

Goodness of Fit Based Non-parametric Spectrum Sensing for Cognitive Radio

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by

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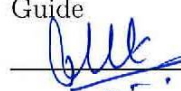
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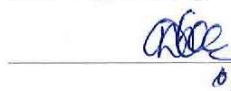
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Publications related to Thesis

International Conference / Journal Publications

- D K. Patel and Y N. Trivedi. *Non-parametric Spectrum Sensing Based on Censored Observations in Quasi-static Fading Channel for Cognitive radio* in 76th meetings of the Wireless Innovation Forum (WINNF), Proceedings of SDR-WinnComm-Europe 11-13 June 2013, pp. 108-111. [Presented at : Rohde and Schwarz , Munich, Germany]
- D K. Patel and Y N. Trivedi. *Non-parametric Blind Spectrum Sensing based on Censored Observations for Cognitive radio*. Journal of Signal Processing Systems (Springer), Volume 78, Issue 3, pp 275-281, March 2015,
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Abstract

The opportunistic spectrum access based on cognitive radio (CR) plays an important role to improve spectral efficiency in wireless communications. To utilize the spectrum efficiently, many spectrum sensing schemes have been proposed in the literature of cognitive radio. For a quick detection of primary user (PU), CR should perform the sensing task at lower number of received observations. In addition to this, the detection performance also depends upon channel conditions and transmitted PU signal. Some sensing schemes have assumed that information of PU and channel state information is known a priori. However, in actual practice, it is difficult to have this a priori information. Therefore, detection of null hypothesis (absence of PU), using goodness of fit (GoF) based non-parametric scheme, is of interest wherein no information about PU and channel is required at CR. In this thesis, we focus on GoF based sensing for achieving better detection performance at lower number of received observations, false alarm probability and signal to noise ratio (SNR).

In the category of non-parametric sensing, energy detection (ED) based sensing is the simplest one for spectrum sensing due to its low complexity. To improve the performance of ED sensing, antenna diversity is used. However, the assumption of having perfect information about distribution of noise at CR becomes very crucial at the low SNR of the PU signal. In case of having imperfect variance of noise, the performance of the ED degrades drastically and results in SNR wall. Therefore, it is of interest to develop a non-parametric sensing algorithm, which gives better performance at low SNR with less number of observations and false alarm probabilities.

Recently, some GoF based sensing schemes have been proposed in the category of non-parametric sensing. In this kind of sensing, empirical cumulative distribution function (ECDF) is determined from the received observations, denoted by F_n . This ECDF is compared with known CDF of noise (F_0) or we test the null hypothesis

($F_n=F_0$). The deviation of ECDF from the known CDF of noise (F_0) decides presence or absence of PU. The methods based on this concept are called as Goodness of Fit (GoF) based non-parametric sensing methods. The prevailing methods are Anderson Darling (AD) sensing, Kolmogorov-Smirnov (KS) sensing, Student- t sensing and Order statistics (OS) based sensing. These methods have used all observations to determine the ECDF. However, the distance of the CDF and ECDF is higher especially at the right tail, even at null hypothesis, due to less number of observations. This results in degradation of the performance, especially, at low SNR. To alleviate this problem, in this thesis, a concept of Type-II right censoring is used. In this approach, we drop some observations in the right tail and determine the statistics using retained observations. We call it as Censored Anderson Darling (CAD) sensing scheme. This proposed CAD scheme makes receiver simple and also outperforms the ED sensing and OS sensing at lower values of SNR in receiver operating characteristics (ROC). Further, we have assumed imperfect value of variance of noise in CAD sensing, called as Blind-CAD (B-CAD), and shown the performance.

The above-mentioned GoF sensing schemes have assumed PU as a constant signal. However, the performance of AD sensing with different PU signals such as independent and identically distributed (*i.i.d*) Gaussian and single frequency sine signals is degraded. Hence, we propose a Likelihood Ratio Statistics (LRS- G^2) sensing based on a likelihood ratio statistic (G^2) using robust normality test, which outperforms all the prevailing GoF based sensing along with ED in various scenarios such as different structures of PU, different channel conditions and unknown variance of noise.

Till now, the background noise or thermal noise is modelled using Gaussian distribution. However, in radio channel, it may be non-Gaussian noise (NGN) due to a mixture of man-made and natural electromagnetic sources. Unfortunately, sensing schemes, designed for additive Gaussian noise, do not perform well in NGN environment. Therefore, we assume narrow band interference as NGN at CR which is modelled using Middleton Class-A interference model. The proposed LRS- G^2 sensing is also used assuming this Middleton Class-A NGN environment and we show that the effect of Gaussian noise in ROC is worst compared to non-Gaussian noise.

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List of Abbreviations

AD	Anderson Darling
AWGN	Additive White Gaussian Noise
B-CAD	Blind Censored Anderson Darling
BER	Bit Error Rate
CAF	Cyclic Autocorrelation Function
CDF	Cumulative Distribution Function
CFD	Cyclostationary Feature Detector
CR	Cognitive Radio
CSD	Cyclic Spectrum Density
CvM	Cramer-von-Misses
DTV	Digital Television
DVB-T	Digital Video Broadcasting for Terrestrial
ECDF	Empirical Cumulative Distribution Function
ED	Energy Detection
EMI	Electro-Magnetic Interference
FCC	Federal Communication Commission
GoF	Goodness of Fit
HOS	Higher Ordered Statistics
i.i.d	Independent and Identically Distributed
ISM	Industrial Scientific Medical Band
JB	Jarque-Bera sensing
KS	Kolmogorov-Smirnov
LED	Light Emitting Diodes
LMS	Least Mean Square
LRS	Likelihood Ratio Statistics
MFD	Matched Filter Detection
MFSK	Minimum Frequency Shift Keying
MIMO	Multiple Input Multiple Output
NGN	Non-Gaussian Noise
OSA	Opportunistic Spectrum Access
PDF	Probability Density Function
PIT	Probability Integration Transform
QoS	Quality of Service
ROC	Receiver Operating Characteristics
SA	Sign Algorithm
SDR	Software Defined Radio
WBD	Wavelet based detection

Chapter 1

Introduction

In the last couple of years, the wireless networks have been the fastest growing industry in the field of telecommunications. A large number of users are trying to connect seamlessly with one another via several applications with mobile devices. With ever increasing demand of bandwidth and limited onboard resources, the existing wireless services are struggling to provide higher data rate, Quality of Service (QoS), ubiquitous connectivity, higher speed, machine intelligence, scalability etc. These are the major requirements to meet the future wireless networks like Machine-to-Machine communication and Internet of Things (IoT), where we expect in the order of 10^6 devices may share the resources (Wu et al.). The constraints for fulfilling the demands are spectrum and power.

While transmission power may be limited for some wireless systems, the spectrum is even more limited, and is in fact becoming a scarce resource. With the rapid progress in wireless communication services, the majority of spectrum has already been licensed or designated for some special applications. The fixed amount of spectrum is not a feasible option for growing number of wireless applications. This shortage of spectrum becomes a severe bottleneck for the development of emerging wireless systems.

In the recent report (FCC White Report), the Federal Communication Commission (FCC) published that the majority of the spectrum, which has been licensed, remains under-utilized i.e. majority of the spectrum resource has not been used at all the time in all the places. Recently, in India also, the spectrum measurement campaign has been initiated and actual spectrum occupancy in UHF band of 470

MHz to 590 MHz has been measured, and the similar trend is reported (Naik et al.). Therefore, optimum utilization of spectrum becomes a thrust area in research community.

An unused portion of frequency band of primary users (licensed users) is called spectrum hole (Tandra, Mishra, and Sahai). One of the possible ways to exploit the spectrum hole is via opportunistic spectrum access (OSA). It allows cognitive radio users (CR) or unlicensed users to reuse the spectrum which is generally licensed to primary users. However, the priority of the primary users for the spectrum must be observed to avoid the interference. The OSA calls for new radio technology for the CR. Simultaneously, a new radio concept, cognitive radio, was proposed based on Software Defined Radio (SDR) (Mitola). The CR supports the idea of OSA to resolve the problem of spectrum under utilization.

The basic idea of CR is for the radio to have cognition capability, to learn from the environment and adapt its transmissions from the environment (Haykin). To support OSA, the CR has to conduct spectrum sensing to locate spectrum hole. Furthermore, it chooses the available frequency band to use automatically, especially from the licensed users. In this way, CR can enhance the utilization of spectrum and give solution for spectrum scarcity for future wireless communications. However, to maintain priority of PU, i.e. least interference to PU, is a challenging task. This motivates us to work on spectrum sensing algorithms in the field of CR.

1.1 Motivation

The cognitive radio user (CR) has to detect the unused channels of primary user (PU) and initiate the communication between CR in licensed band without creating interference to PU. This operation is called as spectrum sensing (Haykin). The spectrum sensing function is suffered by multipath fading, receiver uncertainty, interference etc. Therefore, design of a spectrum sensing algorithm for future wireless communications is a challenging problem in research community. In last couple of years, many efforts are initiated by researches to provide spectrum access in opportunistic way. There are different local spectrum sensing techniques such as cyclo-stationary detection, feature detection, matched filtering etc (Akyildiz et al.)(Yucek and Arslan)(Axell et al.).

All the spectrum sensing techniques can be classified in two categories: para-

metric and non-parametric sensing. In the parametric methods, certain parameters of PU are assumed to be known at the CR terminal. However, inaccurate assumptions of the known parameters lead towards degradation in detection performance. Hence, non-parametric spectrum sensing is matter of interest, where no information about PU is assumed but statistics of background noise is assumed to be known. In this category, Energy Detection (ED) (Urkowitz) (Digham, Alouini, and Simon) (Chen) is the simplest method. However, when there is in accurate information of statistics of noise, ED is susceptible to low signal-to-noise ratio (SNR). The detection performance of ED degrades significantly under noise uncertainty conditions (Tandra and Sahai) and introduces SNR wall.

Recently, some goodness of fit (GoF) based sensing schemes have been proposed in the category of non-parametric sensing. In this kind of sensing, empirical cumulative distribution function (ECDF) is determined using the received observations, denoted by F_n . This ECDF is compared with known cumulative distribution function (CDF) of noise (F_0) or we test the null hypothesis ($F_n=F_0$). The deviation of ECDF from the known CDF of noise (F_0) decides presence or absence of PU. The methods, based on this concept, are called as GoF based sensing.

The GoF based different prevailing sensing schemes are Anderson Darling sensing (Wang et al.), two sample Kolmogorov-Smirnov (KS) sensing (Zhang et al.), one sample KS sensing (Arshad, Briggs, and Moessner), sensing based on Jarque-Bera test (Lu, Wu, and Iyengar), Cramer-von-Misses sensing (Kieu-Xuan and Koo), Student t -sensing (Arshad and Moessner), Higher Order statistics (Denkovski, Atanasovski, and Gavrilovska) and Order statistics based sensing (Rostami, Arshad, and Moessner). All these GoF based sensing methods outperform ED sensing methods considering additive background noise as white Gaussian. The focus of this thesis is GoF based non-parametric sensing.

In summary, based on the survey on GoF, following are identified as the key motivations.

- (a) In determination of ECDF, the received observations of higher amplitudes are very few. These observations may be avoided which reduces complexity of the detection algorithms.

- (b) The prevailing GoF based sensing methods perform well in Additive White Gaussian Noise (AWGN) or quasi-static channels. It is of interest to observe the performance of these methods in the real time scenario of time varying channels.
- (c) The prevailing GoF based sensing schemes assume PU signal as a constant. However, in the real time scenario, it may be random or deterministic as a single tone sine signal.
- (d) In practice, high detection probabilities are expected for lower number of received observations (n), i.e. for less sensing time. The prevailing GoF based sensing methods have obtained higher detection performance with the range of $n = 10^3$ to $n = 10^6$. As per the time bandwidth product, the sensing time will be higher in such conditions. Hence, achieving higher detection performance at lower observations in the range of $n = 14$ to $n = 50$ is a matter of interest.
- (e) The majority of the sensing schemes have chosen the value of false alarm probability (P_f) as 0.1 based on TV white space standard IEEE 802.22. This value of P_f shows upper maximum limit. In actual practice, it should be low enough to reduce the false detection of PU and reducing the total error at CR. Hence, it is of the interest to develop non-parametric sensing scheme which works under the lower range of false alarm probabilities of 10^{-3} to 0.05.
- (f) Generally, for mathematical tractability, the gaussian noise with known variance is considered under H_0 . In actual practice, noise variance is not known a priori. Hence, complete blind detection scheme, i.e unknown PU signal as well as noise variance also, is more realistic approach to detect PU at CR.
- (g) The noise due to different sources (man-made noise, co-channel interference, microwave oven etc) and environmental perturbances gives impulsive characteristics. Hence, to consider Gaussian noise (thermal noise or background noise) under H_0 is not sufficient. Under the non-Gaussian noise environment, the performance of GoF based sensing is also of interest.

1.2 Thesis Contributions

In this thesis, the major contributions are:

First, to alleviate the problem of utilizing all observations at cognitive radio user (CR), a novel non-parametric sensing scheme based on the concept of Type-II right censoring (Lawless) is proposed. In this approach, we drop some observations in the right tail and determine the statistics using retained observations. We call it as Censored Anderson Darling (CAD) sensing scheme. This proposed CAD scheme is evaluated under AWGN channel, quasi-static channel and time varying channel using first order autoregressive model (AR).

Second, to make sensing operation realistic, we have assumed that the noise variance is unknown under H_0 . With the imperfect value of the variance in CAD sensing, a new scheme is proposed called as Blind-CAD (B-CAD). We have presented the comparative performance with other non-parametric schemes such as Blind AD sensing and ED sensing.

Third, the existing GoF based sensing schemes give better detection performance with the assumption of constant PU signal. However, for different PU signals such as independent and identically distributed (*i.i.d*) Gaussian and single frequency sine signals, ED sensing outperforms the GoF sensing schemes. Hence, we develop a novel scheme called as Likelihood Ratio Statistic (LRS- G^2) sensing based on a robust normality test for the gaussian noise environment. The proposed scheme outperforms all the prevailing GoF based sensing along with ED in various scenarios such as different structures of PU, different channel conditions and unknown variance of noise. Fourth, till now, the background noise or thermal noise is modeled using Gaussian distribution. However, in radio channel, the actual noise may be non-Gaussian noise (NGN) due to a mixture of man-made and natural electromagnetic sources. Unfortunately, sensing schemes, designed for additive Gaussian noise, do not perform well in NGN environment. Therefore, we assume Middleton Class-A noise model under H_0 as NGN at the CR. The proposed LRS- G^2 sensing is evaluated under NGN assumption.

1.3 Organizations of the Thesis

This thesis is organized and structured as follows:

Chapter 2 gives the overview spectrum sensing schemes in cognitive radio, focussing on non-parametric sensing schemes. This chapter describes the basics of goodness of fit technique to formulate the problem of primary user detection using null hypothesis testing (H_0).

Chapter 3 proposes a novel Censored AD (CAD) spectrum sensing scheme in the non-parametric category. The concept of Type-II right censoring is introduced to develop the proposed scheme. The detection performance of the proposed scheme is evaluated in different channel conditions such as AWGN, quasi-static and time varying channel which is modeled using autoregressive process.

Chapter 4 evaluates the performance of CAD sensing algorithm assuming noise uncertainty conditions. This blind detection scheme is proposed using student- t distribution. The performance of this Blind CAD (B-CAD) scheme is compared with the existing Blind-AD and Energy detection scheme.

Chapter 5 proposes robust LRS- G^2 sensing scheme assuming different types of PU signals and fading channels. The detection performance is presented and compared with prevailing GoF based sensing taking low false alarm probability and less number of received observations at CR.

Chapter 6 investigates the performance of LRS- G^2 sensing scheme in non-gaussian noise (NGN) environment. This NGN environment is modelled using Middleton Class-A model. Under such assumption, the detection performance is evaluated with approximate analysis and simulation both. Furthermore, the effect of different parameters in Middleton Class-A model such as Gaussian to non-Gaussian ratio (Γ) and impulsive index (A) on the detection probabilities are presented.

Chapter 7 concludes the thesis with summary of contributions and suggests recommendations for the further research. Also, some challenges are discussed.

Chapter 2

Literature Survey

Due to transition from voice-only communications to multimedia type applications, demand of higher data rates is increasing day by day. However, due to limitations of the natural frequency spectrum, the current static frequency allocation schemes can not accommodate the requirements of an increasing number of higher data rate devices. Therefore, we need to device some innovative techniques to exploit the available spectrum efficiently. One of the techniques is Cognitive Radio (Mitola). It is a new promising technology focussed to reduce the problem of spectrum scarcity in wireless communications. The unlicensed users or cognitive radio users (CR) are allowed to access the unused spectrum of licensed users or primary users (PU). These frequency bands are assigned such that they does not affect the quality of service (QoS) of the licensed network (Haykin).

The research in cognitive radio has been encouraged by the measurements of the Federal Communications Commission (FCC), which have revealed that there is a significant amount of licensed spectrum which remains largely under utilized in vast temporal and geographic dimensions. For instance, a field spectrum measurement results, taken in New York city, has shown that the maximum total spectrum occupancy is only 13.1% for 30 MHz to 3 GHz. Recently, measurement of TV white space in the 470-590 MHz band, carried out across India, (Naik et al.) has shown similar results. Due to under utilization of the licensed spectrum all over the world, the FCC has recently issued Notice of Proposed rule making regarding cognitive radio that requires rethinking of the wireless communication architectures so that emerging radios can share spectrum with PUs without causing harmful interference to them

(FCC White Report). The FCC has also allowed the access of unlicensed users to the broadcast television spectrum at locations where that spectrum is not being used by licensed services.

The key element of cognitive radio is the spectrum sensing. Spectrum sensing methods are classified by two ways, parametric and non-parametric. The focus of this thesis is on non-parametric sensing. In this chapter, a spectrum sensing problem as a binary hypothesis testing is presented. We also present survey of some parametric and non-parametric sensing methods. In the non-parametric sensing methods, we present GoF based sensing, where a spectrum sensing problem is considered as a null hypothesis testing. Finally, we define some problem statements of our interest.

2.1 Binary Hypothesis Testing

In cognitive radio networks, one important function of the cognitive users (CR) is to detect the presence of primary users utilizing the channel, and to access the channel in such a way that it causes a little performance degradation to the primary users. Designing fast and accurate spectrum sensing algorithm is a challenging task. In general, spectrum sensing is a problem of detection theory, which is considered as a binary hypothesis testing for the following hypotheses:

$$\begin{aligned} H_0 &: \text{There is only noise.} \\ H_1 &: \text{There is a signal transmitting.} \end{aligned} \tag{2.1}$$

After collecting some samples, a statistic y can be calculated for each sensing method, and compared with some threshold to make a decision.

The algorithms for spectrum sensing seek to balance the conflicting goals of minimizing interference to the PU while maximizing the throughput of the CR. Therefore, performance of a sensing algorithm is typically characterized in terms of the probability of detection P_d , i.e. to sense the existence of the PU and the probability of false alarm P_f , i.e. falsely declaring that the PU is active and thus missing a spectrum opportunity.

The probability of false alarm is defined as

$$P_f = \mathbb{P}\{y > \lambda | H_0\} \tag{2.2}$$

and the probability of detection as

$$P_d = \mathbb{P}\{y > \lambda | H_1\} \quad (2.3)$$

Thus, P_f is related to the throughput of the cognitive radio system, while P_m is related to the interference to the primary system, where $P_m = 1 - P_d$ is probability of miss detection. The tradeoff between P_d and P_f is a crucial task. The plot of probability of detection (P_d) against probability of false alarm (P_f) is called Receiver Operating Characteristic (ROC) curve.

2.2 Parametric Spectrum Sensing

In parametric sensing method, the CR uses some available information of PU in the detection. Some parameters of the transmitted signal are known a priori at CR. Under this category many spectrum sensing schemes are proposed. Few of them are discussed in brief such as Matched filtering based detection, Feature detection, Wavelet based detection and Waveform-Based Sensing. The exhaustive survey on spectrum sensing has been presented in (Yucek and Arslan), (Zeng et al.), (Wang and Liu), (Axell et al.) and (Lu et al.).

2.2.1 Matched Filter based Detection

This is an optimum sensing scheme (Proakis). It is derived based on the correlation with the received signal. A matched filter based detection (MFD) gives the maximum detection performance. However, it has very strict assumption that the transmitted PU signal must be known a priori at the CR. In actual practice, the PU signal is not known. The MFD maximize the output SNR in the presence of background noise. This sensing scheme can be applicable only in the scenario where the PU signal is known completely.

The major advantage of the MFD is that the highest detection probabilities can be achieved at lower false alarm probability and reduced sensing time. On the other hand, the drawback is that the MFD can be implemented for the particularly one PU signal only. Hence, multiple radios should be implemented to accommodate more number of PU signals which increases the complexity. In addition to this, MFD requires perfect synchronization. The detection performance of MFD is also severely

degraded due to errors in received pilot frames or preambles due to the time varying wireless channel.

In digital video broadcasting for terrestrial operations (DVB-T), the pilot based MFD has been proposed in (Lv et al.). It is exclusively designed for the PU signal as the DVB-T signal. Furthermore, the entropy based MFD has been proposed in (Nagaraj), where the MFD output in the form of estimated entropy is compared with the threshold.

2.2.2 Waveform Based Detection

In the transmitted PU signals, some known patterns are transmitted for the synchronization purpose. The known patterns may be spreading sequences, midambles, preambles, pilot patterns etc. Such sequences are effectively utilized in current GSM and CDMA wireless networks. Hence, sensing can be done by correlating the received PU signal with a known copy of the pattern as proposed in (Tang). The waveform based sensing scheme can be applicable only when, the CR knows signal patterns of transmitted signal a priori. Therefore, it is also called as coherent sensing. The transmitted signal is deteriorated by the multipath fading, so the signal patterns also become erroneous. In such scenario, the detection performance of coherent sensing degrades quickly due to synchronization error at the CR. Let us consider,

$$\begin{aligned} H_0 : x_j &= n_j \\ H_1 : x_j &= s_j + n_j, \end{aligned} \tag{2.4}$$

where $j = 1, 2, \dots, N$ and s_j are samples of known pattern of PU and n_j are sample of noise signal. Using waveform based sensing or coherent sensing, decision statistics y can be expressed as

$$y = \text{Re} \left[\sum_{j=1}^N x_j s_j^* \right], \tag{2.5}$$

where $(*)$ denotes conjugate operation. In the absence of the PU i.e. under H_0 , the statistics will be

$$y = \text{Re} \left[\sum_{j=1}^N n_j s_j^* \right], \tag{2.6}$$

In the presence of the PU i.e. under H_1 , the statistics will be

$$y = \sum_{j=1}^N |s_j|^2 + Re \left[\sum_{j=1}^N n_j s_j^* \right], \quad (2.7)$$

The decision on the presence of a primary user signal can be made by comparing the decision metric y against a fixed threshold λ .

The waveform based sensing outperforms ED in the context of reliability and convergence time (Tang). Furthermore, as discussed in (Cabric, Tkachenko, and Brodersen), the waveform based sensing needs lower sensing time to detect PU signal. It is also shown that the higher detection performance can be achieved as the length of the known signal pattern increases. In (Geirhofer, Tong, and Sadler), the preambles of IEEE 802.11b are utilized for analyzing the performance of WLAN.

2.2.3 Feature Detection

The feature detector is also known as cyclostationary feature detector (CFD). The CFD utilizes the cyclostationary feature of the transmitted PU signals. The statistical properties of PU signal such as mean and autocorrelation changes with time which helps to design efficient sensing algorithm. It can be realized using cyclic autocorrelation function (CAF) of the received PU signal. The fourier series expansion of CAF gives the cyclic spectrum density (CSD). The CSD generates spikes, when the PU signal is present. It generally happens when the cyclic frequency (carrier frequency or symbol or chip code) and fundamental frequency of the transmitted PU signal matches. In the presence of noise signal, no spikes are generated due to non cyclostationary behaviour of noise signal. Hence, based on the CSD, a CFD decides the presence or absence of PU signal.

The main advantage of CFD is that it can distinguish the noise and transmitted PU signal at very low SNR i.e a weak signal can be detected using CFD. However, the PU signals are required to be oversampled with respect to the symbol rate or the chip rate to use the periodicity features of the CFD. Hence, the computational complexity of such detectors is always high.

An optimum multicycle spectral correlation based feature detector has been proposed in (Gardner). This detector requires phase information of the PU signal in advance. The CFD based on cyclic frequencies is presented in (Gardner and Spooner).

The CFDs for CDMA signal in DVB-T and universal mobile telecommunication system (UMTS) signal have been proposed in (Goh, Lei, and Chin) and (Marques, Bastos, and Gameiro) respectively. The CFD for cognitive radio application has been proposed in (Sutton, Nolan, and Doyle).

2.2.4 Wavelet Based Detection

Wavelet transform is a multi-resolution analysis, where an input signal is decomposed into different frequency components, and then each component is studied with resolutions matched to its scales. The wavelet transform uses irregularly shaped wavelets as basic functions and thus represents sharp changes and local features. For signal detection over wide-band channels, the wavelet approach offers advantages in terms of both implementation cost and flexibility in adapting the dynamic spectrum, as opposed to the conventional use of multiple narrow-band bandpass filters.

A Wavelet based detection (WBD) is a filter bank approach for spectrum sensing is proposed in (Tian1 and Giannakis). The input signal has to passed through a bank of filters. The output power of the each filter is measured as an estimate of the spectral power over the associated sub-band. The accurate temporal and frequency analysis can be done with high and low frequency components of wavelets respectively as discussed in (Tian2 and Giannakis). The wavelet has good property for describing the singularities which help in spectrum sensing. It is specially helpful for detecting sharp band edges in OFDM signals.

2.3 Non-parametric Spectrum Sensing

In case of non-parametric sensing methods, CR does not use any information of PU in detection. The various non-parametric spectrum sensing schemes are discussed in this section. First, we discuss Energy Detection (ED) (Urkowitz) based sensing. Then, we consider spectrum sensing as Null hypothesis testing problem, which is known as Goodness of Fit (GoF) based sensing. Few of them are Anderson Darling sensing (Wang et al.), Kolmogorov-Smirnov sensing (Arshad and Moessner), Order statistics based sensing (Rostami, Arshad, and Moessner), Cramer-von-Misses sensing (Kieu-Xuan and Koo) and Jarque-Bera sensing (Lu, Wu, and Iyengar).

2.3.1 Energy Detection

The energy detector (ED) is one of the most commonly employed spectrum sensing schemes, since it does not require any prior knowledge about the structure of the PU signal as proposed in (Urkowitz) and (Digham, Alouini, and Simon). In ED, non-coherent detection method is used in which the energy of the received signal is used to determine the presence of primary signals. Energy detection is essentially based on the difference between the energy of the transmitted signal and that of the interfering noise. The energy of the samples is compared with predefined threshold (λ). If the energy exceeds the threshold, we say that PU is present otherwise it is absent. For the AWGN channel, let the received symbols are represented as (Urkowitz),

$$x_j = \sqrt{\gamma_a} s_j + n_j, \quad j = 1, 2, \dots, n \quad (2.8)$$

where s_j is j^{th} BPSK symbol of PU and $s_j \in \{-1, 1\}$, received at the CR terminal. In (2.8), γ_a indicates average SNR of a symbol and n_j represents real AWGN i.e. $n \sim \mathcal{N}(0, 1)$. For energy detection, we have the test statistics,

$$y = \sum_{j=1}^n x_j^2 \quad (2.9)$$

Here, y is the decision variable. If $y > \lambda$, the PU signal is present and $y \leq \lambda$, the PU signal is absent. For a given hypothesis H_0 , y is a central chi-square distributed variable with n degrees of freedom, and the probability density function

$$p_{Y|H_0}(y|H_0) = \frac{y^{n/2-1} e^{-y/2}}{2^{n/2} \Gamma(n/2)}, \quad y \geq 0, \quad (2.10)$$

where $\Gamma(\cdot)$ is the gamma function. Now,

$$\begin{aligned} P_f &= \mathbb{P}\{y > \lambda | H_0\} \\ &= \int_{\lambda}^{\infty} p_{Y|H_0}(y|H_0) dy \\ &= 1 - \Gamma(n/2, \lambda/2), \end{aligned} \quad (2.11)$$

where $\Gamma(a, x)$ is incomplete gamma function. For a given hypothesis H_1 , the distribution of y follows a non-central chi-square distribution with n degrees of freedom and the probability density function

$$p_{Y|H_1}(y|H_1) = \frac{1}{2} \left(\frac{y}{\gamma_c} \right)^{\frac{n-2}{4}} e^{-(y+\gamma_c)/2} I_{n/2-1}(\sqrt{y\gamma_c}) \quad (2.12)$$

Now,

$$\begin{aligned}
P_d &= \mathbb{P}\{y > \lambda | H_1\} \\
&= \int_{\lambda}^{\infty} p_{Y|H_1}(y|H_1) dy \\
&= \mathbb{Q}_{N/2}(\sqrt{N\gamma_a}, \sqrt{\lambda}),
\end{aligned} \tag{2.13}$$

where $\mathbb{Q}_N(a, b)$ is the generalized Marcum Q-function.

Although the energy detection approach can be implemented without any prior knowledge of the PU signal, it has poor performance under low SNR conditions. This is because the noise variance is not accurately known at the low SNR and the noise uncertainty may render the energy detection useless. Furthermore, Energy detector is unable to differentiate the interference from other CR users sharing the same channel and the PU.

2.3.2 Goodness of Fit based Sensing

The performance of the ED is degraded at low SNR or in the presence of imperfect information of noise variance. Now, we discuss about other type of non-parametric sensing, in which spectrum sensing problem is considered as null hypothesis testing instead of binary hypothesis testing (Wang et al.).

In Null hypothesis testing based sensing, the detection of PU signal is done based on the known CDF of noise and the empirical CDF (ECDF) of received observations. If the deviation between them is higher than a specific threshold, we declare PU is present otherwise PU is absent. Detailed procedure for this GoF based sensing is as follows.

Let $Y = \{Y_i\}_{i=1}^n$, represents n real valued observations available at the CR. The $Y_1, Y_2 \dots Y_n$ are received samples drawn from the noise distribution in the absence of the PU signal. We assume that noise samples are independent with known cumulative distribution function CDF $F_0(y)$. If PU signal is present, then the CDF of the received observations (i.e. ECDF) is deviated from the CDF of noise. Thus, the presence or absence of PU is equivalent to test the null hypothesis (H_0),

$$H_0 : Y \text{ is an i.i.d sequence drawn with distribution } F_0(y) \tag{2.14}$$

It is important to note that no information about the PU signal is required a priori.

Let $F_Y(y)$ be the empirical distribution of the received observations Y , which can be determined as

$$F_Y(y) = |\{i : Y_i \leq y, 1 \leq I \leq n\}|/n \quad (2.15)$$

Where, $|\cdots|$ shows cardinality. When the PU signal is not present, the $F_Y(y)$ converges to $F_0(y)$ with unity probability as n approaches infinity. On the other hand, in the presence of PU signal, the $F_Y(y)$ deviates from $F_0(y)$ and we reject the null hypothesis (H_0). To measure the distance between the $F_Y(y)$ and to $F_0(y)$, various null hypothesis tests called as goodness of fit (GoF) test such as Anderson Darling test, Kolmogorov-Smirnov test, Order statistics, Cramer-von-Misses test and Jarque-Bera test have been proposed in the literature.

Anderson-Darling Sensing

The Anderson-Darling (AD) sensing has been proposed in (Wang et al.) for the AWGN channel. The AD test is conducted assuming PU signal as unity. The brief procedure for detecting the PU signal is as given below:

- The critical value (threshold), denoted by t , is determined at a desired P_f using limited distribution given in (Anderson and Darling),

$$\begin{aligned} \lim_{n \rightarrow \infty} P_r(A_c^2 < t | H_0) &= \frac{\sqrt{2\pi}}{t} \sum_{j=0}^{\infty} a_j (4j+1) \exp\left(-\frac{(4j+1)^2 \pi^2}{8t}\right) \\ &\times \int_0^{\infty} \exp\left(\frac{t}{8(w^2+1)} - \frac{(4j+1)^2 \pi^2 w^2}{8t}\right) dw \end{aligned} \quad (2.16)$$

Where, $a_j = \frac{(-1)^j \Gamma(j+0.5)}{\Gamma(0.5)j!}$ and Γ denotes gamma function. It is observed that computation of the threshold (t) from the (2.16) is not straightforward. Hence, pre-defined statistical tables are utilised to get the value of threshold as given in (Anderson and Darling).

- Let Y be the n received observations, where $Y = \{Y_1, Y_2, \dots, Y_n\}$. Without loss of generality, we assume that all the observations are in ascending order, i.e $Y_1 \leq Y_2 \leq Y_3 \dots \leq Y_n$.

- The test statistic A_c^2 is computed as,

$$\begin{aligned} A_c^2 &= n \int_{-\infty}^{\infty} (F_Y(y) - F_0(y))^2 \Phi(F_0(y)) dF_0(y) \\ &= - \sum_{i=1}^n \frac{(2i-1)(\ln Z_i + \ln(1 - Z_{n+1-i}))}{n} - n, \end{aligned} \quad (2.17)$$

where, $Z_i = F_0(Y_i)$ and $\Phi(F_0(y))$ represents the weighting function. The $F_0(y)$ and $F_Y(y)$ represent the CDF under hypothesis H_0 and ECDF respectively. For a special case of AWGN channel and, they may be defined as

$$F_0(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{x^2}{2}} dx \quad (2.18)$$

and,

$$F_Y(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{(x-\sqrt{\rho})^2}{2}} dx \quad (2.19)$$

where ρ is received SNR.

- Finally, compare AD Test statistics A_c^2 as defined in (2.17) with threshold (2.16). Declare a PU is present if $A_c^2 > t$, otherwise declare PU is absent.

The AD sensing gives a higher detection performance as compared to ED sensing at the low number of received observations with assumption of constant PU and AWGN channel. However, for the different structure of PU signal such as Gaussian or single tone sine signal, the performance of AD sensing is degraded compared to ED sensing (Nguyen-Thanh, Kieu-Xuan, and Koo). Furthermore, the performance of AD sensing has been presented, assuming wireless channels, in (Lei, Wang, and Shen).

Kolmogorov-Smirnov Sensing

The Kolmogorov-Smirnov (KS) sensing has been proposed in (Arshad, Briggs, and Moessner) and (Arshad and Moessner), which is based on the KS test. The KS sensing has been proposed for the rician fading environment and constant PU signal. The decision process at CR is similar to AD sensing process. However, the KS statistic is computed in different manner. If, out of n , the i^{th} received observation is denoted by y_i , then KS statistic is defined as,

$$\begin{aligned} \mathbb{T}_{KS} &= \sup \{ |F_n(y_i) - F_0(y_i)| : -\infty < y_i < +\infty, 1 \leq i \leq n \} \\ &= \max(D^+, D^-) \end{aligned} \quad (2.20)$$

where, $D^+ = \max_i \left\{ \frac{i}{n} - Z_{(i)} \right\}$ and $D^- = \max_i \left\{ Z_{(i)} - \frac{i-1}{n} \right\}$. The $Z_i = F(y_i)$ is defined using Probability Integral Transform (PIT) i.e computing CDF from the received observations.

The threshold for KS sensing, i.e. λ_{KS} , for a given P_f can be calculated as,

$$P_f = 1 - F(\mathbb{T}_{KS} | H_0; \lambda_{KS}) \quad (2.21)$$

The detection of PU is done at CR if the $\mathbb{T}_{KS} > \lambda_{KS}$, than Hypothesis H_0 is rejected or PU is present.

The above-mentioned test is considered as one sample test, where we determine ECDF of noise. However, two samples KS test has been proposed (Zhang et al.), where ECDF is computed for both noise and received signal. Here, KS sensing and sequential KS sensing are used. This test is considered in 2×2 MIMO system model to evaluate the performance and compared with ED sensing (Digham, Alouini, and Simon) and covariance based detection (Zeng and Liang2).

Order Statistic Sensing

The Order Statistic (OS) based sensing has been proposed in (Glen, Leemis, and Barr),(Rostami, Arshad, and Moessner) for AWGN channel. Similar to AD and KS sensing methods, the OS sensing is applicable to any noise distribution under H_0 . However, the noise distribution should be known a priory. Furthermore, the OS sensing is based on the quantiles of ordered observations in the distributions. These quantiles are presented as ρ vector and used to compute the test statistic. The extreme values in the ρ -vector indicates poor fit with the noise distribution under H_0 . A brief overview of this sensing algorithm is discussed below:

- Let $\mathbf{y} = [y_1, y_2, \dots, y_n]$ be received observations at CR terminal.
- Based on the PIT elements (z_i) of received observations y_i are, $z_i = F_0(y_i)$, $i \in S$, where $\mathbf{z} = [z_1, z_2, \dots, z_n]^T$ and $S = 1, 2, 3 \dots n$.
- Without loss of generality, we will assume that all the elements in \mathbf{z} are in ascending order, i.e. $z_1 \leq z_2 \dots \leq z_n$.
- The ρ -vector is computed as,

$$\rho_i = \beta(z_i; i, n - i + 1), i \in S, \quad (2.22)$$

where $\beta(y; \alpha, \beta)$ denotes beta CDF with α and β as shape parameters of the distribution.

- Without loss of generality, the authors (Rostami, Arshad, and Moessner) have assumed that the elements in ρ are in ascending order, i.e the ρ is $\rho = [\rho_1, \rho_2, \dots, \rho_n]^T$.
- The test statistic τ_{os} is calculated according to the formula given below:

$$\tau_{os} = \sum_{i \in S} \left| \rho_i - \frac{i}{(n+1)^2} \right| \quad (2.23)$$

- The threshold (λ_{os}) in the detection rule is approximated as

$$\lambda_{os} = 2.599 + 0.8228n - 30.79P_{FA} + 73.79P_{FA}^2 - 49.08P_{FA}^3 - 0.6466P_{FA}n \quad (2.24)$$

- Finally, compare OS Test statistic (τ_{os}) is compared with threshold λ_{os} . A PU is declared present if $\tau_{os} > \lambda_{os}$, otherwise PU is absent.

The OS sensing outperforms AD sensing and ED sensing both in AWGN channel and lower number of observations.

Cramer-von-Mises sensing

The Cramer-von-Mises (CvM) sensing has been proposed in (Kieu-Xuan and Koo) for AWGN channel. It is also a type of null hypothesis testing. It gives better detection performance than ED sensing and KS sensing. However, the AD sensing outperforms CvM sensing. In (Kieu-Xuan and Koo), lower bound on detection probability has been derived.

Jarque-Bera Sensing

The first CR standard is defined for TV white space detection and it is IEEE 802.22. In this standard, DTV signal should be detected effectively at required false alarm probability. With this background, Jarque-Bera (JB) sensing has been proposed in (Lu, Wu, and Iyengar). The PU signal is considered as DTV signal. The detection performance is compared with the higher ordered statistics (HOS) based sensing as proposed in (Denkovski, Atanasovski, and Gavrilovska). The JB sensing outperforms HOS based sensing at low SNR regime.

2.4 Conclusion

The spectrum sensing is a key element in cognitive radio technology. In this chapter, we discuss mainly about two types of spectrum sensing techniques such as parametric and non-parametric sensing. In non-parametric sensing, we define the problem of spectrum sensing as a null hypothesis testing, known as GoF based sensing. Further, we present different GoF based sensing such as AD, KS, OS, CvM and JB sensing methods. In the next chapter, we study AD sensing with an interest of less computational complexity by processing less number of observations. We call this scheme as Censored AD sensing.

Chapter 3

CAD Sensing using Type-II Right Censoring

In Goodness of Fit (GoF) based sensing, the detection problem is formulated as Null hypothesis testing instead of binary hypothesis testing. In this case the CDF of noise or CDF under null hypothesis (H_0) is perfectly known at cognitive radio user (CR). Based on the deviation of this CDF with the empirical CDF of received observations, the presence or absence of PU is determined. The different GoF based sensing such as AD, OS, CvM and KS sensing are discussed in Chapter 2.

All the above-mentioned GoF based sensing methods have used all observations to determine ECDF. However, the deviation of the CDF and ECDF is higher especially at the right tail due to less number of observations. This incomplete information of CDF on the right tail introduces an error in determining statistics in GoF based sensing, especially at low SNR. To alleviate this problem, we have used the concept of censored data which has already been used in survival analysis (Lawless). In view of this, we drop some observations in the right tail, which carry incomplete information of the CDF. Furthermore, we use less number of received observations for processing in case of the proposed CAD sensing compared to the observations in case of AD sensing (Wang et al.). This leads towards reduced processing cost after censoring. In some applications such as wireless sensor network based on cognitive radio, we can save energy of node (CR) by processing less number of observations. In this chapter, we present receiver operating characteristics (ROC) of the CAD sensing in AWGN and time varying channel, which is modelled by first order autoregressive

(AR1) process.

3.1 System Model

Let $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$ be the received signal vector at CR, where n denotes total number of observations. We assume received observations are i.i.d real valued and each y_i is represented as,

$$y_i = \sqrt{\rho}hs + w_i, \quad i = 1, 2, 3, \dots, n, \quad (3.1)$$

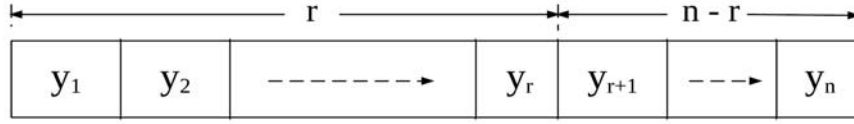
where $s \in \{0, 1\}$, ρ is the received SNR, h represents the channel fading coefficient, which is assumed to be random variable with the standard normal distribution. We also assume that the channel is quasi-static. In (3.1), w_i , where $1 \leq i \leq n$, denotes Gaussian noise samples. In (3.1), $s = 1$ and 0 denote presence and absence of PU respectively.

3.2 GoF Based Type-II Right Censoring

The GoF test is a statistical test for identifying the presence of certain distribution (D'Agostino). More specifically, all received observations are independent and identically distributed (i.i.d) random variables with cumulative distribution function (CDF), denoted by F . In this kind of sensing, the test of the null hypothesis ($F = F_0$) against the alternative hypothesis ($F \neq F_0$) has been done, where F_0 is available CDF of noise. For performing any GoF test, the empirical CDF (ECDF) is determined from the received observations. This ECDF is compared with the known CDF (F_0) under the null hypothesis. The distance of the ECDF from the CDF decides whether PU is present or absent.

We assume that all n observations are in ascending order after applying sorting operation. Without loss of generality, we assume that $y_1 \leq y_2 \leq \dots \leq y_n$. Now, apply the concept of censoring, first r observations are retained and the last $n - r$ observations are dropped or censored as shown in Fig.3.1. Hence, y_r is the highest valued observation. This method of censoring $n - r$ observations is known as Type-II right censoring. (Lawless).

In this scenario, the problem of spectrum sensing as null hypothesis testing

Figure 3.1: Number of received (n) and censored ($n - r$) observations

problem as GoF testing is defined as (Wang et al.),

$$\begin{aligned} H_0 &: F_Y(y) = F_0(y) \\ H_1 &: F_Y(y) \neq F_0(y) \end{aligned} \quad (3.2)$$

where $F_0(y)$ is known a priori.

For the proposed scheme, called as CAD sensing, the modified Cramer-von Mises GoF statistic is used to measure distance between $F_Y(y)$ and $F_0(y)$. Let $F_n(y)$ be the Empirical Cumulative Distribution Function (ECDF) of the received observations \mathbf{y} , which can be expressed as

$$F_n(y) = \frac{|\{i : y_i \leq y, 1 \leq i \leq n\}|}{n}, \quad (3.3)$$

where $|\cdots|$ indicates cardinality.

In this case, based on the asymptotic distribution of censored observations, statistic can be expressed as (Pettitt and Stephens),

$${}_{q,p}A_n^2 = n \int_q^p \frac{(F_n(y) - F_0(y))^2}{F_0(y)(1 - F_0(y))} dF_0(y), \quad 0 \leq q < p \leq 1, \quad (3.4)$$

where p denotes censoring ratio which can be expressed as

$$p = \lim_{n \rightarrow \infty} \frac{r}{n}.$$

Here, we take $q = 0$ for single censoring i.e Type-II right censoring. The same statistic can be written as,

$${}_pA_n^2 = n \int_0^p \frac{(F_n(y) - F_0(y))^2}{F_0(y)(1 - F_0(y))} dF_0(y) \quad (3.5)$$

The above quadratic statistics ${}_pA_n^2$ can be solved using integration by parts and approximated as (Pettitt and Stephens),

$$\begin{aligned} {}_pA_n^2 &= -\frac{1}{n} \sum_{i=1}^r (2i - 1)(\ln z_i - \ln(1 - z_i)) - 2 \sum_{i=1}^r \ln(1 - z_i) \\ &\quad - \frac{1}{n} [(r - n)^2 \ln(1 - z_r) - r^2 \ln z_r + n^2 z_r], \end{aligned} \quad (3.6)$$

where $z_i = F_0(y_i)$.

Based on censored observations, H_0 is rejected when ${}_pA_n^2 > \lambda$, where λ is the value of threshold. The λ is selected such that the false alarm probability (P_f) under H_0 is at a desired level α ,

$$\alpha = P\{{}_pA_n^2 > \lambda | H_0\} \quad (3.7)$$

To find λ , it is worth to mention that the distribution of ${}_pA_n^2$ under H_0 is independent of the $F_0(y)$. To observe this, apply probability integration transform (PIT) for available observations. Hence,

$${}_pA_n^2 = n \int_0^1 \frac{(F_z(z) - z)^2}{z(1-z)} dz, \quad (3.8)$$

where $z = F_0(y)$ and $F_z(z)$ denotes ECDF of z_i . Here, $z_i = F_0(y_i)$ for $1 \leq i \leq r$.

All statistics of observations up to z_r are independent and uniformly distributed over $[0, p]$, where $p \in [0, 1]$. As shown in (Wang et al.) for AD sensing, the distribution of A_n^2 is independent of the $F_0(y)$. The same is also true for the distribution of ${}_pA_n^2$. As given in (D'Agostino) (Pettitt and Stephens), the value of λ is determined for a specific value of P_f and censoring ratio p . For example, when $P_f = 0.05$ and $p = 0.4$, the value of λ is 1.133.

3.3 CAD Sensing Algorithm in Time-varying Channel using AR-1 Model

In the literature of GoF based sensing, the detection performance of spectrum sensing algorithms has been shown assuming Additive White Gaussian (AWGN) or quasi-static channel. Here, we consider time-varying channel, which is modelled by first order AR process (Gomadam and Jafar). Using Monte Carlo simulations, the receiver operating characteristics (ROC) is presented for different time-varying channel conditions.

3.3.1 Modified System Model

Let us consider a communication link in a time varying and flat fading channels, characterized by a first ordered autoregressive (AR1) model (Gomadam and Jafar)

$$h_i = ah_{i-1} + \sqrt{1-a^2}v_i, \quad (3.9)$$

where $h_i \sim \mathcal{N}(0, 1)$ and v_i denotes independent and identically distributed (*i.i.d*) as Gaussian with mean zero and variance one. In (3.9), a indicates correlation coefficient where $0 \leq a \leq 1$. Here $a = 1$ and $a = 0$ denote quasi-static and independent channel respectively. The value of a will be determined using Jake's autocorrelation function (Gomadam and Jafar). At cognitive radio user (CR), the received observations x_i , for $1 \leq i \leq N$, are real valued and represented as,

$$y_i = \sqrt{\rho}sh_i + w_i, \quad i = 1, 2, 3, \dots, n, \quad (3.10)$$

where $s \in \{0, 1\}$, ρ is the received SNR and additive noise w_i , for $1 \leq i \leq n$, are the samples from standard Gaussian probability distribution function. In (3.10), $s = 1$ and 0 denote presence and absence of PU respectively.

3.3.2 CAD Sensing Algorithm

Let us summarize, the above discussion in the following steps for CAD sensing algorithm:

Step:1 Find the threshold λ for a given probability of false alarm P_f using (3.7).

Step:2 Sort the received observations in ascending order. Without loss of generality, we assume that the received observations are in ascending order.

$$y_1 \leq y_2 \leq \dots \leq y_r \leq y_{r+1} \leq \dots \leq y_n,$$

where $y_{r+1} \leq y_{r+2} \leq \dots \leq y_n$ observations are censored.

Step:3 Calculate the required test statistic ${}_pA_n^2$ for the observations $y_1 \leq y_2 \leq \dots \leq y_r$ as defined in (3.6).

Step:4 If ${}_pA_n^2 > \lambda$, then reject null hypothesis H_0 and decide PU is present.

Step:5 Compute performance metric as Probability of Detection (P_d) with a given value of P_f .

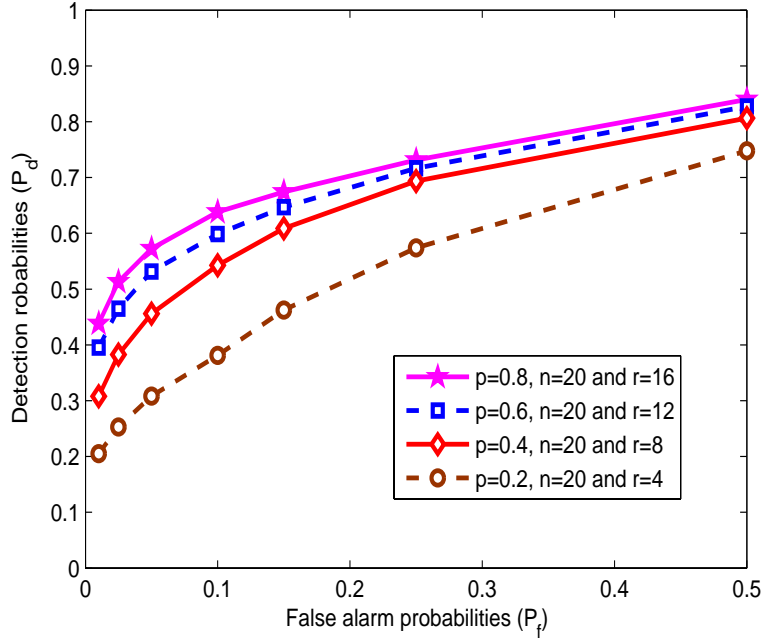


Figure 3.2: ROC for CAD sensing with $n = 20$ at $\text{SNR} = -2\text{dB}$ in quasi-static channel

3.4 Performance Results

In this section, we have shown the performance of the CAD sensing algorithm with receiver operating characteristics (ROC) using simulations for quasi static channel. The ROC curves are obtained for different values of observations (n), censoring ratio (p) and SNR (ρ). We have also presented ROC for ED and OS sensing algorithms with quasi static channel, time varying channel using $AR1$ and compared them with the proposed one.

Fig. 3.2 shows ROC for CAD sensing for different values of p such as 0.2, 0.4, 0.6 and 0.8, and fixed value of n as 20 with an SNR of -2dB. It can be seen that P_d increases with p for a fixed value of P_f . It is expected because higher number of observations improves the detection probability.

Fig. 3.3 shows ROC for CAD sensing at SNR of -6dB in AWGN and quasi-static channels. We have taken $r = 12$ and $n = 20$. It can be seen that higher detection performance is observed in AWGN channel compared to quasi-static fading channel. As we can see from (3.6) that the test statistic (${}_pA_n^2$) depends on empirical CDF ($F_n(y)$). In case of AWGN, the distance between empirical CDF and CDF

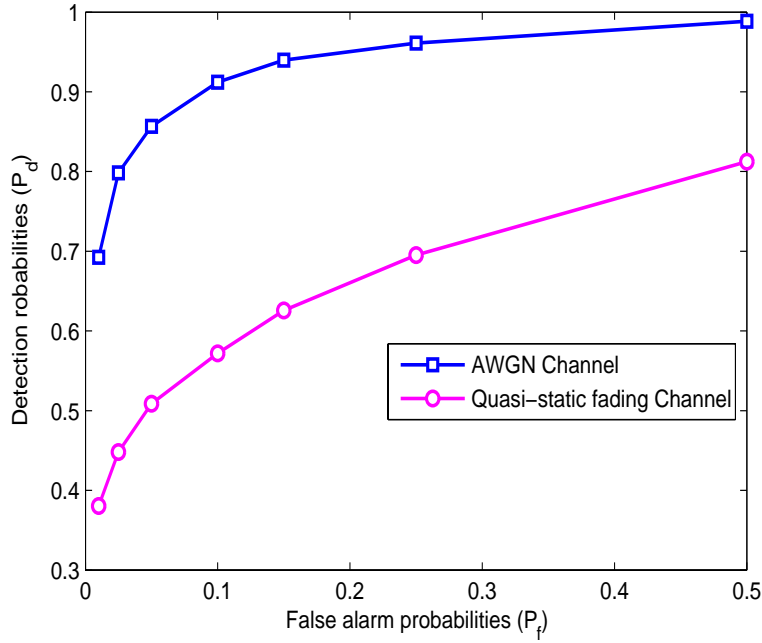


Figure 3.3: ROC for CAD sensing with $n = 20$ and $r = 12$ at $\text{SNR} = -6\text{dB}$ and $p = 0.6$.

(F_0) under H_0 is higher in presence of PU. However, it is varying in case of fading environment. As a result, the improved detection performance in AWGN channel compared to fading channel is expected as shown in Fig. 3.3.

Fig. 3.4 shows ROC for CAD sensing in fading channel for $n = 40$, $r = 24$ and SNR of -5dB . We have also presented ROC for ED and OS sensing algorithms for $n = 40$. It can be seen that the CAD sensing outperforms ED and OS sensing. The detection probabilities are 0.6134, 0.4 and 0.361 for CAD, ED and OS sensing respectively for $P_f = 0.05$. It is to be noted that OS sensing outperforms ED and AD sensing in AWGN channel (Rostami, Arshad, and Moessner). However, in the considered fading channel, the performance of OS sensing is degraded drastically.

For OS sensing (Glen, Leemis, and Barr), in case of AWGN channel and in the presence of PU (i.e. alternate hypothesis), the elements of transformation vector (β) have less amount of variation and most of the elements are extremely high i.e. greater than 0.99. This shows poor fit with the CDF in Null hypothesis. As a result, it leads towards high detection probability. However, in the absence of primary users (i.e.

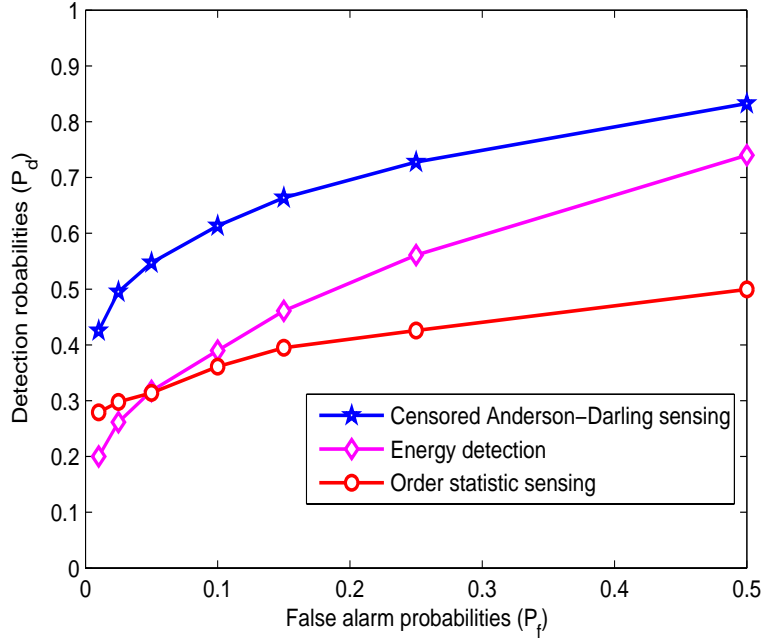


Figure 3.4: ROC for CAD sensing with $n = 40$ and $r = 24$ at $\text{SNR} = -5\text{dB}$, $p = 0.6$ and $P_f = 0.05$ in quasi-static channel.

Null hypothesis), the elements of β vector have more amount of variation and no extreme value is observed in all the elements. This shows good fit with the CDF in Null hypothesis. Using this fact, (Rostami, Arshad, and Moessner) has shown that the OS sensing outperforms AD and ED sensing in AWGN channel. However, in case of quasi-static fading channel, the fading coefficient (h) may take any real value. Due to inclusion of this in the received observations, the elements of β do not follow the above mentioned trends in both the hypotheses. This leads towards degradation in the detection performance.

In, Fig. 3.5, we have shown P_d versus SNR for $P_f = 0.05$, $n = 40$ and $p = 0.6$ for CAD sensing. As SNR increases, P_d increases as per expectation. We have also presented performance of ED and OS sensing methods in the same figure. We can see that $P_d = 0.1412$, 0.26 and 0.4157 for ED, OS and CAD respectively at SNR of -8dB . It can be seen that CAD sensing has almost 6dB gain over ED sensing with $P_d = 0.8344$. We can see significant improvement in P_d compared to OS sensing especially at higher SNR. Thus, the CAD sensing outperforms ED and OS sensing

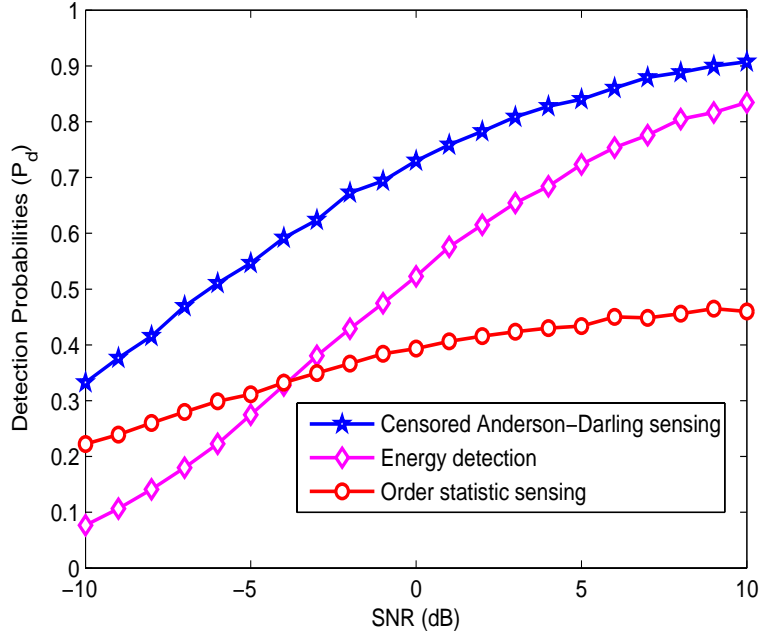


Figure 3.5: P_d vs SNR for $P_f = 0.05$, $n = 40$ and $p = 0.6$ in quasi-static channel.

for the whole range of SNR from -10dB to 10dB .

Now, further the performance of the proposed CAD sensing algorithm is shown in time varying channel which is model using AR1. Fig. 3.6 shows the impact of time varying nature of the channel on ROC of the proposed scheme at -2dB of SNR using different values of correlation coefficient (a) such as 0, 0.9, 0.95, 0.99, 1 taking $n = 20$ and $r = 12$. It means 12 observations are used for the detection of PU to identify its presence or absence. It can be seen that P_d is improved as the value of a increases towards unity. It means the performance is degraded when channel is fast time varying instead of slow time varying.

Fig. 3.7 shows P_d versus SNR for $P_f = 0.05$ for the same values of n , r and a . As SNR increases, P_d increases as per expectation. From the results shown in Fig. 3.6 and Fig. 3.7, we can say that CAD sensing improves P_d , when the channel is quasi-static ($a = 1$). However, as the value of a decreases, the performance degrades. It should be noted that in the considered CAD sensing, test statistic and threshold are dependent upon variance of noise only, not on the signal or channel component.

The ROC of the proposed CAD sensing is compared with the existing GoF based

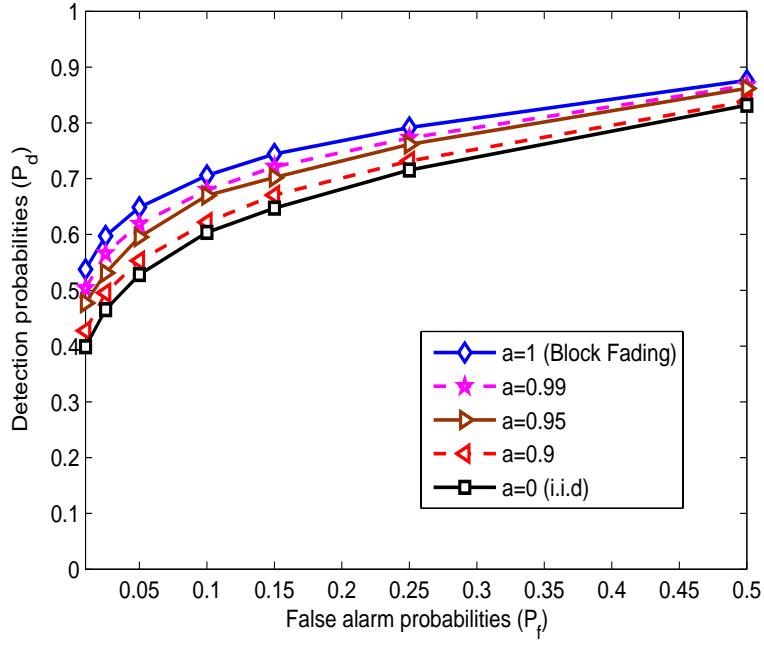


Figure 3.6: ROC for CAD sensing with $n = 20$ and $r = 12$ in time-varying channel.

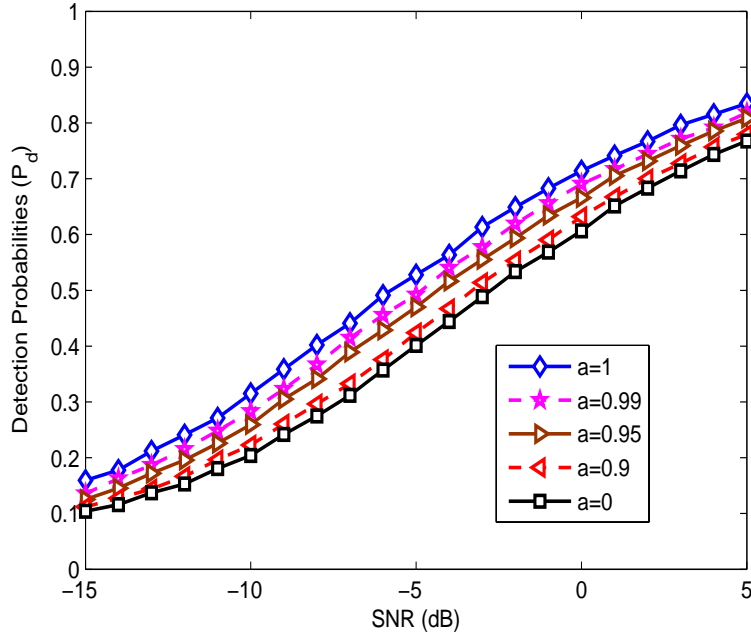


Figure 3.7: P_d vs SNR for $P_f = 0.05$ and $n = 20$ in time-varying channel.

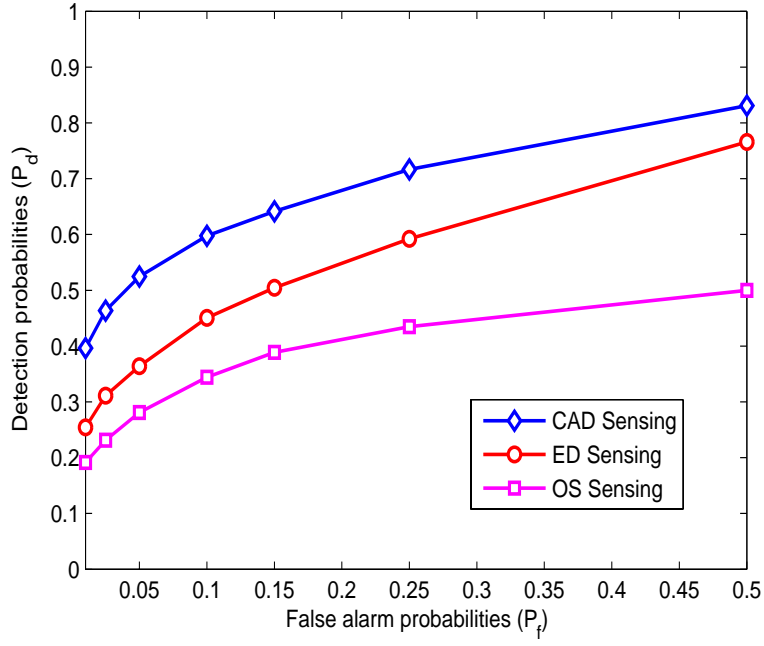


Figure 3.8: ROC for $a = 0.99$, $n = 20$ at SNR = -5 dB in time-varying channel.

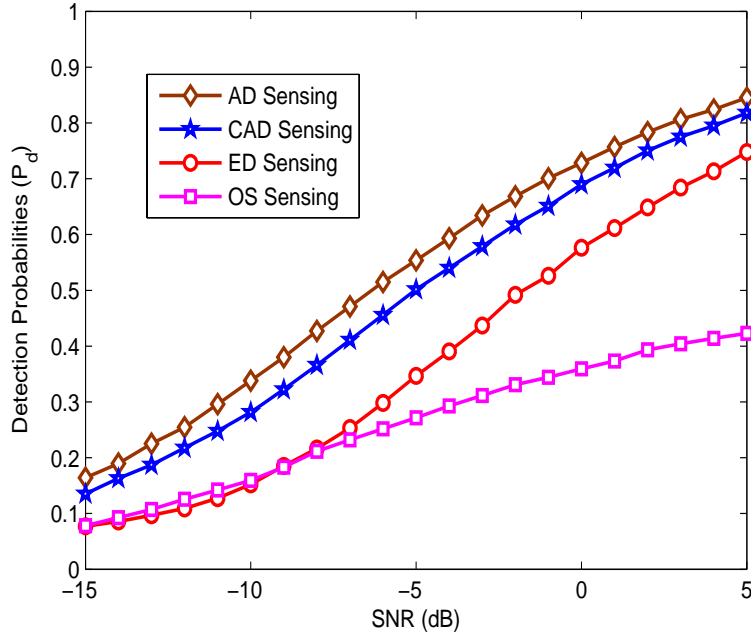


Figure 3.9: P_d vs SNR for $a = 0.99$ and $n = 20$ in time-varying channel.

sensing schemes as shown in Fig. 3.8. We have considered the time-varying channel which is modeled using AR1 model. We take $a = 0.99$, $n = 20$, $\text{SNR} = -5\text{dB}$ and $r = 12$ (for CAD sensing only). It can be seen from the graph that CAD sensing outperforms the other two methods in the whole range of P_f . For $P_f = 0.05$, P_d in CAD sensing is 0.5247 whereas for ED and OS sensing it is 0.3641 and 0.2809 respectively.

Fig. 3.9, shows P_d versus SNR for AD, ED and OS based sensing in the considered time-varying channel and compared them with the proposed CAD sensing for $P_f = 0.05$, $n = 20$, $a = 0.99$ and $p = 0.6$. To present fair comparison with the CAD sensing, the AD sensing is considered without censoring. The $P_d = 0.88, 0.867, 0.81$ and 0.48 are achieved for AD, CAD, ED and OS sensing respectively at SNR of 8dB. Thus, similar trend in P_d can be seen over here for a wide range of SNR.

From Fig. 3.9, it can be seen that the AD sensing has improved detection than CAD sensing. However, CAD sensing uses lower number of observations ($r = 12$) for achieving almost same detection performance as obtained for AD sensing at $n = 20$. So, the CAD sensing helps for saving the processing energy of secondary user and reducing the computational complexity too. The OS based sensing performs poorly in comparison with CAD and ED sensing methods. However, OS sensing outperforms in the AWGN channel only (Rostami, Arshad, and Moessner).

3.5 Conclusion

In this chapter, the problem of spectrum sensing is presented as null hypothesis testing problem for censored observations called CAD sensing under quasi-static channel. Also, the time varying channel is considered which is modelled using AR1 process. The ROC is presented for the CAD sensing and compared with ED and OS sensing methods. The CAD sensing method outperforms ED and OS sensing methods. In this chapter, we have assumed known variance of noise. However, in a real time scenario, it may not be known perfectly. In the next chapter, we will show the effect of noise uncertainty on the performance of CAD sensing.

Chapter 4

Blind CAD Sensing With Noise Uncertainty

The censoring based scheme is proposed in the last chapter. However, the CAD sensing has an assumption that the noise variance is known a priori under the null hypothesis. However, in actual practice due to noise uncertainty the variance is not known. The most realistic conditions at CR are unknown PU signal and the noise variance too. In this chapter, we present a CAD sensing method with assumptions of unknown noise variance, called as Blind-CAD (B-CAD) sensing. For developing the blind sensing scheme, we have used the Student-t distribution test initially proposed by (Shen et al.). The detection performance is shown in flat fading channel. The proposed B-CAD scheme is compared with AD Sensing (Wang et al.), Blind AD sensing (Shen et al.) and Energy detection (ED) (Digham, Alouini, and Simon).

4.1 System Model

Let $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$ be the received observations at CR. where n denotes total number of observations. We assume received observations are i.i.d real valued and each y_i is represented as,

$$y_i = \sqrt{\rho}hs + w_i, \quad i = 1, 2, 3, \dots, n, \quad (4.1)$$

where $s \in \{0, 1\}$, ρ is the received SNR, h represents the fading coefficient, which is assumed to be random variable with the standard normal distribution. We also assume that the channel is quasi-static. In (4.1), w_i , where $1 \leq i \leq n$, denotes noise

samples with mean 0 and variance σ^2 , where σ^2 is unknown. In (4.1), $s = 1$ and 0 denote presence and absence of PU respectively.

4.2 Student- t Distribution Test

The problem of unknown noise variance due to noise uncertainty using Student- t distribution is addressed by (Shen et al.). We have used the same for the CAD sensing based on the Type-II right censoring. The Student- t distribution is generally used for the testing of the normality under null hypothesis specially in the conditions where the standard deviation is not known a priori. The probability distribution function of the noise in H_0 is denoted by the Student- t distribution as

$$T(m-1, t) = \frac{\Gamma(\frac{m}{2})}{\sqrt{\pi(m-1)}} \left(1 + \frac{t^2}{m-1}\right)^{-\frac{m}{2}} \quad (4.2)$$

where $\Gamma(\cdot)$ is the Gamma function of $m-1$ degree and m is factor of total received observations (n). Hence, the cumulative distribution function (CDF) is denoted by $F_{0,m}(y)$,

$$F_{0,m}(y) = \int_{-\infty}^y T(m-1, t) dt \quad (4.3)$$

The (4.3) represents CDF of noise samples in H_0 . It indicates that the spectrum sensing problem is now formulated as testing whether the received observations are derived independently from the Student- t distributions or not. It is important to note that as compared to AD test this GoF sensing problem is different. In the case of AD sensing, the variance must be specified a priori. However, the null hypothesis testing using Student- t distribution is an independent of the noise variance.

4.3 B-CAD Sensing Algorithm

In this section, the Blind CAD (B-CAD) sensing scheme is presented, where uncertainty in the noise variance is assumed as used in (Shen et al.). In (Shen et al.), the spectrum sensing problem as Student- t distribution testing problem has been considered. The summary of the modified algorithm (B-CAD) is as follows:

- Select an integer $m > 1$, where m is a factor of n . In addition to this, the group is obtained by dividing the samples $Y = \{y_i\}_{i=1}^n$ into $g = \frac{n}{m}$ groups. Hence, the m presents the number of received observations per group. For example if $n =$

32 total received observations then using $m = 2$, number of groups are eight and each group contains four observations.

- For the j^{th} group ($j = 1, 2, 3, \dots, g$), calculate T_j ,

$$T_j = \frac{\bar{Y}_j}{S_j/\sqrt{m}}, j = 1, 2, \dots, g \quad (4.4)$$

where \bar{Y}_j is mean and S_j^2 is variance of the received observations in the j^{th} group as (Gosset),

$$\bar{Y}_j = \sum_{k=0}^{m-1} \frac{Y_{mj-k}}{m} \quad (4.5)$$

$$S_j^2 = \sum_{k=0}^{m-1} \frac{(Y_{mj-k} - \bar{Y}_j)^2}{m-1} \quad (4.6)$$

In (4.4), T_j follows the Student- t distribution.

- Select the threshold λ corresponding to probability of false alarm P_f as α

$$\alpha = P\{ {}_pA_n^2 > \lambda | H_0 \}, \quad (4.7)$$

where ${}_pA_n^2$ represents the modified AD test statistics shown in (Pettitt and Stephens).

- Without loss of generality, we assume that the received observations are in ascending order and then apply Type-II right censoring on $\{T_j\}_{j=1}^g$ with individual groups, we get

$$T_1 \leq T_2 \leq \dots \leq T_r \leq T_{r+1} \leq \dots \leq T_g,$$

where $T_{r+1} \leq T_{r+2} \leq \dots \leq T_g$ observations under respective groups are censored.

- Calculate the required test statistic ${}_pA_n^2$ for the individual group observations, $T_1 \leq T_2 \leq \dots \leq T_r$ as defined in (4.4). In continuation with test statistics, take the decision if ${}_pA_n^2 < \lambda$, then failed to reject null hypothesis H_0 i.e If $Y_j \sim N(0, \sigma^2)$, then T_j is Student- t distributed with $m - 1$ degree as defined in (Lenth). It shows the absence of PU.

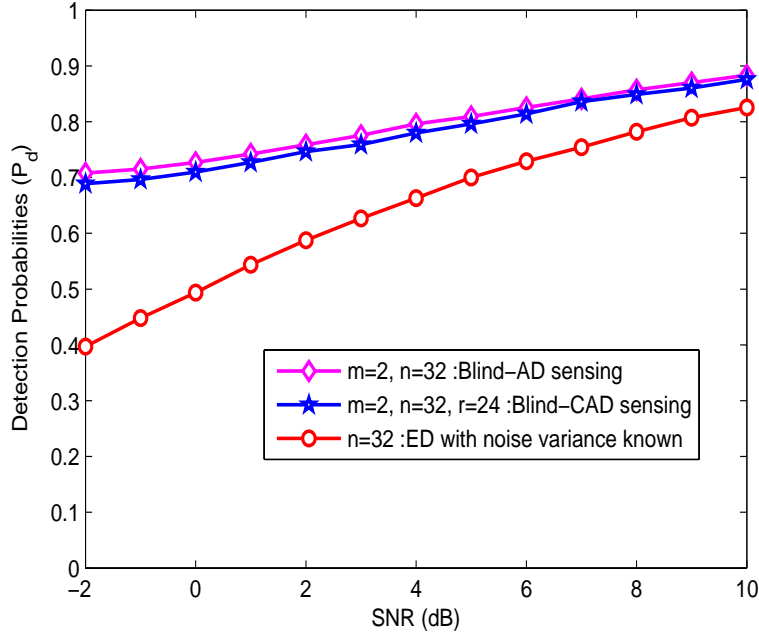


Figure 4.1: P_d vs SNR for $P_f = 0.05$ and $p = 0.8$ in quasi-static channel.

- Finally, compute the performance metric as Probability of Detection (P_d) with a fixed value of P_f as defined in (4.7). The Probability of Detection (P_d) is defined as,

$$P_d = P\{ {}_pA_n^2 > \lambda | H_1 \} \quad (4.8)$$

4.4 Performance Results

In this section, we present performance of the Blind CAD method using probability of detection (P_d) versus SNR (ρ) for different values of received observations (n), Student- t parameter (m), censoring ratio (p) and false alarm probability (P_f). We also present performance of Blind AD sensing (assuming unknown variance of noise) and ED sensing for comparison.

Fig. 4.1 shows P_d versus SNR for the proposed Blind CAD sensing for $n = 30$, $P_f = 0.05$, $m = 2$ and SNR = 7dB. Performance of Blind AD sensing and ED sensing is also presented. In ED sensing, noise variance is known. The value of $P_d = 0.8389$ for Blind AD sensing. The similar detection performance of $P_d =$

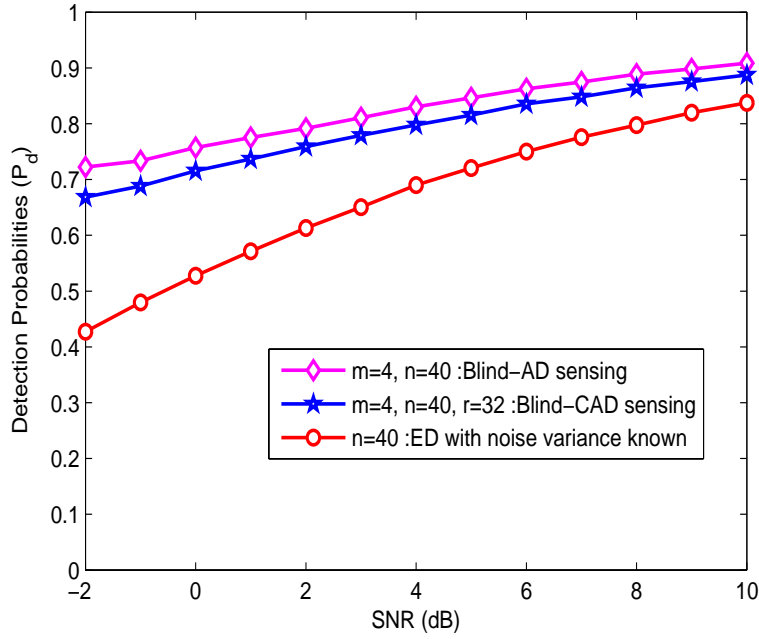


Figure 4.2: P_d vs SNR for $P_f = 0.05$ and $p = 0.8$ in quasi-static channel.

0.8316 is obtained using B-CAD sensing at $r = 24$, $P_f = 0.05$ and $\text{SNR} = 7\text{dB}$. Further, it can be seen that the Blind CAD sensing outperforms ED sensing with a gain of 4dB at $P_d = 0.8$.

Fig. 4.2 shows P_d versus SNR for the proposed Blind CAD sensing for $n = 40$, $r = 32$, $P_f = 0.05$, $m = 4$ and $\text{SNR} = 7\text{dB}$. Similar trend can be seen in this figure also. It can be shown from Fig. 4.1 and Fig. 4.2, B-CAD sensing has almost 3dB gain over ED sensing with known noise power and reduced signal processing cost at SU.

4.5 Conclusion

In this chapter, we have used Blind CAD scheme assuming unknown variance of Gaussian noise. This noise has been modelled by Student- t distribution with parameter m . We observed that B-CAD sensing has a gain in SNR of 4dB over ED sensing at $P_d = 0.8$ and $P_f = 0.05$. In the next chapter, we will propose a new GoF based sensing scheme using Likelihood Ratio Statistics.

Chapter 5

LRS- G^2 Sensing Based on Likelihood Ratio Test

In the Chapters 3 and 4, the objectives were to reduce the processing cost and obtain the similar detection performance at reduced number of observations during the decision process at CR. Now, continuing with the same objective of having less number of received observations and less probability of false alarm with higher detection probability, we propose a new robust GoF based sensing using likelihood ratio statistics (LRS- G^2) and use three different statistics, called Z_a , Z_k and Z_c . We show performance of the proposed LRS- G^2 sensing assuming AWGN, quasi-static and time varying channel, modelled by *AR1* process using simulations. Further, we have considered three different forms of PU such as constant, *i.i.d.* Gaussian and single tone sine wave. We have also considered blind LRS- G^2 assuming unknown noise variance and shown the performance.

Finally, we have presented performance of prevailing GoF based sensing schemes and shown that the proposed one outperforms all the schemes. This is a robust sensing scheme as its performance does not depend much on the structure of channel or primary user. Further, this GoF based sensing scheme does not depend much on the variance of noise. So, its blind version also gives better performance compared to prevailing GoF based sensing schemes.

5.1 Introduction

The Chi-Square (χ^2) test has been proposed by (Pearson). This χ^2 test was then known as χ^2 goodness of fit (GoF) test (Plackett). The χ^2 test gives higher statistical power when the number of observations are high enough. However, the χ^2 statistics in (Pearson) gives constant values for the lower number of observations. Hence, it is difficult to take a decision on the null hypothesis. This fact has been described in (Cochran) and references therein. To solve such problems, the likelihood ratio statistics (G^2) based null hypothesis test has been proposed by (Cressie and Read). The authors have used the power divergence statistic to propose the GoF test for the multinomial distributions.

Based on G^2 test, using parameterization approach, (Zhang) have proposed the powerful GoF test which provides higher statistical power in comparison with traditional GoF tests such as Anderson-Darling (AD) test, Kolmogorov-Smirnov (KS) test and Cramer-von-Mises (CvM) test. The new likelihood ratio statistics (LRS) have been derived based on G^2 test, called as Z_a , Z_k and Z_c . These statistics give the highest statical power at the low number of observations. In addition to this, the same authors, (Zhang and Wu) have proposed an omnibus normality GoF test using LRS for the Gaussian distribution under null hypotheses.

In this chapter, we propose a novel GoF based non-parametric sensing scheme using likelihood ratio statistics (LRS- G^2). The first non-parametric sensing scheme under GoF based sensing is AD sensing (Wang et al.). The AD sensing method outperforms the ED sensing assuming the AWGN channel and constant PU signal. However, (Nguyen-Thanh, Kieu-Xuan, and Koo) has investigated the performance of AD sensing with different PU signals such as independent and identically distributed (*i.i.d*) Gaussian and single frequency sine signals. Under both these PU signals, ED sensing outperforms the AD sensing. Hence, we will show that the proposed LRS- G^2 scheme outperforms ED sensing, when PU is modelled by *i.i.d*. Gaussian and single frequency sine signals. Furthermore, the test is also applied, assuming different types of channels such as additive white Gaussian noise (AWGN) and quasi-static channels. In all these cases,

the proposed scheme outperforms the prevailing GoF based sensing schemes.

The assumption of known variance of noise in GoF based scheme as well as in ED scheme is very crucial. We have seen that it degrades the performance significantly. In (Shen et al.), blind AD sensing scheme has been proposed in quasi-static channel with constant PU signal. This blind AD scheme does not require any information about the variance of noise. This blind AD outperforms ED sensing significantly. Our proposed scheme, without having knowledge of variance of noise, called as Blind LRS- G^2 , outperforms Blind AD and ED based sensing methods.

All GoF based sensing schemes have been used assuming AWGN or quasi-static channel. However, in a practical scenario, the channel is time-varying. Hence, it is of interest to evaluate the performance of the GoF sensing scheme in a time-varying channel. We have shown the performance of the proposed scheme assuming a time-varying channel which is modelled by first order autoregressive (AR1) process. The proposed scheme shows significant improvement in the performance compared to AD and ED sensing in these conditions also.

5.2 System Model

Let $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$ be a vector of n observations of PU, received at CR, where $n \geq 1$. We assume that all the received observations are real as considered in (Wang et al.), (Rostami, Arshad, and Moessner), (Shen et al.), and each y_i is represented as,

$$y_i = \sqrt{\rho} h_i s_i + w_i, \quad i = 1, 2, 3, \dots, n, \quad (5.1)$$

where ρ is the received SNR, h_i represents the channel coefficient. In (5.1), $w_i \sim \mathcal{N}(0, \sigma^2)$, where $1 \leq i \leq n$, denotes samples of gaussian noise and s_i denotes symbol of PU, which can be assumed as constant one or *i.i.d.* Gaussian as $s_i \sim \mathcal{N}(0, 1)$ or single frequency sine signal as defined in (Nguyen-Thanh, Kieu-Xuan, and Koo). The CDF of w_i is denoted by $F_0(w)$. The PU signal as a single carrier frequency (f_c) in the discrete version of sine signal can be

represented as,

$$s_i = \sqrt{2} \sin \left(\frac{2\pi}{k} i + \theta \right), \quad (5.2)$$

where θ is an initial phase and $k = \frac{f_s}{f_c}$ is the ratio of the sampling frequency (f_s) to the carrier frequency (f_c). The value of k is assumed to be six. Without loss of generality, we assume that all n observations are in ascending order. It means $y_1 \leq y_2 \leq \dots \leq y_n$.

We assume three different models for channel coefficient h_i .

- AWGN channel: In this case, h_i is assumed to be one and noise distribution is Gaussian with mean zero and variance σ^2 .
- quasi-static channel: In this case, $h_i \sim \mathcal{N}(0, 1)$, however it remains constant during a block of n symbols.
- Time-varying channel: In this case, $h_i \sim \mathcal{N}(0, 1)$, however it varies with time in a block of n symbols. This channel is generated using first ordered autoregressive (AR1) process,

$$h_i = ah_{i-1} + \sqrt{1 - a^2}v_i, \quad 0 \leq a \leq 1 \quad (5.3)$$

where v_i denotes *i.i.d* as Gaussian with mean zero and variance one. In (5.3), a indicates correlation coefficient between consecutive symbols i.e. $a = E[h_{i-1}^* h_i]$, where $E[\cdot]$ represents expectation operator. Here, $a = 1$ and $a = 0$ denote a constant (quasi-static fading) channel and an independent channel respectively. The value of a is determined using Jake's autocorrelation function (Jakes and Cox) as $a = J_0(2\pi f_d T_s)$, where f_d and T_s denote doppler frequency in Hz and symbol time in seconds respectively.

5.3 Robust Likelihood Goodness of Fit tests

In GoF based sensing, we test the received observations, whether they are drawn from null hypothesis (H_0) or not. We assume that the CDF of Gaussian noise under H_0 is known and denoted by $F_0(t)$, where t represents any continuous random variable. In literature, null hypothesis testing algorithms are classified in two ways, Pearson's Chi-squared test and empirical distribution function (EDF) test. The AD, KS and CvM tests are

under the category of EDF tests. In (Cressie and Read), authors have proposed a new hypothesis test based on power divergence statistics for null-hypothesis testing as,

$$2nI^\lambda = \frac{2n}{\lambda(\lambda+1)} \left\{ F_n(t) \left[\frac{F_n(t)}{F_0(t)} \right]^\lambda + [1 - F_n(t)] \left[\frac{1 - F_n(t)}{1 - F_0(t)} \right]^\lambda - 1 \right\} \quad (5.4)$$

where, λ represents a parameter for selection of goodness of fit test, n and $F_n(t)$ denote number of received observations and empirical CDF respectively.

By selecting $\lambda = 1$, (5.4) reduces to Pearson's Chi-squared test statistics (\mathbb{X}^2) as,

$$\mathbb{X}^2 = \frac{n[F_n(t) - F_0(t)]^2}{F_0(t)[1 - F_0(t)]} \quad (5.5)$$

and $\lambda = 0$, (5.4) reduces to Likelihood Ratio Statistics (LRS) as,

$$G^2 = 2n \left\{ F_n(t) \log \frac{F_n(t)}{F_0(t)} + [1 - F_n(t)] \log \frac{1 - F_n(t)}{1 - F_0(t)} \right\} \quad (5.6)$$

5.3.1 Traditional GoF Tests From the Pearson's Chi-squared Statistics

In (Zhang and Wu), authors have proposed a parametrization approach to construct a generalized omnibus GoF tests for a specified distribution (F_0) under hypothesis H_0 as normal distribution using different weight functions. They have proposed general test statistics called as Z statistics using,

$$Z = \int_{-\infty}^{\infty} z_t w(t) dt, \quad (5.7)$$

where z_t indicates a type of goodness of fit test statistics and $w(t)$ denotes weighting function. The power of any goodness of fit test depends on these two parameters z_t and $w(t)$.

Let $z_t = \mathbb{X}^2$ as shown in (5.5). Then, (5.7) can be expressed as

$$Z = \int_{-\infty}^{\infty} \frac{n[F_n(t) - F_0(t)]^2}{F_0(t)[1 - F_0(t)]} w(t) dt \quad (5.8)$$

Substituting the distinct weighting functions $w(t) = F_0(t)$, $w(t) = n^{-1}F_0(t)[1 - F_0(t)]$ and $w(t) = F_0(t)[1 - F_0(t)]$ in (5.8), the Z statistics represent AD, KS and CvM statistics respectively as discussed in (D'Agostino). Using these AD, KS and CvM statistics, different spectrum sensing schemes have been proposed in (Wang et al.), (Arshad, Briggs, and Moessner), (Arshad and Moessner) and (Kieu-Xuan and Koo).

5.3.2 Omnibus Normality Test from Empirical Distribution Function

The authors of (Zhang and Wu) have proposed powerful omnibus tests. To derive such test, they used LRS- G^2 by substituting (5.6) into (5.7) in place of z_t ,

$$\begin{aligned} Z &= \int_{-\infty}^{\infty} G^2 w(t) dt \\ &= \int_{-\infty}^{\infty} 2n \left\{ F_n(t) \log \frac{F_n(t)}{F_0(t)} + [1 - F_n(t)] \log \frac{1 - F_n(t)}{1 - F_0(t)} \right\} w(t) dt \end{aligned} \quad (5.9)$$

By using different weight functions ($w(t)$) in (5.9) as mentioned below, Z produces Z_k , Z_a and Z_c statistics called as Zhang's omnibus statistics.

For $w(t) = 1$, Z approaches Z_k statistic, which is expressed as

$$\begin{aligned} Z_k &= \max_{1 \leq i \leq n} \left(\left(i - \frac{1}{2} \right) \log \left\{ \frac{i - \frac{1}{2}}{n F_0(y_{(i)})} \right\} \right. \\ &\quad \left. + \left(n - i + \frac{1}{2} \right) \log \left\{ \frac{n - i + \frac{1}{2}}{n \{1 - F_0(y_{(i)})\}} \right\} \right) \end{aligned} \quad (5.10)$$

For $w(t) = F_n(t)^{-1} \{1 - F_n(t)\}^{-1}$, Z approaches Z_a statistic, which is expressed as

$$Z_a = - \sum_{i=1}^n \left[\frac{\log \{F_0(y_{(i)})\}}{n - i + \frac{1}{2}} + \frac{\log \{1 - F_0(y_{(i)})\}}{i - \frac{1}{2}} \right] \quad (5.11)$$

For $w(t) = F_0(t)^{-1} \{1 - F_0(t)\}^{-1}$, Z approaches Z_c statistic, which is expressed as

$$Z_c = \sum_{i=1}^n \left[\log \left\{ \frac{F_0(y_{(i)})^{-1} - 1}{(n - \frac{1}{2}) / (i - \frac{3}{4}) - 1} \right\} \right]^2 \quad (5.12)$$

The sampling distribution of the Zhang statistic (Z_c), is mathematically intractable, so it is unattainable to derive the close form expression of the false alarm probability (P_f) and probability of detection (P_d). Hence, we use extensive Monte Carlo Simulations to evaluate the sensing performance of the proposed scheme. We choose above mentioned statistics and use it for hypothesis testing considering different conditions for channels and PU. The effect of the different Zhang statistics (Zhang and Wu) on the detection performance of CR is discussed in the next section.

5.4 LRS- G^2 Sensing Algorithm

In this section, we consider LRS- G^2 sensing scheme assuming known variance of noise as well as unknown variance of noise.

5.4.1 With Known Variance of Noise

The problem of spectrum sensing as a null-hypothesis testing problem is defined as (Wang et al.),

$$\begin{aligned} H_0 : F_Y(y) &= F_0(y) \\ H_1 : F_Y(y) &\neq F_0(y) \end{aligned} \quad (5.13)$$

For LRS- G^2 sensing, we use statistics Z_c as defined in (5.12) to measure distance between $F_Y(y)$ and $F_0(y)$. Let $F_n(y)$ be the empirical cumulative distribution function (ECDF) of the received observations which can be expressed as,

$$F_n(y) = \frac{|\{i - \frac{1}{2} : y_i \leq y, 1 \leq i \leq n\}|}{n} \quad (5.14)$$

where $|\cdots|$ indicates cardinality.

We assume that the noise variance is known a priori. The noise under H_0 is $w_i \sim \mathcal{N}(0, \sigma^2)$. Here, we assume that $\sigma^2 = 1$.

First, for the detection of PU at the CR, the value of threshold (λ) is selected so that the false alarm probability (P_f) is at a desired level (α) as,

$$\alpha = \mathbb{P}\{Z_c > \lambda | H_0\} \quad (5.15)$$

To find λ , it is worth mentioning that the distribution of Z_c under H_0 is independent of the $F_0(y)$. Hence, after applying the probability integration transform (PIT) for available observations,

$$Z_c = \int_0^1 2n \left\{ F_Z(z) \log \frac{F_Z(z)}{z} + [1 - F_Z(z)] \times \log \frac{1 - F_Z(z)}{1 - z} \right\} z^{-1} \{1 - z\}^{-1} dz, \quad (5.16)$$

where $z = F_0(y)$ and $F_Z(z_i)$ denotes ECDF of the transformed observations z_i , where $z_i = F_0(y_i)$ for $1 \leq i \leq n$. Each of the statistics (Z_a, Z_c, Z_k) of observations is independent and uniformly distributed over $[0, 1]$. As shown in (Wang et al.) for AD sensing, the distribution of A^2 is independent of the $F_0(y)$. The same is also true for the distribution of Z_c . As given in (Zhang and Wu), the value of λ is determined for a specific value of P_f . For example, when $P_f = 10^{-3}$ and $n = 50$, then the value of λ is 31.707. Second, sort all the received observations in ascending order. Then, without loss of generality, we get

$$y_1 \leq y_2 \leq \dots \leq y_n. \quad (5.17)$$

Third, calculate the test statistic (Z_c) using (5.12) as,

$$Z_c = \sum_{i=1}^n \left[\log \left\{ \frac{u_i^{-1} - 1}{(n - \frac{1}{2}) / (i - \frac{3}{4}) - 1} \right\} \right]^2 \quad (5.18)$$

where $u_i = F_0(y_i)$.

At last, compare the value of (5.18) with λ . If $Z_c > \lambda$, then reject the null hypothesis H_0 in favor of the presence of PU signal. Otherwise, declare that the PU is absent. Compute performance metric as Probability of Detection (P_d) with a given value of P_f . Furthermore, (P_d) is computed theoretically as,

$$\begin{aligned} P_d &= \mathbb{P}\{Z_c > \lambda | H_1\} \\ &= 1 - F_{Z_c, H_1}(\lambda) \end{aligned} \quad (5.19)$$

5.4.2 With Unknown Variance of Noise

In this case, LRS- G^2 sensing method is used considering uncertainty in the variance of noise, we call it Blind LRS- G^2 sensing. Recently, (Shen et al.) has proposed the Blind AD sensing method, where noise uncertainty was considered. Authors of the papers have considered the spectrum sensing problem as Student- t distribution testing problem. We have used the same approach by replacing AD test with the proposed Zhang test in LRS- G^2 sensing. The summary of the algorithm is as follows:

Step:1 Select an integer m , where $m > 1$ and it is a factor of n . Divide all the samples $Y = \{y_i\}_{i=1}^n$ into $g = \frac{n}{m}$ groups, where m number of received observations are there in one group (Shen et al.).

Step:2 For the j^{th} group ($j = 1, 2, 3, \dots, g$), calculate T_j ,

$$T_j = \frac{\overline{Y}_j}{S_j/\sqrt{m}}, j = 1, 2, \dots, g \quad (5.20)$$

where \overline{Y}_j is mean and S_j^2 is variance of the received observations in the j^{th} group,

$$\overline{Y}_j = \sum_{k=0}^{m-1} \frac{Y_{mj-k}}{m} \text{ and } S_j^2 = \sum_{k=0}^{m-1} \frac{(Y_{mj-k} - \overline{Y}_j)^2}{m} \quad (5.21)$$

Step:3 Find the threshold λ for a given probability of false alarm P_f using (5.15).

Step:4 Sort T_j in ascending order. Then, without loss of generality, we get

$$T_1 \leq T_2 \leq \dots \leq T_g$$

Step:5 Calculate the required test statistic Z_c for each group as shown in (5.18) by replacing y_i by T_j .

Step:6 If $Z_c < \lambda$, then reject null hypothesis H_0 i.e If $T_j \sim N(0, \sigma^2)$, then T_j is Student- t distributed variable with $m - 1$ degrees. It shows the absence of PU. Compute P_d for the fixed value of P_f . Repeat the above-mentioned steps for other values of P_f .

5.5 Performance Results

In this section, receiver operating characteristics (ROC) is presented i.e. plot of P_d versus P_f for different values of SNR for the proposed LRS- G^2 sensing method using simulations. We have also presented P_d versus SNR for lower values of P_f . The three types of channels are considered such as AWGN, quasi-static and time-varying channels using auto regressive process (AR1) model. We have also considered three types of PU signals such as constant, single frequency sine wave and *i.i.d* Gaussian with mean zero as defined in (Nguyen-Thanh, Kieu-Xuan, and Koo).

In AWGN channel environment, Z_c , Z_k and Z_a provide similar detection performance. So, we choose the Z_c statistic for taking decision at secondary user (SU). However, in fading channel, Z_k statistic provides better performance. Therefore, we choose Z_k statistic for quasi-static and time varying channel. Furthermore, we have considered the noise uncertainty and shown its effect on detection performance by varying SNR. Finally, we have compared all our results with prevailing GoF based sensing such as AD, KS, OS and ED schemes.

Fig. 5.1 shows the ROC for the proposed LRS- G^2 method in comparison with prevailing GOF sensing schemes at SNR = -4dB, $n = 30$ and constant PU signal. It can be seen that the proposed technique outperforms all under AWGN channel. To observe the performance of the proposed scheme at lower value of P_f such as 0.01, we have shown P_d versus SNR with $n = 30$ under AWGN channel in Fig. 5.2. At SNR = -8dB, the detection probabilities of 0.7293, 0.5505, 0.4026, 0.3206 and 0.0195 are achieved for LRS- G^2 , KS, OS, AD and ED sensing respectively.

Considering the PU signal as a discrete sinusoidal signal or independent and identically distributed Gaussian signal (Nguyen-Thanh, Kieu-Xuan, and Koo), Fig. 5.3 shows ROC for the proposed scheme along with AD and ED sensing at an SNR of -5dB and $n = 30$. It can be seen that the proposed scheme outperforms both the AD and ED sensing in both the PU signals. Furthermore, it can be seen that the ED sensing outperforms

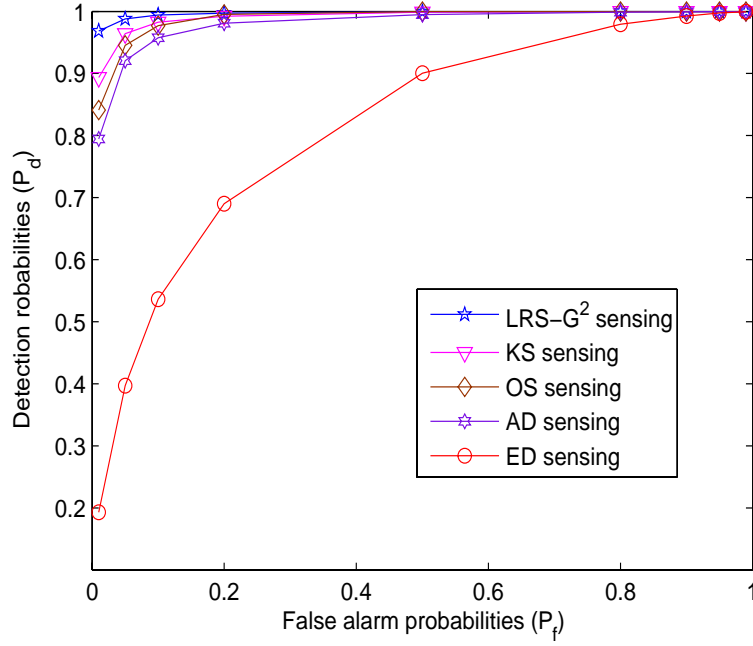


Figure 5.1: ROC for different sensing schemes in AWGN channel for constant PU signal at $\text{SNR} = -4\text{dB}$ and $n = 30$.

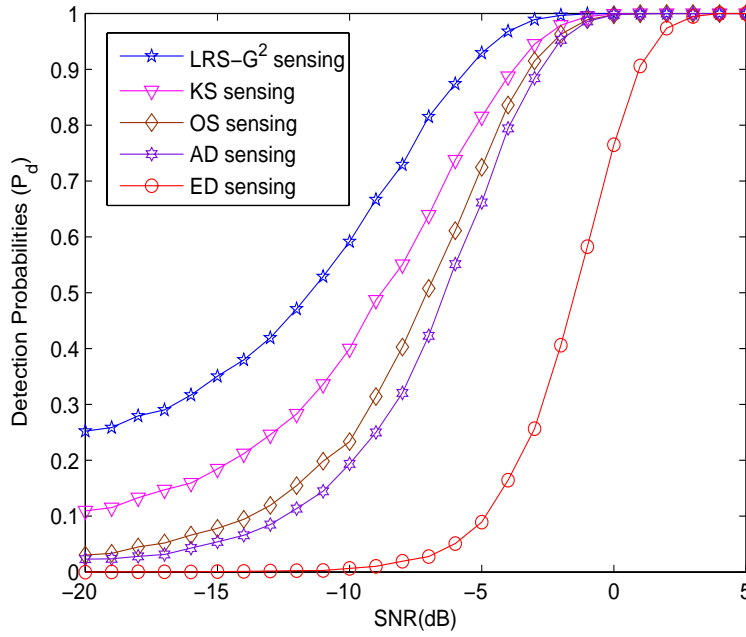


Figure 5.2: Detection probability (P_d) versus SNR for different sensing schemes in AWGN channel for constant PU signal at $P_f = 0.01$ and $n = 30$.

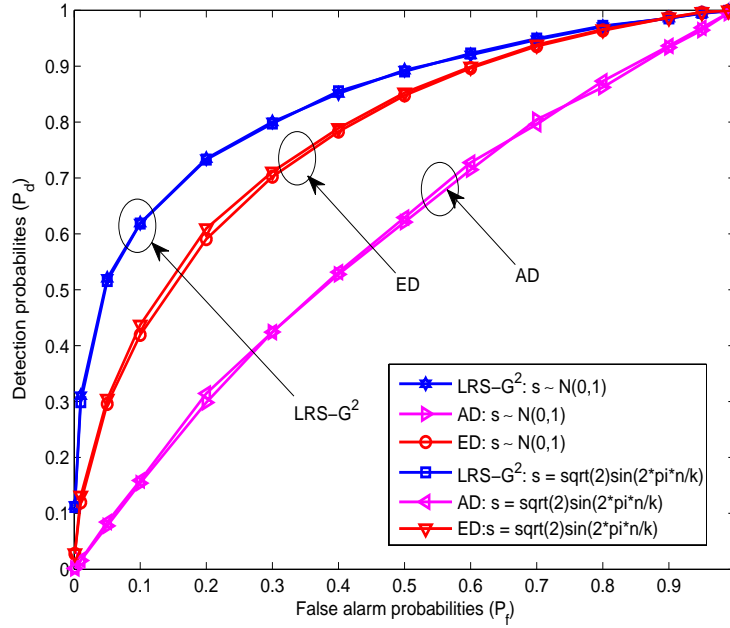


Figure 5.3: ROC for various sensing schemes in AWGN channel with different types of PU signals at SNR = -5 dB and $n = 30$.

GoF based AD sensing, however proposed GoF based LRS- G^2 scheme outperforms ED sensing. It proves that the LRS- G^2 scheme is robust against the nature of PU signal.

So far, we have shown performance of the proposed scheme in AWGN channel with different PU signals. Further, in Fig. 5.4, the detection performance of LRS- G^2 is shown under quasi-static channel with PU signal as single frequency sine signal with $n = 30$. We have also presented performance for LRS- G^2 sensing taking all Zhang test statistics as derived in (Zhang and Wu). The ED outperforms AD sensing. Interestingly, we can observe that the LRS- G^2 with Z_k , Z_a and Z_c outperform ED and AD sensing under fading environment.

Now, we consider blind LRS- G^2 with uncertainty in noise, i.e. the noise variance (σ^2) is unknown. We assume that the channel (h) is quasi-static and PU signal is constant (Shen et al.). In Fig.5.5, we have shown P_d versus SNR for $P_f = 0.05$ with $m = 4$ and $n = 32$. It can be seen that

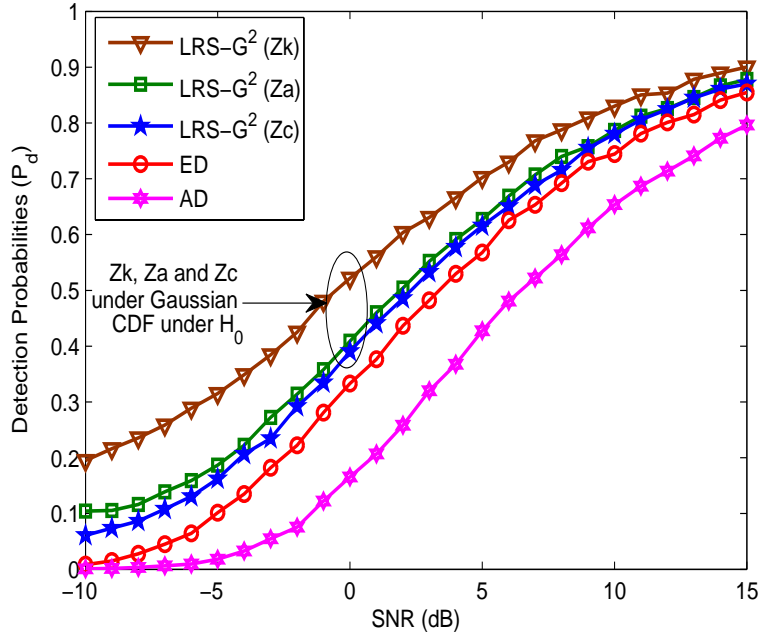


Figure 5.4: P_d versus SNR in quasi-static channels with PU Signal as single frequency sine signal at $P_f = 10^{-3}$

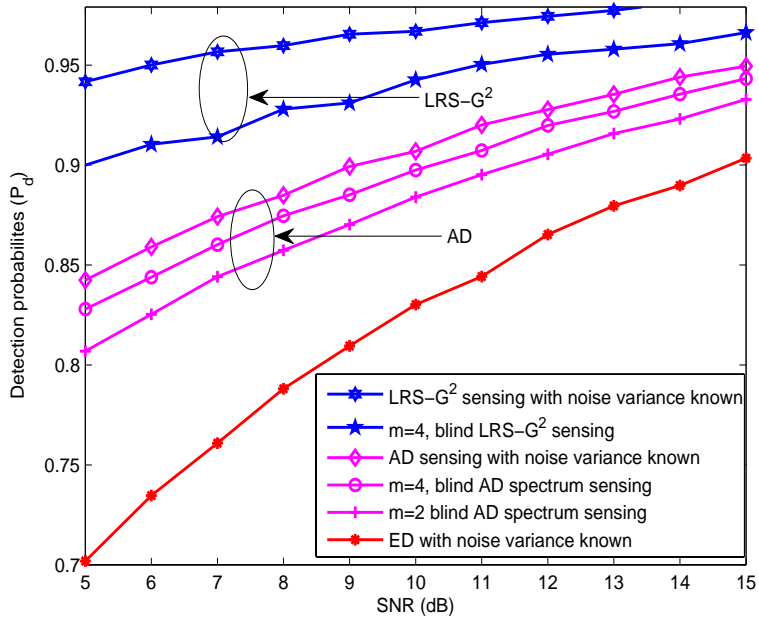


Figure 5.5: P_d versus SNR in quasi-static channels with noise uncertainty for constant PU signal at $P_f = 0.05$ and $n=32$ in quasi-static channel

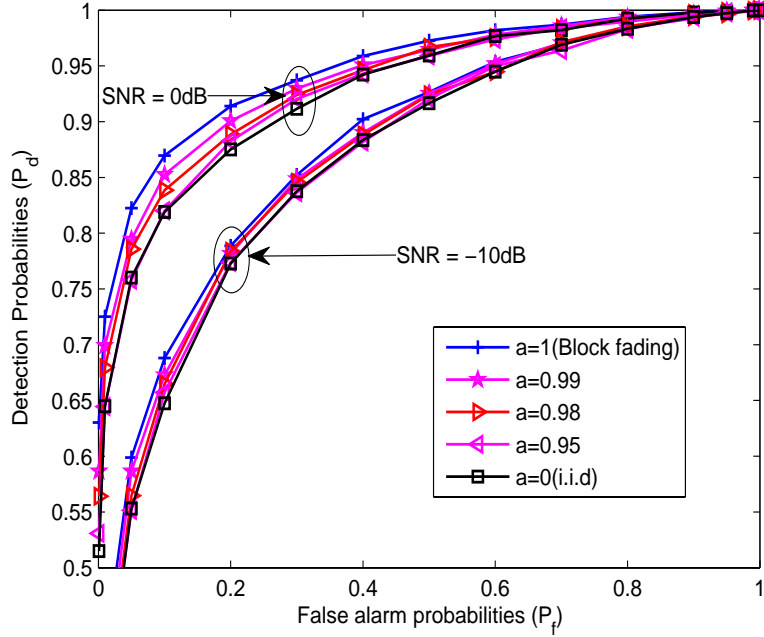


Figure 5.6: ROC for LRS- G^2 sensing with different correlation coefficient (a) at $n = 30$ in time-varying channel.

uncertainty in noise degrades the performance as expected. We have also presented performance of AD sensing and blind AD sensing (for $m = 4$ and $m = 2$) along with performance of ED sensing with known variance of noise. It can be seen that the blind LRS- G^2 outperforms AD and ED sensing with known variance also.

In Fig. 5.6, we have shown ROC for the proposed scheme assuming PU signal as a single frequency sine signal and channel is time-varying modelled by $AR1$ process. The ROC for LRS- G^2 sensing is presented for different values of correlation coefficient (a) such as 1, 0.99, 0.98, 0.95, 0 at $n = 30$ and SNR of 0dB and -10 dB. It can be seen that performance improves as the value of a increases towards unity. In Fig. 5.7, we have shown P_d versus SNR for $P_f = 0.05, 0.001$ for the same values of n and a . From the results, shown in Fig. 5.6 and Fig. 5.7, we can say that LRS- G^2 sensing improves P_d when the channel is quasi-static ($a = 1$). However, as the value of a decreases, the performance degrades as the channel becomes time-varying

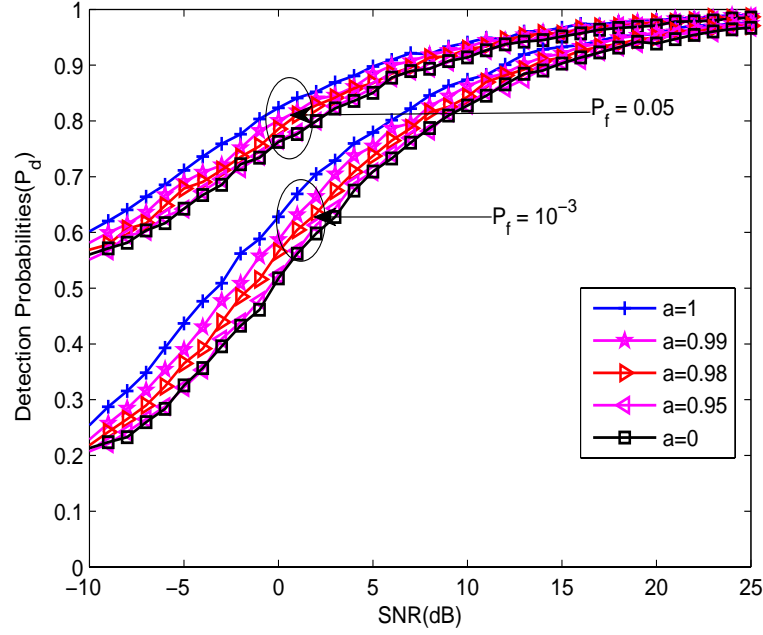


Figure 5.7: P_d versus SNR for LRS- G^2 sensing with different values of correlation coefficient (a) at $n = 30$ in time-varying channel.

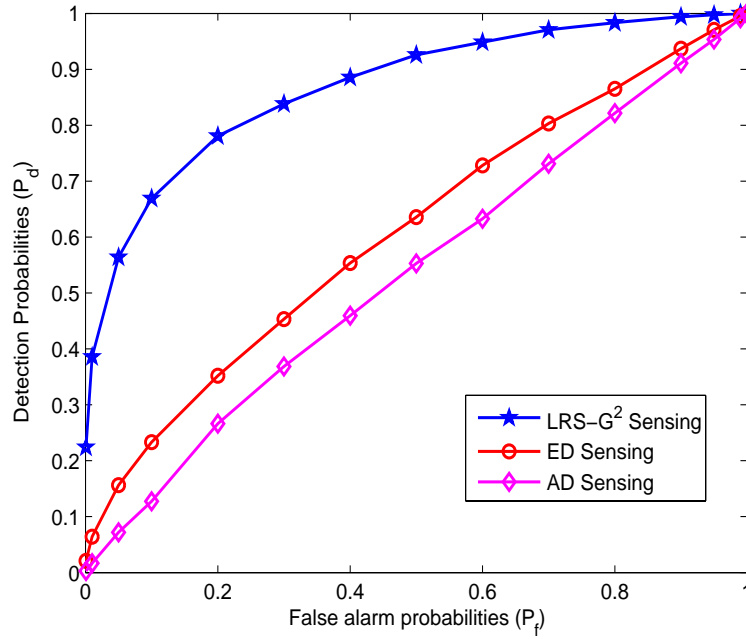


Figure 5.8: ROC for different sensing with $n = 30$, $a = 0.99$, SNR = -10 dB in time-varying channel.

In Fig. 5.8, we present ROC for ED and AD sensing in the considered time-varying channel and compared them with the proposed LRS- G^2 sensing. We take $a = 0.99$, $n = 30$ and $\text{SNR} = -10\text{dB}$. It can be seen from the graph that LRS- G^2 sensing outperforms the other non-parametric methods in the whole range of P_f .

5.6 Conclusion

A novel non-parametric spectrum sensing scheme based on likelihood ratio statistics using goodness of fit test has been proposed. The detection performance is presented using ROC assuming various types of primary user signals as well as different channel conditions. Furthermore, the adverse effect of noise uncertainty is also shown on the performance. The ROC for ED and prevailing GoF based sensing schemes such as AD, OS and KS are compared with the proposed one.

The ED based sensing usually outperforms traditional GoF based sensing schemes when PU signal is not constant. However, the proposed GoF based scheme outperforms ED as well as all these GoF based sensing. In case of time-varying channel, the performance of the proposed scheme degrades as the channel changes from slow time varying to fast time varying.

Chapter 6

LRS- G^2 Sensing with Middleton Class-A Non-Gaussian Noise

Cognitive radio user (CR) assumes additive background noise in null hypothesis (H_0) as thermal noise. For this noise, Gaussian distribution, with zero mean and some variance, is assumed. In the previous chapters, we have shown detection performance of GoF based spectrum sensing schemes with different values of variance, i.e. changing SNR. We have noticed that imperfect information about this variance degrades the performance significantly in blind sensing schemes. In today's scenario, thermal noise is not only the source of background noise (due to less noisy hardware), but it includes man-made and natural electro-magnetic interference (EMI), multi-user interferences and narrow-band or wideband interferences. These sources of noise make the background noise non-Gaussian noise (NGN). The resultant noise may be characterized by Middleton distributions.

In this chapter, LRS- G^2 sensing scheme is evaluated under non-Gaussian noise which is modelled by Middleton Class-A distribution. The sampling distribution of the proposed test statistic is derived and the detection performance is shown using Monte Carlo simulations. We present the results and conclude that the performance is degraded if Gaussian component in

the Middleton noise is higher than non-Gaussian component.

6.1 Modeling of Non-Gaussian noise

The real time environment consists many sources of radiations. It affects the transceiver operations. The different sources are microwave ovens (Kanemoto, Miyamoto, and Morinaga), automobile ignitions (Middleton1), co-channel interference in wireless networks (Gulati et al.) and many others.

Based on the above discussion, we need accurate non-Gaussian model for the development of the non-parametric sensing algorithm. Based on the different environmental conditions and noise bandwidth, Middleton has proposed a statistical-physical way to model the non-Gaussian noise radiated by man-made and natural sources (Middleton1). It is used in various fields such as SONAR, telecommunications, radar and optical communication (Middleton2). There are mainly two types of Middleton models based on the response of the receiver to the noise as proposed in the literature: Middleton Class-A and Middleton Class-B as discussed next. In this chapter, we have used Middleton Class-A model to characterize the actual non-Gaussian environment and evaluated the performance of LRS- G^2 sensing scheme.

6.1.1 Middleton Class-A noise

This type of Middleton noise is used when the bandwidth of the receiver (ΔB_R) is larger than the noise bandwidth. The noise model using this condition has very narrow spectral components. The important condition for such noise as defined in (Middleton3),

$$t_r \Delta B_R \gg 1,$$

where t_r is the time duration of radiation from source and ΔB_R represents the bandwidth of the receiver.

Let the amplitude of the Class-A noise is denoted as $W(t)$ and the pdf of

its samples is given as,

$$f(w) = \sum_{m=0}^{\infty} \frac{A^m}{m!} e^{-A} \left[\frac{1}{\sqrt{2\pi}\sigma_m^2} \right] e^{\frac{-w^2}{2\sigma_m^2}}, \quad (6.1)$$

where $\sigma_m^2 = \frac{m+\Gamma}{1+\Gamma}$, A and Γ represent impulsive rate and the ratio of Gaussian to non-Gaussian noise component respectively.

6.1.2 Middleton Class-B noise

This noise model has opposite characteristic as compared to the Middleton Class-A noise. The bandwidth of receiver is smaller than the noise bandwidth. Similar to the Class-A noise, the condition can be defined (Middleton3) as,

$$t_r \Delta B_R \ll 1.$$

The closed form expression of the Middleton Class-B is not available. It can be shown using different types of function for different values of noise amplitudes. The detailed description of the parameters are available in (Middleton1) and (Middleton4). The different methods of estimation of these parameters are available in (Middleton5). This Middleton model is used for characterising the natural noise sources like lightning discharges in atmosphere and man-made noises like automobile ignition etc (Middleton1).

6.2 Noise and Interferences: Modelled using Middleton Class-A

The different types of noises and interferences are accurately modelled using (6.1). The brief summary is presented in this section.

6.2.1 Microwave Oven

In recent years, the number of wireless devices, generally operated in ISM bands, are increased drastically. These devices operate in frequency ranges of 1GHz to 3GHz. In such type of devices, a microwave oven (MWO) is one of the devices who significantly generate a high level of interference.

This affects the performance of existing communications systems. The statistical characteristic of an interference from the MWO is significantly different than the characteristics of Gaussian noise. Hence, the communication systems, designed for Gaussian noise, perform poorly in the presence of high frequency interference. With this motivation, the authors (Kanemoto, Miyamoto, and Morinaga) have proposed the statistical model of microwave oven based on Middleton Class-A model. In this study, two types of MWO are considered, one is trans MWO and the other is switching MWO. The performance of the digital communication radio is evaluated under such interference environment. The first order statistical characteristic has been presented using Middleton Class-A model. Furthermore, the performance of optimum receiver with and without interleave scheme has been presented.

6.2.2 Co-channel Interference

The future wireless systems require higher bandwidth for accommodating a more number of users. In existing wireless systems, it is achieved by the reuse of spectrum resources. However, it causes the co-channel interferences. The performance of the wireless communication system degrades significantly because of such interference. Hence, the physical-statical modeling of co-channel interference has been proposed by (Gulati et al.) in the field of Poisson-Poisson clustered distributed interferers. In wireless networks, the Poisson-Poisson process is generally used to model the distribution of interferer clustering. The authors have proposed the unified framework for different wireless networks such as WiFi ad hoc network, cellular network and femtocell network. Furthermore, statistics of co-channel interference are derived, which perfectly modelled using Middleton Class-A model.

6.2.3 Micro-cellular Mobile Radio

Micro-cellular Mobile Radio To meet the capacity requirements and achieve an efficient use of the spectral resources, the Micro-cellular mo-

mobile radio is designed. This mobile system provides better signal strength because of line-of-sight transmission and the shorter radio paths. The authors (Prasad, Kegel, and de Vos) have proposed a new system model for evaluating the performance of Micro-cellular Mobile Radio system with Rayleigh-faded co-channel interference and impulsive non-Gaussian noise which is modelled using Middleton Class A noise. The differential phase shift keying modulation was considered. The bit error rate (BER) and spectral efficiency were derived for non-gaussian noise interference with different values of the impulsive index (A) and the ratio of Gaussian to non-Gaussian components (Γ) in Middleton Class-A model. Furthermore, the selection diversity has been used to improve the system performance in the case of fast multipath fading and impulsive non-Gaussian noise environment.

6.2.4 Impulsive noise to Wi-Fi Transceivers

The desktop computers and laptops are the fundamental necessity of our daily life. These devices contain many hardware components, for example clock generator and controllers. These generate radio frequency interference for the IEEE 802.11 a/b/g/n Wi-Fi transceivers. This interference has an impulsive nature. Hence, the authors of (Nassar et al.) have modelled such interference using Middleton class-A distribution and addressed the methods for the reduction of the interference to wireless transceivers. The detection performance under Middleton Class A interference has been analyzed for different types of receiver structure such as correlation receiver, Wiener filter, Bayes hypothesis testing, and myriad filtering.

6.2.5 LED Light Bulb: New Interferer for Future Radio System

It is now well known that the traditional incandescent bulbs have been replaced by light-emitting diodes (LEDs) because of their lower power consumption and much longer operating life. The LED bulb uses a switching power-supply circuit for achieving better lighting characteristics. This cir-

cuit generates a pulsed current for driving the LED. During the ON time, this current flows with power line to the LED and further causes unwanted radiation of broadband electromagnetic (EM) noise (Kanno and Akiyama). This noise contains high frequency harmonics in order of 100MHz. It covers the bandwidth of many wireless communications services.

A number of LED bulbs will become a major source of interference to existing communications systems. Recently, authors of (Matsumoto et al.) have proposed the statistical modelling of such radiated electromagnetic radio interference using Middleton Class-A noise characteristics. They have stated that the probability density function of such noise is characterized by Middleton class-A model which provides a good approximation.

6.3 Signal Processing Algorithms using Middleton Class-A model

Middleton Class A model is useful in modeling of non-gaussian noise in different signal processing algorithms as discussed below.

6.3.1 Channel Estimation

As the wireless channel is time-varying, the fast time-varying frequency-selective fading channels require continuous estimation of channel state vector. Most of the channel estimation algorithms have been proposed with the assumption that the received signal is corrupted with background Gaussian noise. However, in actual practice, the received signal is corrupted by non-Gaussian noise. Hence, the author (El-Mahdy1) has proposed an adaptive detection of signal in time varying channel with non-Gaussian noise assumption which is modelled using the class-A model.

The channel state vector has been estimated using adaptive estimation algorithms such as least mean square (LMS) and sign algorithm (SA). The authors have justified that the LMS algorithm is not the right choice when the noise impulsiveness becomes stronger. It happens because the performance of the LMS algorithm depends on the squared error function which

is sensitive to impulsive samples. The SA is a better choice which is based on clipping the error signal. Furthermore, the theoretical performance analysis has been presented for the proposed detector. The performance of the detector has been affected by the noise impulsive index.

6.3.2 MIMO Channel Equalization

For higher data rate applications, MIMO wireless communication is a preferred technology. However, it suffers from inter symbol interference and co-channel interference. Due to this, the process of equalization becomes more tedious.

The Kalman filter is used to equalize the time varying MIMO channels with decision feedback equalization with thermal Gaussian noise assumption, which has been proposed by (Enescu, Sirbu, and Koivunen). However, the received signal is significantly degraded by the impulsive noise sources as discussed in the previous section. With non-Gaussian noise environment, the Kalman filter is not the right option. Hence, the particle filter based MIMO channel estimation has been proposed in (Zheng-cong, Bin, and Ke). The authors have taken care of non-Gaussian noise which is modelled using Middleton Class-A noise. The MIMO channel estimation has been done using particle filter and a decision feedback equalizer which is derived based on MMSE criterion. Furthermore, the comparison of bit error rate performance comparison was done for Kalman Filter and Particles Filter method in channel tracking with NGN environment.

6.3.3 Modulation Classification

The future wireless networks will rely on heterogeneity i.e different types of wireless network users will share the resources with one another. In such scenario, the modulation classification is an important task for taking decision on signal confirmation, identification of various interferences and selection of demodulation scheme based on the received signal. In (El-Mahdy2), author has proposed the classification of minimum frequency shift keying (MFSK) signals transmitted in time varying and flat correlated

fading channels under impulsive noise environment, which is modelled using Middleton Class-A noise. A Karhunen-Loeve expansion has been used for the correlated fading process. The classifier had been proposed for different waveforms. Furthermore, the analytical expression for the classifier metric derived which is more sensitive to impulsive index. The proposed classifier can be used in an environment where dense impulsive interferers are present.

6.4 System Model

Let $\mathbf{y} = [y_1, y_2, \dots, y_n]^T$ be a vector of n observations of PU, received at secondary user, where $n \geq 1$. We assume that all the received observations are real and independent, and each y_i is represented as,

$$y_i = \sqrt{\rho} h_i s_i + w_i, \quad i = 1, 2, 3, \dots, n, \quad (6.2)$$

where ρ , h_i and w_i denote received SNR, channel coefficient and sample of Middleton class-A noise respectively. In (6.2), s_i denotes symbol of PUs, which is assumed as i.i.d. Gaussian with mean zero and variance one. Without loss of generality, we assume that all n observations are in ascending order. It means $y_1 \leq y_2 \leq \dots \leq y_n$. The channel is assumed to be quasi-static, $h_i \sim \mathcal{N}(0, 1)$, and it remains constant during a block of n symbols.

6.5 LRS- G^2 Sensing scheme under Middleton Class-A Noise

First, let $F_n(y)$ be the Empirical Cumulative Distribution Function (ECDF) of the received observations \mathbf{y} (Zhang),

$$F_n(y) = \frac{|\{i - \frac{1}{2} : y_i \leq y, 1 \leq i \leq n\}|}{n} \quad (6.3)$$

where $|\dots|$ indicates cardinality.

The problem of spectrum sensing as a null-hypothesis testing problem is

defined as (Wang et al.),

$$\begin{aligned} H_0 : F_n(y) &= F_0(w) \\ H_1 : F_n(y) &\neq F_0(w). \end{aligned} \quad (6.4)$$

In (2.13), the $F_0(w)$ represents any continuous CDF of noise. Here, we consider Middleton Class A noise as defined in (6.1).

Second, for computing the difference between $F_n(y)$ and $F_0(w)$, the omnibus Zhang's test statistics is used (Zhang), as

$$Z_c = \sum_{i=1}^n \left[\log \left\{ \frac{u_i^{-1} - 1}{(n - \frac{1}{2})/(i - \frac{3}{4}) - 1} \right\} \right]^2 \quad (6.5)$$

where $u_i = F_0(y_i)$.

Third, for the detection of PU, the value of threshold (γ) is selected such that false alarm probability (P_f) is α . It means

$$\alpha = \mathbb{P}\{Z_c > \gamma | H_0\} \quad (6.6)$$

The value of γ can be determined for a specific value of α using (Zhang). For example, when $\alpha = 10^{-3}$ and $n = 30$, the value of γ is 31.707.

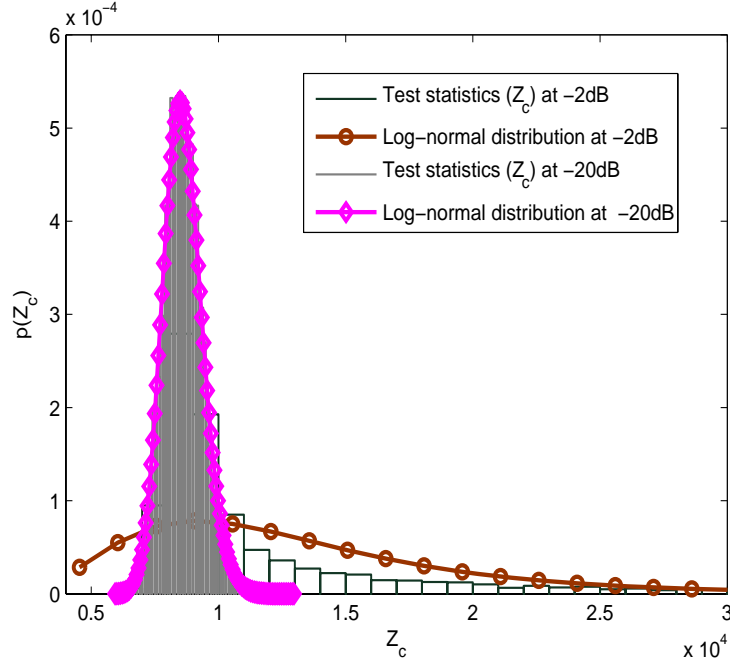
At last, compare the value of (6.5) with γ . If $Z_c > \gamma$, then reject the null hypothesis H_0 in favor of the presence of PU signal. Otherwise, declare that the PU is absent. Compute performance metric as Probability of Detection (P_d) with a given value of P_f .

6.5.1 Sampling Distribution of Z_c

The sampling distribution of Z_c is not mathematically tractable. Hence, we use distribution fitting technique to approximate the probability density function (pdf) of Z_c . The approximated pdf of Z_c , at low SNR, is closely matching with Log-normal pdf as shown in 6.1, which can be expressed (Proakis), as

$$p(z_c) = \frac{1}{z_c \sqrt{(2\pi)\sigma}} e^{\frac{-(\ln z_c - \mu)^2}{2\sigma^2}} \quad (6.7)$$

It is to be noted that mean (μ') and standard deviation (σ') of Z_c can be expressed as $e^{\mu + \frac{\sigma^2}{2}}$ and $(e^{\sigma^2} - 1)e^{2\mu + \sigma^2}$ respectively. In Table 6.1, we


 Figure 6.1: The approximated Log-normal distribution for the test statistics(Z_c)

show μ' and σ' of Z_c , while approximating it with Log-normal pdf for total samples of $n = 50000$. We have also shown errors in μ' and σ' as $(\epsilon_{\mu'})$ and $(\epsilon_{\sigma'})$ respectively for SNR of -2 dB, -6 dB and -20 dB. It can be seen that the errors are very low and decreasing as SNR reduces further.

Now, the theoretical detection probability(P_d) can be derived as,

$$\begin{aligned}
 P_d &= \mathbb{P}\{Z_c > \gamma | H_1\} \\
 &= 1 - F_{Z_c, H_1}(\gamma) \\
 &= 1 - \left\{ \frac{1}{2} + \frac{1}{2} \operatorname{erf} \left\{ \frac{\ln \gamma - \mu'}{\sqrt{2}\sigma'} \right\} \right\} \\
 &= \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left\{ \frac{\ln \gamma - \mu'}{\sqrt{2}\sigma'} \right\}
 \end{aligned} \tag{6.8}$$

In the next section, we present results using simulations and this analytical expression of P_d .

6.6 Performance Results

In this section, for different values of SNR (ρ), the receiver operating characteristics (ROC), i.e. plot of P_d versus P_f , are presented for the proposed

Table 6.1: The parameters of approximated Log-Normal distribution for Z_c with $n = 50000$.

SNR = -2dB			
μ'	σ'	$\epsilon_{\mu'}$	$\epsilon_{\sigma'}$
10.823	0.286126	0.0016635	0.0011763
SNR = -6dB			
10.7524	0.188694	0.000988	0.000698
SNR = -20dB			
10.6707	0.0395586	0.000194401	0.000137465

scheme using simulations. As the considered Middleton Class A noise is a combination of Gaussian and non Gaussian components, the dominance of Gaussian component can be expressed by two parameters; impulsive index (A) and the ratio of gaussian to non-gaussian noise component (Γ). We have shown the effect A and Γ on P_d .

The ROC is shown in Fig. 6.2 for the proposed scheme at SNR of -2 dB and -6 dB with received observations $n = 30$ in quasi-static channel with PU signal as *i.i.d* Gaussian. It can be seen that analytical and simulation results are closely matching as SNR reduces from -2 dB to -6 dB. Assuming same noise environment, ROC for AD sensing (Wang et al.) and Chi-square (Teguig, Le Nir, and Scheers) sensing are presented. It can be seen that the proposed scheme outperforms both the prevailing schemes.

In Fig. 6.3, the P_d is presented for different values of Γ with SNR = 2 dB, -6 dB and -12 dB, taking $n = 30$, $A = 0.2$ and $P_f = 0.05$. Higher values of Γ represent dominance of Gaussian noise. It can be seen that P_d decreases as Γ increases. Thus, the performance is worst for the Gaussian noise.

In Fig. 6.4, ROC is presented for different values of A with SNR = -10 dB, $n = 50$, and $\Gamma = 0.4$. In this case also, higher values of A represent

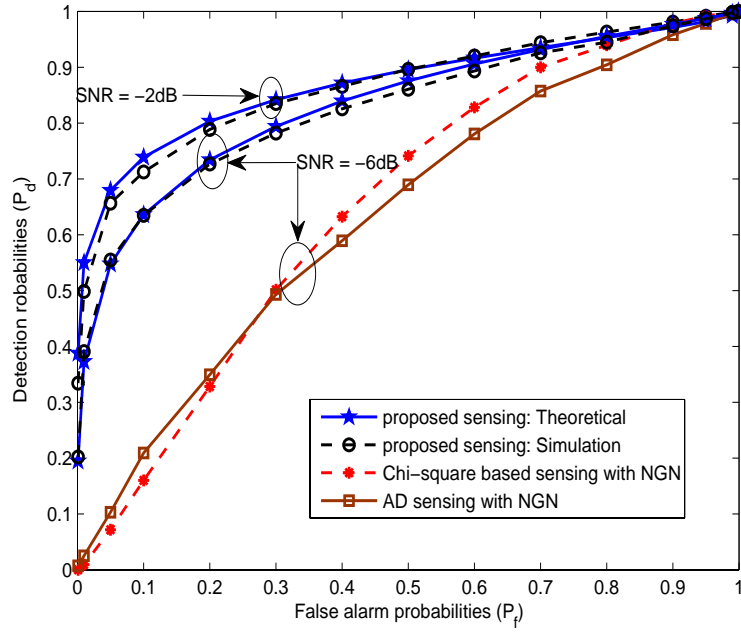


Figure 6.2: ROC with $n = 30$, $A = 0.2$, $\Gamma=0.5$ in quasi-static channel with Middleton Class A noise.

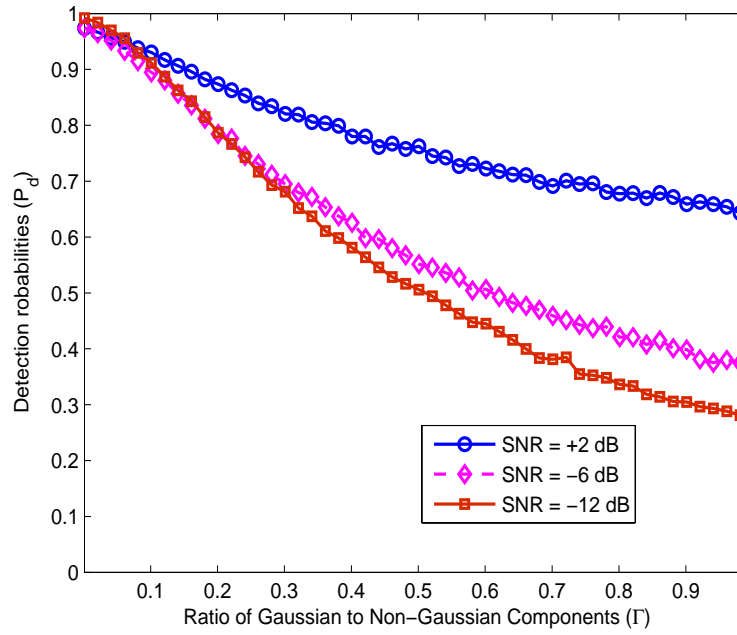


Figure 6.3: P_d versus Γ for different SNR, $P_f = 0.05$, $n = 30$ with $A = 0.2$ with Middleton Class A noise in quasi-static channel.

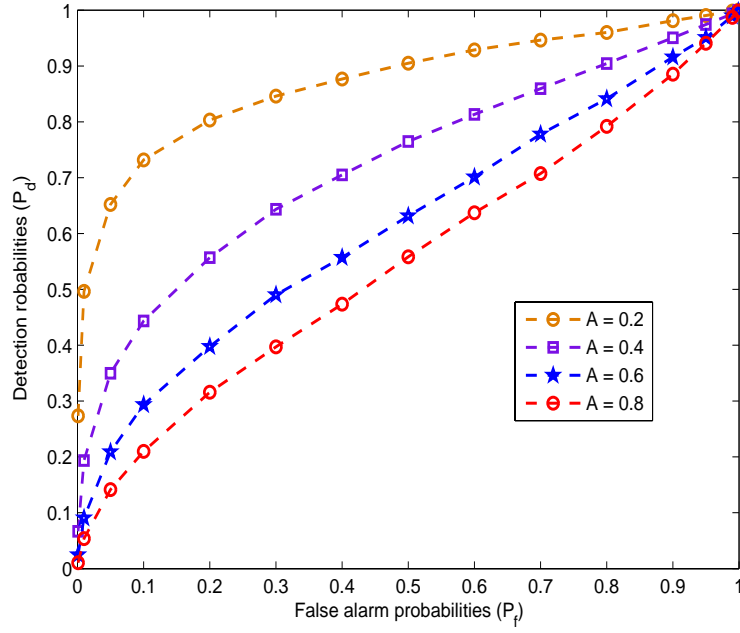


Figure 6.4: ROC at $\Gamma = 0.4$, $\text{SNR} = -10\text{dB}$ and $n = 50$ with Middleton Class A noise in quasi-static channel.

dominance of Gaussian noise. It can be seen that performance is degraded as A increases. Thus, the performance is worst for the Gaussian noise.

6.7 Conclusion

The non-parametric sensing scheme was proposed for non Gaussian environment modelled by Middleton class-A noise. The pdf of test statistics was approximated as Log-normal and an expression of detection probability P_d was derived. The simulation results are closely matching with the analytical counterparts for lower SNR. The proposed scheme outperforms prevailing schemes in the considered noise environment. It was also observed that the performance is degraded as the contribution of Gaussian components is higher than that of the non Gaussian components.

Chapter 7

Conclusion

The Cognitive radio (CR) is a key enabling technology for solving the problem of spectrum scarcity because of fixed allocation of spectrum in existing wireless services. The CR provides legacy right to use the spectrum resources assigned to licensed or primary users (PU) without interfering to it. This can be implemented using an opportunistic spectrum access (OSA) at CR. The OSA can be successfully implemented when CR is able to detect the PU when it is not utilizing its allocated channel. The detecting or identifying the available channel from the PU is called as spectrum sensing. In spectrum sensing, goodness of fit (GoF) based non-parametric algorithms are realistic as they do not require a priori information about the transmitted signal or channel state information. A few GoF based sensing schemes have been proposed in the literature for spectrum sensing. However, majority of them have considered a very high number of received observations, which results in high sensing time.

In this thesis, we have proposed GoF based sensing methods with a view to reduce number of received observations without compromising detection performance for low SNR and low probability of false alarm. Furthermore, we have shown performance of the proposed schemes for different PU signals, channel conditions and non-Gaussian noise as a background noise, which was modelled by Middleton Class A noise. In the subsequent sections, we show major conclusions and future work.

7.1 Conclusion

The concluding remarks are as follows.

- a. In the proposed Censored AD sensing (CAD sensing), with less number of received observations, the detection performance was presented using ROC. It is almost similar to the performance of AD sensing with all received observations. This reduces complexity of the CR terminal in determination of statistics in decision rule.
- b. In wireless communication, the performance is usually degraded due to fading compared to the performance of AWGN channel. We have shown performance of the proposed CAD sensing in AWGN, quasi-static channel and time varying channel modelled by $AR1$ process. We conclude that the performance is degraded when the channel is converted from slow to fast time varying. We have also presented performance of prevailing schemes such as ED and OS based sensing. The proposed CAD sensing outperforms these prevailing schemes in fading channels.
- c. In GoF based sensing schemes, we do not require information about PU, however we know about statistics of noise. Imperfect information of this statistics may deteriorate the performance. We propose Blind CAD sensing in which distribution of noise is modelled by using Student t-distribution. We conclude that the proposed Blind CAD sensing is more robust in this condition than prevailing GoF based sensing.
- d. Majority of GoF based schemes have been evaluated assuming PU signal as constant ($= 1$) in literature. However, for different PU signals such as independent and identically distributed (i.i.d) Gaussian or single frequency sine signals, the performance of GoF schemes such as AD, KS, OS and CvM degrades. Hence, we proposed a novel robust non-parametric sensing scheme $LRS-G^2$ based on likelihood ratio statistic derived from the empirical distributions for the Gaussian noise environment. We have shown that the proposed $LRS-G^2$ outperforms

all the prevailing schemes.

- e. The proposed LRS- G^2 sensing outperforms ED sensing and other traditional GoF sensing schemes at less number of received observations and lower value of false alarm probability. Furthermore, not only for different types of PU signals or for different channel conditions such as AWGN channel or quasi-static channel or Time varying channel, LRS- G^2 outperforms ED sensing and traditional GoF sensing schemes. In addition to this, Blind LRS- G^2 , proposed for noise uncertainty conditions, outperforms Blind-AD sensing significantly with reduced number of observations.
- f. The additive noise is usually assumed as Gaussian. However, when we consider the mixed heterogeneous network environment, the additive noise cannot follow the Gaussian distribution. Hence, we designed LRS- G^2 sensing algorithm for non-Gaussian noise (NGN) environment which is modelled using Middleton Class-A model. Under such scenario of NGN, the proposed scheme outperforms recently proposed Chi-square based sensing and AD sensing. The test-statistic of the proposed scheme is approximated using log-normal distribution with minimum mean square error. Based on this, the approximate closed form expression of probability of detection (P_d) is derived for GoF based sensing under non-Gaussian noise environment.

7.2 Future scope

In the above discussion of non-parametric sensing, we have used SISO systems, however it can be extended for multiple input multiple output (MIMO) systems also. The performance of the proposed schemes can be evaluated in MIMO systems assuming different channel conditions, different structure of PU signals or non Gaussian noise environment. In this thesis, we have considered PU with real signals, however it can be extended for complex signals also. In parametric sensing, cooperative spectrum sensing has several advantages. The cooperative spectrum sensing can also be considered assuming the proposed GoF based sensing. The proposed re-

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