Static Finite Element Analysis of Piezoelectric Laminated Composite Plates

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NIRMA UNIVERSITY

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Major Project

Submitted in Partial Fulfillment of the Requirements

For the degree of

Master of Technology in Civil Engineering

(Computer Aided Structural Analysis & Design)

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Declaration

This is to certify that

a. The major project comprises my original work towards the Degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis and Design) at Nirma University and has not been submitted elsewhere for a degree.

b. Due acknowledgement has been made in text to all other material used.

Akash L. Jivani

Certificate

This is to certify that Major Project entitled **"Static and Dynamic Finite element Analysis of Piezoelectric Laminated composite Plate"** submitted by **Akash L. Jivani (13MCLC27)**, towards the partial fulfillment of the requirements for the degree of Master of Technology in Civil Engineering (Computer Aided Structural Analysis & Design) of Nirma University of Science and Technology, Ahmedabad is the Record of work carried out by him under my supervision and guidance. In my opinion, the submitted work has reached a level required for being accepted for examination. The results embodied in this major project, to the best of my knowledge, haven't been submitted to any other university or institution for award of any degree or diploma.

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Abstract

Piezoelectric Laminated Composite Plates are used to construct intelligent structures. They consist of composite laminates embedded with layers of Piezo-electric materials.

The present study covers the analysis of laminated composite plates embedded with plates of piezo-electric materials, using Finite Element Analysis. Specially, the study includes the behaviour of plate under transverse normal stresses and strains, and electric field.

A displacement model with two mechanical degrees of freedom and one electric degree of freedom (i.e. voltage) is considered. Finite element formulation is derived using 8-node isoperametric (serendipity) element based on displacement function. Study of electromechanical coupling effect is done from direct piezo-electric effect (sensing action) and converse piezo-electric effect (actuation action).

Results of plate deflection and internal stresses of several examples are compared with the results reported in available literatures. Computer programs are developed for automatic mesh generation of Piezo-electric Laminated Composite plate for static deflection and interlaminar stresses under uniform transverse loads. The program supports any number of elements, geometry, support conditions, mechanical loading and electrical loading.

In addition, the analysis of piezolaminated plates is done in Finite Element Analysis software to enhance the knowledge of software applications on Piezo-electric Laminated Composite Plates.

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I would like to thank my guide **Dr. Paresh V. Patel**, whose keen interest and knowledge base helped me to carry out the major project work. His constant support and guidance during my project work equipped me with a great understanding of different aspects of the project work. He has shown keen interest in this work right from beginning and has been a great motivating factor in outlining the flow of my work.

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-Akash L. Jivani

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Abbreviation, Notation and Nomenclature

x,y,z .... Global coordinate system

$1, 2, 3$ .... Lamina reference coordinate system

E_L, E_T ...... Longitudinal modulus of elasticity of lamina in the direction of Fibre

E_1, E_2, E_3 Transverse modulus of elasticity of lamina in the cross direction of Fibre

$\sigma_1, \sigma_2, \sigma_3$ Normal stresses at any point on the laminate with reference to the lamina axes

$\sigma_x, \sigma_y, \sigma_z$ Normal stresses at any point on the laminate with reference to the global axes

$\epsilon_1, \epsilon_2, \epsilon_3$ .. Normal strains at any point on the laminate with reference to the lamina axes

$\epsilon_x, \epsilon_y, \epsilon_z$ .. Normal strains at any point on the laminate with reference to the global axes

ν_{ij} .. Poissons ratio giving the strain in the jth direction caused by strain in the ith direction

G_{12}, G_{23}, G_{13} .... Shear modulus of lamina

$\tau_{12}, \tau_{23}, \tau_{13}$ ...... Shear stresses at any point on the laminate with references to the lamina axes

$\tau_{xy}, \tau_{yz}, \tau_{xz}$ Shear stresses at any point on the laminate with references to the global axes

$\gamma_{12}, \gamma_{23}, \gamma_{13}$ Shear strains at any point on the laminate with references to the lamina axes

$\gamma_{xy}, \gamma_{yz}, \gamma_{xz}$ ...... Shear strains at any point on the laminate with references to the lamina axes

S_{ij}, Q_{ij} ... Component of the matrix represents the relationship between strain and stress

$[T]$ .... Transformation matrix for coordinate system

$[R]$ .... Represent the relationship between engineering and tensor strain vector u,v,w .... Displacements of a point along x, y and z directions

$[\mu]$ .... Dielectric Constant matrix

$[J]$ .... Jacobian matrix

W_a, W_b .... Weighting factors corresponding to Gauss sampling points

$[C]$ .... Constitutive law model for a mechanical stress-strain

d_{ij} .... Electromechanical coupling coefficient

$[e]$ .... Piezo-electric strain constant matrix

E .... Vector of Applied Electric field

N_i .... Shape function at i node

NN .... Number of nodes

NG .... Number of Gauss points

NL .... Number of layers in laminate

ζ, η .... Natural coordinate system

$[K_{uu}]$ .... Stiffness matrix corresponding to the mechanical degrees of freedom

$[K_{uE}]$ .... Stiffness matrix due to electromechanical coupling

$[K_{EE}]$ .... Stiffness matrix due to electrical degrees of freedom

F_e .... Element load vector

q_e .... Element charge vector

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Chapter 1

Introduction

1.1 General

The word composite in composite material signifies that two or more materials are combined on a macroscopic scale to form a useful material and the individual materials are easily distinguishable. The first high performance composite material is as old as man as himself, for it is the human body: the bones, muscle tissues that are multidirectional fibrous laminate etc.

Current developments are pointed towards combination of unusually strong, high modulus fibers and organic, ceramic, or metal matrices. Such materials promise to be far more efficibent than any structural materials known previously.[7]

The advantage of composites is that they usually exhibit the best qualities of their constituents and often some qualities that neither constituent possesses. The properties that can be improved by forming a composite material include:

- Strength
- Stiffness

- Corrosion resistance
- Wear resistance
- Attractiveness
- Weight
- Fatigue
- Temperature dependent behaviour
- Thermal insulation
- Acoustical insulation

Naturally, not all of the above properties are improved at the same time, not usually there is any requirement to do so.

Plates made up of isotropic or orthotropic laminae are widely used in a variety of structures and machines. A multiphase or two material laminae consists of a stiff filament material embedded in a compatible matrix material. Examples of filaments are glass, boron, carbon, graphite and steel, whereas matrix materials have included phenotics, polyesters, aluminum, and epoxies.

However, unlike isotropic materials, composite materials exhibit low out-of-plane moduli relative to in-plane moduli. This may result in quite considerable transverse shear deformations and transverse normal strains, which can significantly influence the response and the failure mechanisms of laminated anisotropic composite plate.

An accurate analysis of these fiber reinforced composite material components is often necessary for the safe, efficient and optimum design for specified structures.

1.2 Composite Laminate

1.2.1 History

Composite materials have very long history of usage. Their beginnings are unknown, but all recorded history contains references to some form of composite material. "Although the concept of fiber reinforced materials can be traced back to the use of straw as reinforcement in bricks manufactured by the Israelites in 800 B.C., and in more recent times to the use of short glass fiber reinforcement in cement in the United States in the early 1930's, fiber reinforced resin matrix materials (or fiber reinforced composites as we know them today) were not developed until the early 1940's".

After World War II, US manufacturers began producing fiberglass and polyester resin composite boat hulls and radomes (radar cover). The automotive industry first introduced composites into vehicle bodies in the early 1950's. Because of the highly desirable lightweight, corrosion resistance, and high strength characteristics in composites; research emphasis went into improving the material science and manufacturing process. That effort led to the development of two new manufacturing techniques known as filament winding and pultrusion, which helped advance the composite technology into new markets. There was a great demand by the recreation industry for composite fishing rods, tennis rackets, ski equipment and golf clubs. The aerospace industry began to use composites in pressure vessels, containers, and non-structural aircraft components. The US Navy applied composites in mine sweeping vessels, crew boats and submarine parts. The domestic consumers began installing composite bathtubs, covers, railings, ladders and electrical equipment. The first civil application in composites was a dome structure built in Benghazi in 1968, and other structures followed slowly[26].

1.2.2 Civil Structural Application

- Composite materials have more environmental resistance than traditional civil engineering materials such as steel, concrete, masonry, and plaster. Degradation in strength and stiffness for steel structures due to the corrosion problem requires frequent inspection, maintenance and repair. Similarly, stress cracking due to the warm/cold weathering limits the service life of concrete structures. Timber is susceptible to moisture-swelling problems and paste attack[7].

- Most important of all, these traditional construction materials are relatively inefficient in earthquake and fatigue resistance. Composite materials are as stiff as steel, but weigh approximately 80% less and have stiffness-to-weight ratio higher $approx.5times$ than that of steel and so less susceptible to be in resonance with the ground motion of an earthquake. For this reason alone, composite materials structures are safer and can minimize property and life loss induced by earthquake.

- Currently, composite materials are being used to retrofit andor reinforce existing infrastructures.

- Flat composite laminates have been bonded to the exterior surface of reinforced concrete deck to increase its bending stiffness.

- Composite materials are suitable for construction of highway bridges, power transmission towers, office/residential buildings, retaining walls, etc.

1.2.3 What is Lamina?

A lamina is a flat (sometimes curved in a shell) arrangement of unidirectional fibers as shown in Fig.1.1 in matrix. The fibers are the principal reinforcing or load-carrying agent. They are typically strong and stiff. The matrix can be organic, ceramic, or metallic. The function of matrix is to support and protect the fibers and to provide a means of distributing load among and transmitting load between the fibers.

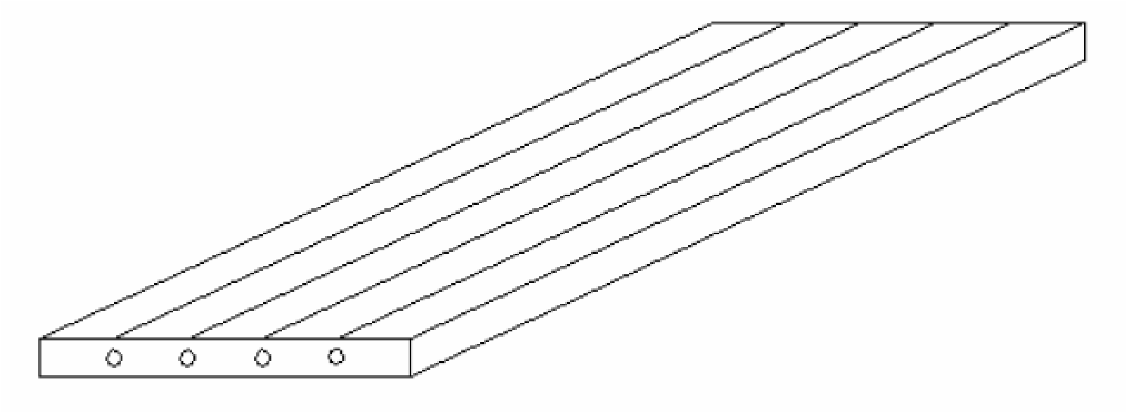


Figure 1.1: Lamina with Unidirectional Fibers

1.3 What is Laminate?

A laminate is a stack of laminae with various orientations of principal material directions in the laminae as in Fig.1.2. The layers of a laminate are usually bound together by the same matrix material that is used in the laminae. Laminate can be composed of plates of different materials or in the present context, layers of fiber-reinforced laminae. A laminated circular cylindrical shell can be constructed by winding resin-coated fibers on a mandrel first with one orientation to the shell axis then another, and so on until the desired thickness is built up.

A major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element. Laminates are uniquely suited to this objective since the principal material directions of each layer can be oriented according to need.

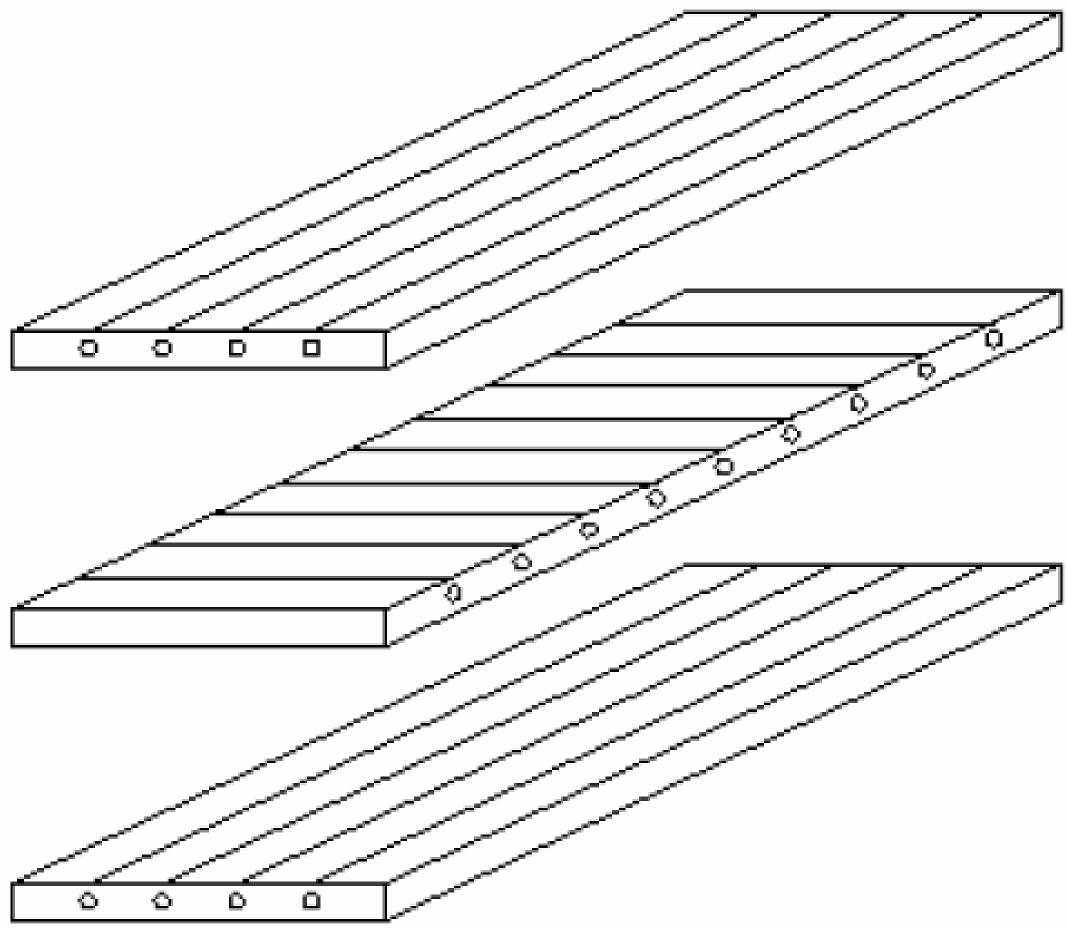


Figure 1.2: A 3=ply Laminate construction

A potential problem in the construction of laminates is the introduction of shearing stresses between layers. The shearing stresses arise due to the tendency of each layer to deform independently of its neighbours because all may have different properties (at least from standpoint of orientation of principal material directions). Such shearing stresses are largest at the edges of a laminate and may cause delamination there.

1.3.1 Characteristics of Composite

The mechanical properties of composites depend on many variables such as fiber types, orientations and architecture. The fiber architecture refers to the preformed textile configurations by braiding, knitting or weaving. Composites are anisotropic materials with their strength being different in any direction. Their stress-strain curves are linearly elastic to the point of failure by rupture. The polymeric resin in a composite material which consists of viscous fluid and elastic solids, responds viscoelastically to applied loads. Although the viscoelastic material will creep and relax under a sustained load, it can be designed to perform satisfactorily. Composites have many excellent structural qualities and some examples are high strength, material toughness, fatigue endurance and lightweight. Other highly desirable qualities are high resistance to elevated temperature, abrasion, corrosion and chemical attack.

Some of the advantages in the use of composite structural members include the ease of manufacturing, fabrication, handling, and erection. Project delivery time can be short. It took the Russell county engineer one day to install the deck panels in the first vehicular composite bridge. Composites can be formulated and designed for high performance, durability and extended service life. They have excellent strength-toweight ratios. If durability can be proven to last 75 years, composites can be economically justified using the life - cycle cost method.

Some of the disadvantages in the use of composites in bridges are high first cost, creep, and shrinkage. The design and construction require highly trained specialists from many engineering and material science disciplines. The composites have a potential for environmental degradation, for examples, alkalis' attack and ultraviolet radiation exposure. There are very little or nonexistent design guidance and/or standards. There is a lack of joining and/or fastening technology. Because of the use of thin sections, there are concerns in global and local buckling. Although the lightweight feature may be an advantage in the response to earthquake loading, it could render the structure aerodynamically unstable. In manufacturing with the hand lay-up process, there is a concern about the consistency of the material properties.

1.3.2 Manufacturing Process

There are three basic manufacturing techniques in producing composite structural products with many variations and patented processes:

a. The **Pultrusion** process involves a continuous pulling of the fiber roving and mats through a resin bath and then into a heated die. The elevated temperature inside the die cures the composite matrix into a constant cross-section structural shape.

b. The **Filament winding** process can be automated to wrap resin-wetted fibers around a mandrel to produce circular or polygonal shapes.

c. The **Lay-up** process engages a hand or machine buildup of mats of fibers that are held together permanently by a resin system. This method enables numerous layers of different fiber orientations to be built up to a desired sheet thickness and product shape.

1.4 Laminated Composite with Piezoelectric Material

1.4.1 Introduction

Piezoïs the Greek word for Pressure. When any electric voltage is applied to certain materials they experience a dimensional change; such materials are known as

P̈iezoelectric Material: Because of converse effect, they generate electricity when pressure is applied. The Piezoelectric effect was discovered in 1880 by Curie brothers Pierre Curie and Jacques Curie.
The nature of piezoelectric material is closely linked to the significant quantity of electrical dipoles within these materials.

Dipole: A dipole is a separation of positive and negative charges. The simplest example of this is a pair of electric charges of equal magnitude but opposite sign.

The centers of the negative and positive charges of the each molecule co-inside. The external effects of the charges are reciprocally canceled. As a result, an electrically neutral molecule appears as shown in Fig.1.3. After exerting some pressure on the material, the internal structure is deformed, that causes the separation of the positive and negative centers of the molecules. As a result, little dipoles are generated as shown in Fig.1.3.

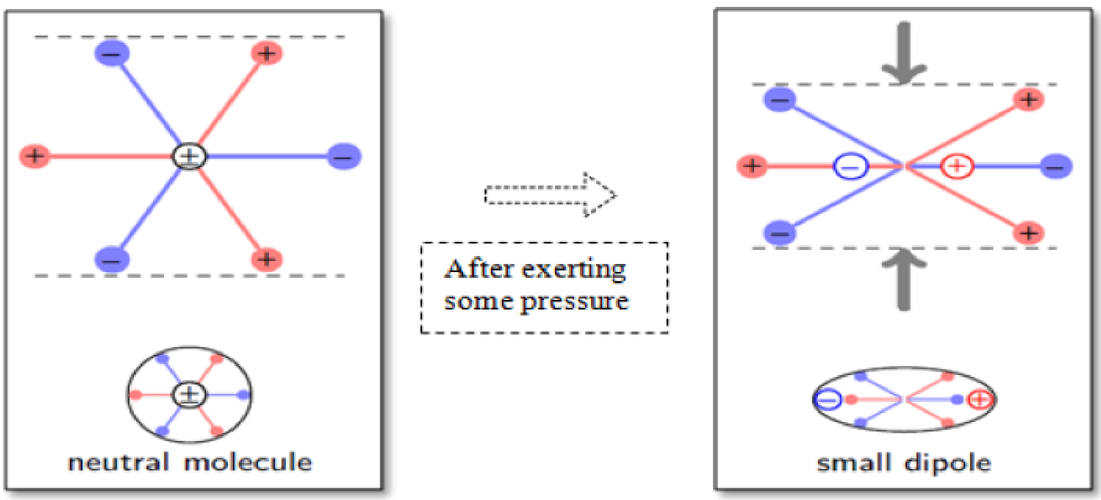


Figure 1.3: Diapole Effect

A dipole is a vector with direction and value in accordance to electric charge around. These dipoles are generally randomly oriented, and they altogether form regions called Weiss domains as shown in Fig.1.4.

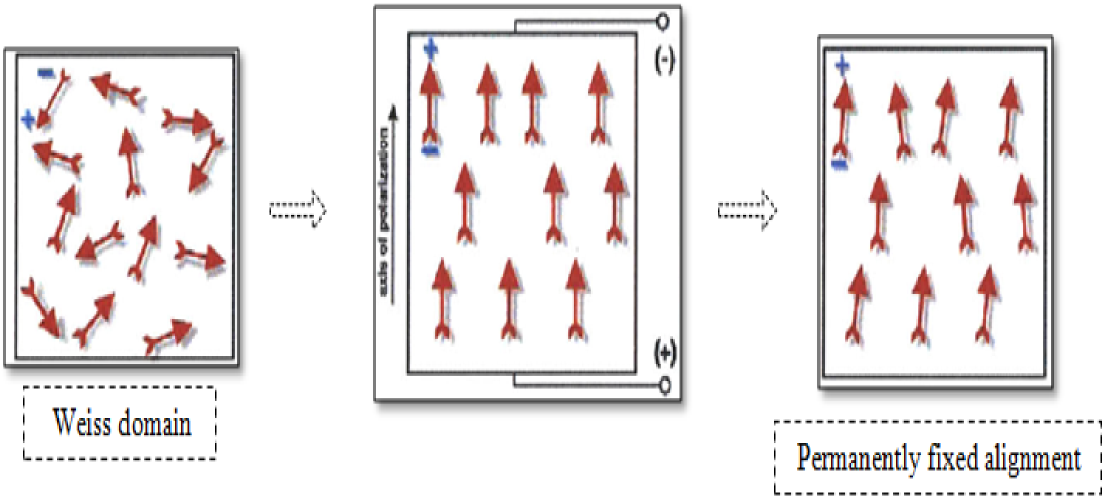


Figure 1.4: Response of molecules under electric field

A piezoelectric material has a characteristic Curie temperature. When it is heated above this temperature, the dipoles can change their orientation in the solid phase material. Then applying strong electric field, the dipoles shift into alignment with the direction of this field. Now the alignment of dipoles is permanently fixed as shown in Fig.1.4.

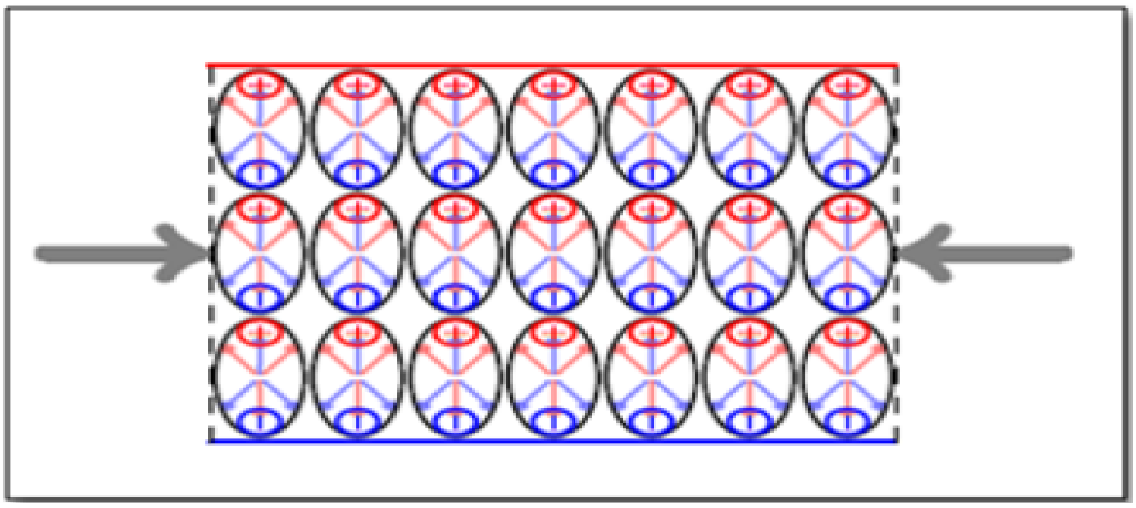


Figure 1.5: Converse effect

Converse effect: By applying external pressure, the polarization generates an electric field and can be used to transform the mechanical energy of the material's deformation into electrical energy.

1.4.2 Piezoelectric Materials

The most known material is Quartz (SiO2), but there are other piezoelectric materials such as[27],

- Lead Zirconate Titanate PZT
- Polyvinylidene Fluoride $PVDF$
- Berlinite $AlPO4$
- Gallium orthophosphate $GaPO4$
- Tourmaline
- Barium Titanate $BaTiO3$
- Zinc Oxide ZnO
- Aluminum Nitride AlN

1.4.3 Need of Piezoelectric Material

To enhance the response control and measurement of structural element laminated with composite plates under applied mechanical and electrical field by embedding actuator and sensor.

1.5 Objective of Study

The objectives of present study are:

- To understand the constitutive law for the piezoelectric materials.
- To derive the formualtion for static finite element analysis of Piezoelectric Laminated Composite Plates.
- Modelling of Piezoelectric Laminated Composite Plates.
- Generation of computer program.
- To study the different problems in Piezoelectric Laminated Composite Plates.
- Comparision of results obtained with available results in literature.

1.6 Scope of work

The scope of work in this study are:

- The formulation of static finite element analysis of piezolaminated composite plates by plane stress method.
- The computer program generation for the finite element analysis of piezolaminated composite plates in C language.
- The analysis of piezolaminated composite plates using ANSYS Workbench FEA software.
- Comparision of obtained results with the results reported in literature.n

1.7 Organisation of report

The major project is devided in various chapters as follows:

Chapter 1 includes introduction of composite plates and its application in structural engineering, introduction of lamina and laminate, characteristic of composites and its manufacturing process. Introduction of piezo-electric laminated composite plate includes process of piezo-electricity, piezo-electric materials. This chapter includes objectives of this project.

Chapter 2 includes literature survey of different journal papers about piezo-electric laminated composite plates.

Chapter 3 includes basic mechanics of composite plates, constitutive relations of piezo-electric materials, finite element formulation for the static analysis. The procedure for the static analysis of piezo-electric laminated composite plates is also described.

Chapter 4 includes process for developement of computer program. In this chapter, features of program, flow of program, parameters of program, process of program is discussed.

Chapter 5 includes introduction of FEM software for the analysis of piezo-electric laminated composite plates. The procedure steps are discussed for analysis of piezo laminated composite plate in ANSYS Workbench software.

Chapter 6 includes results of static analysis and discussion of results. The problem discretisation is also discussed in this chapter.

Chapter 7 includes summary of the whole project and conclusions obtained from the results of static analysis. Future scope of work is presented in this chapter.

Chapter 2

Literature Review

2.1 General

Literature review related to static and dynamic Finite Element Analysis of Piezoelectric Laminated Composite Plates is presented in this chapter. Various research papers have been refereed to understand theoretical formulation for analysis of piezoelectric laminated composite plates.

2.2 Literature

Rango and *et al* [1] presented the formulation of an enriched plate element based on FSDT$First order Shear Deformation Theory$. This quadrilateral four-node laminated plate finite element has been formulated and generalized for being applied to anisotropic plates, with symmetric lamination scheme, and it is tested through the static and dynamic analysis in some numerical examples, producing very good results. It is possible to achieve very good accuracy in the results using a low number of polynomials and without densify the mesh in plates with general quadrilateral planform, which can be divided into a small number of macro-elements. The presented example provides evidence of the versatility and capability of the current method.

YIN and *et al* [2] presented a new formulation of piezoelectric laminated plates based on the linear piezoelectricity theory and the layerwise theory is developed. A meshless collocation method based on the radial basis function is used to discrete the derived Euler-Lagrange equations. It only needs nodes distributed on the upper and lower surfaces of each layer, which avoids the troublesome in generating 3D mesh or 2D background mesh and saves much preprocess time. The method yields excellent results. Its suitable for thin and thick laminated plates.

Jayakumar and *et al* [3] have analysed piezoelectric laminated composite plates under uniformly distributed transverse load. Nonlinearityisconsideredintermsof-moderatelylarge deflections. A modified Galerkins method is employed to obtain the solution of the problem. In the proposed general formulation, it is possible to control the deflection of uniformly loaded piezo- laminated plates with appropriate electric field and position of piezoelectric layer.The proposed approach gives direct solution of the problem. It is preferable to use continuum methods if closed form solution methods for such a system are not possible. Simple continuum solution methods can provide not only a check against the numerical approaches like finite element method, but also a means by which the effect of a parameter change on a system can be readily gauged, which is useful in design process.

Neto and *et al.* [5] have developed one finite element for static and dynamic analysis of composite smart structures. The SDST (smart discrete shear triangular) element is a degenareted plate element that is based on the ReissnerMindlin theory, and it uses the Allman formulation for the inplane strains together with the generalization of the discrete Kirchhoff technique to include the transverse shear strains. The smart finite element model combines 2D single-layer elements with finite 3D piezoelectric sub-layer elements. The element was implemented directly into FEAP program and can be used for the modeling of laminated smart or adaptive structures that use embedded piezoelectric sensors andor actuators. The elements accuracy is tested in a number of standard static benchmark beam, plate and shell problems. The numerical results obtained using SDST element correlated well with other published results.

Huang and *et al* [4] studied the static and dynamic response of rectangular laminated composite plates with piezoelectric layer as a sensor and actuator by emplouing the first-order shear deformation theory. The fundamental unknowns, such as the displacement components and the electric potential, are assumed to be the functions of the plate thickness coordinate. The Fourier series method is successfully applied to find the analytical solution. Numerical results show that the plate aspect ratio, plate thinness ratio, lamination scheme, fiber orientations, and piezoelectric coupling significantly influence the static and dynamic responses of the plate. It is observed that piezoelectric coupling is sensitive to the lamination scheme of the plate and fiber orientations in the composites. These results could provide us with insight into how a piezolaminated composite will perform and would be helpful in understanding the electroelastic behavior of the composite. Also, the method presented here can be equally applied to design piezoelectric actuators and sensors in MEMS applications.

Phung-Van and *et al.* [6] presented a simple and effective approach based on the combination of IGA and HSDT for the static, free vibration analyses and dynamic control of composite plates integrated with piezoelectric sensors and actuators. In the piezoelectric composite plates, the mechanical displacement field is approximated according to the HSDT using isogeometric elements based on NURBS and featuring at least C1-continuity, whereas the electric potential is assumed to vary linearly through the thickness for each piezoelectric sub-layer. A displacement and velocity feedback control algorithm is used for the active control of the static deflection and of the dynamic response of the plates through a closed-loop control with bonded or embedded distributed piezoelectric sensors and actuators. Several numerical examples are performed to analyze the static deflection, natural vibration mode and dynamic control of piezoelectric laminated plates with different stacking schemes.

Hwang and *et al.*[32] have studied the vibration control of laminated plate with piezoelectric sensors/actuators. For dynamic analysis equation of motion was formulated using Classical Laminate Theory and Hamilton's principle. The plate was discretized using 4-node quadrilateral plate bending element with 12 degrees of freedom per node and one electrical degree of freedom per element. The piezoelectric sensor was distributed and integrated because output voltage was dependent on the strain rate. For validation static responses of bimorph beam were calculated. For vibration control of plate the responses of plate under given displacements and external loads were obtained using direct time integration of equation of motion using newmark-β method. From Finite element formulation code was developed.

Chen and *et al.*[33] presented the studies on vibration control of intelligent structure using Finite element analysis. Finite element formulation for vibration control and suppression of intelligent structures with a new piezoelectric plate element was carried out. A method of active vibration control and suppression for intelligent structures was developed based on a negative velocity feedback control law. For Finite element formulation, 4-node isoparametric Finite element with 12-degree of freedom (*mechanical*) and one electrical degree of freedom per node was considered. For validation of the presented method two examples were calculated using bimorph beam and intelligent plate.

Shiyekar and *et al.*[27] presented a studies on a complete analytical solution for cross- ply composite laminates integrated with piezoelectric Fiber-reinforced composite (PFRC) actuators under bi-directional bending. A higher order shear and normal deformation theory (HOSNT12) had been used to analyze such hybrid or smart laminates subjected to electromechanical loading. In the presented studies the electrostatic potential had been assumed to be layer wise (LW) linear through the thickness of PFRC. Navier's technique and principle of minimum potential energy had been used to obtain the equations of equilibrium. Transverse shear stresses were presented at the interface of PFRC actuator and laminate under the action of electrostatic potentials. They also observed that actuating effects were more in case of thick than thin laminates. They compared the results with first order shear deformation theory (FOST) and exact solution.

Neto and *et al.*[28] presented the studies on static and dynamic analysis of smart laminated structures. To analyse they presented the three node finite element with piezoelectric coupling. The element was a continuum-based degenerated plate element based on the Reissner-Mindlin theory with six mechanical degrees of freedom per node and one electrical degree of freedom per finite element. The electric field was assumed constant across the thickness of each piezoelectric layer. The bending and membrane consistent mass matrices had been derived for applications on structural dynamics. Since this finite element was firstly developed to allow the active vibration control of the exible multibody components The numerical results obtained by this finite element correlated well with other published results.

Kogl and *et al.*[29] presented the studies on analysis of smart laminates using piezoelectric MITC plate and shell elements. In the presented studies, recently developed piezoelectric MITC plate and shell elements had been employed for the modelling of multi-layer smart structures. The piezoelectric MITC elements were free of locking and yield very accurate and reliable results. The formulation allowed the incorporation of layers of arbitrary material properties such as viscoelastic bonding layers. By means of numerical examples, the accuracy of the solutions and the suitability of the approach for the modelling of smart structures were demonstrated. Finally, the excellent performance of the MITC-P shell elements was confirmed by investigating harmonic vibrations of an elastic cylindrical shell with two piezoelectric actuators attached to it.

Fukunaga and *et al.*[30]presented the finite element model for analyzing the composite laminates containing the piezoelectrics statically and dynamically. A simple higher order plate theory was used, which can satisfy the free conditions of transverse shear strains on the top and bottom surfaces of the plates. To develop C_0 type FEM scheme, two artificial variables in the displacement field had been introduced to avoid the higher-order plate theory. Also a generalized coupling FEM model for the mechanical and electric fields from the variational framework was proposed. Finally various examples studied in many previous researches had been employed to verify the justification, accuracy and efficiency of the presented model.

Chapter 3

Static Analysis

3.1 Introduction

In this report, Finite Element Method (FEM) is used for analysis of plates. Finite element method is based on principle of discretization. By assuming properties of a single element, global matrices are formed. By solving equilibrium equations, response of piezo-electric Laminated Composite Plates under static condition can be obtained. Element properties can be derived from stress-s c c train relationship and strain displacement relationship. When a piezoelectric material is subjected to mechanical strain or stress, it develops electric polarization which generates electric charge. This occurrence is called as direct piezoelectric effect. Conversely when piezoelectric material is electrically polarized by applying electric field, it will experience strain. This action is called converse or reciprocal piezoelectric effect. Here elastic deformation can be an expansion or contraction in either direction according to the sign and magnitude of applied electric field. So as piezoelectric material exhibit both direct and converse effects, the same structural element can be used as an actuator or sensor or both simultaneously. The converse effect enables actuation and the direct effect accelerates sensing of structural element vibrations.

3.2 Basic Mechanics

3.2.1 Generalize Hook 's Law

Normal stress and strain, Uniaxially applied force:

- For isotropic materials, the stress strain relationship is free from the direction of force, so only one elastic constant ($Modulus of elasticity$), is required to discuss the relationship between stress and strain for a uniaxially applied force.
- For a nonisotrapic material, at least two elastic constants are needed to explain the stress strain behavior of the material. The schematic diagram of an isotropic and an unidirectional fiber-reinforced material is shown in fig

The stiffness of plate mode from in isotropic material can be described by one value, the modulus at elasticity, E of the material, independent of the direction of load. For or isotropic material, the stress strain relationship can be represented as,

$$\sigma \equiv E \cdot e \tag{3.1}$$

The stiffness of plate from an orthotropic plate has to be described by two values, one along longitudinal direction of the fiber defined as EL and one along transverse direction of the fiber defined as Et. Subscripts 1 and 3 can be used such that $E_L = E_1 and E_T = E_2$. Now these indices must be added to the stress, strain and modulus of elasticity to describe the direction of the applied force. For an orthotropic material, direction be specified and represented as,

$$\sigma_1 = E_1 e_1 or \sigma_2 = E_2 e_2 \tag{3.2}$$

3.2.2 Constitutive relations of Pizoelectric Laminated Composite Plates:

In generics, generally, plate experiences stress in more than one direction with in the plate. This type of condition id called plane stress condition. In this condition Poisson's ratio will became more important. It is the ratio of the strain, perpendicular to a given loading direction, to the strain parallel to given loading direction. Poisson's ratio, for loading along the fibers,

$$\nu_{12} = \frac{E_T}{E_L} \tag{3.3}$$

For Loading perpendicular to fibers,

$$\nu_{21} = \frac{E_L}{E_T} \tag{3.4}$$

3.3 Constitutive Equations

The constitutive equations describing the piezoelectric property are based on the assumption that the total strain in the transducer is the sum of mechanical strain induced by the mechanical stress and the controllable actuation strain caused by the applied electric voltage. The axes are identified by numerals rather than letters. In Figure 3.1, 1 refers to the x axis, 2 corresponds to the y axis, and 3 corresponds to the z axis. Axis 3 is assigned to the direction of the initial polarization of the piezoceramic, and axes 1 and 2 lie in the plane perpendicular to axis 3. This is demonstrated more clearly in Figure 3.2.

The describing electromechanical equations for a linear piezoelectric material can be written as:

$$\epsilon = {S_{ij}}^E \sigma_j + d_m E_m \tag{3.5}$$

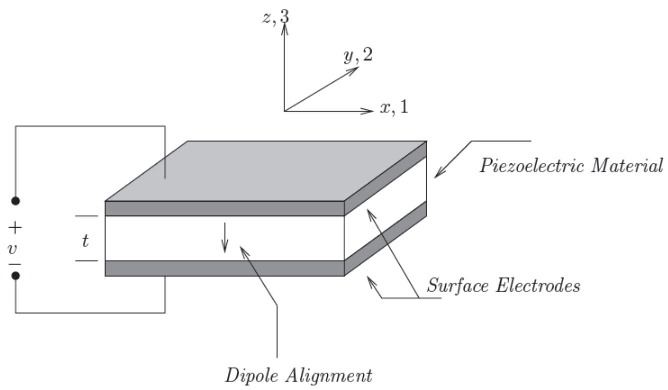
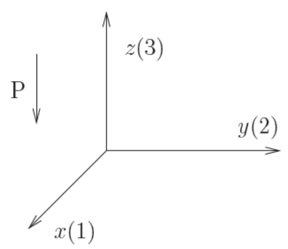


Figure 3.1: Schematic diagram of a piezoelectric transducer

$$D_m = d_{mi}\sigma_i + \xi_{ik}{}^{\sigma} E_k \tag{3.6}$$

where the indexes i, j = 1, 2, . . . ,6 and m, k = 1, 2, 3 refer to different directions within the material coordinate system, as shown in Figure 3.2.,
σ...stress vector N/m^2,
ϵ...strain vector m/m,
E...vector of applied electric field V/m,
ξ...permitivity F/m,
d...matrix of piezoelectric strain constants m/V,
S...matrix of compliance coefficients m^2/N,
D...vector of electric displacement C/m^2.



#	Axis
1	x
2	y
3	z
4	Shear around x
5	Shear around y
6	Shear around z

Figure 3.2: Axis Nomenclature

In matrix form, above equations can be written as:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \tau_{23} \\ \tau_{31} \\ \tau_{12} \end{bmatrix} + \begin{bmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \\ d_{15} & d_{25} & d_{35} \\ d_{16} & d_{26} & d_{36} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

and

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\ d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\ d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} + \begin{bmatrix} e^{\sigma}_{11} & e^{\sigma}_{12} & e^{\sigma}_{13} \\ e^{\sigma}_{21} & e^{\sigma}_{22} & e^{\sigma}_{23} \\ e^{\sigma}_{31} & e^{\sigma}_{32} & e^{\sigma}_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}.$$

3.4 Finite Element Formulation

From constitutive law relationship which represents the electromechanical coupling, Finite element formulation is derived using 8-node isoparametric Finite element based on considered displacement Fields. Constitutive law relationship is given by:

$$\begin{Bmatrix} \{\sigma\} \\ \{D\} \end{Bmatrix} = \begin{bmatrix} Q & -e \\ e^T & \mu \end{bmatrix} \begin{Bmatrix} \{\epsilon\} \\ \{E\} \end{Bmatrix} or \bar{\sigma} = \left[\bar{Q}\right] \bar{\epsilon} \tag{3.7}$$

Above equation is simplified as follows:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{23} \\ \sigma_{13} \\ D_1 \\ D_2 \\ D_3 \end{Bmatrix} = \begin{bmatrix} \mathrm{Q}_{11} & Q_{12} & Q_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} \\ Q_{21} & Q_{22} & Q_{23} & 0 & 0 & 0 & 0 & 0 & -e_{32} \\ Q_{31} & Q_{32} & Q_{33} & 0 & 0 & 0 & 0 & 0 & -e_{33} \\ 0 & 0 & 0 & Q_{44} & 0 & 0 & 0 & -e_{24} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} & 0 & -e_{15} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & Q_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & e_{15} & 0 & u_{11} & 0 & 0 \\ 0 & 0 & 0 & e_{24} & 0 & 0 & 0 & u_{22} & 0 \\ e_{31} & e_{32} & e_{33} & 0 & 0 & 0 & 0 & 0 & u_{33} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{12} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ E_1 \\ E_2 \\ E_3 \end{Bmatrix} \tag{3.8}$$

3.4.1 Transformation of Stress and Strain:

The stiffness of lamina changes with the change of orientation in composite laminates. To solve this problem a particular axis system is chosen, this system is known as the reference axis. In fiber reinforced composites, axis system which is parallel and perpendicular calculation of material properties of lamina. Now, to get response of lamina in global direction, the transformation of stress and strain in global or reference axis system is needed.

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} \mathrm{T}_{11} & 0 \\ 0 & T_{22} \end{bmatrix} \tag{3.9}$$

$$\begin{bmatrix} \mathrm{T}_{11} \end{bmatrix} = \begin{bmatrix} \mathrm{C}^2 & S^2 & 0 & 0 & 0 & -2CS \\ S^2 & C^2 & 0 & 0 & 0 & 2CS \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C & S & 0 \\ 0 & 0 & 0 & S & C & 0 \\ CS & -CS & 0 & 0 & 0 & C^2 - S^2 \end{bmatrix} \tag{3.10}$$

$$\begin{bmatrix} \mathrm{T}_{22} \end{bmatrix} = \begin{bmatrix} \mathrm{C}^2 & S^2 & 0 \\ S^2 & C^2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.11}$$

Here θ is an angle of Fibre orientation. So consitutive model can be written in global x-y-z direction as follows:

$[\sigma] = [T]^T \begin{bmatrix} \mathrm{Q} & \text{-e} \\ \mathrm{e}^T & \mathrm{u} \end{bmatrix} \begin{bmatrix} \mathrm{T} \end{bmatrix}$ (3.12)

and

$$\begin{bmatrix} \varepsilon \end{bmatrix} = \begin{bmatrix} \bar{Q} & -\bar{e} \\ \bar{e}^T & \bar{u} \end{bmatrix} \tag{3.13}$$

From above equation the expanded form can be written as follows:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \\ D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} \bar{Q_{11}} & \bar{Q_{12}} & \bar{Q_{13}} & 0 & 0 & 0 & 0 & 0 & -\bar{e_{31}} \\ \bar{Q_{21}} & \bar{Q_{22}} & \bar{Q_{23}} & 0 & 0 & 0 & 0 & 0 & -\bar{e_{32}} \\ \bar{Q_{31}} & \bar{Q_{32}} & \bar{Q_{33}} & 0 & 0 & 0 & 0 & 0 & -\bar{e_{33}} \\ 0 & 0 & 0 & \bar{Q_{44}} & 0 & 0 & 0 & -\bar{e_{24}} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q_{55}} & 0 & -\bar{e_{15}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{Q_{66}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{e_{15}} & 0 & \bar{u_{11}} & 0 & 0 \\ 0 & 0 & 0 & \bar{e_{24}} & 0 & 0 & 0 & \bar{u_{22}} & 0 \\ \bar{e_{31}} & \bar{e_{32}} & \bar{e_{33}} & 0 & 0 & 0 & 0 & 0 & \bar{u_{33}} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{xy} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ E_x \\ E_y \\ E_z \end{Bmatrix} \tag{3.14}$$

The coefficients of $[\bar{Q}]$ and $[-\bar{e}]$ are given by:

$\bar{Q}_{11} = Q_{11}C^4 + 2(Q_{12} + 4Q_{44})S^2C^2 + Q_{22}S^4$

$\bar{Q}_{12} = (S^4 + C^4)Q_{12} + (Q_{11} + Q_{22} - 4Q_{44})S^2C^2$

$\bar{Q}_{13} = C^2Q_{13} + S^2Q_{23}$

$\bar{Q}_{14} = (Q_{11} - Q_{12} - 2Q_{44})C^3S + (Q_{12} - Q_{22} + 2Q_{44})S^3C$

$\bar{Q}_{22} = Q_{11}S^4 + Q_{22}C^4 + (2Q_{12} + 4Q_{44})S^2C^2$

$\bar{Q}_{23} = C^2Q_{23} + S^2Q_{13}$

$\bar{Q}_{24} = (Q_{11} - Q_{12} - 2Q_{44})S^3C + (Q_{12} - Q_{22} + 2Q_{44})C^3S$

$\bar{Q}_{33} = Q_{33}$

$\bar{Q}_{34} = (Q_{13} - Q_{23})SC$

$\bar{Q}_{44} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{44})S^2C^2 + (C^4 + S^4)Q_{44}$

$\bar{Q}_{55} = C^2Q_{55} + S^2Q_{66}$

$\bar{Q}_{56} = (Q_{66} - Q_{55})SC$

$\bar{Q}_{66} = S^2Q_{55} + C^2Q_{66}$

$\bar{e}_{31} = (e_{31}C^2 + e_{32}S^2)$

$\bar{e}_{32} = (e_{31}S^2 + e_{32}C^2)$

$\bar{e}_{33} = e_{33}$

$\bar{e}_{14} = (e_{15}C^2S + e_{24}CS^2)$

$\bar{e}_{24} = (e_{24}C^3 + e_{15}S^3)$

$\bar{e}_{15} = (e_{15}C^3 + e_{24}S^3)$

$\bar{e}_{25} = (e_{24}C^2S + e_{15}CS^2)$

$\bar{\mu}_{12} = C^2S^2(\mu_{11} + \mu_{22})$

$\bar{\mu}_{22} = \mu_{22}C^4 + \mu_{11}S^4$

$\bar{\mu}_{33} = \mu_{33}$

In present work for finite element formulation the 8-node isoparametric element is shown in Fig **Displacement Model:** The element will have two mechanical

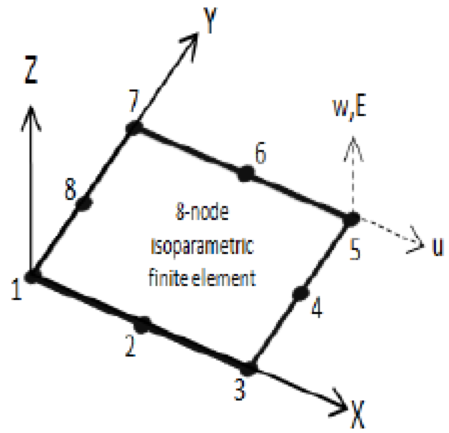


Figure 3.3: 8-node isoparametric element with displacement fields at each node

degrees of freedom $U(x, y, t)$ and $W(x, y, t)$ respectively in x and z directions and one electrical degree of freedom $E_z(x, y, t)$ in z-direction per node as shown in fig.3.1 therefore element will have 24 degrees of freedom. The displacement related at any point within the element is:

$$u(x, y, z) = \sum_{i=1}^{8} N_i(\xi, \eta)u_i(t), w(x, y, t) = \sum_{i=1}^{8} N_i(\xi, \eta)w_i(t), E_z(x, y, t) = \sum_{i=1}^{8} N_i(\xi, \eta)E_{zi}(t) \tag{3.15}$$

Here ζ and η are the isoparametric coordinates and $u_i\,(t)$; $w_i\,(t)$ are the mechanical nodal degrees of freedom, and $E_{zi}\,(t)$ is the electrical nodal of freedom. Strain-displacement relationship is given by:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ 2\varepsilon_{xz} \\ E_z \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial_x} & 0 & 0 \\ 0 & \frac{\partial}{\partial_z} & 0 \\ \frac{\partial}{\partial_z} & \frac{\partial}{\partial_x} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} u \\ w \\ D_z \end{Bmatrix} \tag{3.16}$$

By substituting equation in above equation, will allow to express strain in terms of nodal displacement vector $\{q\}_e$ and electrical field vector $\{E_z\}_e$:

$$\{q\}_e = \{u_1\;\, w_1\;\, u_2\;\, w_2\;\, u_3\;\, w_3\;\, u_4\;\, w_4\;\, u_5\;\, w_5\;\, u_6\;\, w_6\;\, u_7\;\, w_7\;\, u_8\;\, w_8\}^T$$

$$\{E_z\}_e = \{E_1\;\, E_2\;\, E_3\;\, E_4\;\, E_5\;\, E_6\;\, E_7\;\, E_8\}^T \tag{3.18}$$

Now from above relation $[B]$ matrix for mechanical field $[B_u]$ and electrical field $[B_E]$ can be written as:

$$[B]_u = \sum_{i=1}^{NN} \begin{vmatrix} \frac{\partial_{N_i}}{\partial_x} & 0 \\ 0 & \frac{\partial_{N_i}}{\partial_z} \\ \frac{\partial_{N_i}}{\partial_z} & \frac{\partial_{N_i}}{\partial_x} \end{vmatrix}_{(3\times16)} \begin{vmatrix} u_i \\ w_i \end{vmatrix}_{(16\times1)} \tag{3.19}$$

$$[B]_E = \sum_{i=1}^{NN} \Big|N_i\Big|_{(1\times8)} \Big|E_i\Big|_{(8\times1)} \tag{3.20}$$

where, NN = number of nodes.

For static analysis,

$$\begin{bmatrix} [K_{uu}] & [K_{uE}] \\ [K^T{}_{uE}] & [K_{EE}] \end{bmatrix} \begin{Bmatrix} \{u\}_e \\ \{E_z\}_e \end{Bmatrix} = \begin{bmatrix} \{F\}_e \\ \{q\}_e \end{bmatrix} \tag{3.21}$$

Where, the $[K_{uu}]$ is the stiffness matrix corresponding to the mechanical degrees of freedom. $[K_{uE}]$ is the stiffness matrix due to electromechanical coupling. $[K_{EE}]$ is the stiffness matrix due to electrical degrees of freedom. F_e is the element load vector and q_e is the element charge vector. These all matrices are given by:

$$[K_{uu}] = t\int_{-1}^{+} 1 \int_{-1}^{+} 1[B_u]^T[\bar{Q}][B_u]|J|\partial_\xi\partial_\eta \tag{3.22}$$

$$[K_{uE}] = -t\int_{-1}^{+} 1 \int_{-1}^{+} 1[B_u]^T[\bar{e}][B_E]|J|\partial_\xi\partial_\eta \tag{3.23}$$

$$[K_{EE}] = t\int_{-1}^{+} 1 \int_{-1}^{+} 1[B_E]^T[\bar{\mu_{33}}][B_E]|J|\partial_\xi\partial_\eta \tag{3.24}$$

Element load and charge vector are given by:

$$F_e = F_c + \int_{s1} [N]^T F_s dS_1 \tag{3.25}$$

$$q_e = - + \int_{s2} [N]^T D_s dS_2 \tag{3.26}$$

To solve Integration form 2×2 Gauss Integration scheme is used for both stiffness matrix due to mechanical and electromechanical coupling.

$$\begin{aligned}
[\mathrm{K}_{uu}] &= \sum_{a=1}^{NG}\sum_{b=1}^{NG} t[B_u]^T[\bar{Q}][B_u]|J|W_aW_b \\
[\mathrm{K}_{uE}] &= \sum_{a=1}^{NG}\sum_{b=1}^{NG} -t[B_u]^T[\bar{e}][B_E]|J|W_aW_b \\
[\mathrm{K}_{EE}] &= \sum_{a=1}^{NG}\sum_{b=1}^{NG} t[B_E]^T[\bar{\mu_{33}}][B_E]|J|W_aW_b \\
[\mathrm{F}_e] &= \sum_{a=1}^{NG}\sum_{b=1}^{NG} F_s[N_i]^T|J|W_aW_b
\end{aligned} \tag{3.27}$$

After evaluating stiffness matrices and load vectors individually, they are assembled to obtain overall stiffness matrix and load vector. Subsequently incorporating boundary conditions the are solved for displacement and voltage. In sensing problem for a given mechanical loading, developed voltage across the smart patch is determined. First mechaniacal displacement due to applied load is calculated. Then from obtained mechanical displacement the developed voltage across the patch is determined. This sensing problem is exhibits a direct effect of piezoelectric material. In actuation problem for a given electric field, developed strain across the smart patch is determined. If an arbitrary value of E_z is specified, the problem comes under the category of open-loop control and if the value of E_z comes from sensor output that is fed back to the controller, then the control scheme is reffered to as closed-loop control. This actuation problem exhibits a converse effect of piezoelectric material.

3.5 Static Analysis:

General: Static Analysis is done to calculate the displacements along x,y and z direction and interlaminar stresses in piezoelectric Laminated composite plate under different types of static loads. After obtaining global stiffness matrix and load vector form governing equations, static response can be obtained using numerical methods.

Assembly and Solution: After obtaining the element stiffness matrices and modal load vectors matrices, they are assembled according to the element-node relationship for a lamina. After that they are added to obtain global stiffness matrix and final load vector for piezoelectric laminated composite plates.

Boundry Condition: The above finite element formulation is developed based on assumed displacement function. These displacement functions also involve the electric field condition. So that along with displacement boundary conditions, electrical boundary condition is also specified.

3.5.1 Governing Equation:

Solution Procedure: For the solution, the standard finite element solution procedure is followed. In this procedure, in the first element stiffness matrices and the consistant load vectors are assembled to obtain a global stiffness matrix and the global load vector. Incorporation of a minimum boundary conditions gives the system of governing equation as per below:

$$
\begin{aligned}
[\mathrm{K}_{uu}] * u_e + [K_{ue}] * E_{ze} &= F \\
[\mathrm{K}_{uu}] * u_e &= F - [K_{ue}] * E_{ze} \\
[\mathrm{K}_{uu}] * u_e &= F^*
\end{aligned}
\tag{3.28}
$$

Chapter 4

Compute Program Developement

4.1 Introduction

In this chapter, the study of computer program developed in C language environment for the analysis of Piezoelectric Laminated Composite Plates is discribed. For the analysis, the program is divided into two parts.

a. **Input Program:** This part is able to perform a meshing of Piezolaminted plate and to generate an input data file for main analysis.

b. **The Core Program:** From this part, the analysis of Piezoelectric Laminated Composite Plate using finite element formulation based on plane stress problem can be done.

4.2 Features of Computer Program

Features of computer program are discussed as follows:

a. Mesh generation.

b. Generation of element mechanical stiffness matrix.

c. Generation of load vector.

d. Generation of element electromechanical coupling stiffness matrix.

e. Generation of overall stiffness matrix, load vector and electromechanical coupling stiffness matrix in banded form.

f. Incorporating Boundry Condition.

g. Solution of governing equation.

h. Calculation of Stresses.

4.3 Flow of Computer Program

Mesh Generation with Input Data: The meshing of composite laminate is performed automatically. The laminated plate is divided in number of elements by assigning the number of divisions in along length and width direction. Numbering to the elements is assigned automatically moving in the direction from left to right and bottom to top. Each element has 8 nodes, which are numbered sequentially from left to right and bottom to top as shown in Fig.4.1.

After numbering the required input data for analysis are given. The required data for analysis are as follows:

a. Plate Dimensions

b. Number of Materials with properties

c. Number of Laminates

d. Laminate ID with angle of orientation and thickness

e. Assigning of load

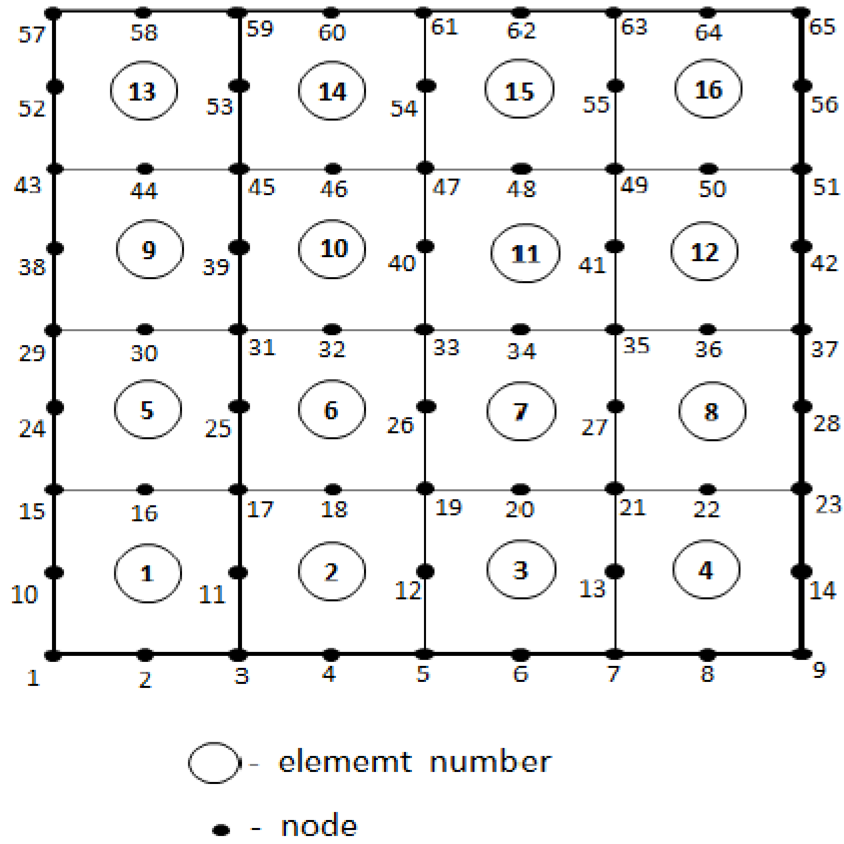


Figure 4.1: Meshing of Plate

After material properties the element incidences and element coordinates are evaluated automatically. And based on support condition the boundary condition is assigned to the each node. Flow chart is shown in Fig.4.3.

Constitutive Law Matrix: A formulation of constitutive law matrix is executed in computer program based on the finite element formulation discussed in previous section. Constitutive law matrix is formulated using the entered material properties, which exhibits the coupling of mechanical and electrical field.

Strain-Displacement relationship Matrix: Matrix is formulated using shape functions and curvature - nodal displacement relationship. Here two $[B]$ matrices are derived, one is for mechanical field $[B_u]$ and other one is for electrical field $[B_E]$

as shown in Flow chart Fig

Overall Stiffness Matrix and Load Vector: The element stiffness matrix of mechanical field for the laminate is generated by laminate constitutive relation $\left[\bar{Q}\right]$ and $[B_u]$ matrix. Similarly stiffness matrix for electrical field is generated by piezoelectric coefficient matrix $[\bar{e}]$, $[B_u]$ and $[B_E]$ matrix. Then generated element stiffness matrices for the laminate are assembled in banded form. The integration for stiffness matrix is evaluated by 2×2 Gauss integration scheme.

Load Vector and Electrical Charge Vector: For load vector each element load vector is generated by reading loading type and value from input data. Then element load vectors are assembled in banded form. For charge vector also at each node it is obtained from applied voltage and thickness of laminate. Then element charge vectors are assembled in banded form.

Incorporating Boundary conditions: In input data file at each node based on displacement model and according to degrees of freedom 0 or 1 value is assigned to each degree of freedom. From this data file boundary condition is assigned to all overall stiffness matrices in core program.

Solution Using Governing Equation: After assigning boundary condition to overall stiffness matrices from counterpart of global assembled governing equations, displacement is calculated using subroutine based on Gauss Elimination method. The procedure of solution is as follows:

$$\begin{aligned} [\mathrm{K}_{uu}] * u_e + [K_{ue}] * E_{ze} &= F \\ [\mathrm{K}_{uu}] * u_e &= F - [K_{ue}] * E_{ze} \\ [\mathrm{K}_{uu}] * u_e &= F^* \end{aligned} \tag{4.1}$$

Output: Output file will consist of following data and results:

a. Plate Dimensions

b. Number of elements nodes and material

c. Material Properties

d. Laminate data : id, angle of orientation, thickness

e. Element Incidences

f. Joint Coordinates

g. Boundary conditions

h. Lamina Thickness

i. Overall mechanical, electromechanical stiffness matrix

j. Overall load vector

k. Overall field intensity vector

l. Displacement vector

m. Stresses various elements

Flow Chart for the program of Meshing of laminated composite plate with Input Data in Fig.4.2 and Flow Chart for the static analysis program is shown in Fig.4.3iiijjjiijjn:

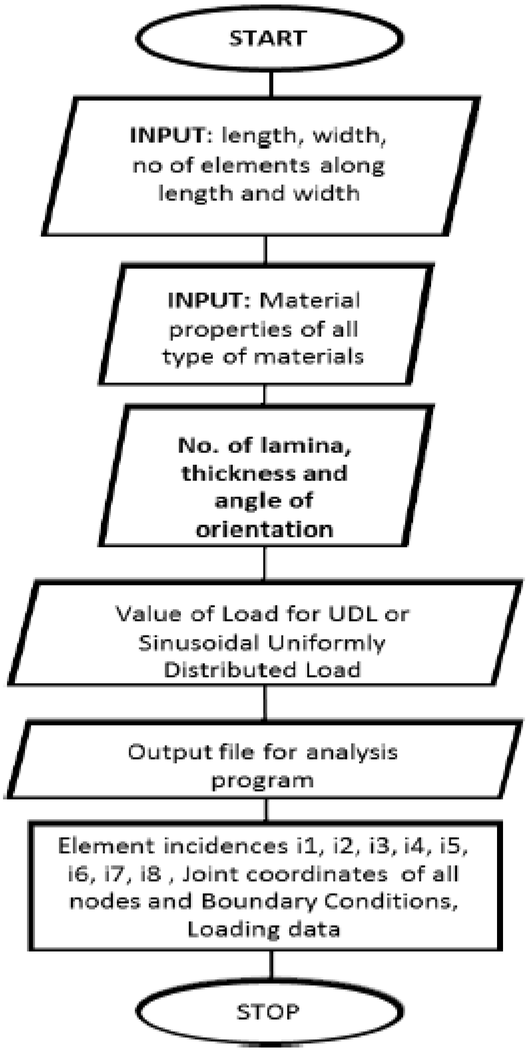


Figure 4.2: Input data with meshing of Laminated Plate

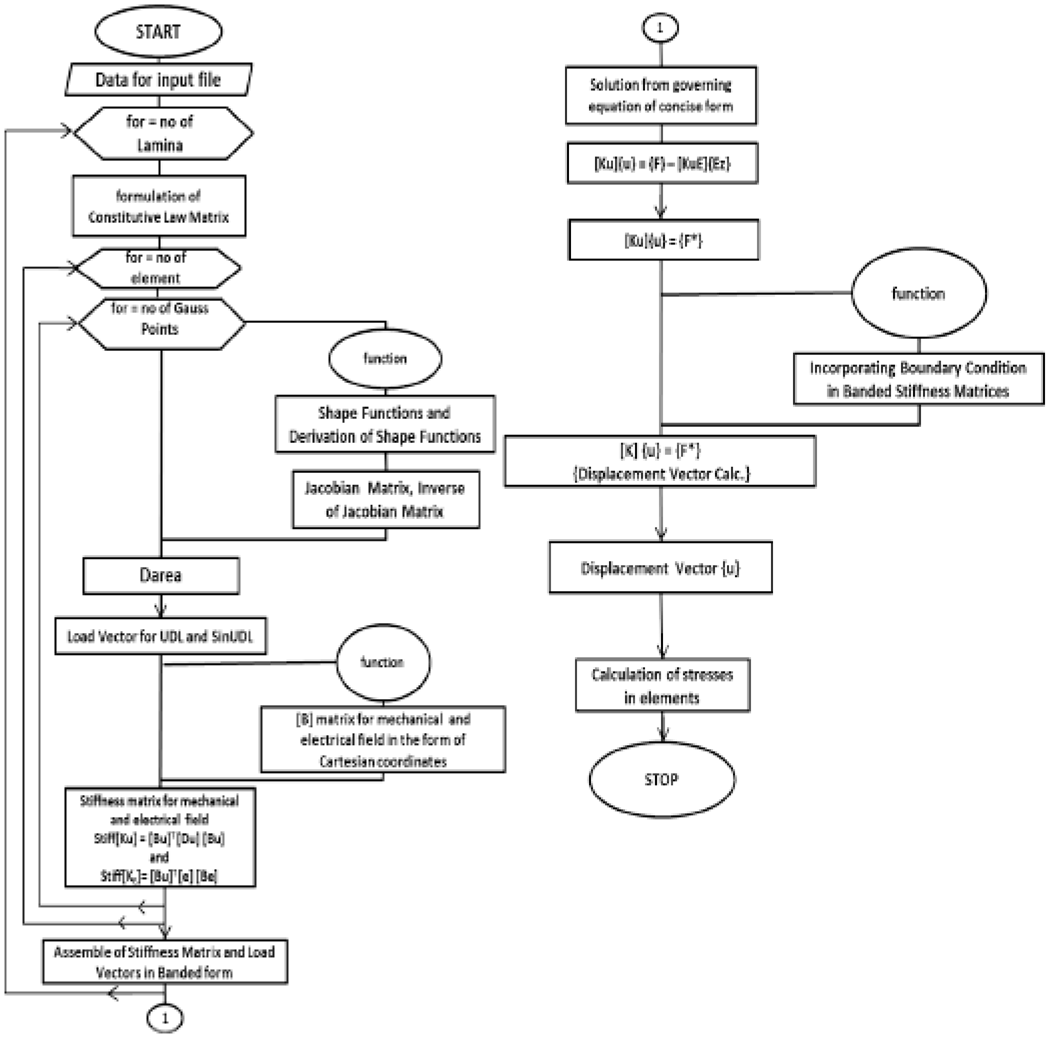


Figure 4.3: Analysis of composite laminate embedded with smart patches of piezo-electric

Chapter 5

Software Applications

5.1 General

There are many commercial software packages available for Finite Element Analysis ex. ANSYS, ABAQUS, NASTRAN, $SAP2000$ etc. Here ANSYS software package is used for introduction analysis of Piezo-electric Laminated Composite Plates. In this chapter, steps are provided to analyse Piezo-electric Laminated Composite Plates.
Software Package: ANSYS Inc.
Version: ANSYS 15.0.1
Software: ANSYS Workbench 15.0
Extension: ACT Piezo $R150$ $v8$

5.2 Steps for Analysis

Steps for analysis of Piezo-electric Laminated Composite Plates in ANSYS Workbench are as per below:

a. To define geometry, there are different ways. One is by using cross-section available in software and extruding as per requirement. Some body operations also need to be done. Another way is importing the geometry from AutoCAD.

The Fig. 5.1 shows Piezo-electric Laminated Composite Plate geometry generated in ANSYS Workbench design-modeller.

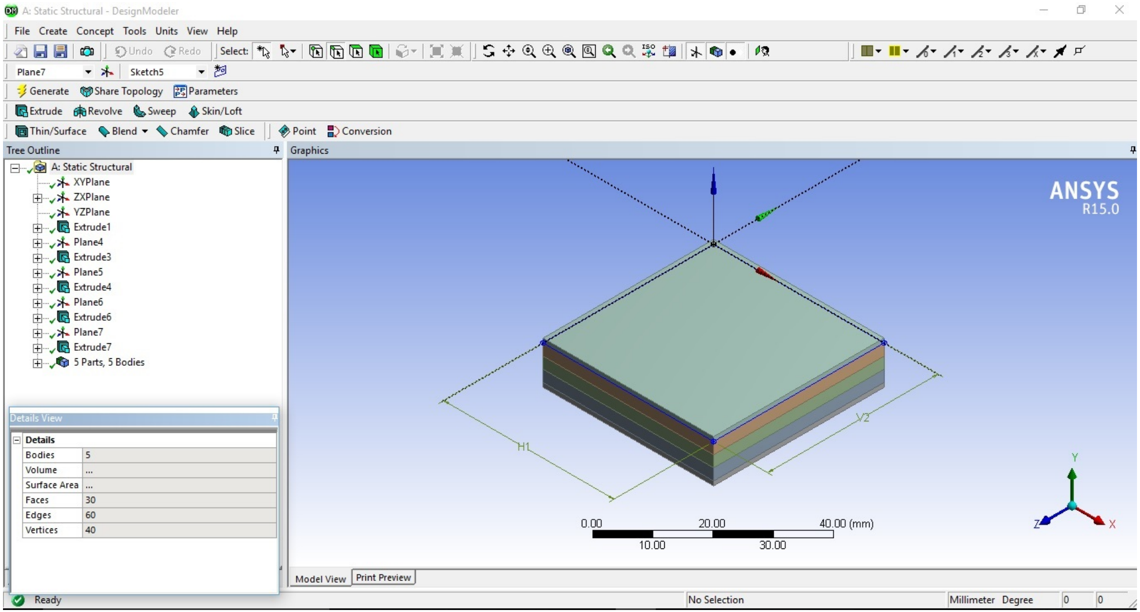


Figure 5.1: Geometry of Piezo-electric laminated composite Plates in ANSYS

b. The support condition of plate is simply supported plate. So in software, at supports displacement in X and Y direction are restrained.

c. The meshing is most important parameter in any FEM software. The accuracy of solution is directly dependent on meshing size. Finer the meshing size, more accurate the solution. Fig 5.2 shows meshing parameter in ANSYS Workbench.

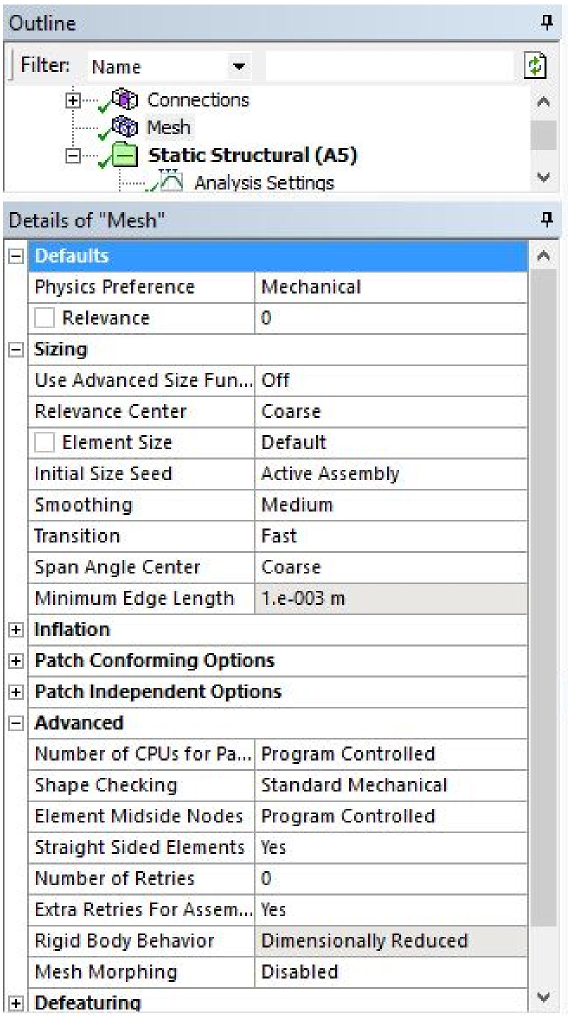


Figure 5.2: Meshing parameters

d. After the application of meshing, model must be checked for connectivity of nodes between meshes of different layers of plate. This connectivity ensures the proper distribution of load/ stresses between different layers of composite plate. For this, layers of composite plate need to be defined as one body with five parts. Fig.5.3 shows proper model connectivity between meshes of different layers of Piezo-laminated Composite Plates.

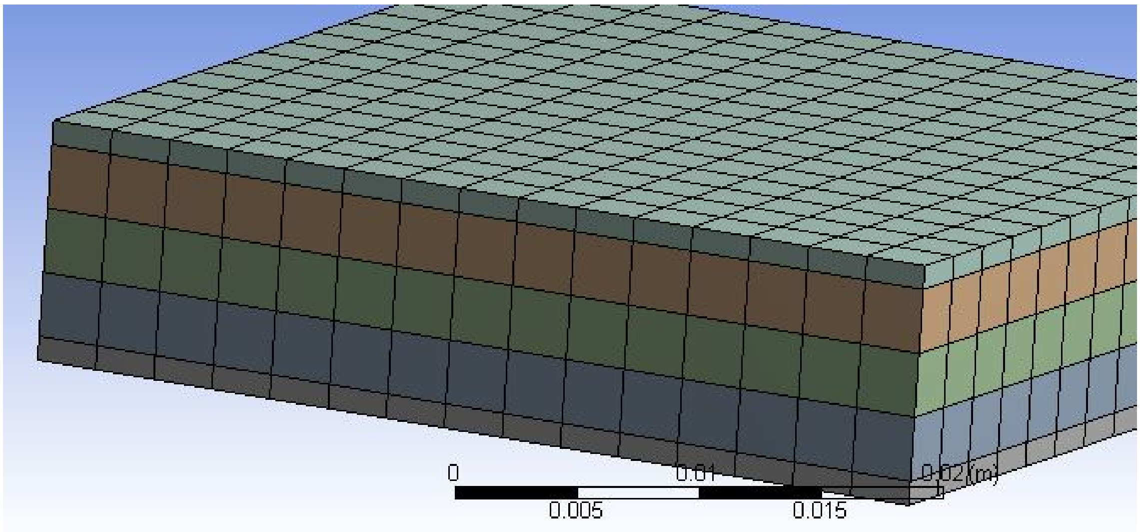


Figure 5.3: Modal connectivity of meshes

e. For Piezoelectric part, first of all in Workbench home section, ACT Piezo $R150$ $v8$ (ExtPiezo) extension needs to be activated. Then in Mechanical section of Workbench, one toolbar for Piezo-electric property declaration is activated. But elastic properties of Piezo-electric material need to be declared earlier in Engineering data section of Workbench along with elastic properties of composite material. Fig.5.4 shows declaration of Piezo-electric properties of the Piezo-electric material layer.

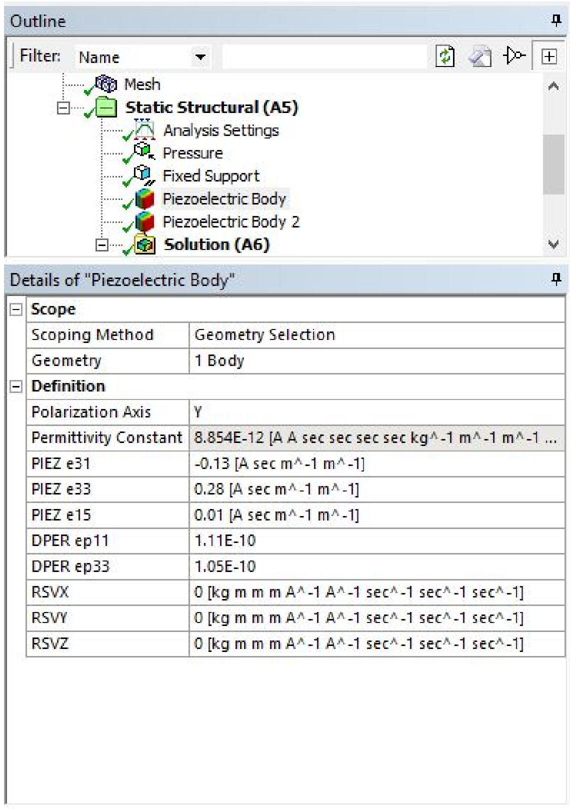


Figure 5.4: Piezo-electric Properties

f. Then pressure is applied on the top of composite plate. It is show in Fig.5.5.

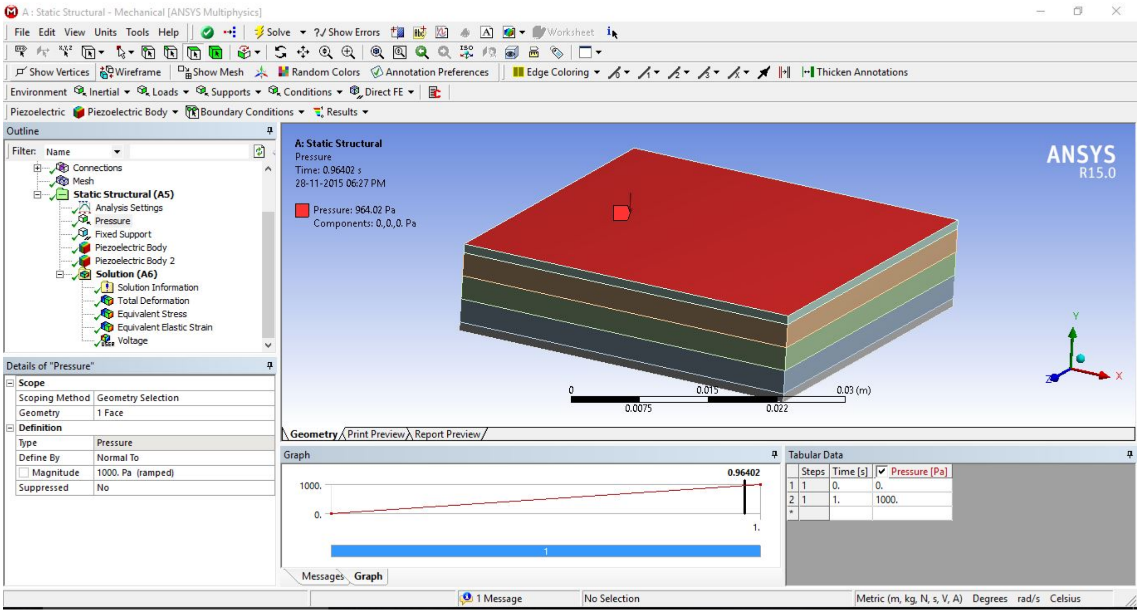


Figure 5.5: Application of pressure

g. After that solution is generated. The elements are automatically assigned to it. As shown in Fig.5.6 $SOLID226$ is assigned to all the layers of composite plate. The degrees of freedom of element depend upon type of element.

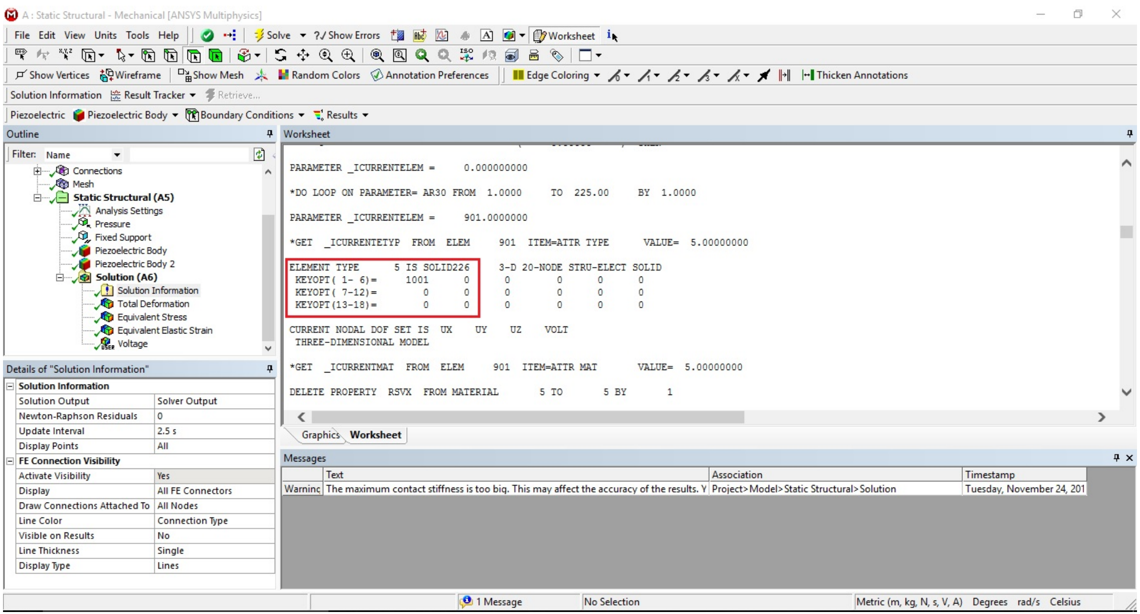


Figure 5.6: Assigned element

5.3 Results

5.3.1 Total Deformation

Fig.5.7 shows deformation diagram of Piezoelectric Laminated Composite Plates.

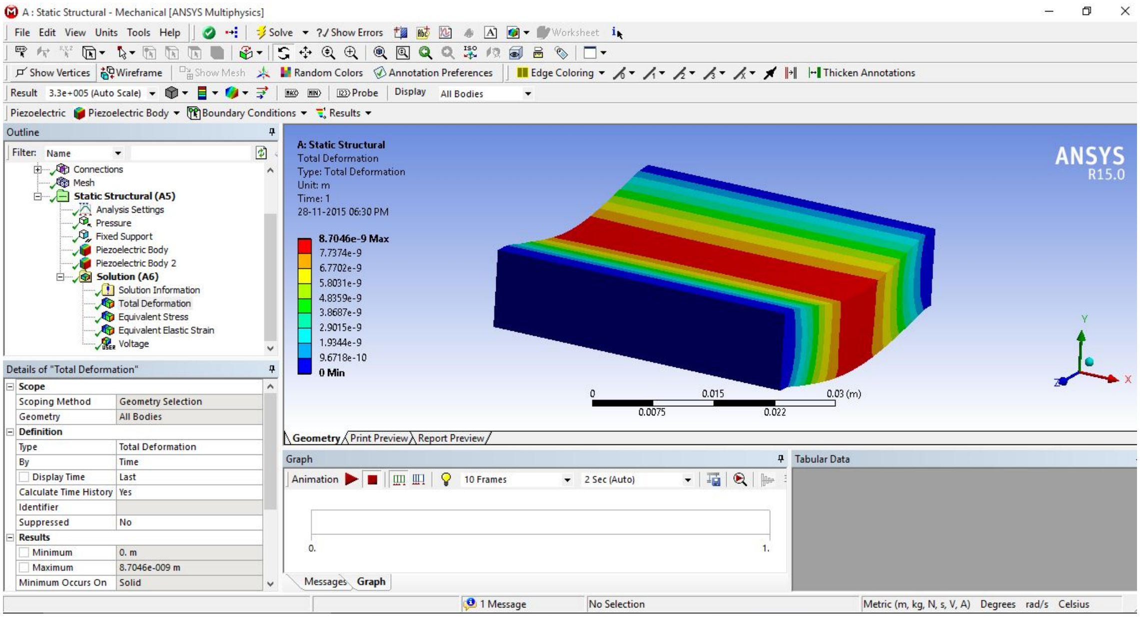


Figure 5.7: Total Deformation

5.3.2 Equivalent Stress

Fig.5.8 shows Stress diagram of Piezoelectric Laminated Composite Plates.

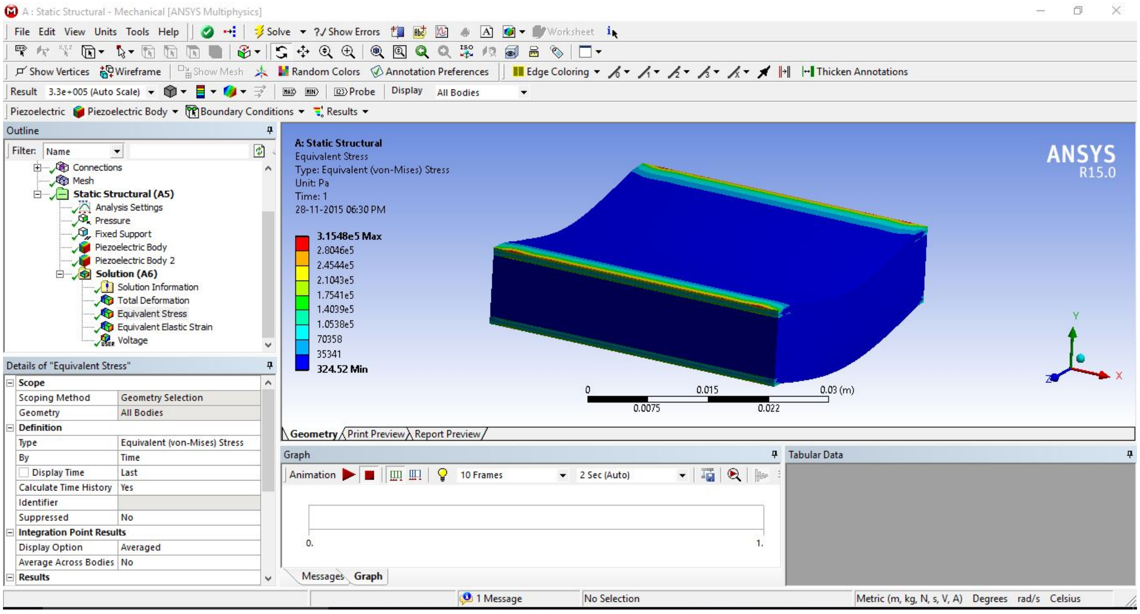


Figure 5.8: Equivalent stress

Chapter 6

Results and Discussion

6.1 General

The Finite Element Method formulation based on Plane stress theory is discussed in **Chapter 3**. Computer program is developed for static analysis of Piezo-electric Laminated Composite Plates as per Finite Element Modelling discussed in **Chapter 4**. In this chapter, FEM is employed for analysis of Piezo-electric Laminated Composite Plate under different static loadings with different support conditions, material anisotropy using computer program solution is done and displacements and stresses are calculated. To validate the results of this finite element formulation, different examples available in literature are solved and discussed.

6.2 Problem Discretisation

From convergence study in general[7], the plate is devided in 16 elements in the quarter part of the plate as shown in Fig.6.1. The maximum deflection and stresses are found out at centre point of plate by using computer program for uniformly distributed loading as shown in Fig.6.2 having different support conditions as shown in Fig.6.3.

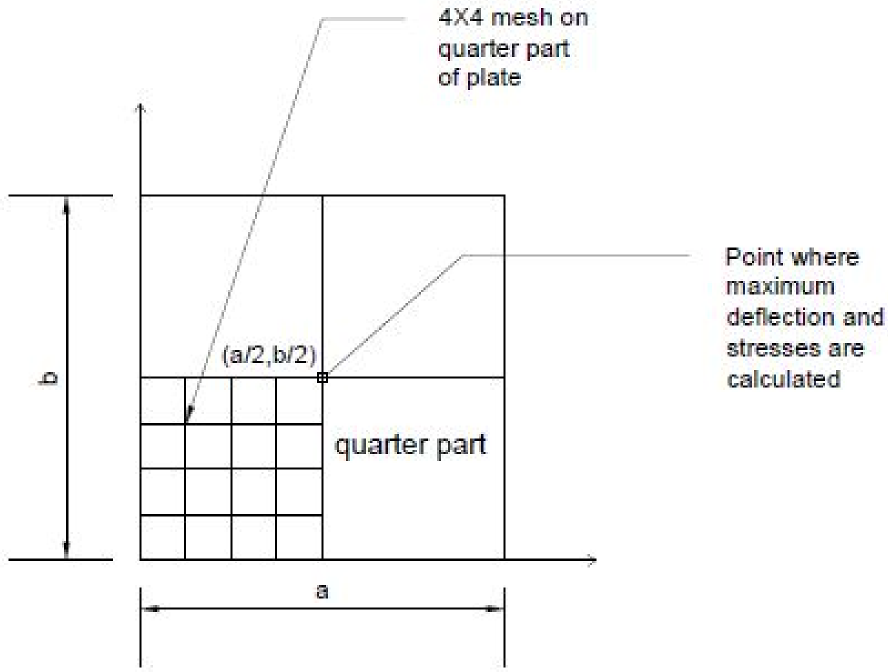


Figure 6.1: 4X4 mesh on quarter part of plate

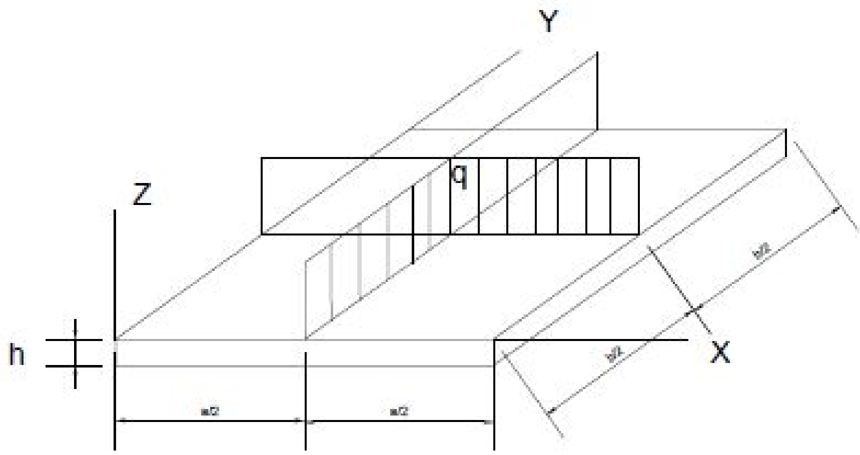


Figure 6.2: Uniformly distributed load on plate

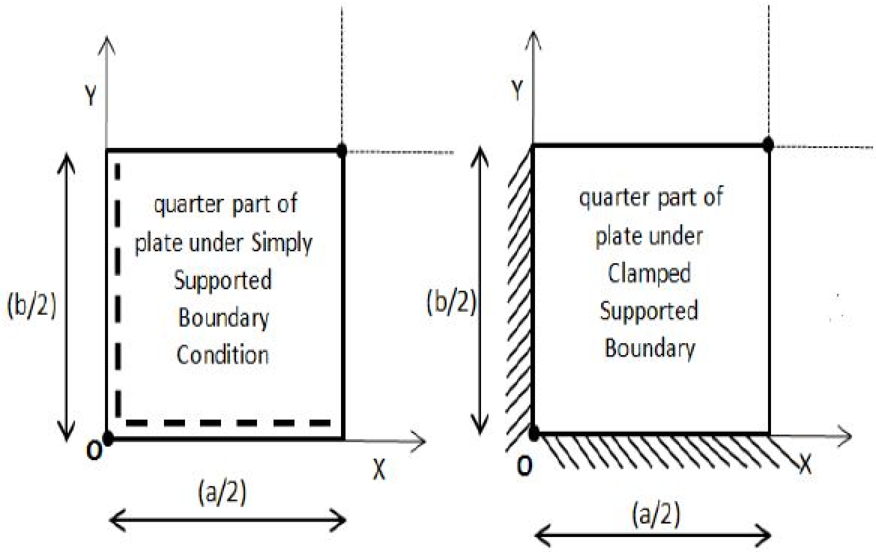


Figure 6.3: Support Conditions

Boundary Conditions : Simply supported on four sides

$$u_0 = 0 \text{ at } x = 0 \text{ and } x = a$$
$$v_0 = 0 \text{ at } y = 0 \text{ and } y = b$$

The nondimentional results of displacements due to different bondings are defined as per following formula:

$$m_1 = \frac{100 * E_2 * h^3}{P_0 * a^4} \tag{6.1}$$

Non-dimensional displacement $\left(\bar{W}\right) = m_1 * Actualdisplacement\,(W_0)$.
The non-dimensional stresses,

$$Non-dimensionalstresses(\bar{\sigma_x}, \bar{\sigma_y}, \bar{\tau_x}) = \frac{h^2}{P_0 * a^2} * Actualstresses(\sigma_x, \sigma_y, \tau_x) \tag{6.2}$$

Material Set-1(PVDF):

C$_{11} = 238.0GPa, C_{22} = 23.6, C_{33} = 10.6, C_{12} = 3.98, C_{13} = 2.19, C_{23} = 1.92, C_{44} = 2.15, C_{55} = 4.40, C_{66} = 6.43$.
$e_{31} = -0.13(C/m^2), e_{32} = -0.14, e_{33} = 0.28, e_{24} = e_{15} = 0.01$.
$\chi_{11} = 1.1 * 10^-10(F/m), \chi_{22} = \chi_{33} = 1.05 * 10^-10$.

Material Set-2(Graphite-epoxy):

C$_{11} = 134.9GPa, C_{22} = 14.35, C_{33} = 14.35, C_{12} = 5.156, C_{13} = 5.156, C_{23} = 7.133, C_{44} = 3.606, C_{55} = 5.654, C_{66} = 5.654$.

Material Set-3(PZT-5A):

E$_1 = 63GPa, E_2 = 63GPa, G_{12} = 24.2GPa, G_{23} = 24.2GPa, \nu_{12} = 0.3, d_{31} = -154 * 10^-12(m/V), d_{32} = -154 * 10^-12(m/V)$.

Material Set-4:

$E_1 = 150GPa, E_2 = 9GPa, G_{12} = 7.1GPa, G_{23} = 2.5GPa, \nu_{12} = 0.3$

6.3 Comparision of Results

The following cases are considered to validate the derived Finite Element Formulation with different types of orientation of lamina as shown in Fig.6.4 and 6.5.

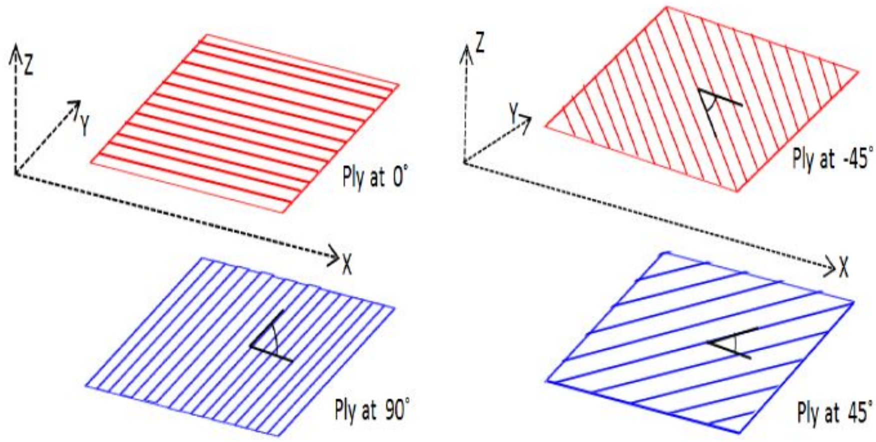
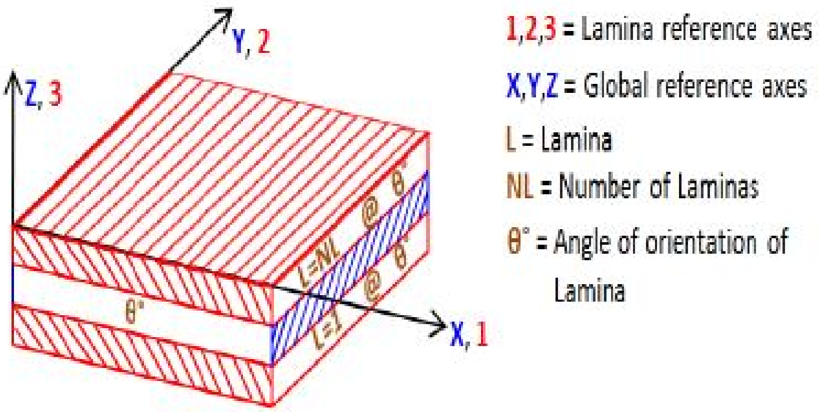


Figure 6.4: Orientation of lamina (a)Cross-ply (b)Angle-ply



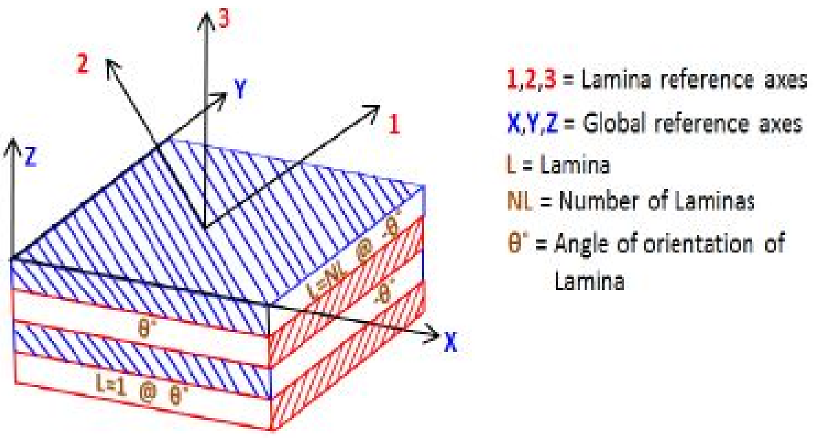


Figure 6.5: Laminated plates (a)Cross-ply (b)Angle-ply

Case-1:

In this case, the study of equally layered piezo-electric laminated composite plate is carried out. In this study, simply supported boundary condition is considered. The central deflection of piezo-electric laminated composite plates under different uniformly distributed loads and voltages are obtained and compared with results reported in literature. Different orientations are considered. In the literature central difflection is calculated as per following formula:

$$q_{max} = \frac{\alpha * h}{\delta} * \Delta_{max} * (1 + \delta_1 * \Delta_{max} + \delta_2 * {\Delta_{max}}^2) \tag{6.3}$$

Fig.6.6 shows non-dimentional displacements for predefined potential difference for three different orientations for a=b=100 mm and h=2.2 mm. The results are compared with MSC Nastran software results and results from the theory developed in literature. The comparision shows that variations in results are little higher so that another theory needs to be considered for finite element formulation.

Table 6.1: Non-dimensional displacements for predefined potential difference for different orientations using material set 3,4

$\delta\phi_u$ (kV)	q_{max}(MPa) equivalent	Δ_{max}		
		Present FEM	Theory in Literature[3]	MSC Nastran[3]
[PZT/0/0/0/0/PZT]				
1.1	0.0063	0.0258	0.0206	0.0205
5.3	0.0536	0.2083	0.168	0.165
10.7	0.1905	0.6019	0.474	0.462
16	0.4018	0.9368	0.7435	0.7177
[PZT/30/30/-30/-30/PZT]				
1.1	0.0063	0.0256	0.0206	0.0196
5.3	0.0536	0.2122	0.1712	0.1643
10.7	0.1905	0.6285	0.4949	0.4819
16	0.4018	0.9825	0.7798	0.7613

Case-2:

To verify the developed finite element analysis, the square piezo-electric laminated composite plate is analyse. The plate has equally thick layers with cross-ply orientaion. Results are obtained for different aspect ratios S for $q = 10N/m^2$ loading.

Fig.6.7 shows comparision of results with developed finite element formulation in literature exact solution from literature[20].

Table 6.2: Non-dimentional displacements and stresses for different aspect ratios and predefined potential difference

S(a/H)	v	Source	u (0,b/2,H/2)	σ_x (a/2,b/2,H/2)	σ_y (a/2,b/2,H/2)	σ_{xy} (0,0,H/2)
6*4	0	Present FEM	0.0118	-0.9576	-0.6468	0.0636
		Ref. FEM[18]	0.0095	-0.798	-0.539	0.053
		Exact Sol.[18]	0.0094	-0.754	-0.557	0.049
	100	Present FEM	-0.1869	14.689	6.27	-0.921
		Ref. FEM[18]	-0.1508	12.04	5.36	-0.749
		Exact Sol.[18]	-0.1465	11.78	5.21	-0.728
6*10	0	Present FEM	0.009	-0.716	-0.338	0.0348
		Ref. FEM[18]	0.00732	-0.607	-0.282	0.029
		Exact Sol.[18]	0.00737	-0.589	-0.288	0.0288
	100	Present FEM	-0.0229	1.824	-0.359	-0.0696
		Ref. FEM[18]	-0.0185	1.52	-0.29	-0.058
		Exact Sol.[18]	-0.0186	1.48	-0.261	-0.058

6.4 Summary

In this chapter, the static analysis of piezo-electric laminated composite plate is described. Non-dimensional deflections and stresses are calculated using developed computer program. In solution process, 4X4 mesh on quater part of plate is considered. To verify FEM formulation, results are compared with available results reported in literature. The comparision of results is done having different aspect ratios, orientation, layer thickness etc.

Chapter 7

Summary and Conclusion

7.1 Summary

This study includes the intrduction of fibre reinforced composites, introduction of laminated composite plates, introduction of piezo-electricsity with special focus on piezo-electric laminated composite plates, different types of piezo-electric materials, its applications in structural engineering.

General mechanics of piezo-electric laminated composite plate is studied and presentaed. The finite element formulation is discussed using Plane Stress problem. A displacement model having three degrees of freedom i.e. two mechanical degrees of freedom and one electrical degree of freedom is presented. Using this displacement model, properties of eight-node isoparamatric (Serendipity) element i.e. stiffness matrix and load vector are derived.

For static analysis of piezo-electric laminated composite plates, a computer program is developed. Analysis of piezo-electric laminated composite plate with different support conditions, width to thickness ratios and mechanical anisotropy can be done by this computer program. In solution, nondimensional displacements and stresses are calculated. A computer program is validated by comparing results with that reported in available literature.

For introduction, analysis of piezo-electric laminated composite plates is done in FEM software. For the analysis, ANSYS Workbench is used. Different steps are discussed for the analysis of piezo laminated composite plates in ANSYS Workbench. Software generated deflection diagram and stress diagram is also shown after analysis.

7.2 Conclusion

In classic plate theory, a transverse displacement is assumed constant throughout the thickness. Here to perform analysis, plane stress theory is used. Based on static finite element analysis of plate using 8-noed isoparamatric elements with 24 degrees of freedom, following conclusions are obtained.

a. In analysis of square piezo laminated composite plate due to symmetry, the computation efforts can be redused.

b. In the analysis of piezo laminated composite plate for the maximum displacement under predefined potential differences and uniformly distributed load with different orientations of lamina configuration, results obtained from present finite element formulation varry by 25% from the results reported in literature.

c. In analysis of piezo laminated composite plate having same aspect ratio and different potential differences with constant uniform loading, results obtained from present finite element formulation varry 28% from results reported in literature.

d. From results, the higher order theory is need to studied for the finite element formulation for better accuracy of the results.

7.3 Future Scope of Work

This present work can be further extended by incorporating following objectives:

a. Developement of computer program for dynamic analysis of piezo-electric laminated composite plates.

b. Static and dynamic analysis of funcutionally graded composite laminted plates.

c. Static and dynamic analysis of piezo-electric laminated compoiste plates having different shapes and sizes.

d. Static and dynamic analysis of piezo-electric laminated compoites plates with cut outs.

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