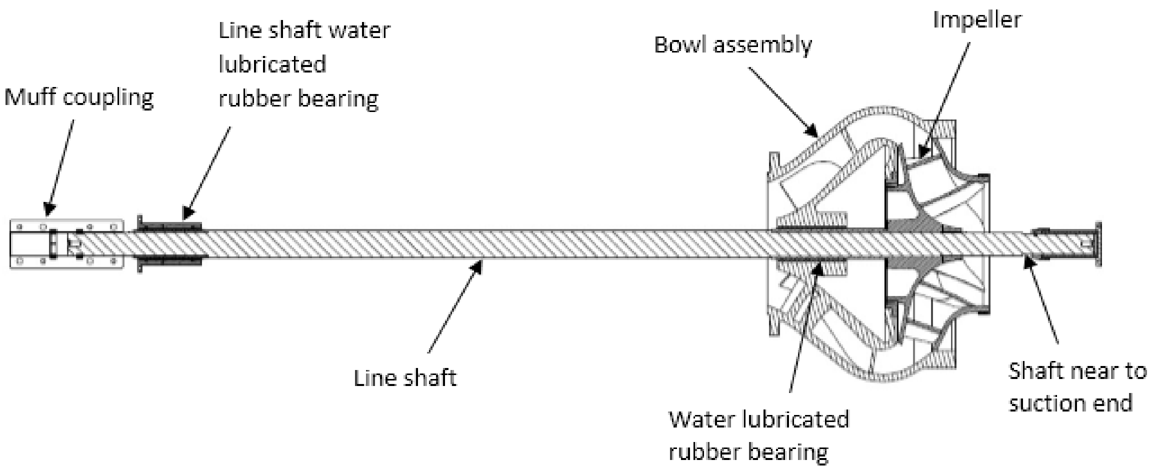

Figure 4.1: CAD model of single stage of Vertical Turbine Pump

4.2 Materials for various parts of VTP

As the complete assembly of VTP consists of various components with different material compositions, it is inevitable to identify related mechanical properties for studying its vibration characteristics through FE analysis.

The following table4.1 shows the materials for different components and table4.2 shows the essential mechanical properties of used material.

Sr.No.	Parts	Material
1	Pump shaft, Muff coupling	SS410 - stainless steel
2	Motor shaft, Spacer coupling	SS410 - stainless steel
3	Impeller	Bronze IS:318 Gr LTB2
4	Flexible coupling	Cast steel 280-520N
5	3 phase squirral cage induction motor rotor parts	FE 410 WA IS 2062 steel

Table 4.1: Materials for different components of VTP

Sr.No.	Material	Properties	Values
1	SS410-stainless steel	Density,ρ	7750 Kg/m^3
		Young's modulus,E	200 GPA
		Shear modulus, G	7.69×10^{10}N/m^2
		Poission ratio,μ	0.3
2	IS:318 Gr LTB2	Density	9250 Kg/m^3
3	Cast steel 280-520N	Density	7800 Kg/m^3
		Young's modulus,E	210 GPA
4	FE 410 WA IS 2062 steel	Density	7850 Kg/m^3
		Tensile strength	410 MPA

Table 4.2: Essential mechanical properties of used material for VTP

4.3 Identification of lumped parameters for VTP

For vibration analysis, the complex VTP model is to be converted into simple lumped mass model.

Figure4.2 shows complete lumped parameter model of VTP system. Various locations are shown with location numbers in figure4.3 to 4.6 starting from the left end(suction end) of the complete model as shown in figure4.2.

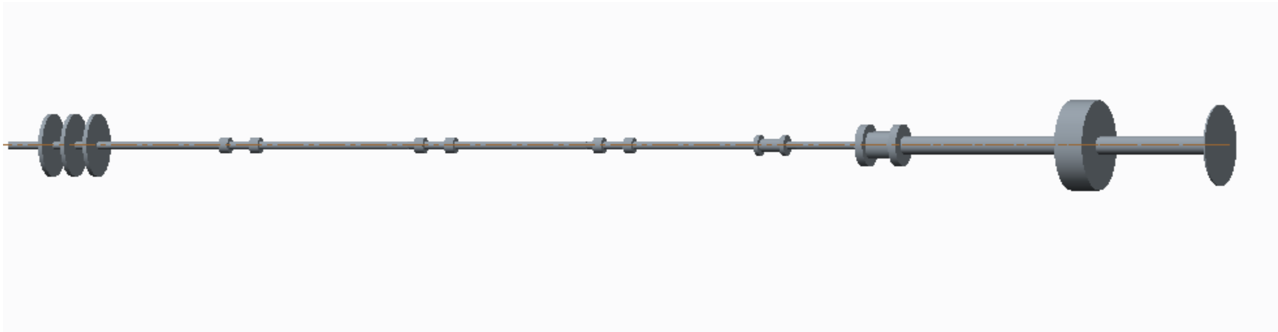

Figure 4.2: Complete lumped parameter model of VTP

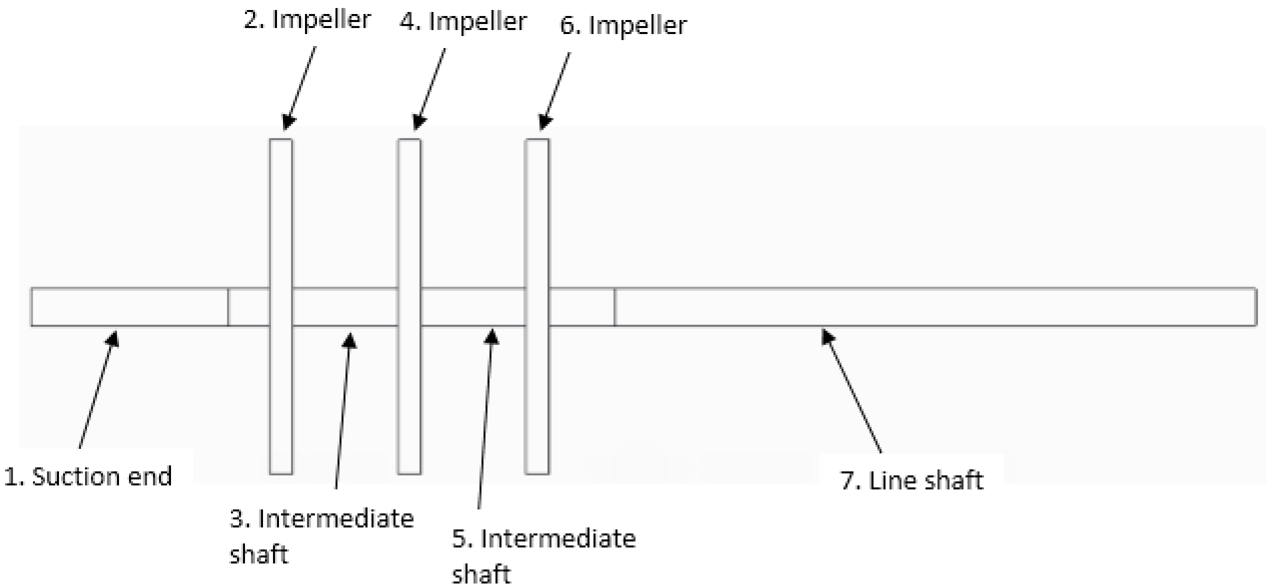

Figure 4.3: Various components of VTP

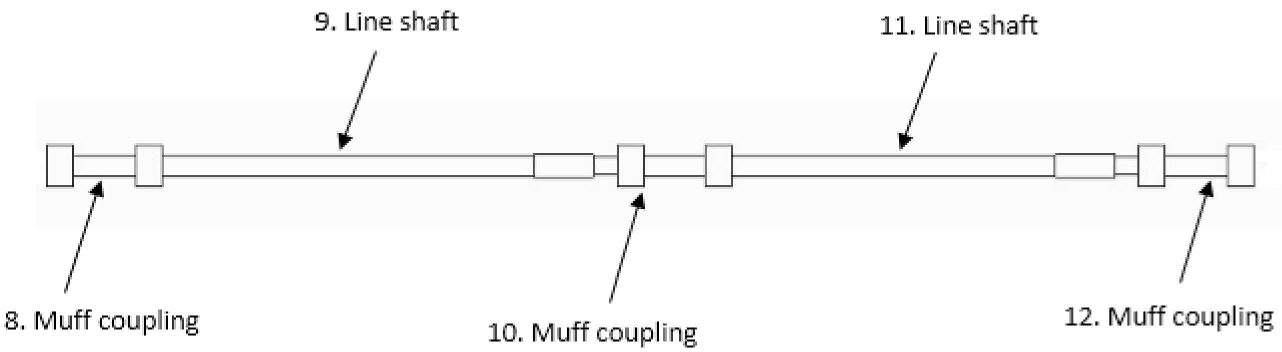

Figure 4.4: Various components of VTP

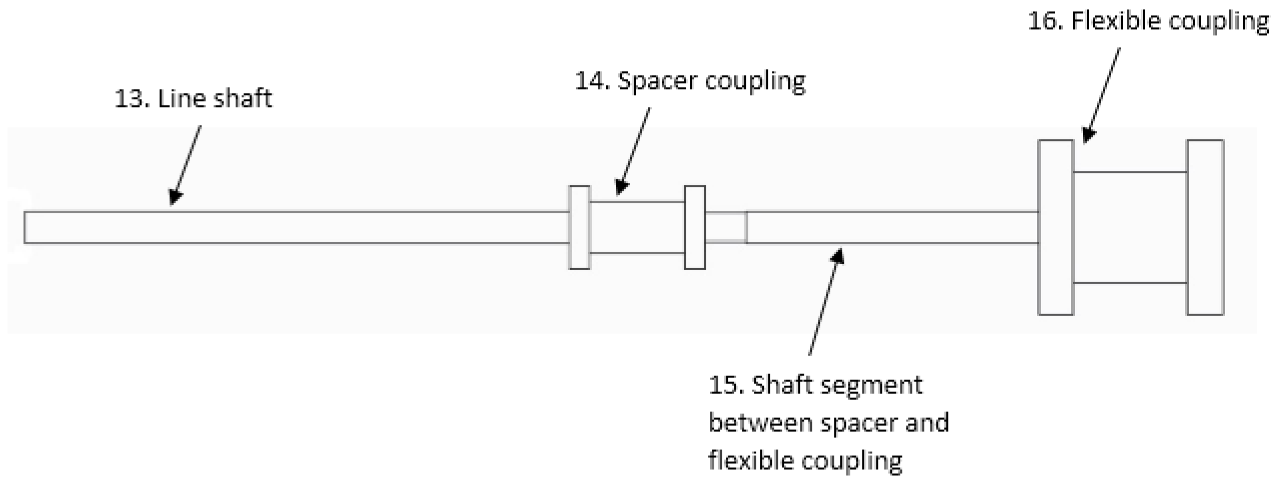

Figure 4.5: Various components of VTP

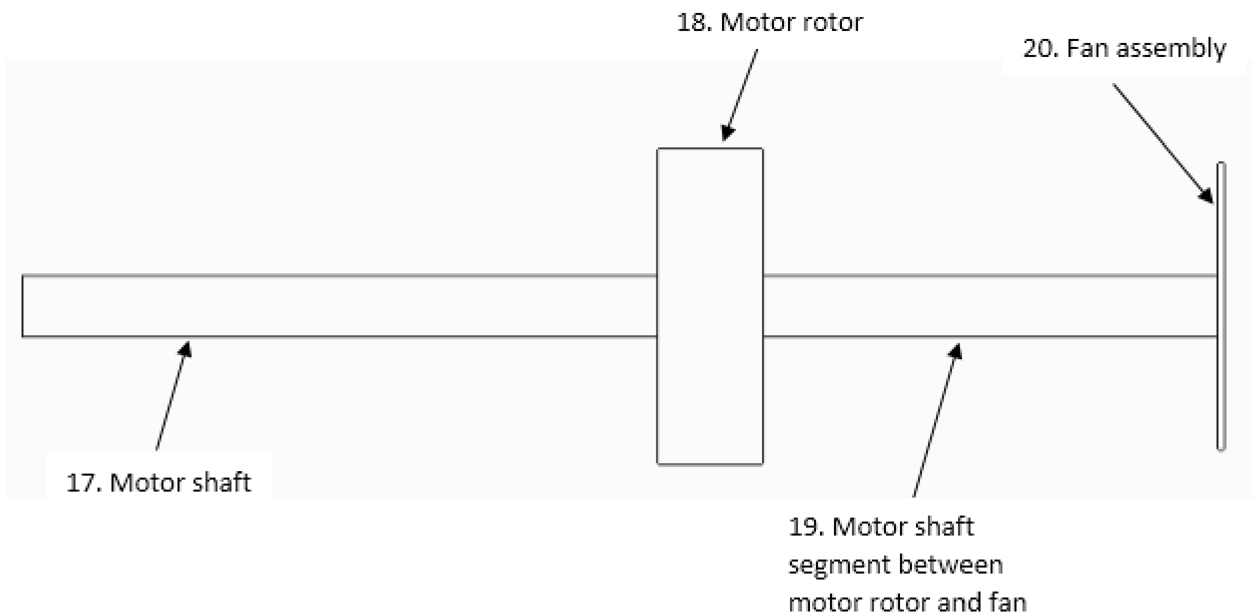

Figure 4.6: Various components of VTP

The assignment of lumped parameters are summerized into the following table.

Sr.No.	Component	Lumped parameter
1	suction end	torsional stiffness
2	impeller	mass moment of inertia
3	intermediate shaft	torsional stiffness
4	impeller	mass moment of inertia
5	intermediate shaft	torsional stiffness
6	impeller	mass moment of inertia
7	line shaft	torsional stiffness
8	muff coupling	torsional stiffness &mass moment of inertia
9	line shaft	torsional stiffness
10	muff coupling	torsional stiffness &mass moment of inertia
11	line shaft	torsional stiffness
12	muff coupling	torsional stiffness &mass moment of inertia
13	line shaft	torsional stiffness
14	spacer coupling	torsional stiffness &mass moment of inertia
15	pump shaft segment	torsional stiffness
16	flexible coupling	torsional stiffness &mass moment of inertia
17	motor shaft	torsional stiffness
18	motor rotor	mass moment of inertia
19	motor shaft segment	torsional stiffness
20	fan assembly	mass moment of inertia

Table 4.3: Assignment of lumped parameters of VTP

4.4 Calculation of Stiffness and mass moment moment of inertia of system

1. Suction end- shaft element:

The stepped shaft is synthesize into equivalent parameter model. So that the stiffness is calculated by following equation given;

$$K_e = \frac{GJ_e}{L_e} \tag{4.1}$$

Where G=7.69×10^{10}

L_e= Equivalent length of the shaft

d_e=Equivalent diameter of the shaft

J_e= Equivalent polar moment of inertia of the shaft, m^4

$$J_e = \frac{\pi}{32} d_e^4 \tag{4.2}$$

For shaft element 1:

d_1=0.085m , L_1=0.150 m

d_2=0.1 m, L_2= 0.055 m

d_3=0.070 m, L_3= 0.223m

d_4=0.111m, L_4=0.067 m

d_5=0.078 m, L_5= 0.0875 m

Equivalent diameter of the shaft, d_e= 0.075m

Equivalent moment of inertia,

$$J_e = \tfrac{\pi}{32}(0.075)^4 = 3.1063111 \times 10^{-6} m^4 \tag{4.3}$$

Equivalent length

$$L_e = J_e \left\{ \tfrac{L_1}{J_1} + \tfrac{L_2}{J_2} + \tfrac{L_3}{J_3} + \tfrac{L_4}{J_4} + \tfrac{L_5}{J_5} \right\} \tag{4.4}$$

$$L_e = 3.1063111 \times 10^{-6} \left\{ \frac{0.150}{\frac{\pi}{32}(0.085)^4} + \frac{0.055}{\frac{\pi}{32}(0.1)^4} + \frac{0.223}{\frac{\pi}{32}(0.07)^4} + \frac{0.067}{\frac{\pi}{32}(0.111)^4} + \frac{0.0875}{\frac{\pi}{32}(0.078)^4} \right\}$$

$$L_e = 0.490954234m$$

So, the equivalent stiffness of shaft 1 is

$$K_e = \tfrac{7.69 \times 10^{10} \times 3.1063111 \times 10^{-6}}{0.490954234}$$

$$K_e = 4.86553138 \times 10^5 Nm/rad \tag{4.5}$$

2. Impeller:

t = 0.04662 m

R_o = 0.3375 m

R_i= 0.039 m

$$I = \tfrac{1}{2} \times \pi \times \rho \times t \times (R_o^4 - R_i^4) \tag{4.6}$$

$$I = \tfrac{1}{2} \times \pi \times 9250 \times 0.04662 \times (0.3375^4 - 0.039^4)$$

$$I = 8.555640171 Kg.m^2 \tag{4.7}$$

Impeller disk having location no. 4 and 6 have same radius and thickness so mass moment of inertia of all 3 impellers are same.

3. Intermediate shaft:

Since intermediate shaft having location no.3 is stepped shaft, same procedure can be followed as in case of suction end-shaft.

d_e= 0.078m

L_e= 0.3599m

K_e= 7.76466634$\times10^5$Nm/rad.

Intermediate shaft having location no.5 have same L_e and d_e so torsional stiffness K_e is also same.

4. Line shaft:

Line shaft having location no.7 is stepped shaft so same procedure can be followed for calculating torsional stiffness.

d_e= 0.075m

L_e= 1.37754m

K_e= 1.734071653$\times10^5$Nm/rad.

5. Muff coupling:

Since same muff couplings used at location no. 8,10 and 12 the lumped parameters are same for all three components.

Coupling should be modelled with shaft element having stiffness equal to coupling stiffness and disk having one half of coupling's total mass moment of inertia at each end.

Stiffness of muff coupling given: 1.558$\times10^6$Nm/rad.

Total mass moment of inertia: 0.12287038 kg.m^2

Mass moment of inertia of each disk= $\frac{0.12287038}{2}$= 0.06143519 kg.m^2

6. Line shaft:

All line shaft segments are stepped shaft so same procedure can be used as shown above.

Line shaft having location no.9 has

d_e=0.08m

L_e=1.4369187m

K_e=2.152058×10^5Nm/rad

Line shaft having location no.11 has

d_e=0.078m

L_e=1.160943m

K_e=2.407097958×10^5Nm/rad

Line shaft having location no.13 has

d_e=0.085m

L_e=1.43299131m

K_e=2.750162654×10^5Nm/rad

7. Spacer coupling:

Spacer coupling is to be modelled same as muff coupling.

Stiffness of spacer coupling given: 8.038×10^6Nm/rad.

Total mass moment of inertia: 0.407064976 kg.m^2

Mass moment of inertia of each disk= $\frac{0.407064976}{2}$= 0.203532488 kg.m^2

8. Shaft segment between spacer and flexible coupling:

Since it is also stepped shaft(location no.15),

d_e=0.078m

L_e=0.8212375m

K_e=3.402795714×10^5Nm/rad

9. Flexible coupling:

Flexible coupling is to be modelled same as muff coupling.

Stiffness of flexible coupling given: 1.87×10^8Nm/rad.

Total mass moment of inertia: 6.083773282 kg.m^2

Mass moment of inertia of each disk= $\frac{6.083773282}{2}$= 3.041886641 kg.m^2

10. Motor shaft:

Since it is stepped shaft(location no.17),

d_e= 0.190m

L_e= 1.960881m

K_e= 5.017523319×10^6Nm/rad.

11.Motor rotor:

Radius, R=0.484940342m

Thickness, t=0.323131685m

$$I = \frac{1}{2} \times \pi \times \rho \times t \times R^4 \tag{4.8}$$

$$I = \frac{1}{2} \times \pi \times 7850 \times 0.321313 \times 0.4849403^4$$

$$I = 220.35471 kg.m^2$$

12. Motor shaft segment between motor rotor and fan:

Since it is also stepped shaft(location no.19),

d_e=0.190m

L_e=1.4002m

K_e=7.0267$\times 10^6$Nm/rad

13. Fan assembly:

Radius, R=0.4403167m

Thickness, t=18.97mm

$$I = \frac{1}{2} \times \pi \times \rho \times t \times R^4 \tag{4.9}$$

$$I = \frac{1}{2} \times \pi \times 7850 \times 0.01897 \times 0.44031667^4$$

$$I = 8.7918222 kg.m^2$$

Hence lumped parameters of various components of VTP are calculated and summerized in table4.4.

Sr.No.	Part	Torsional stiffness(Nm/rad)	Mass moment of inertia(kg.m^2)
1	suction end	4.865531381×10^5	-
2	impeller	-	8.555640171
3	intermediate shaft	7.764666342×10^5	-
4	impeller	-	8.555640171
5	intermediate shaft	7.764666342×10^5	-
6	impeller	-	8.555640171
7	line shaft	1.734071653×10^5	-
8	muff coupling	1.558×10^6	0.12287038
9	line shaft	2.152058×10^5	-
10	muff coupling	1.558×10^6	0.12287038
11	line shaft	2.407097958×10^5	-
12	muff coupling	1.558×10^6	0.12287038
13	line shaft	2.750162654×10^5	-
14	spacer coupling	8.038×10^6	0.407064976
15	pump shaft segment	3.4027957×10^5	-
16	flexible coupling	1.87×10^8	6.083773282
17	motor shaft	5.017523319×10^6	-
18	motor rotor	-	220.35471
19	motor shaft segment	7.0267×10^6	-
20	fan assembly	-	8.7918222

Table 4.4: Lumped parameters of various components of VTP

4.5 Water Lubricated Line shaft Bearing Stiffness[17, 18, 19, 21, 22, 24, 25]

Rotor system in VTP is vertically oriented and have shaft which transmitt axial and torsional loads. Axial loads are produced by hydraullic forces from impellers at botom of system and balanced by thrust bearings at the top of the system. Between these two points of the tensile load of the rotor system, radial bearings provide position control and stability to rotating system at various intervals along the length of the system. These bearings are water lubricated rubber bearings in vertical turbine pump.[17]

It can be stated that infinitely long hydrodynamic bearing theory is capable of generating reliable performance characteristics for bearing with L/D$\geq$2[21]. In this analysis the bearing have L/D$\approxeq$2.

Since for FE lumped parameter models, the stiffness of supports play major role in governing vibration behaviour, it is inevitable to calculate the stiffness to be considered in the lumped parameter model.

Stiffness of the water lubricated bearing is calculated on basis of Hydrodynamic bearing theory considering following parameters;

Length of the bearing, l=200mm

Radius of the shaft, r=45mm

Radius of the bearing,R=45.25mm

Radial clearence=0.25mm

Dynamic viscosity of water(60^{o}C), μ= 0.4658×10^{-9}Ns/mm^2

Shaft speed, n$_s$=960 rpm

Unit bearing pressure, p=0.6 N/mm^2

Radial force[25],

$$W = 2 \times p \times r \times l \tag{4.10}$$

$$W = 2 \times 0.6 \times 45 \times 200$$

$$W = 10800N$$

Sommerfeld number (dimensionless),

$$S = \left(\frac{r}{c}\right)^2 \times \frac{\mu n_s}{p} \tag{4.11}$$

$$S = 0.024147072$$

For S=0.024147072, The eccentricity ratio (ε) = n$_0$ = 0.7648[25].

Now, from the hydrodynamic bearing theory and Raynold's equation the expressions for non dimensional stiffness and damping coefficients as function of the eccentricity ratio of bearing are[18, 19, 22, 24]:

$$\left(K_{zz}\right)_c = f_1\left(n_0\right)\left[9.8696 + 41.8696n_0^2 + 12.2608n_0^4\right] \tag{4.12}$$

$$\left(K_{yz}\right)_c = f_2\left(n_0\right)\left[-7.75157 + 15.50314n_0^2 + 4.8148n_0^4\right] \tag{4.13}$$

$$\left(K_{zy}\right)_c = f_2\left(n_0\right)\left[7.75157 + 32.884314n_0^2 + 9.6296n_0^4\right] \tag{4.14}$$

$$\left(K_{yy}\right)_c = f_3\left(n_0\right)\left[19.7392 + 6.1304 n_0^2\right] \tag{4.15}$$

$$\left(C_{zz}\right)_c = f_2\left(n_0\right)\left[15.50314 + 44.39195 n_0^2 + 15.50314 n_0^4\right] \tag{4.16}$$

$$\left(C_{yz}\right)_c = \left(C_{zy}\right)_c = f_3\left(n_0\right)\left[19.7392 + 7.47842 n_0^2\right] \tag{4.17}$$

$$\left(C_{yy}\right)_c = f_2\left(n_0\right)\left[15.50314 - 9.6296 n_0^2 - 5.87354 n_0^4\right] \tag{4.18}$$

Where,

$$f_1\left(n_0\right) = \frac{4}{\left(1-n_0^2\right)\left[16 n_0^2 + \pi^2\left(1-n_0^2\right)\right]^{1.5}} \tag{4.19}$$

$$f_1\left(n_0\right) = \frac{4}{\left(1-0.7648^2\right)\left[16\left\{0.7648\right\}^2 + \pi^2\left(1-0.7648^2\right)\right]^{1.5}} = 0.1952463$$

$$f_2\left(n_0\right) = \frac{4}{n_0\sqrt{\left(1-n_0^2\right)}\left[16 n_0^2 + \pi^2\left(1-n_0^2\right)\right]^{1.5}} = 0.164475 \tag{4.20}$$

$$f_3\left(n_0\right) = \frac{4}{\left[16 n_0^2 + \pi^2\left(1-n_0^2\right)\right]^{1.5}} = 0.081043 \tag{4.21}$$

Now put the values of $f_1(n_0)$, $f_2(n_0)$ & $f_3(n_0)$ in above equations for stiffness and damping coefficients,

$\left(K_{zz}\right)_c$= 7.527665, $\left(K_{yz}\right)_c$= 0.487471, $\left(K_{zy}\right)_c$= 4.980436

$\left(K_{yy}\right)_c$= 1.890327, $\left(C_{zz}\right)_c$= 7.693, $\left(C_{yy}\right)_c$= 1.292952

$\left(C_{yz}\right)_c$=$\left(C_{zy}\right)_c$=1.95383

Now bearing stiffness and damping values are:

$$K_{zz} = K_{22} = \frac{\left(K_{zz}\right)_c \times W}{c} \tag{4.22}$$

$$K_{zz} = K_{22} = \frac{7.527665 \times 10800}{0.25} = 3.252 \times 10^8 N/m$$

$$K_{yy} = K_{11} = \frac{\left(K_{yy}\right)_c \times W}{c} = 8.1662 \times 10^7 N/m \tag{4.23}$$

$$K_{yz} = K_{12} = \frac{(K_{yz})_c \times W}{c} = 2.106 \times 10^7 N/m \tag{4.24}$$

$$K_{zy} = K_{21} = \frac{(K_{zy})_c \times W}{c} = 2.15155 \times 10^8 N/m \tag{4.25}$$

$$C_{yy} = C_{11} = \frac{(C_{yy})_c \times W}{c \times \omega} = 5.5561 \times 10^5 N.s/m \tag{4.26}$$

$$C_{zz} = C_{22} = \frac{(C_{zz})_c \times W}{c \times \omega} = 3.305834 \times 10^6 N.s/m \tag{4.27}$$

$$C_{yz} = C_{zy} = C_{12} = C_{21} = \frac{(C_{yz})_c \times W}{c \times \omega} = 8.396 \times 10^5 N.s/m \tag{4.28}$$

The above stiffness and damping values are used in formulation of FE for support at bearing locations. The figure shows schematic of the element COMBI214.

As per notation K_{11}=K_{xx}, K_{22}=K_{zz}, K_{12}=K_{xz}, K_{21}=K_{zx}, C_{xx}=C_{11}, C_{22}=C_{zz}, C_{xz}=C_{zx}=C_{12}=C_{21}

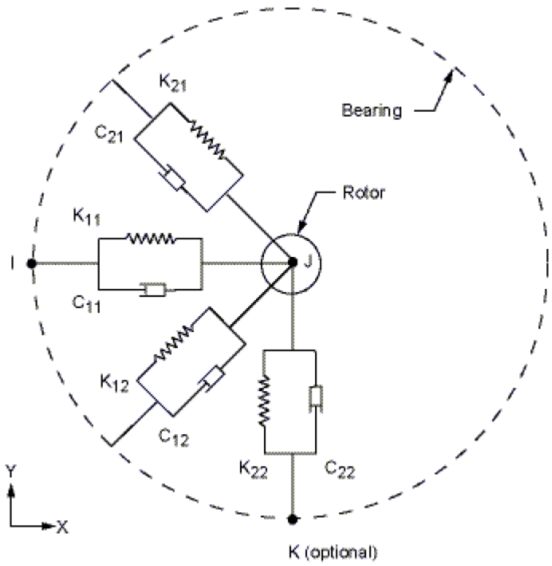

Figure 4.7: COMBI214 element.[29]

4.6 Deep groove ball bearing (6328M) stiffness[26, 19]

Rotor system in VTP is vertically oriented and have shaft which transmitt axial and torsional loads. Axial loads are produced by hydraullic forces from impellers at botom of system and balanced by thrust bearings at the top of the system. At the top of the system, between spacer coupling ans flexible coupling thrust spherical roller bearing is used to balance axial load and near to that single row deep groove ball bearing is used to balance moderate radial load and/or light axial load. Here the calculation of radial stiffness of deep groove ball bearing is given below.

From SKF catalogue[26][19],

Bore dia, d=140mm

Outside dia, D=300mm

Width, B=62mm.

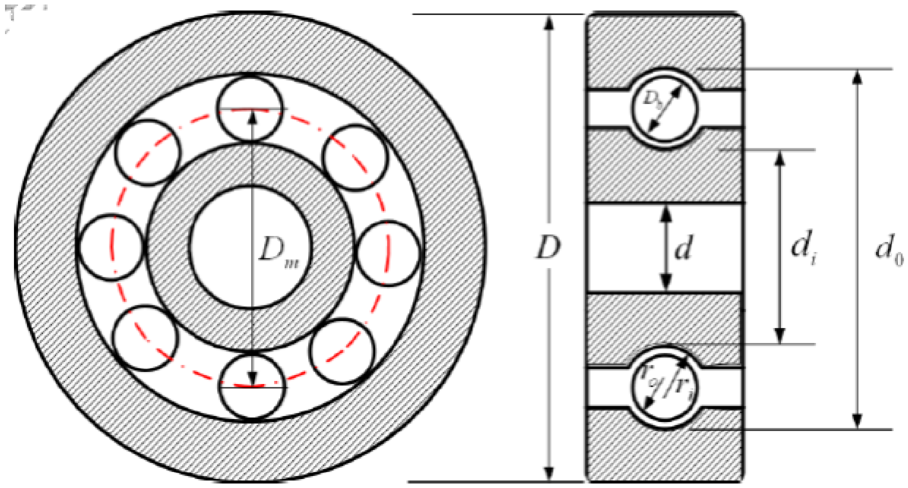

Figure 4.8: SKF Bearing 6328M[19]

Pitch Diameter, $D_m = 0.5\,(d + D)$= 220 mm.

Dia of ball,D_b=0.3$(D - d)$=48mm

No. of balls, Z=2.9$\left(\frac{D+d}{D-d}\right)$=7.975 say 8.

Radial thickness of rings, S=0.15$(D - d)$=24mm

Inner & outer ring groove radius of curvature, r_i=r_o=0.515D=154.5mm

From SKF catalogue, for bore 140 mm radial clearance is 18-48μm.

take maximum clearance, hence g=48$\times10^{-3}$mm.

For zero radial clearance, inner and outer ring dia at raceway:

d_o=220+48=268mm & d_i=220-48=172 mm.

$\gamma=\frac{D_b}{D_m}$=0.21818, $f_i=\frac{r_i}{D_b}$=3.21875, $f_o=\frac{r_o}{D_b}$=3.21875

$$\Sigma\rho_i = \frac{1}{D_b}\left(4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma}\right) = 0.08848mm^{-1} \tag{4.29}$$

$$\Sigma\rho_o = \frac{1}{D_b}\left(4 - \frac{1}{f_i} - \frac{2\gamma}{1+\gamma}\right) = 0.0694mm^{-1} \tag{4.30}$$

$$F\left(\rho\right)_i = \frac{\frac{1}{f_i} + \frac{2\gamma}{1-\gamma}}{4 - \frac{1}{f_i} + \frac{2\gamma}{1-\gamma}} = 0.204892 \tag{4.31}$$

F(ρ)	a*	b*	δ*
0.0000	1.0000	1.000	1.000
0.1075	1.0760	0.9318	0.9974
0.3204	1.2623	0.8114	0.9761
0.4795	1.4556	0.7278	0.9429
0.5916	1.6440	0.6687	0.9077
0.6716	1.8258	0.6245	0.8733
0.7332	2.0110	0.5881	0.8394
0.7948	2.2650	0.5480	0.7961
0.8350	2.4940	0.5186	0.7602
0.8737	2.8000	0.4863	0.7169
0.9100	3.2330	0.4499	0.6636
0.9366	3.7380	0.4166	0.6112
0.9574	4.3950	0.3830	0.5551
0.9729	5.2670	0.3490	0.4960
0.9838	6.4480	0.3150	0.4352

Table 4.5: Dimensionless contact parameters[19]

From table4.5, for $F(\rho)_i$=0.204892, δ_i=0.98765

$$F\left(\rho\right)_o = \frac{\frac{1}{f_o} + \frac{2\gamma}{1+\gamma}}{4 - \frac{1}{f_o} - \frac{2\gamma}{1+\gamma}} = 0.200799 \tag{4.32}$$

From figure4.5, for $F(\rho)_o$=0.200799, δ_o=0.988065

The load deflection constants at inner and outer raceway contacts:

$$K_{pi} = 2.15 \times 10^5 \left(\Sigma\rho_i\right)^{-\frac{1}{2}} \left(\delta_i\right)^{-\frac{3}{2}} = 7.36395 \times 10^5 N/mm^{1.5} \tag{4.33}$$

$$K_{po} = 2.15 \times 10^5 \left(\Sigma \rho_o\right)^{-\frac{1}{2}} \left(\delta_o\right)^{-\frac{3}{2}} = 8.3096 \times 10^5 N/mm^{1.5} \tag{4.34}$$

K_{pio}=Coefficient of propotionality depending on geometry & material properties of bearing.

$$K_{pio} = \frac{1}{\left(\frac{1}{K_{pi}}\right)^{\frac{2}{3}} + \left(\frac{1}{K_{po}}\right)^{\frac{2}{3}}} = 4.2416 \times 10^3 N/mm^{1.5} \tag{4.35}$$

Bearing stiffness, K(x)=a+bx^2, a=K(0) & b=$\frac{K(g)+K(0)}{g^2}$

ψ_i= angle between lines of action of radial load(direction of displacement of moving ring) and radius passing through center of i_{th}ball.

x=displacement of ring in radial direction.

$$K\left(x\right) = K_{pio} \times \sum_{i=1}^{Z} \left(g - xcos\psi_i\right)^{1.5} \left(cos\psi_i - \frac{C}{1.5B} sin\psi_i\right) cos\psi_i \tag{4.36}$$

where, ψ_i=$\frac{\pi}{Z}(2i-1)$, i=1,2,...,Z

so for Z=8, ψ_i=$\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

$$B = \sum_{i=1}^{Z} \left(g + xcos\psi_i\right)^{0.5} sin^2\psi_i \tag{4.37}$$

$$C = \sum_{i=1}^{Z} \left(g + xcos\psi_i\right)^{0.5} sin\psi_i cos\psi_i \tag{4.38}$$

So, at x=0,

B=0.876 & C=0.

Therefore,

$$K\left(0\right) = K_{pio} \sum_{i=1}^{8} \left(g\right)^{0.5} cos^2\psi_i = 3.7156 \times 10^3 N/mm \tag{4.39}$$

At, x=g=0.048,

B=0.84033 & C=0

Therefore,

$$K\left(g\right) = K_{pio} \sum_{i=1}^{8} \left(g + gcos\psi\right)^{0.5} cos^2\psi_i = 3.17055 \times 10^3 N/mm \tag{4.40}$$

For diametric clearance, P_d<g, K(x)=a+bx^2

a=K(0)=3.7156×$10^3 N/mm$

b=$\frac{K(g)+K(0)}{g^2}$=-2.3656×$10^5 N/mm$

So,

$$K\left(x\right)=3.7156\times10^{3}-2.3656\times10^{5}x^{2} \tag{4.41}$$

K(x) is negative for 236.56$(x)^2$>3.7156

so, x>0.1253mm.

At, x=g

$$K\left(x\right)=3.7156\times10^{3}-2.3656\times10^{5}g^{2}$$

$$K\left(x\right)=3.170566\times10^{7}N/m$$

• Stiffness of cylindrical roller bearing NU338 is given, 8.73726×$10^8 N/m$ from manufecturer's catalogue. The cylindrical roller bearing is used in the electric motor at the end near flexible coupling. There is also one deep groove ball bearing is mounted in electric motor at other end that is near to fan assembly.

• Stiffness of thrust spherical roller bearing SKF 29426 E is given, 9.27135×$10^9 N/m$ from manufecturer's catalogue. The large axial load and very light radial load are balanced by spherical roller thrust bearing which is mounted at the top portion of VTP between spacer coupling and flexible coupling near to deep groove ball bearing.

Chapter 5

Critical Speed Analysis of Vertical Turbine Pump Using FEA

This chapter deals with developement of FE model and perform vibration analysis of VTP using ANSYS. The material properties of rotor, impellers, couplings and bearing properties are already shown in chapter 4. This chapter includes the selection of elements, modelling procedure of VTP in ANSYS, results obtained from ANSYS and its validation using analytical Holzer's method. For performing analysis, the methodology adopted in chapter 3 is used for FE formulatiom.

5.1 Selection of elements

For vibration analysis of VTP using ANSYS, elements should be chosen based upon criteria that should support gyroscopic effect. The elements are used to build up the FE model of VTP are:

5.1.1 BEAM188[12, 29]

BEAM188 element is developed based on Timoshenko beam theory. Hence the element includes shear deformation effects. BEAM188 element is two node beam element in 3D with torsion, bending, compression and tension capabilities as shown in figure5.1. When KEYOPT(1)=0 is used(default) the element has 6 degrees of freedoms; translation in nodal x,y,z directions and rotation about nodal x,y,z axes. This element is associated with sectional library which consist of different sectional shape. So, this element can be modelled with desired section shape and real constants for the chosen section are included automatically[29, 12].

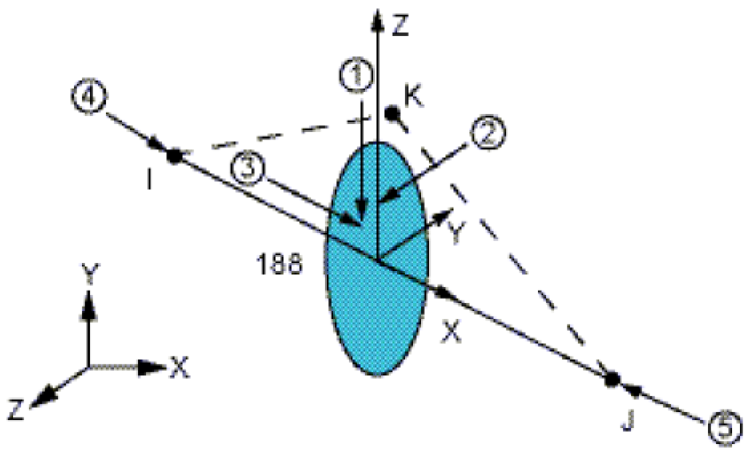

Figure 5.1: BEAM188 element[29]

5.1.2 MASS21[12, 29]

MASS21 is a point element and defined by single node as shown in figure5.2. It has six degrees of freedoms; translational in nodal x,y,z and rotation about nodal x,y,z axes. Rotery inertia effect can be included or excluded with KEYOPT(3). If element has only one mass input, it is assumed that mass acts in all coordinate directions[29, 12].

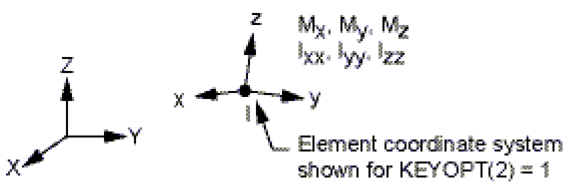

Figure 5.2: MASS21element[29]

5.1.3 COMBI214[12, 29]

It is 2D spring damper bearing element with longitudinal tension and compression capabilities. It is defined by 2 nodes and it has two degrees of freedoms at each node; translational in any two nodal directions. The COMBI214 element has stiffness(K) and damping (C) characteristics that can be defined in straight terms(K_{11},K_{22},C_{11},C_{22}) as well as in cross coupling terms(K_{12},K_{21},C_{12},C_{21}). The geometry of element is shown in below figure[29, 12].

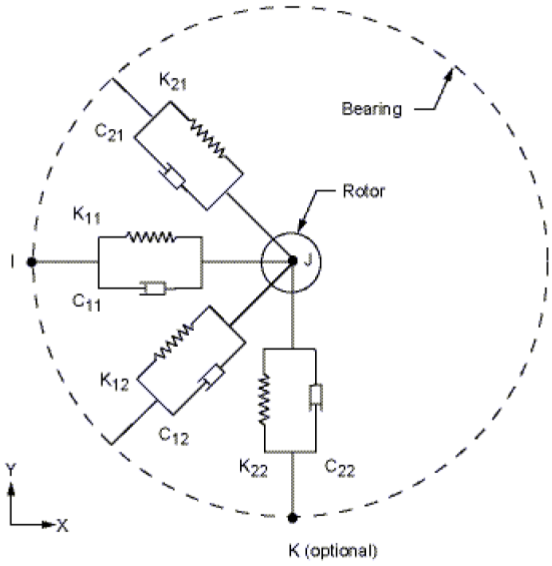

Figure 5.3: COMBI214element[29]

5.2 FE modelling for critical speed analysis

The modelling procedure is given in three sections; modelling of shaft, modelling of impeller and coupling mass and modelling of bearings. This section explains how to decide appropriate keyoption, real constant and beam section through sets of commands in APDL.

5.2.1 FE Modelling of rotor

Shaft segments add only stiffness to the model in this analysis. The rotor is to be modelled with BEAM188 element. The KEYOPT(3)=2 is chosen for that.KEYOPT(3) represents the element behaviour and value 2 denotes that the element is based upon quadratic shape function.

Commands used to defined element and keyoption are:

ET, element type-ID, BEAM188

KEYOPT, element type-ID,3,0 ! linear shape

KEYOPT, element type-ID,3,2 ! Quadratic shape[12].

Beam section consideration is important to resemble the shape of real life rotor system into FE model. VTP system has number of solid cylindrical sections. BEAM188 element is associated with section library which consist of some predefined section shape[29]. Followings are the APDL commands used to defined beam section;

SECTYPE, section-ID,BEAM, CSOLID

SECDATA, R, N, T

TYPE, element type-ID

MAT, element type-ID

where, R=radius of cylinder

N=numbers of divisions around circomference(default=8)

T=numbers of divisions through the radius(default=8)[29].

5.2.2 FE Modelling of impeller and coupling mass

For vibration analysis, impeller and coupling add mass to the model. Impeller do not contribute anything related to the stiffness. Masses are included in the FE model using concentrated point mass. These point masses are modeled using MASS21 element having its rotery inertia option activated. The real constants include masses in x, y, z directions, polar moment of inertia, $I_p(I_{xx})$ and diametral moment of inertia, $I_d(I_{yy}\&I_{zz})$.

Following APDL commands are used to create this element at specified node;

ET, element type-ID, MASS21

KEYOPT, element type-ID, 3,0 ! 3D with rotary inertia

! Real constants

R, real-ID, mass-x, mass-y, mass-z, I_{xx}, I_{yy}, I_{zz}

TYPE, element type-ID

REAL, real-ID

! create element at specified node

EN, element-ID, node number[29].

5.2.3 FE Modelling of bearing

The bearings are modelled as linear isotropic bearing using COMBI214 element. These elements are defined in the plane parallel to XZ plane. So degrees of freedoms of these elements are in UX and UZ directions[29].

Followings are the APDL(ANSYS parametric design language) commands used to model the bearings;

ET, element type-ID, COMBI214

! Define element in XZ plane

KEYOPT, element type-ID, 2, 2

! Real constants

R, real-ID, K_{11},K_{22},K_{12},K_{21},C_{11},C_{22}

TYPE, element type-ID

REAL, real-ID

EN, element-ID, node number[12].

FE model is shown in figure5.4.

Figure 5.4: FE model of VTP system

5.3 Boundary condition

Figure5.5 shows the FE model with boundry conditions. Shaft nodes are constrained in axial and torsional direction and bearing nodes at the base are fixed in all direction. Gravity is applied in negative y direction.

Commands used to apply boundry conditions are;

D,ALL,UY ! NO TRACTION & NO TORSION

D,ALL,ROTY

D,79,ALL

D,80,ALL

D,81,ALL

D,82,ALL

D,83,ALL

D,84,ALL

D,86,ALL

D,88,ALL

D,87,ALL

D,85,ALL

Acel,0,-9.81,0

Alls,all,all

Figure 5.5: FE model of VTP system with boundry condition

5.4 FE modal analysis

The natural frequencies with out spin condition are shown in table5.1. To print the values SET,LIST comand is used.

Mode no.	Frequency(Hz)
1	3.7232
2	16.168
3	17.454
4	36.699
5	37.274
6	40.706
7	45.948
8	46.203

Table 5.1: Frequencies at zero speed

5.4.1 Mode shapes

To plot the mode shapes the following comands are used;

SET, Load step, sub step

PLDISP

Mode shapes for the frequencies in table5.1 are shown below;

Figure 5.6: Mode 1 and 2

Figure 5.7: Mode 3 and 4

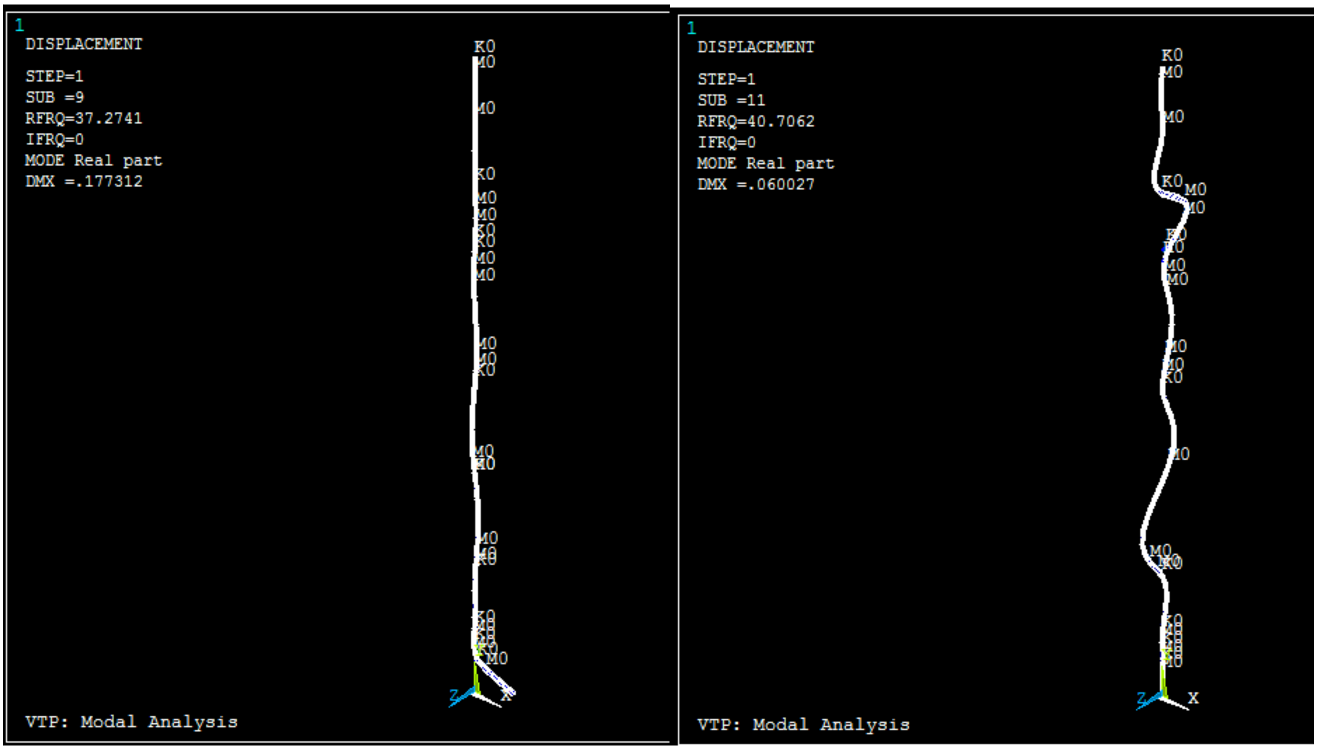

Figure 5.8: Mode 5 and 6

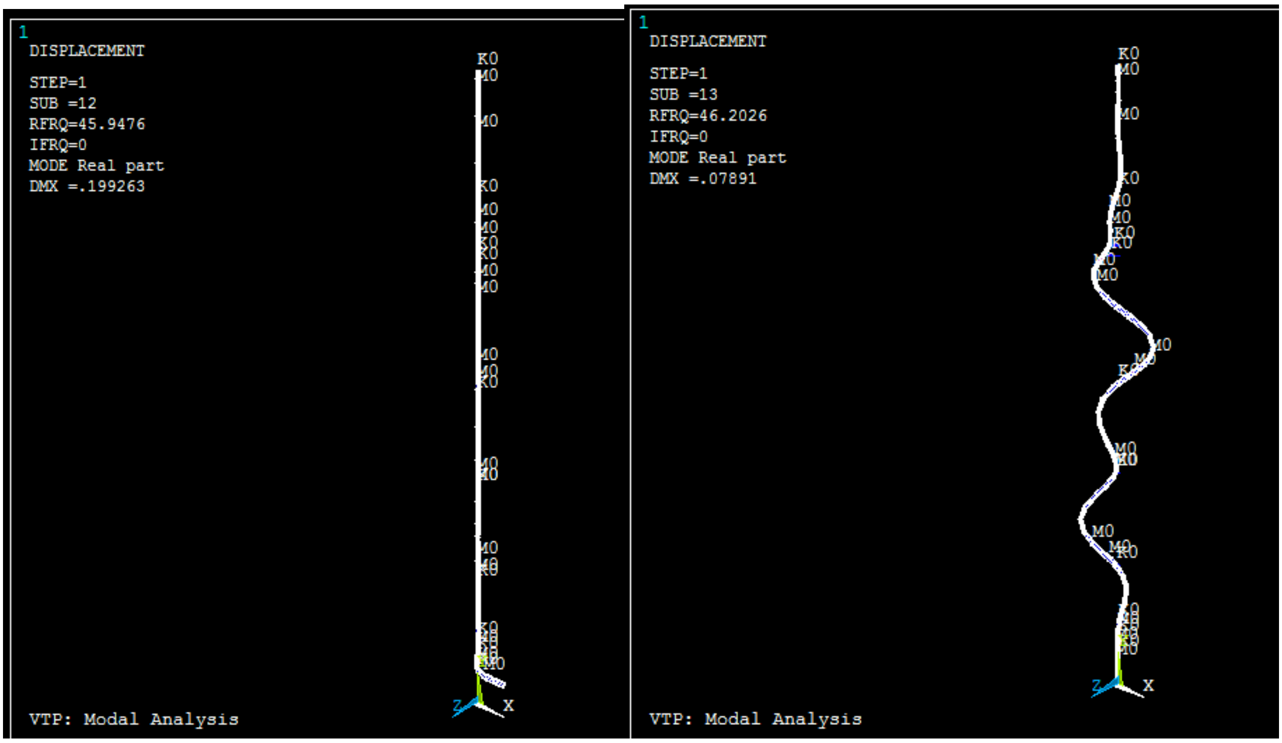

Figure 5.9: Mode 7 and 8

5.5 Critical speed analysis

Critical speed analysis of VTP is performed with spin range of 0 to 2000 rpm with increment of 50 rpm. Since VTP operating speed is 960 rpm, here frequencies and mode shapes are shown at 950 rpm.

5.5.1 Frequencies at spin 950 rpm

The frequencies at spin 950 rpm obtained from ANSYS are shown in table5.2

Mode No.	Frequencies(Hz)	Whirl
1	3.6516	BW
2	3.7754	FW
3	7.8463	BW
4	17.348	BW
5	17.431	FW
6	33.267	BW
7	35.941	BW
8	36.798	FW
9	37.248	BW

Table 5.2: Frequencies at 950 rpm

5.5.2 Mode shapes

Mode shapes for first 6 frequencies in table5.2 are shown below;

Figure 5.10: Mode 1 and 2

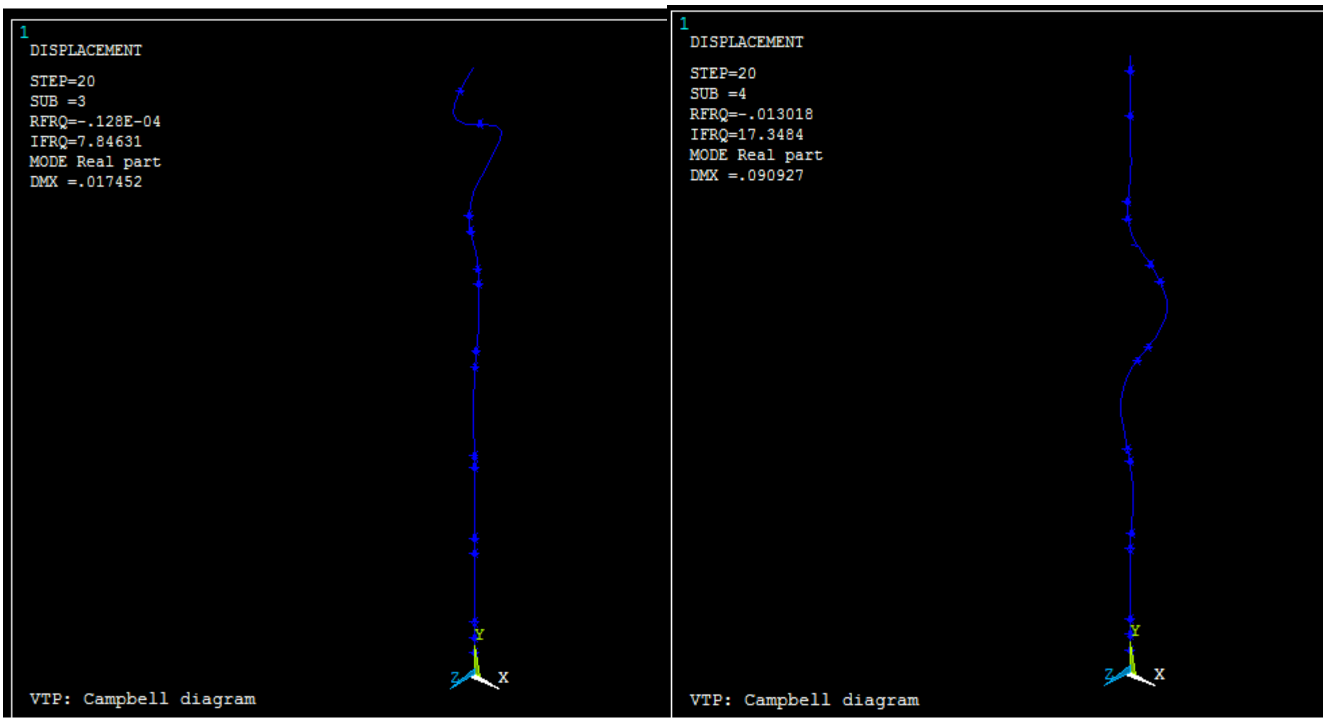

Figure 5.11: Mode 3 and 4

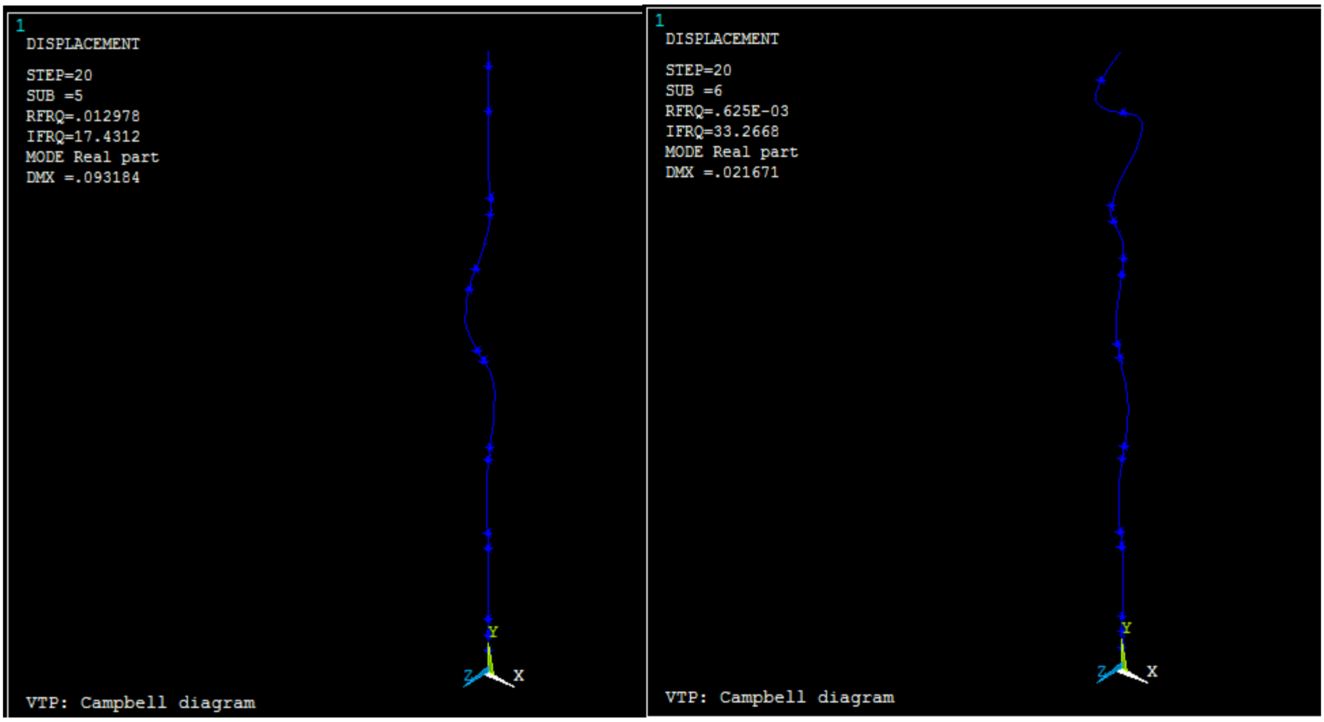

Figure 5.12: Mode 5 and 6

5.5.3 Campbell diagram identification

As mentioned in literature, the campbell diagram is graphical representation of natural frequency versus excitation frequencies as a function of rotational speed.

In the Campbell diagram, Whirl frequencies(Hz) are plotted along Y axis and rotational speeds(rpm) are plotted along the X axis. Upward slopping lines are excitation line. Intersection of frequency line and upward slopping line gives critical speed. Campbell diagram check the stability and whirl of each mode.

The variation of eigen frequencies between 0 rpm to 2000 rpm corrosponding to different rotational velocity are plotted in campbell diagram. The campbell diagram for VTP system obtained from the ANSYS is shown in figure5.13. Critical speeds are determined for the excitation slope 1 that means excitation due to unbalance. Comand used to plot the diagram is;

/POST1

PLCAMP,0,1,RPM,0,ROT_PART,0

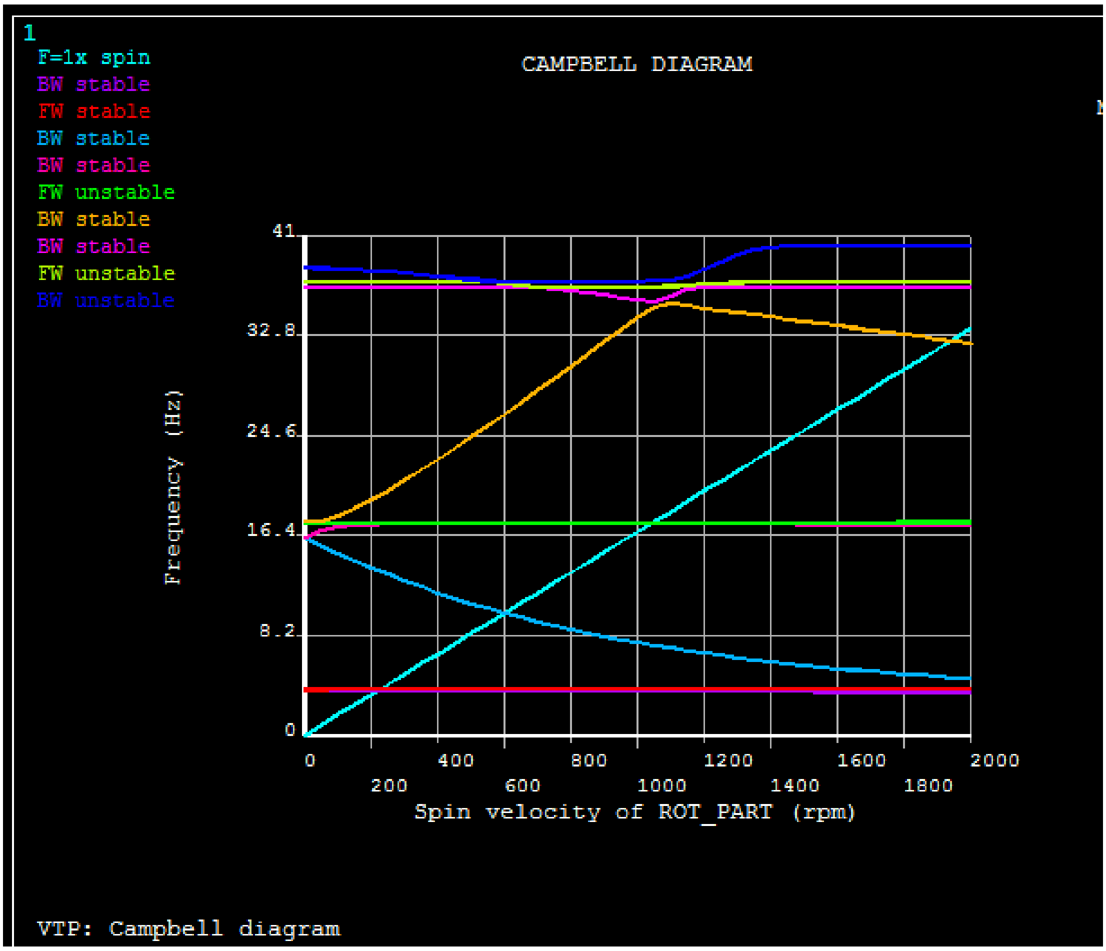

Figure 5.13: Campbell diagram for VTP system

Critical speeds obtained from ANSYS between 0rpm to 2000rpm are shown in table5.3. Comand used to print critical speeds is;

PRCAMP,0,1,RPM,0,ROT_PART,0

Sr.No.	Critical speed(rpm)
1	222.540
2	224.192
3	600.543
4	1040.720
5	1046.081
6	1942.942

Table 5.3: Critical speeds for VTP system

5.5.4 Orbital motion of shaft

Whirl orbit for frequencies shown in table5.2 are shown below in figure5.14 to 5.16.

To display the orbit of each rotating node as well as deformed shape at time t=0(real part of solution), PLORB command is used. The PRORB command prints out the orbit characteristics semi major axis(A), semi minor axis(B), PSI(angle between local Y axis and major axis), PHI(angle between initial position{t=0} and major axis), YMAX and ZMAX(maximum displacement along Y and Z axis respectively). This commands PLORB and PRORB gives the node movemennt and orbit characteristics that are necessory for various analysis. For example, Harmonic response analysis we can visualize the maximum amplitude location by plotting and printing orbits. Also in case of multipool system, we can visualize orbital motion and orbit characteristics of each pool separately.

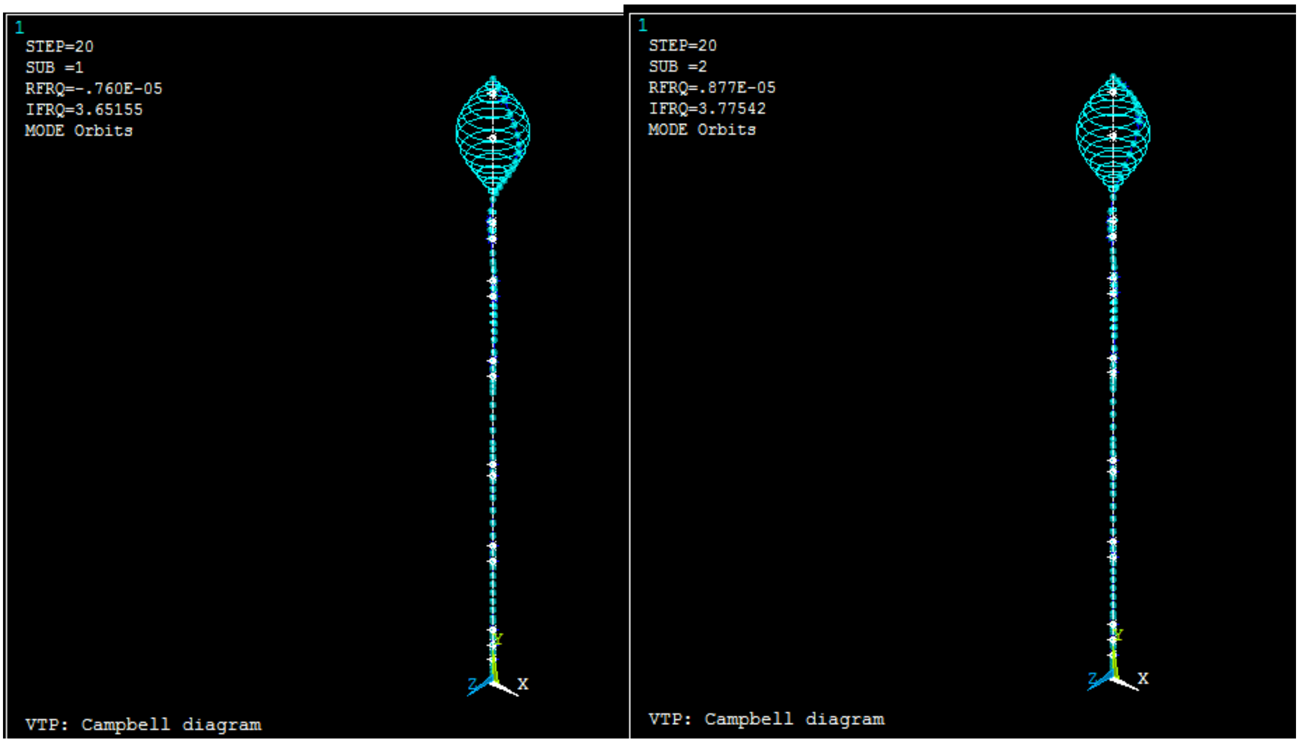

Figure 5.14: Whirl orbit for 3.6516Hz & 3.7754Hz

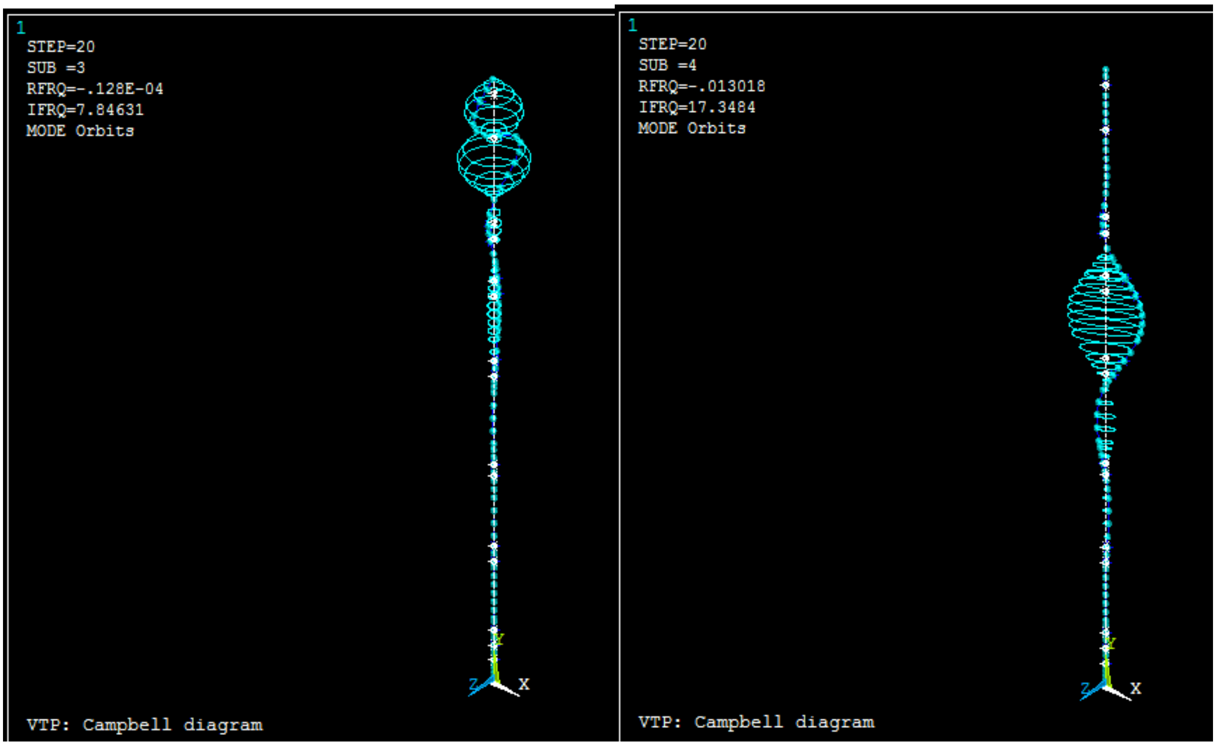

Figure 5.15: Whirl orbit for 7.8463Hz & 17.348Hz

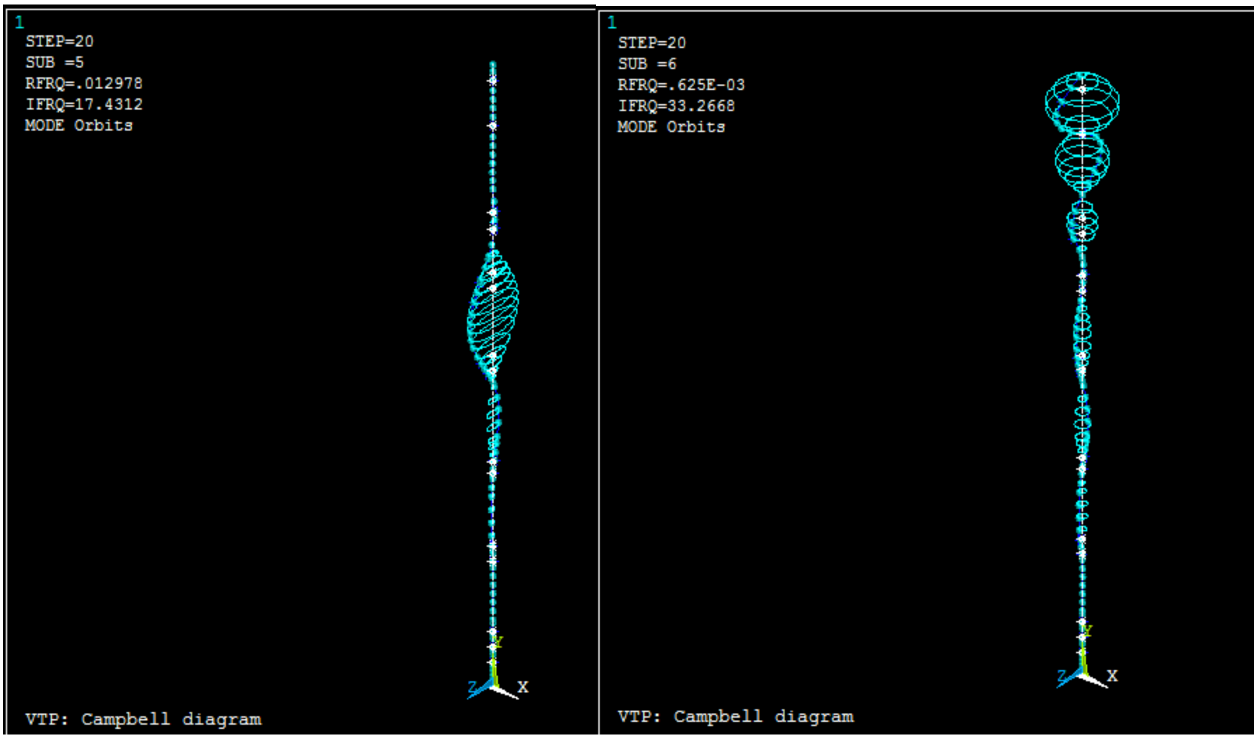

Figure 5.16: Whirl orbit for 17.431Hz & 33.267Hz

5.6 Natural frequencies by analytical holzer's method

With the help of C program shown in appendix, the natural frequencies at zero spin by holzer's method are shown below table5.4.

Program gives values in rad/sec.

1 Hz = 6.2831853 rad/sec.

Algorithm for Holzer,s method is shown in figure5.18.

Sr.No.	Freq.(rad/sec)	Freq.(Hz)
1	23.970980	3.8151
2	99.418841	15.823
3	108.001672	17..189
4	232.597236	37.019
5	236.83210	37.693
6	272.01794	43.293
7	292.31891	46.524
8	384.744568	61.234

Table 5.4: Natural frequancies of VTP system by Holzer's method

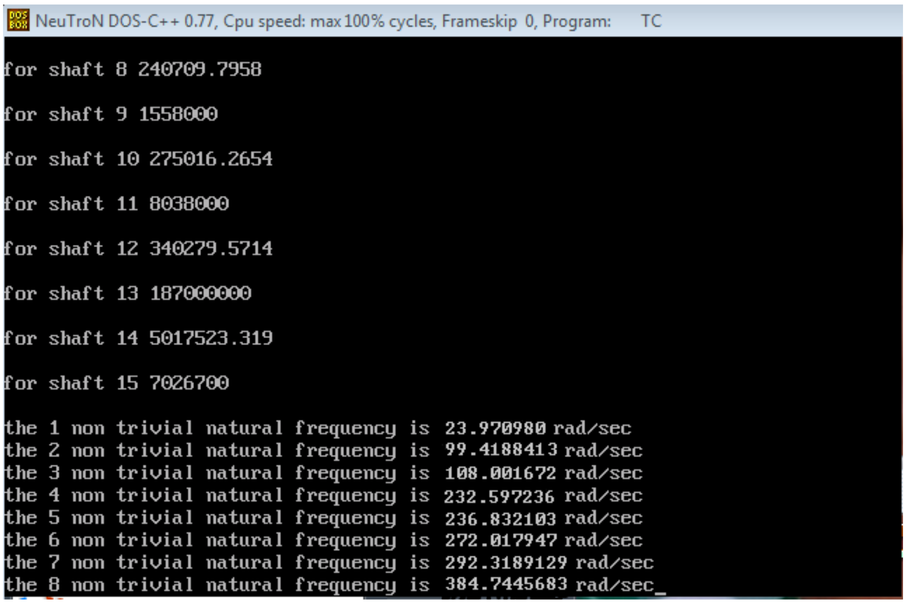

Figure 5.17: Result of Holzer's method

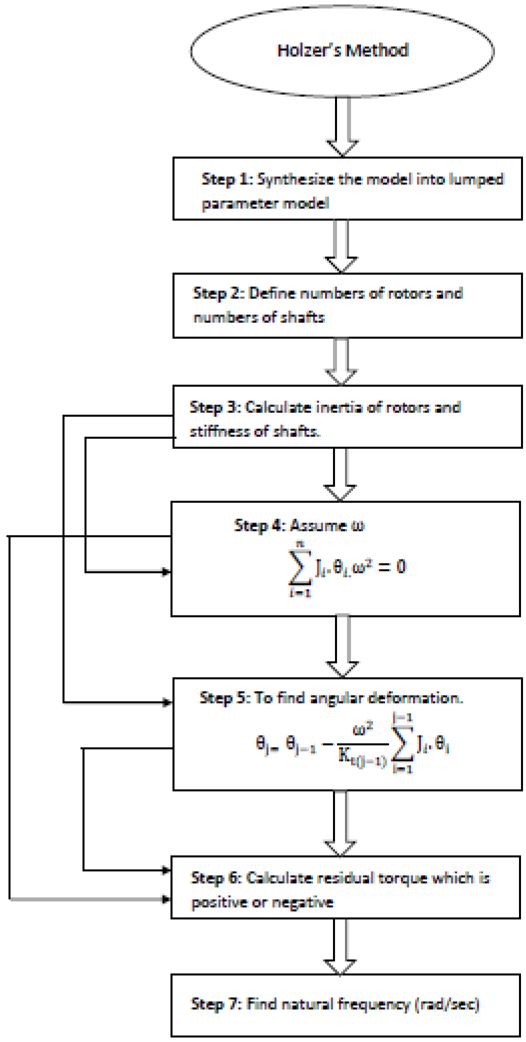

Figure 5.18: Algorithm for Holzer's method

5.7 Results and comparison

Hence, Critical speeds of VTP are obtained by plotting Campbell diagram in ANSYS. The result of modal analysis by FE & analytical Holzer's method are shown in table5.5. Both are close to each other. Also frequencies at rotating condition(950 rpm) are shown in same table with nature of whirl that is forward whirling(FW) and backward whirling(BW).

Sr.No.	FEA(modal,Hz)	Analytical(Hz)	FEA(950 rpm,Hz)	Critical Speed(rpm)
1	3.7232	3.8151	3.6516 (BW)	222.540
2	16.168	15.823	3.7754 (FW)	224.192
3	17.454	17.189	7.8463 (BW)	600.543
4	36.699	37.019	17.348 (BW)	1040.720
5	37.274	37.693	17.431 (FW)	1046.081
6	40.706	43.293	33.267 (BW)	1942.942
7	45.948	46.524	35.941 (BW)	-
8	46.203	61.234	36.798 (FW)	-
9	53.413	-	37.248 (BW)	-
10	60.495	-	-	-

Table 5.5: Results of vibration analysis of VTP system

The first critical speed(222.540 rpm) is close to fundamental natural frequency(3.7232Hz). The percentage of margin between any natural frequency at rotating condition(950 rpm) and operating speed(950 rpm) is not less than $\pm 10\%$. So all frequencies are within the acceptable region as per API610 guidelines.

Figure5.19 shows bar chart for comparison of modal analysis by FE and analytical Holzer's method.

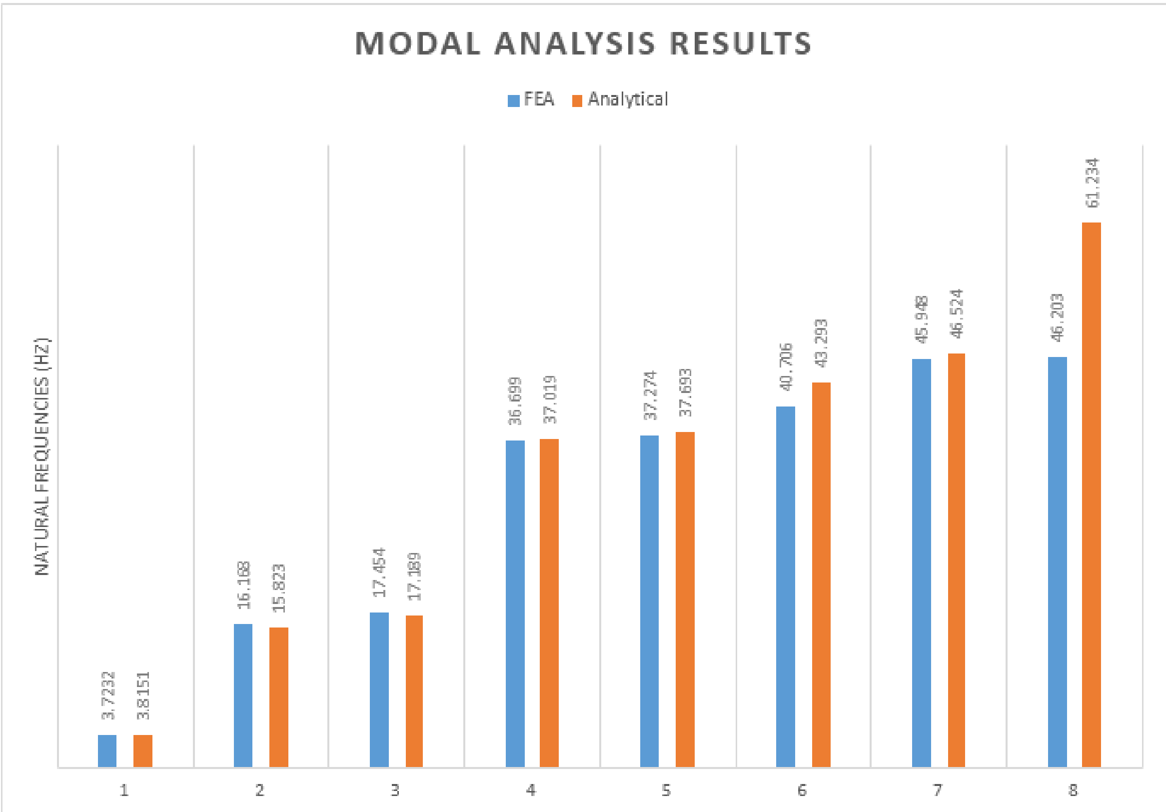

Figure 5.19: Bar chart for comparison of result-Modal Analysis

Chapter 6

Conclusion and Future Scope

6.1 Conclusion

This dissertation aimed for performing vibration analysis of vertical turbine pump operating at 960 rpm. The finite element methodology is used to predict critical operating speeds for VTP. The bench mark study using single vertical shaft rotor assembly has been performed using FEA to establish methodology. The effect of gravity load on natural frquencies are also been studied and found that gravity has no profound effect on results. The Campbell diagram is obtained and critical speeds for bench mark case has been predicted. The vibration study of VTP has been carried out using lumped parameter formulation. The complex VTP assembly has been resolved into various lumped parameter either stiffness or mass moment of inertia. The methodology obtained through bench mark study has been applied to lumped VTP model. The Campbell diagram is obtained through which critical operating speed with various associated parameters like whirling direction, orbital motion etc. are obtained. The critical operating speeds are found not to lie nearer to operational speed. Hence, the design will remain safe during operation.

Also the natural frequencies of VTP lumped model has been obtained analytically using Holzer method and it is found to be in close confirmation with FEA results.

6.2 Future scope

For evaluating the support bearing formulation for FEA, damping can also be considered which may alter the performance. Moreover for critical cases where in the operating speed falls in the $\pm 10\%$ range of any of critical speed, the various response analysis like harmonic, transient etc. can be performed to evaluate the performance.

Appendix

C Program for Holzer Method

```
#include<stdio.h>
#include<conio.h>
void main()
{
float sum,w,t,tprev;
int n,i,l,j,x;
float I[100],k[100],theta[100];
theta[0]=k[0]=I[0]=0;
//int i=0;
clrscr();
printf("please enter the number of rotors attached to the shaft(min 2) ");
scanf("%d",&n);
printf("\nenter the inertia for each rotor starting from the free end ");
for(i=1;i<=n;i++)
{
printf("\nfor rotor %d ",i," ");
scanf("%f",&I[i]);
}
printf("\nenter the torsional stiffness for the intermediate shafts starting from the free end ");
i=1;
for(i=1;i<=n-1;i++)
```

```
{
printf("\nfor shaft %d ",i," ");
scanf("%f",&k[i]);
}
w = 0.0001;
theta[1]=1;
l=1;
for(l=1;l<n;l++)
{
int flag=1;int slag=1;
while(flag!=0)//t=scalar sum of torque on individual rotors,for equilibrium it should tend
to zero
{
t=0; i=1;
for(i=1;i<=n-1;i++)
{
sum=0; j=1;
for(j=1;j<=i;j++)
{
sum+=I[j]*theta[j];
}
theta[i+1]=theta[i]-((w*w)/k[i]*sum);
}
x=1;
for(x=1;x<=n;x++)
{
t+=I[x]*theta[x]*w*w;
}
w=w+0.0001;
if(slag==1)
```

```
{ tprev = t; slag = 0;
}
else
{
if((t/tprev)<0)
{ flag=0;
}
tprev=t;
}//end of else
}//end of while
printf("\nthe %d non trivial natural frequency is %f rad/sec",l,(w-0.0001));
}//end of outer If
getch();
}//end of program
```

Bibliography

[1] Mark A. Corbo & Stanley B. Malanoski, Proceeding of the twenty fifth turbo machinery symposium, Practical design against torsional vibration

[2] Martin Gula, Peter Hudk,Frantisek Trebuna, Peter Frankovsky. Numerically computed dynamics rotor using ansys software. Modeling of Mechanical and Mechatronics System,the 4th International conference, 2011.

[3] Ramana podugu, b.V. Ramana murthy, International Journal of Engineering Science and Technology (IJEST), A Model approach for vibration and condition monitoring of centrifugal pump.

[4] Naveena M & Dr. Suresh P M, International Journal of Innovative Research in Science, Engineering and Technology Vol.2, August 2013, Lateral critical speed analysis of multistage centrifugal pump using FEA.

[5] M.Chouksey, J.k. Dutt, Mechanism and Machine Theory 48 (2012) Elsevier, Modal analysis of rotor-shaft system under the influence of rotor-shaft material damping and fluid film forces.

[6] Erik Swanson, Chris D. Powell and Sorin Weissman, A Practical Review of Rotating Machinery Critical Speeds and Modes, Sound and Vibration/ May 2005.

[7] William D. Marscher, Proceeding of the Twenty -Seventh International Pump Users Symposium, An end-user's guide to centrifugal pump rotor dynamics.

[8] M.H. Sadeghi, S. Jafari & B. Nasseroleslami, IUST International Journal of Engineering Science Vol.19, 2008, Modal analysis of turbo pump shaft.

[9] Kenneth E. Atkins, James D. Tison & J.C. Wachel, Proceeding of the second international pump symposium, Critical speed analysis of an eight - stage centrifugal pump.

[10] Nagaraju Tenali and Srinivas Kadivendi, International Journal of Mechanical Engineering And Robotics Research,Vol.3, January 2014, Rotor Dynamic Analysis of Steam Turbine Rotor Using ANSYS.

[11] Ion NILA, Radu BOGATEANU, Marcel STERE, Daniela BARAN, INCAS BULLETIN, Volume 4, Issue 2/ 2012, Modal analysis of small vertical axis wind turbine.

[12] Rotor Dynamic Analysis Guide, ANSYS.

[13] Singiresu S. Rao, Mechanical Vibrations, Fifth edition, Pearson.

[14] Fred. R. Szenasi J. C. (Buddy) Wachel. Analysis of torsinal vibration in Rotating Machinery.

[15] http://www.wisegeek.org/what-is-a-turbine-pump.htm

[16] http://brownbros.co.nz

[17] Shibing Liu, Bingen Yang, Proceeding of the Twenty-Ninth International Pump Users Symposium, October 1-3, 2013, Texas, Vertically Suspenden Pumps With Water Lubricated Rubber Bearings- Experimental Identification of Dynamic Stiffness Coefficients.

[18] Uugur Yucel, Journal of Engineering Sciences 2005 11(3) 335-343, Calculation of Dynamic Coefficients for Fluid Film Journal Bearins.

[19] R.Tiwari, Chapter 3, January 2010, IIT Guwahati, Rotor Dynamic Parameters of Bearings, Seals and Dampers.

[20] M Varun Kumar, B Ashiwini Kumar, International Journal of Engineering and Advanced Technology(IJEAT) Volume-2, April 2013, Modal Analysis of Axially Symmetric Linear Rotating Structures.

[21] Marco Tulio C.Faria, Proceeding of the World Congress on Engineering 2014 Vol-2, On The Hydrodynamic Long Journal Bearing Theory.

[22] Prof. Rajiv Tiwari, Module 7, Instability in Rotor System: Bearings, IIT Guwahati, Theory and Practice of Rotor Dynamics.

[23] Malcolm E. Leader, Senior Engineer, Central Engineering Division, Monsanto Fibers and Intermediates Company, Texas city, Texas, Introduction to Rotor Dynamics of Pumps Without Fluid Forces.

[24] J.S.Rao, Rotor Dynamics, Third Revised Edition, New Age International(P) Ltd.

[25] V.B.Bhandari, Design of Machine Elements, Third Edition, McGraw Hill Education(India) Private Limited.

[26] www.skf.com

[27] http://www.schaeffler.co.in/content.schaeffler.co.in/en/products_services/inafagproducts

[28] C.E. Agostini, E.A.Capello Sousa, 15th international conference on experimental mechanics, Complex Modal Analysis of a Slender Vertical Rotor by Finite Elements Method.

[29] ANSYS help documentation, Version ANSYS 14.5.