

# Modeling and Analysis of DFIG based Power System

## Major Project Report

*Submitted in Partial Fulfillment of the Requirements*

*for the degree of*

**MASTER OF TECHNOLOGY**

**IN**

**ELECTRICAL ENGINEERING  
(ELECTRICAL POWER SYSTEMS)**

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# CERTIFICATE

This is to certify that the Major Project Report (Part-II) entitled “**MODELING AND ANALYSIS OF DFIG BASED POWER SYSTEM**” submitted by **Mr. Nirav R. Maheta (14MEEE08)**, towards the partial fulfillment of the requirements for Semester-IV of Master of Technology (Electrical Engineering) in the field of Electrical Power Systems of Nirma University is the record of work carried out by him under my supervision and guidance. The work submitted has in my opinion reached a level required for being accepted for examination. The results embodied in this major project work to the best of my knowledge have not been submitted to any other University or Institution for award of any degree or diploma.

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I, **Nirav R. Maheta** (Roll.No.14MEEEE08), give undertaking that the Major Project entitled “**Modeling and Analysis of DFIG based Power System**” submitted by me, towards the partial fulfillment of the requirements for the degree of Master of Technology in Electrical Power Systems of Nirma University, Ahmedabad, is the original work carried out by me and I give assurance that no attempt of plagiarism has been made. I understand that in the event of any similarity found subsequently with any published work or any dissertation work elsewhere; it will result in severe disciplinary action.

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# Abstract

Doubly Fed Induction Generator(DFIG) based wind penetration in the power system is increasing every day. Many issues like inertia reduction, reduced Low voltage ride through capability, system stability etc. are also to be handled while increasing the wind generation penetration. It is meaningful model DFIG based power system and perform various analysis on the same. Among all the issues that has been written above mainly inertia reduction and stability related issues has been discussed in this thesis. Considering the inertia reduction issue, the controller of the system functions to change the torque set point, adjusting pitch compensation and maximum power, so that the converter responds to the changes in the frequency during transient conditions in order to increase the effective inertia of the system. In order to check the stability of any system eigenvalue analysis is a tool to employ and has been discussed here. Synchronous generators based SMIB is modelled mathematically and the obtained eigenvalues are verified by finding the eigenvalues from in the standard software. Further, one synchronous machine is replaced by equivalent rating of DFIG (farm) in SMIB system and WSCC 9 bus standard test system. The system eigenvalues are determined analytically and validated through software.

## Abbreviations

DFIG	.....	Doubly Fed Induction Generator
PMSG	.....	Permanent Magnet Synchronous Generator
WECS	.....	Wind Energy Conversion System
SMIB	.....	Single Machine Infinite Bus System
LVRT	.....	Low Voltage Ride Through
ROCOF	.....	Rate of Change of Frequency

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# Chapter 1

## Introduction

### 1.1 General Description

Wind has been used as a source of power since centuries. Wind energy is extracted from the blowing air using wind turbines to generate the electrical energy. It is the fastest growing energy domain among all renewable energy sources. The kinetic energy of the wind is used to rotate the blades of turbines which are then used to generate electricity.

Doubly fed Induction generator (DFIG) is most widely used for wind power generation due to its various advantages over fixed speed machine and Permanent Magnet Synchronous Generator (PMSG). The important advantage of DFIG over PMSG is that the convertor used with DFIG is of low rating compared to PMSG and over fixed speed machine is that DFIG increases the efficiency by operating it near to its optimal turbine efficiency.

Apart from advantages, there are lots of issues with DFIG based Wind Energy Conversion System (WECS) when connected to the grid. The work done mainly focuses on the modeling of DFIG based power system, which is simulated in MATLAB and PowerWorld Simulator software and then the necessary analysis are performed.

## 1.2 Significance of the work

In present scenario, wind power penetration in conventional power system is increasing rapidly. When a DFIG based WECS is connected to the grid there are many issues, few major issues are reduction in the fault ride through capability, reduction of low voltage ride through capability, reduction in the inertia of system and stability issue. So, to understand these issues proper model of the grid-connected variable speed WECS and also some standard system with DFIG should be established first and then necessary analysis would be carried out.

## 1.3 Literature Survey

Literature survey is carried out to develop the basic understanding and modeling of DFIG based WECS, it also helps us to understand the issues that occur when DFIG based WECS is connected to the grid. Most of the below listed literature is about eigenvalue analysis and reduction in inertia of DFIG based power system.

In [1], authored by J.S. Lather, S.S Dhillon and S.Marwaha, includes the issues related to DFIG based WECS connection to the grid. It explains about the issues like LVRT, Stator harmonic current control, reduced system inertia, etc. and gives the possible solution related to it. Most of the issues discussed in this paper are based on frequency response and on the inertia of the system.

In [2], authored by Oleksandra Dudurych and Michael Conlon discuss about the impact of reduce system inertia due to high wind power penetration. With this reduced system inertia, rate of change of frequency due to major system events is increased. This will cause unintended tripping of anti islanding protection systems. This literature also suggests controllers for controlling the rate of change of frequency and for increasing the damping in the system.

In [3], authored by Durga Gautam, Vijay Vittal and Terry Harbour, gives control

strategy to mitigate impact of reduced system inertia. In this paper main idea is to design a controller such that the inertia supplied by the DFIG increases. In this control strategy mainly three things are focused; changing the torque set point of the DFIG with change in frequency, the parameter of the PI controller of the pitch compensator is selected such that the pitch compensator will not increase the pitch angle during transient condition when the system is about to loose the generation and change in maximum power supply.

In [4],[5],[6] authors explain different strategy to emulate inertia and improve the frequency response.

In [7], authored by Indrajit Koley , Swarnankur Ghosh et al, explains the DFIG, AC to DC and DC to AC converter control and also explains about the simulated results for isolated as well as grid connected DFIG.

In [8], authored by Hector A. Pulgar-Painemal, and Peter W. Sauer described two machine system. In this paper, model analysis of DFIG based power system has been performed and also active and reactive power controllers are modeled. Basically in this paper system behavior is analyzed when system is connected to synchronous generator and when connected to negative load.

In [9], authored by Françoise Mei and Bikash C. Pal explained the modeling and performed small signal analysis of grid connected DFIG.

In [10], authored by Sushanta kumar Senapati derived detail mathematical model (including power grid, aerodynamic system of wind turbine, rotor mechanical system, DFIG, and the overall control system) of DFIG based WECS connected to grid.

In [11], authored by M. Jazaeri<sup>1</sup>, A.A. Samadi, H.R. Najafi and N. Noroozi-Varcheshme modeled DFIG based WECS connected to grid and performed eigenvalue analysis during normal, sub synchronous and super synchronous modes and then all eigenvalues

had been checked for dynamic stability.

In [12], authored by L.M. Fernandez a, C.A. Garca, J.R. Saenz, F. Jurado has focused on procedure for Equivalent models of wind farms by using aggregated wind turbines.

In [13], Luis M. Ferna ndeza, Francisco Juradob, Jose Ramo n Saenza has focused on procedure for Aggregated dynamic model for wind farms with doubly fed induction generator.

In [14], Luis M. Ferna ndeza, Francisco Juradob, Jose Ramo n Saenza explain the modeling of dfig based SMIB, explain the procedue for eigenvalue analysis of DFIG based power system and also optimize the controller parameters.

In [15], Peter W. Sauer, M.A.Pai explain the modeling of SMIB, and also explain the procedue for eigenvalue analysis of WSCC 9bus sytem.

## **1.4 Problem Identification**

During the literature survey it is observed that when a DFIG based WECS is connected to the grid, some problems are observed and one such problem is that due to high wind power penetration, system inertia is reduced. It is clearly observed that most of the issues are directly or indirectly dependent on frequency response and hence on the system inertia. Stability related problem have also been reported in various literature [1],[2],[3].

## **1.5 Objectives of Work**

Based on literature survey, it is evident that upon connecting DFIG based WECS to the grid, numerous issues are emerging. A few among them is inertia reduction and stability related issues. The main objective of the dissertation is to model SMIB

system and WSCC 9 bus system consisting of DFIG as one of the source and to perform inertia reduction related observations and eigenvalue analysis in order to check DFIG behaviour in both steady state as well as transient condition. Based on the analysis, some corrective actions may be possible to suggest.

## **1.6 Scope of Work**

Due to high wind power penetration, system inertia (or kinetic energy stored in rotor) is not available during transient period, hence rate of change of frequency increases and frequency response becomes poor. The controller on the rotor side converter operates with speed deviation and it will not respond to frequency deviation. So in this first model, the additional controller is designed which responds to changes in the frequency and also exhibit hidden inertia of the blades during transient condition. Hence, frequency response becomes better.

In conventional SMIB system, Eigenvalue analysis is performed. Eigenvalue analysis is used to design controller and also used to check the stability of the system.

In SMIB system consisting of DFIG as a source, eigenvalue analysis is performed. Eigenvalue analysis is used to optimize controller parameters and also used to check the stability of the system.

In fifth model, Eigenvalue analysis is performed on WSCC 9bus system consisting one source as DFIG. It is validated through Power World Simulator software(demo version-19). Eigenvalue analysis is used to design controller and also used to check the stability of the system.

## **1.7 Methodology / Project Planning**

- Literature survey
- Identification of the problem

- Methodology to be adopted
- Requisite of the software
- Simulation and mathematical modeling
- Results
- Conclusion

## **1.8 Organization of Thesis**

Chapter 1 introduces requirement of DFIG based WECS, literature survey, reason for selection of this topic, objective of the work and scope of the work.

Chapter 2 introduces the system, its related simulation and mathematical modeling.

Chapter 3 presented results based on simulation and mathematical modeling done.

Chapter4 includes conclusion and future work.



# Chapter 2

## Simulations and Mathematical Modeling

In this chapter four systems are discussed.

DFIG based Power System with proposed controller is shown in Fig 2.2. The system is taken from "www.mathworks.com" and its analysis is done, results and related discussion is presented in chapter 3.

Conventional SMIB system is shown in Fig 2.5. The eigenvalue analysis is performed in PSAT tool of MATLAB software as well as in PowerWorld simulator software. Related results and discussion is shown in chapter 3.

SMIB consisting of DFIG as a source is shown in Fig.2.6. The eigenvalue analysis is performed in PowerWorld Simulator software. Related results and discussion is shown in chapter 3.

Modified WSCC 9 Bus system consisting DFIG machine as one of the source is shown in Fig.2.7. The eigenvalue analysis is performed in PowerWorld Simulator software. Related results and discussion is shown in chapter 3.

MATLAB and PowerWorld Simulator (demo version-19) software are used for simu-

lation of above systems.

## 2.1 Inertia Control Model in DFIG based Power System

Generally DFIG operate on power speed characteristics as shown in Fig.2.1

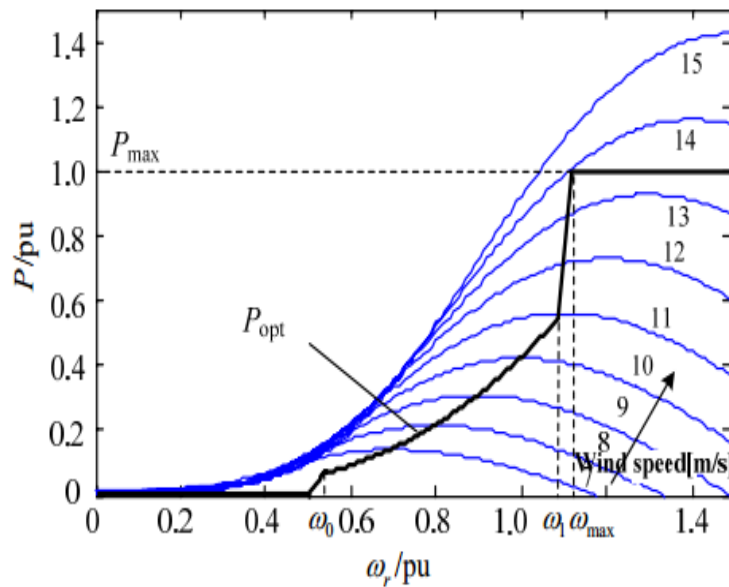


Figure 2.1: Power speed characteristics of DFIG

The torque set point is an input for the converter control that realizes the torque by controlling the generator currents. The conventional controller gives a torque set point that is based on measured speed and power and it will not respond to the change in the frequency. So the controller is modified such that it will respond to the speed change as well as to the change in the frequency and rate of change of frequency by providing additional damping in to the system.

The modified controller is shown in a fig. 2.2.

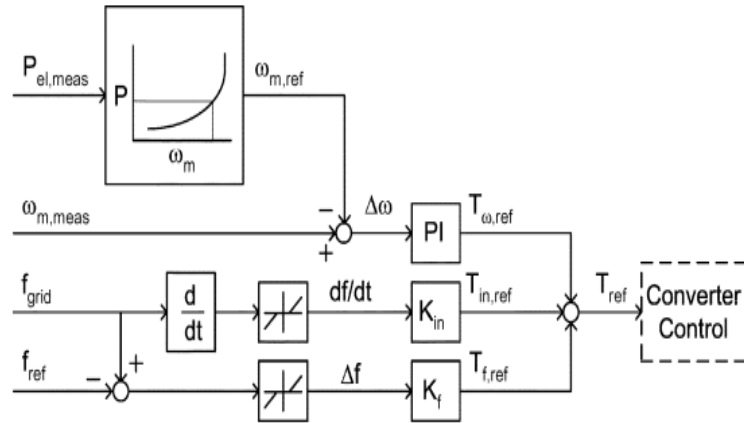


Figure 2.2: Modified controller of DFIG

The DFIG based power system with modified controller is shown in fig. 2.3. There are mainly two area in a system. The total load of system is 1600 MVA. Area 1 consist of 800 MW of load, one 900 MVA synchronous generator and one DFIG based wind farm consists of different number of wind turbines based on the level of wind power penetration. Area 2 also consist of 800 MW of load and two synchronous machine of 900 MVA. Area 1 and Area 2 is connected to bus 1 and bus 2 respectively. All three synchronous generator in system are set to a power reference of 0.535pu.

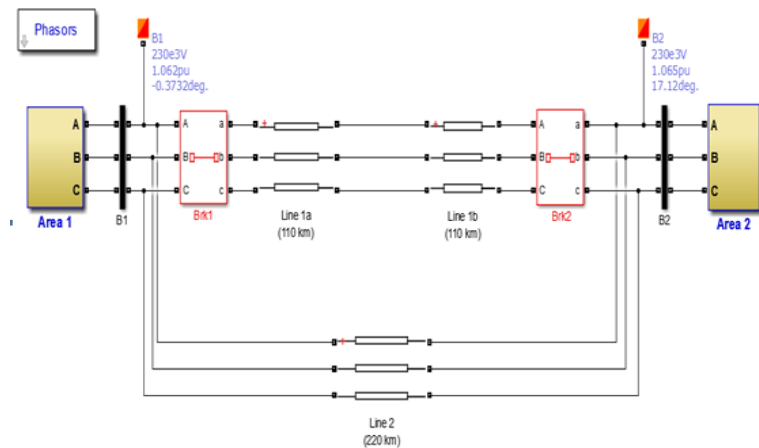


Figure 2.3: DFIG based Power System model (courtesy : [www.mathworks.com](http://www.mathworks.com))

Fig. 2.4 shows the Area 1 of the system. The 163 MW load is added at 30 s to bus 1. At this time comparative studies of network frequency responses with and without the modified controller following load sudden changes is done. The results and related discussions are carried out in chapter 3.

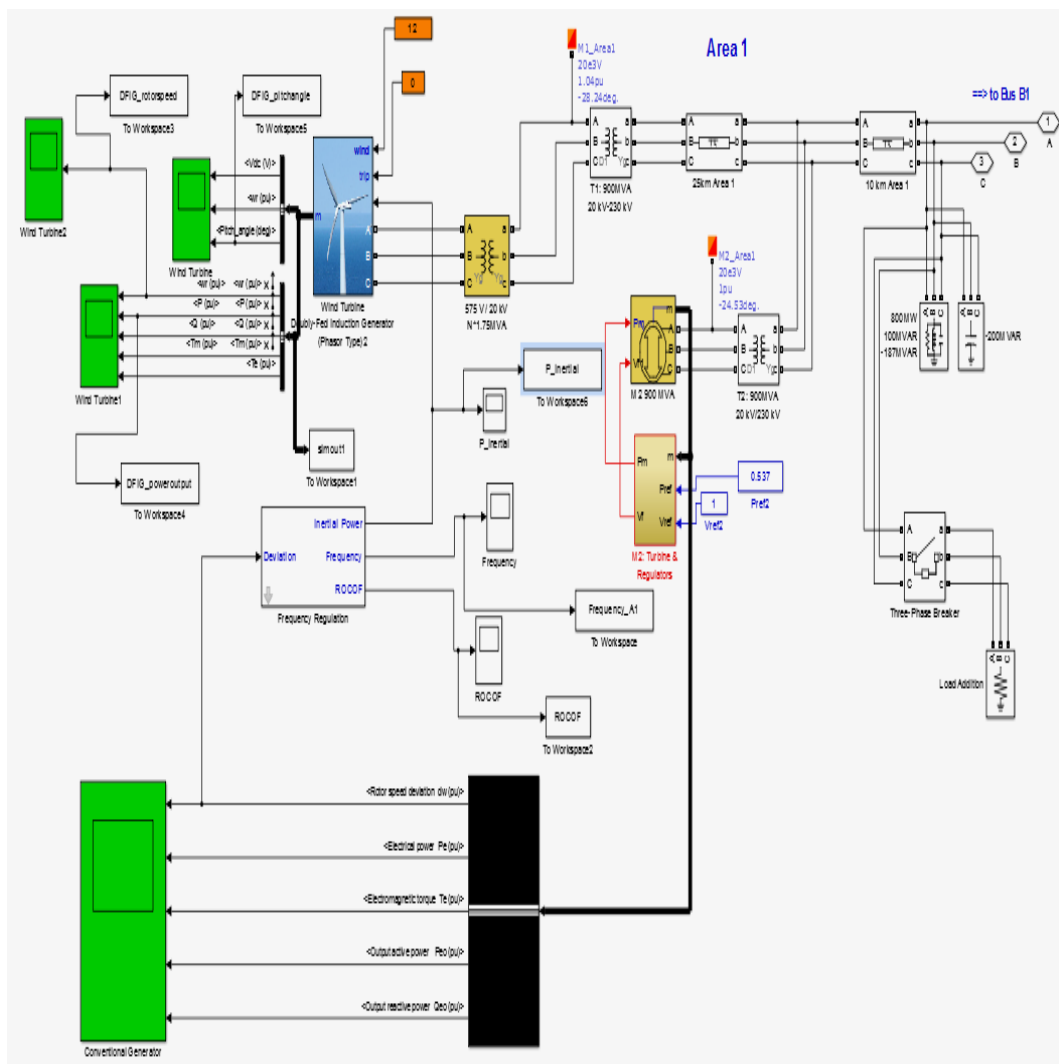


Figure 2.4: Area 1 - a subsection of DFIG based Power System model (courtesy : [www.mathworks.com](http://www.mathworks.com))



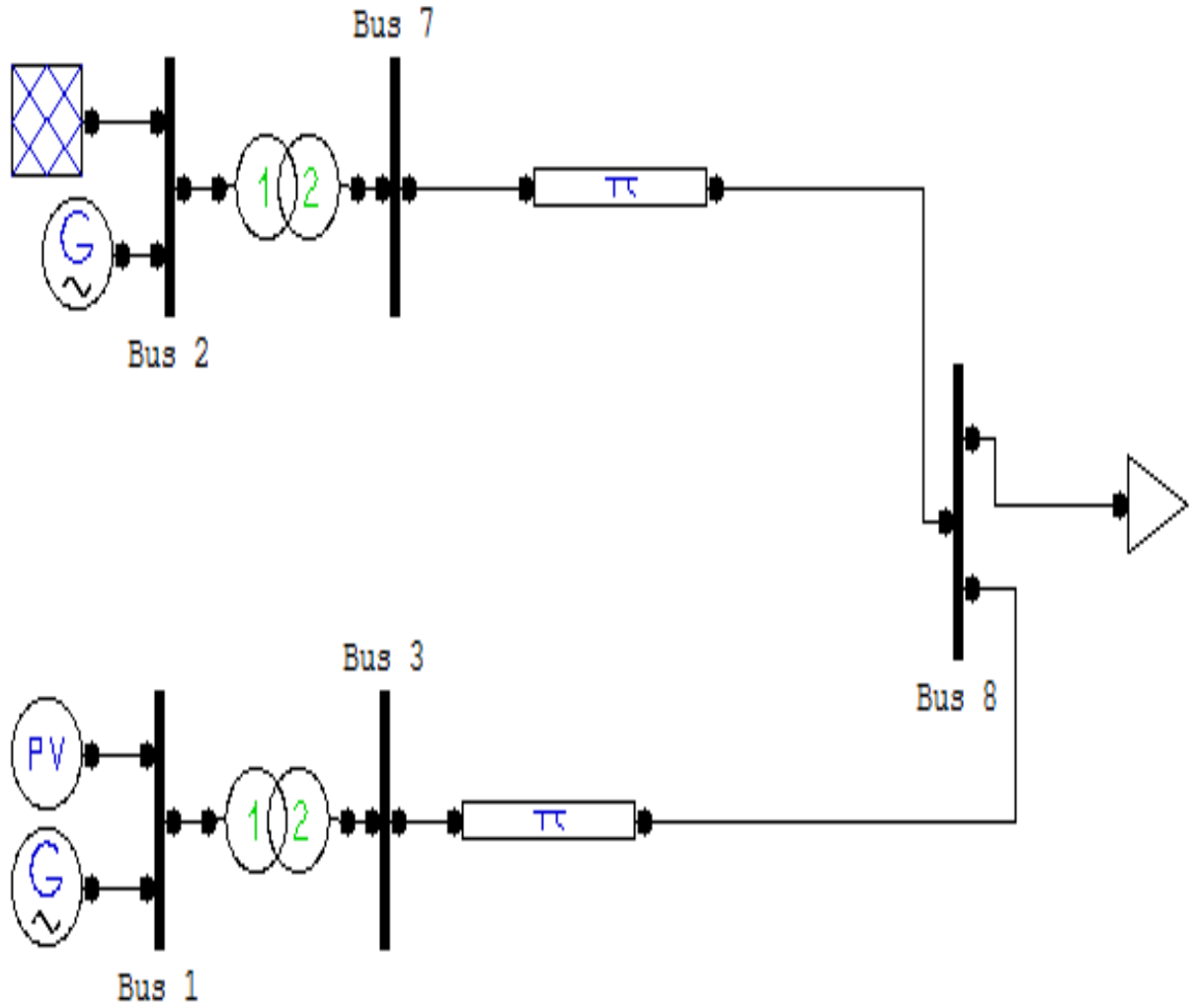


Figure 2.6: Conventional SMIB system PSAT Model

$$X'_{d1} = 1.81, X_{tr1} = 0.15, X_{line1} = 0.6, X'_{d2} = 0.019, X_{tr2} = 0.0062, X_{line2} = 0.007, E' = 1,$$

$$E_0 = 1, \delta_0 = 0, 2H = 3.5 \text{ MWs/MVA.}$$

The eigenvalue calculation of the system Fig 2.6 is shown below:

$$X_{total} = \left( X'_{d1} + X_{tr1} + X_{line1} \right) \parallel \left( X'_{d2} + X_{tr2} + X_{line2} \right) \quad (2.1)$$

=0.098pu

$$K_s = \frac{E' E_0}{X_{total}} \cos \delta_0 \quad (2.2)$$

=11.052 pu torque/rad

$$A = \begin{bmatrix} \frac{-K_d}{2H} & \frac{-K_s}{2H} \\ W_0 & 0 \end{bmatrix} \quad (2.3)$$

$$= \begin{bmatrix} 0 & -3.5 \\ 376.8 & 0 \end{bmatrix} \quad (2.4)$$

$$A - \lambda I = \begin{bmatrix} 0 - \lambda & -3.5 \\ 376.8 & 0 - \lambda \end{bmatrix} \quad (2.5)$$

$$\lambda = \pm 34.49i$$

The eigenvalues obtained are: 34.49i and -34.49i.

The same model is prepared in PowerWorld Simulator software because PSAT tool of MATLAB software does not give accurate eigenvalues. Whenever we run this model it shows different eigenvalues every time.

The model shown in Fig.2.7 consist of 10 MVA synchronous machine which is coupled with AVR and PSS block but during calculation the effect of AVR and PSS is not considered. 100 MVA is taken as the base for the calculation.

Eigenvalue calculation of the system has been shown below: The required parameter values is shown below:

**Parameters used for the calculation and in a software are:(all units are in pu)**

$$X'_d = 0.1, X_{tr} = 0.2, X_{line1} = 0.2, X_{line2} = 0.3, E' = 1.053, E_0 = 1, \delta_0 = 6.53, 2H = \text{MWS/MVA}.$$

$$X_{total} = X'_{d1} + X_{tr1} + \{(X_{line1} \times X_{line2}) \div (X_{line1} + X_{line2})\} \quad (2.6)$$

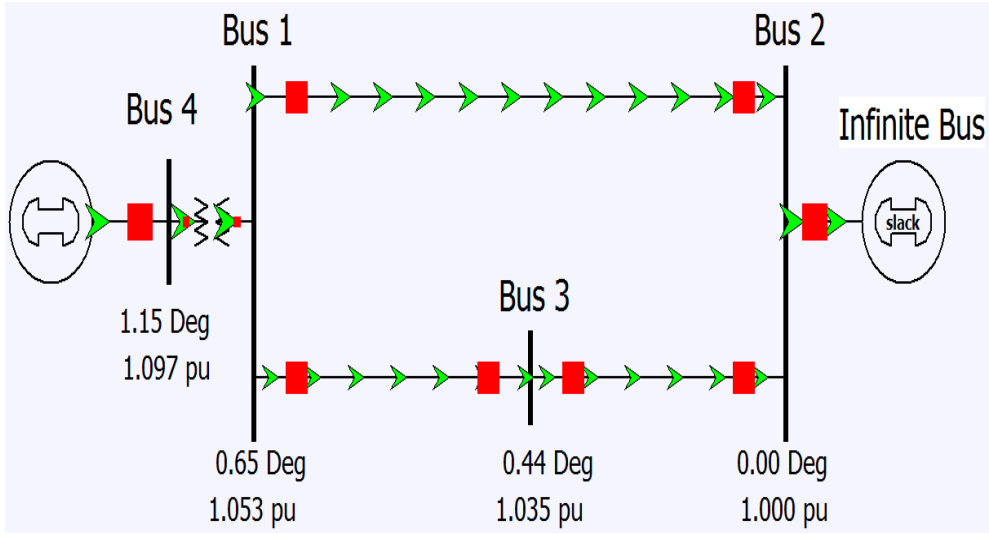


Figure 2.7: Conventional SMIB system Power World Simulator model

$$=0.42 \text{ pu}$$

$$K_s = \frac{E' E_0}{X_{total}} \cos \delta_0 \quad (2.7)$$

$$=2.558 \text{ pu torque/rad}$$

$$A = \begin{bmatrix} \frac{-K_d}{2H} & \frac{-K_s}{2H} \\ W_0 & 0 \end{bmatrix} \quad (2.8)$$

$$= \begin{bmatrix} 0 & -0.42647 \\ 376.8 & 0 \end{bmatrix} \quad (2.9)$$

$$A - \lambda I = \begin{bmatrix} 0 - \lambda & -0.4264 \\ 376.8 & 0 - \lambda \end{bmatrix} \quad (2.10)$$

$$\lambda = \pm 12.0i$$

The eigenvalues obtain are +12i and -12i.



## 2.3 SMIB consisting DFIG as a source

The model in Fig.2.8 shows the SMIB system consist of DFIG. This model is prepared in PowerWorld Simulator software. This system consist of one DFIG generator of 2 MVA and for modeling purpose 100 MVA generator is connected to bus which can be treated as infinite bus.

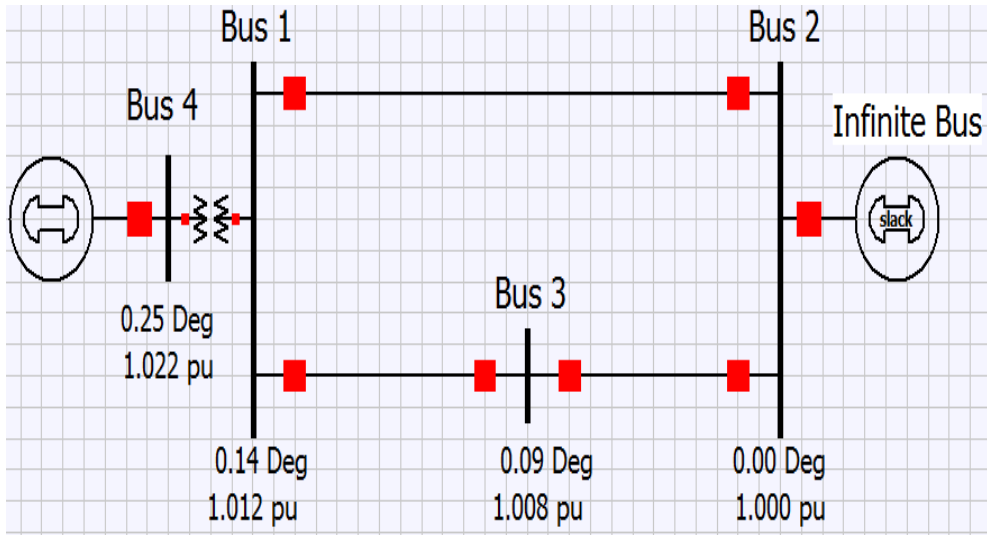


Figure 2.8: SMIB system consisting of DFIG as a source

For system modelling refer equation from [14]. The equations are mentioned in Appendix A. The system matrix and calculation of eigenvalues are shown below:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & A_{36} & A_{37} & A_{38} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & A_{46} & A_{47} & A_{48} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & A_{56} & A_{57} & A_{58} \\ A_{61} & A_{62} & A_{63} & A_{64} & A_{65} & A_{66} & A_{67} & A_{68} \\ A_{71} & A_{72} & A_{73} & A_{74} & A_{75} & A_{76} & A_{77} & A_{78} \\ A_{81} & A_{82} & A_{83} & A_{84} & A_{85} & A_{86} & A_{87} & A_{88} \end{bmatrix} \quad (2.11)$$

Where, A is a system matrix.

$$A = \begin{bmatrix} -17.91 & 6969.9 & -112.07 & -6527.6 & -377.52 & 2019.7 & 0 & 100.98 \\ -6962.6 & -19.281 & 6527.6 & -112.07 & 2046.5 & 0 & 2019.7 & 0 \\ -17.405 & 6750.2 & -114.82 & -6311.3 & -386.81 & 2069.3 & 0 & 103.46 \\ -6742.6 & -18.812 & 6311.3 & -114.82 & 2149.2 & 0 & 2069.3 & 0 \\ 0.3913 & -0.112 & -0.3733 & -0.0311 & 0 & 0 & 0 & 0 \\ 0.2503 & 0.0417 & -10 & 0 & 0 & 0 & 0 & 10 \\ 0.6867 & 0.1143 & 0 & -10 & 12.621 & 0 & 0 & 0 \\ -0.6926 & 0.1153 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.12)$$

$$A - \lambda I = \begin{bmatrix} -18 - \lambda & 6969 & -112 & -6527 & -377 & 2020 & 0 & 101 \\ -6963 & -19 - \lambda & 6527 & -112 & 2046 & 0 & 2020 & 0 \\ -17 & 6750 & -115 - \lambda & -6311 & -387 & 2069 & 0 & 103 \\ -6742 & -19 & 6311 & -115 - \lambda & 2149 & 0 & 2069 & 0 \\ 0.391 & -0.11 & -0.37 & -0.03 & 0 - \lambda & 0 & 0 & 0 \\ 0.250 & 0.042 & -10 & 0 & 0 & 0 - \lambda & 0 & 10 \\ 0.69 & 0.11 & 0 & -10 & 13 & 0 & 0 - \lambda & 0 \\ -0.69 & 0.11 & 0 & 0 & 0 & 0 & 0 & 0 - \lambda \end{bmatrix} \quad (2.13)$$

The eigenvalues obtained analytically are:

Table 2.1: Analytically obtained eigenvalues for SMIB system consisting DFIG as a source

No.	Eigenvalues obtain Analytically
1.	-0.22
2.	-0.67
3.	-21.1-68i
4.	-21.1+68i
5.	-42.1-204i
6.	-42.1+204i
7.	-69.7 + 520i
8.	-69.7 - 520i

## 2.4 Modified WSCC 9 bus system Consisting of DFIG

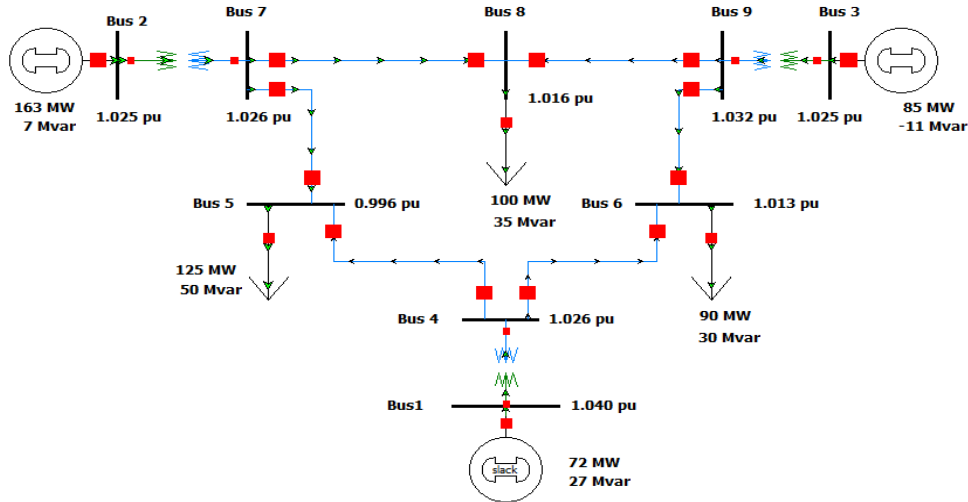


Figure 2.9: WSCC system consisting of one source as DFIG

The WSCC 9 bus system is used for preparing this model. In this model one 85 MW generator is replaced by equal rating of one DFIG generator. For modelling, refer equation from [14] and [15]. For this model the required parameter and matrix

equations are referred from Appendix A. The eigenvalue calculation of this system shown below: equation (2.14) represents A1 which is a machine matrix for generator connected to bus 1, equation(2.15) represents A2 which is a machine matrix for generator connected to bus 2 and equation(2.16) represents A3 which is a machine matrix for generator connected to bus 3.

$$A1 = \begin{pmatrix} -0.11 & 0 & 0 & 0 & 0.11 & 0 & 0 \\ 0 & -3.23 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.19 & 0 & 3.19 \\ 0 & 0 & 0 & 0 & 0.514 & -2.86 & 0 \\ 0 & 0 & 0 & 0 & -18 & 100 & -5 \end{pmatrix} \quad (2.14)$$

$$A2 = \begin{pmatrix} -0.167 & 0 & 0 & 0 & 0.167 & 0 & 0 \\ 0 & -1.87 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3.19 & 0 & 3.19 \\ 0 & 0 & 0 & 0 & 0.51 & -2.86 & 0 \\ 0 & 0 & 0 & 0 & -18 & 100 & -5 \end{pmatrix} \quad (2.15)$$

$$A3 = \begin{pmatrix} -17.91 & 6969.9 & -112.07 & -6527.6 & -377.52 & 2019.7 & 0 & 100.98 \\ -6962.6 & -19.28 & 6527.6 & -112.07 & 2046.5 & 0 & 2019.7 & 0 \\ -17.41 & 6750.2 & -114.82 & -6311.3 & -386.81 & 2069.3 & 0 & 103.46 \\ -6742.6 & -18.81 & 6311.3 & -114.82 & 2149.2 & 0 & 2069.3 & 0 \\ 0.3913 & -0.112 & -0.3733 & -0.0311 & 0 & 0 & 0 & 0 \\ 0.6867 & 0.1143 & 0 & -10 & 12.621 & 0 & 0 & 0 \\ -0.6926 & 0.1153 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.16)$$

The system matrix(A) is obtained by:

$$A = \begin{pmatrix} A1 & 0 & 0 \\ 0 & A2 & 0 \\ 0 & 0 & A3 \end{pmatrix} \quad (2.17)$$

By using the equation  $A - \lambda * I = 0$ , The eigenvalues obtained analytically are shown in Table 2.2

Table 2.2: Analytically obtained eigenvalues for Modified WSCC 9 buS system consisting DFIG as one source

No.	Eigenvalues obtain by calcualtion
1.	0
2.	-0.1
3.	-0.11
4.	-0.5019
5.	-3.22
6.	-5.2+7.97i
7.	-5.2-7.97i
8.	0
9.	-0.1
10.	-0.2
11.	-0.5
12.	-1.87
13.	-5.2+7.97i
14.	-5.2-7.97i
15.	-0.1
16.	-0.50
17.	-17.7+382i
18.	-17.7-382i
19.	-4.8+10.09i
20.	-4.8-10.09i
21.	-110.08+91.02i
22.	-110.08-91.02i

# Chapter 3

## Results and Discussion

The results of the models have been analysed which were discussed in chapter 2 to show the impact of modified controller on a system and to analyze the stability of the system.

### 3.1 Inertia Control

Fig 2.1 shows the model of DFIG based power system with modified controller which emulate inertia in to the system during large load changes. At 30s, load is increased by 165 MW at bus 2. Then the effect on DFIG speed, frequency response of a system, rate of change of frequency and power output of DFIG has been analyzed. Also, the impact of pitch angle controller at different wind speed has been analyzed.

#### **Influence on DFIG rotor speed due to modified controller**

From the fig.3.1 it is seen that due to increase in load at 30s, the speed of DFIG reduces from 1.2 pu to 1.165 pu during the time interval of 30s to 35s and then rotor speed is stabilized due to modified controller. The drop in rotor speed is opposing normal operation which shows that DFIG is required to deviate from its optimum speed in order to provide the inertial support as shown in Fig.2.1. Due to sudden increase in load the speed of the DFIG drops, which stabilizes after few seconds this shows that modified controller works satisfactorily.

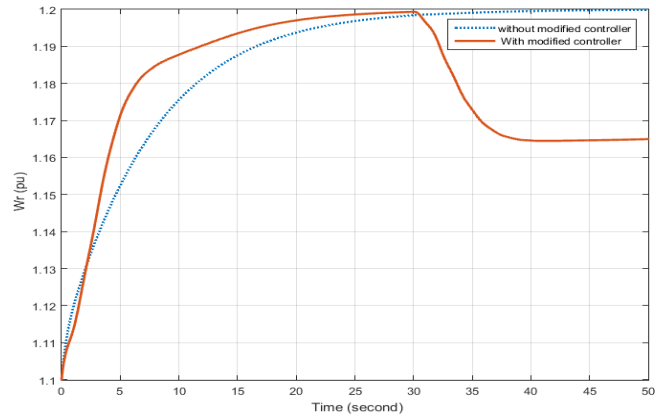


Figure 3.1: Influence on DFIG rotor speed due to modified controller

### Influence on DFIG Power output due to modified controller

The conventional controller works on MPPT control, it is clear from the Fig.3.2 that by using the conventional controller, power output of DFIG is not changed. After using this modified controller only part of the aerodynamic power is transmitted to the grid while remaining power is used to increase the turbine speed to its optimal value. The electrical power output of the DFIG increases after the load increases to provide inertial support for 30 to 37s which shows that controller works satisfactorily.

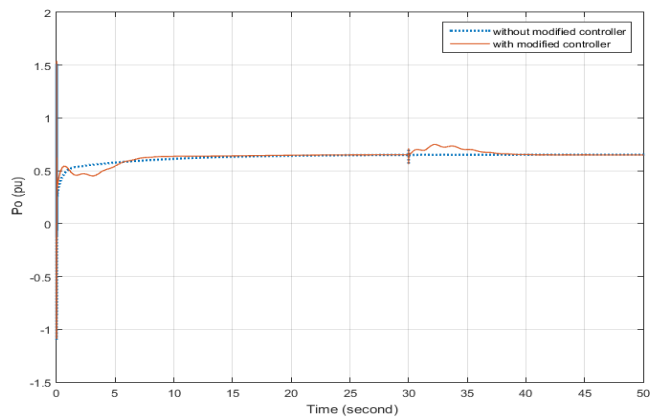


Figure 3.2: Influence on DFIG Power output due to modified controller

### Influence on DFIG output frequency due to modified controller

It can be seen from Fig. 3.3 that the modified controller improves system performance by decreasing the maximum frequency point from around 59.75 Hz to 59.62 Hz. It is also clear that using the modified controller, fast stabilization of the frequency occurs and hence the frequency response of the system is improved.

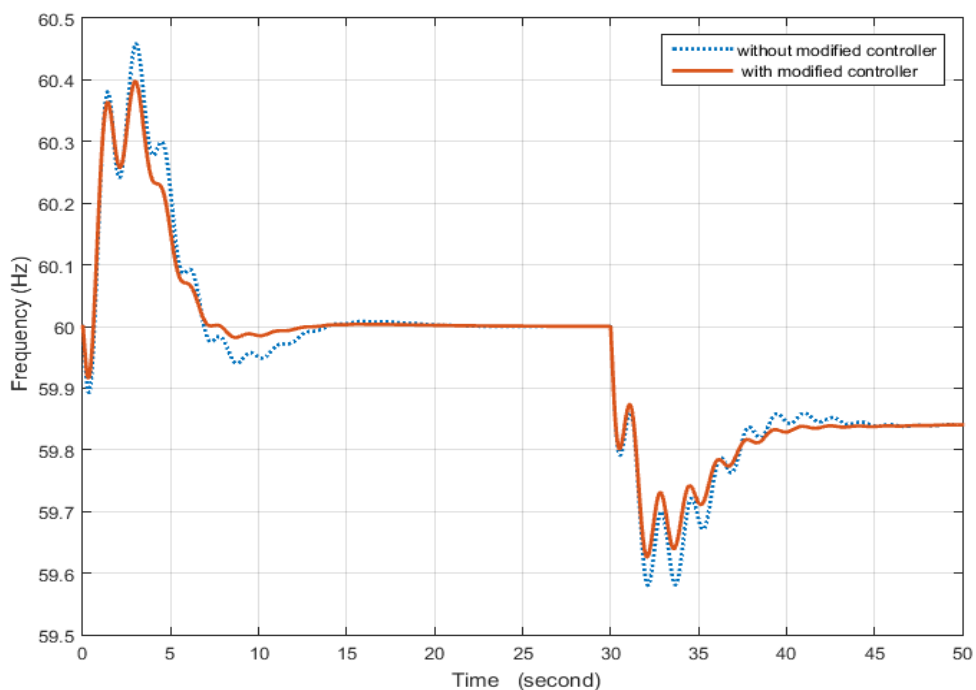


Figure 3.3: Influence on DFIG output frequency due to modified controller



### Influence on rate of change of frequency due to modified controller

Due to increase in the load power imbalance occurs and hence the change in the system frequency takes place. Increase in the rate of change of frequency is particularly important because it is a problem that can lead to additional losses of generated power and increase the risk of a potential system collapse.

Fig 3.4 shows that due to DFIG penetration and increase in load at 30 s, rate of change of frequency increases. In this condition the system is stabilize nearly at 45 s. When using the modified controller rate of change of frequency decreases and stabilize around 40 s.

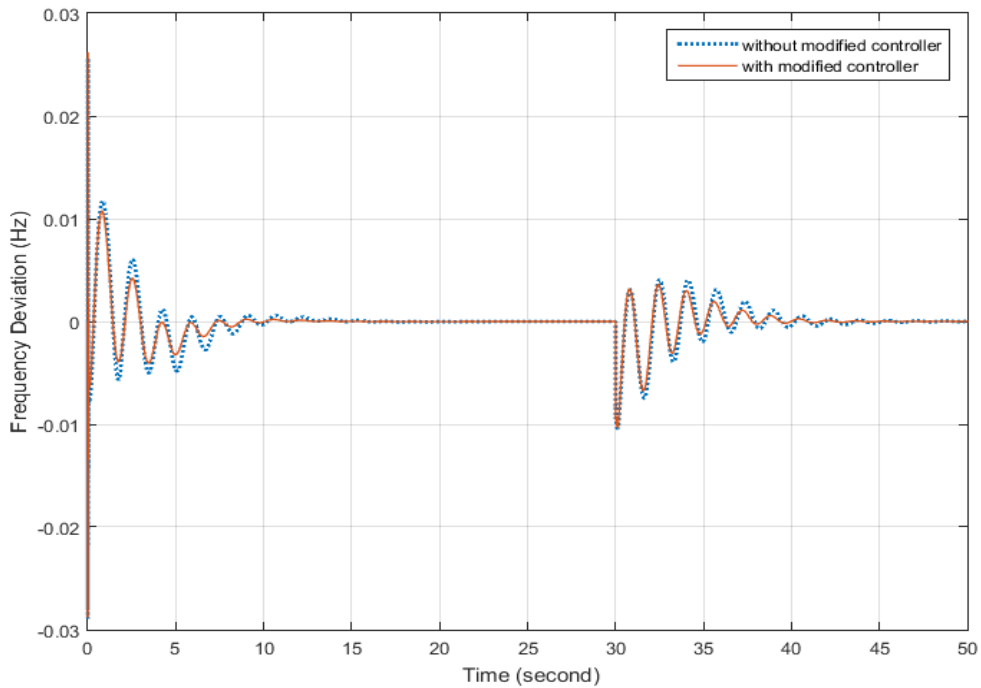


Figure 3.4: Influence on Rate of change of frequency due to modified controller

### Influence on pitch angle of DFIG due to modified controller

The pitch angle control scheme is used to regulate the pitch angle and so the output power is kept at rated value even when the wind fluctuations occur. The pitch angle of DFIG is shown in the Fig.3.5. for different wind speeds. At rated wind speed(12m/s) or at rated power no influence on pitch angle of the DFIG is observed.

Normally the pitch angle controller increases the pitch angle above its rated value. But due to new modified controller it allows DFIG rotor speed and hence its power output to increase up to 20% more than its rated value. As the speed increases from 1.2 pu the pitch angle start increasing to control the rotor speed and hence the output power to a rated value. This shows that modified controller works satisfactorily.

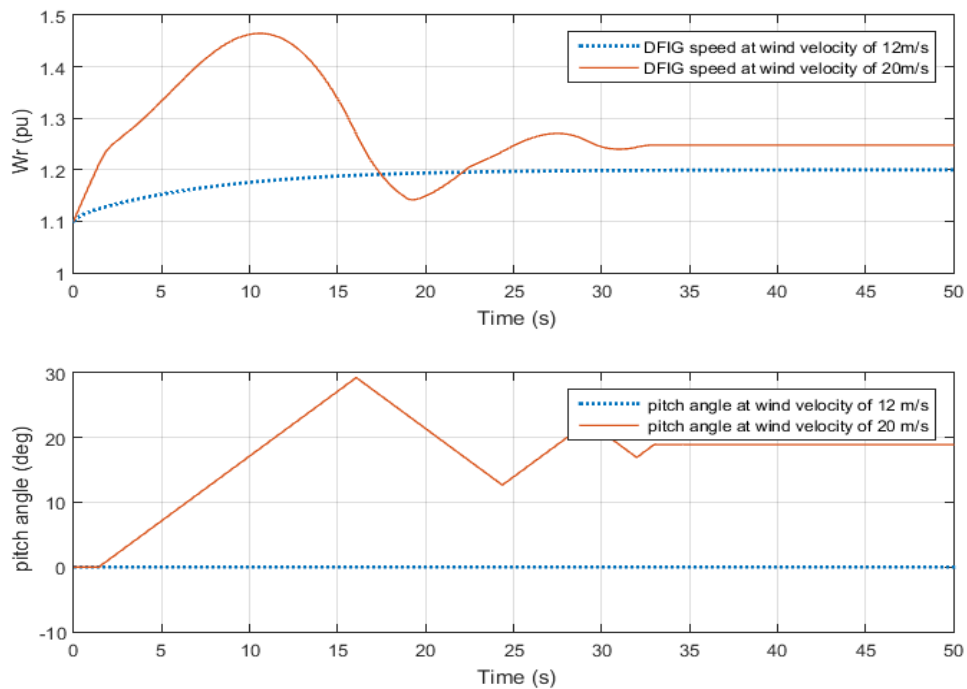


Figure 3.5: DFIG pitch angle at different wind speed

### 3.2 Eigenvalue Analysis PSAT results of Conventional SMIB system

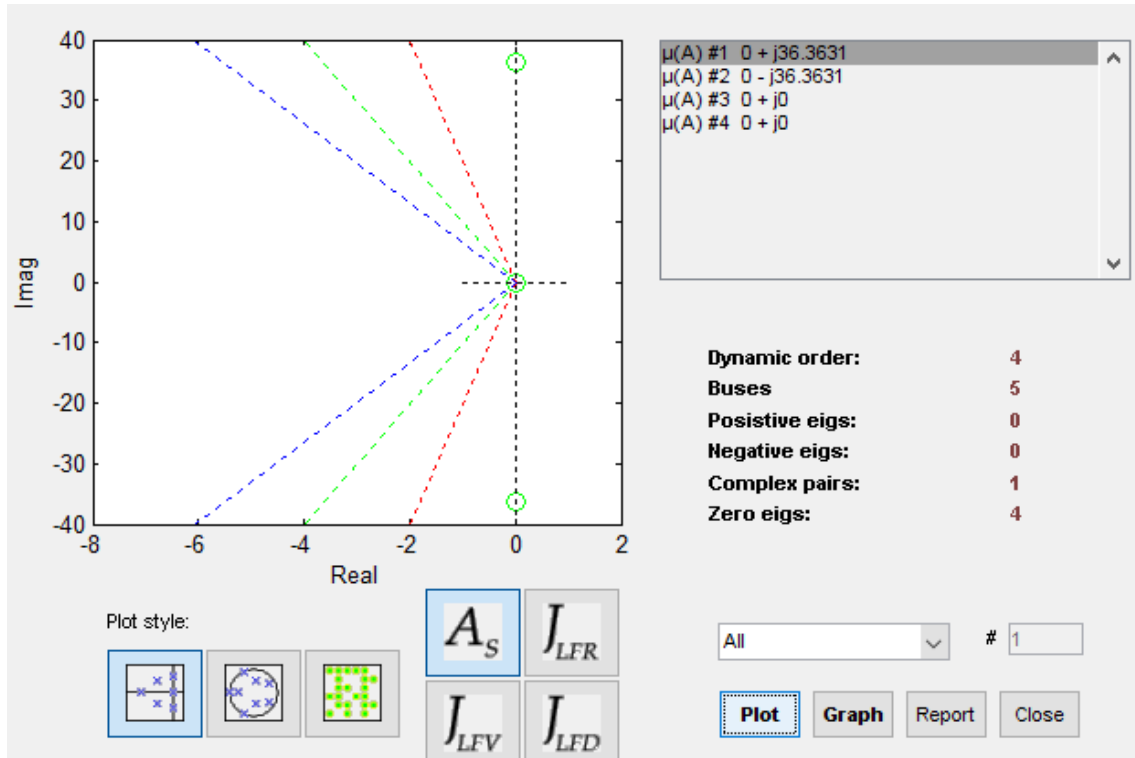


Figure 3.6: Eigenvalue analysis of SMIB with Synchronous Generator

The result of the eigenvalue analysis obtained from PSAT tool of MATLAB software is  $+36i$  and  $-36i$ .

From the equation (2.5), the eigenvalues obtained are  $+34.49i$  and  $-34.49i$ .

The synchronous machine used in conventional SMIB system is coupled with AVR and PSS block but during calculation the effect of AVR and PSS is not considered. Due to which certain variations in the eigenvalues are observed. Even though the calculated eigenvalues and the values obtained analytically are very near to each other, the results are validating each other.

### 3.3 Eigenvalue Analysis PowerWorld Simulator results of Conventional SMIB system

From the equation (2.10), the eigenvalues obtained are  $-0.9+11i$  and  $-0.9-11i$ .

The results of the eigenvalue analysis obtained from PowerWorld Simulator software are  $12i$  and  $-12i$ .

The calculated eigenvalues and the values obtained from power world simulator software are very near to each other and hence validating each other.

### 3.4 Eigenvalue Analysis results of SMIB consisting DFIG as a source

Comparison of eigenvalues of SMIB consisting DFIG as a source has been obtained analytically and by software is shown in Table 3.2.

This table shows that the values obtain mathematically and through software are validating each other.

Table 3.1: Eigen values validation of SMIB system with DFIG

No.	Eigenvalues obtain mathematically	Eigenvalues obtain by software
1.	-0.22	-0.1
2.	-0.67	-0.67
3.	-21.1-68i	-20.77-82.03i
4.	-21.1+68i	-20.77+82.03i
5.	-42.1-204i	-39.11+121.04i
6.	-42.1+204i	-39.11+121.04i
7.	-69.7 + 520i	-46.42+484.29i
8.	-69.7 - 520i	-46.42-484.29i

### 3.5 Eigenvalue Analysis results of Modified WSCC 9 bus system

Comparison of eigenvalues obtained mathematically and by software is shown below in Table 3.1.

Most of the eigenvalues are nearer and hence validating each other.

Table 3.2: Eigenvalues validation of Modified WSCC 9bus system

No.	Eigenvalues obtain by calculation	Eigenvalues obtain by software
1.	0	0
2.	-0.1	-0.1
3.	-0.11	-0.4+0.7i
4.	-3.22	-2
5.	-3.22	-3.22
6.	-5.2+7.97i	-5.16+7.82i
7.	-5.2-7.97i	-5.16-7.82i
8.	0	0
9.	-0.16	-0.1
10.	-0.2	-0.4+0.7i
11.	-0.5	-0.4-0.7i
12.	-1.87	-1.95
13.	-5.2+7.97i	5.85+7.87i
14.	-5.2-7.97i	5.85-7.87i
15.	-0.1	-0.23
16.	-0.5	-0.67
17.	-17.78+382i	-15.2+8.25i
18.	-17.78-382i	-15-8.25i
19.	-4.8+10.09i	-3.36+15.13i
20.	-4.8-10.09i	-3.36-15.13i
21.	-110.08+91.02i	-32
22.	-110.08-91.02i	-50

# Chapter 4

## Conclusion and Future Work

### 4.1 Conclusion

The extract of the work can be concluded as:

- Due to the increasing DFIG based wind power penetration in the system, the effective inertia of the system reduces due to which certain issues are observed like instability, poor frequency response, etc. The simulation results shows that the proposed controller satisfactorily works and exhibit the inertial power when there occurs deviation in the frequency. After the frequency deviation the output power increases for first few seconds and the speed decreases showing that it exhibits the inertia. With the increase in power beyond a certain limit, pitch angle increases to maintain the speed and output power of DFIG at rated value which shows that it works satisfactorily.

The small signal stability analysis of system shall help in observing the system behaviour under small disturbance conditions. The work embodied in this report, focuses on the effect of the wind energy penetration on the conventional system small signal performance. The report presents performance of SMIB and a WSCC 9-bus system when one generator is Doubly Fed Induction Generator (DFIG). The results of both the systems with DFIG included is presented analytically through use of mathematical model equation and validated using the standard software (demo version). It is quite evident that the K<sub>p</sub> and

$K_i$  (controller gain) variations of the system plays significant roles shifting the eigenvalues in softwares, however, model does not cover this factors to the great extent. More of the eigenvalues are found to be matched under specific values of gain and are shown in the report. Hence, it is concluded that the system eigenvalues is significantly affected by the controller parameters and required to be carefully selected.

## 4.2 Future work

The future work can be explained as:

- PI tuning has an impact on the eigenvalues obtained, due to which some variations are also observed. On increasing the precision of PI tuning these variations can be eradicated.
- Further the contingency will be created in Modified WSCC 9 bus system and then eigenvalue will be found out and analyze .
- The possible future work on this research will be to incorporate the controller parameters in the mathematical model so that the effect of these parameters can be evaluated. Also, modeling of multiple DFIG generators / different wind generators in a system will be meaningful to understand system stability.

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# Appendix A

## DFIG parameters

$D_{sh} = 0.01, K_{sh} = 10, H_{tot} = 3.5, H_g = 0.5, H_t = 3, V_{wbase} = 9 \text{ m/s}, \lambda = 8.1, C_p = 0.48, P_{nom} = 84, P_{mec} = 84, P_{nom1} = 84, P_{elebase} = 84, P_{windbase} = 1, C1 = 0.5176, C2 = 116, C3 = 0.4, C4 = 5, C5 = 21, C6 = 0.0068, Pitch_{rate} = 2, Pitch_{max} = 45, K_{opt} = 0.56, K_p = 5, K_i = 25, V_b = 0.69, S_b = 100, F_b = 60, W_s = 1, W_b = 376.8, W_{so} = 1, X_{tr} = 1, R_s = 0.004488, X_{ts} = 0.09241, R_r = 0.00549, X_{lr} = 0.09955, X_m = 3.95279, X_{rm} = 0.02, X_{rr} = 4.04, X_{ss} = 4.05, V_{dsinf} = 0, V_{qsinf} = 1, V_{Asc} = 40, X/R = 10, Z_e = 0.05, R_e = 0.0050, X_e = 0.0498, R_t = 0.0099, X_t = 0.0998, W_{ro} = 1.1, I_{dso} = -0.055, I_{qso} = 0.661, I_{dro} = 0.197, I_{qro} = 0.693, V_{dso} = -0.066, V_{qso} = 0.998, V_{dro} = 0.02, V_{qro} = -0.098, \Delta V_{ds} = -0.0664, \Delta V_{qs} = 1.0010, T_e = 0.6653, V_{so} = 1.0001, \Delta V_s = 1.0032, I_{qref} = 0.6954, K_{p1} = 0.05, K_{i1} = 10.00, K_{p2} = 0.05, K_{i2} = 10, K_{p3} = 7, S_o = 0.2,$

## DFIG matrix equations

$$A_{11} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-R_T X_{rr} + \{(\frac{k_{p2}}{V_{s0}\omega_s})R_T V_{ds0} + X_T V_{qs0}\}] \quad (1)$$

$$A_{12} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(X_{ss}X_{rr} - S_o X_m^2)\omega_s + X_T X_{rr} + \{(\frac{k_{p2}}{V_{s0}\omega_s})R_T V_{qs0} - X_T V_{ds0}\}] \quad (2)$$

$$A_{13} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-X_m(R_r + K_{p2})] \quad (3)$$

$$A_{14} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_m X_{rr} + S_o X_m X_{rr})\omega_s] \quad (4)$$

$$A_{15} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(X_m I_{qso} - X_{rr} I_{qro})X_m] \quad (5)$$

$$A_{16} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} \quad (6)$$

$$A_{17} = 0 \quad (7)$$

$$A_{18} = \frac{\omega_b X_m K_{p2}}{X_{rr}X_{ss} - X_m^2} \quad (8)$$

$$A_{21} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_{ss}X_{rr} + S_o X_m^2)\omega_s - X_T X_{rr} + \{(\frac{k_{p1}k_{opt}X_{ss}\omega_s\omega_{ro}^2}{v_{s0}^3})(R_T v_{ds0} + X_T v_{qs0})\}] \quad (9)$$

$$A_{22} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-R_T X_{rr} + \{(\frac{k_{p1}k_{opt}X_{ss}\omega_s\omega_{ro}^2}{v_{s0}^3})(R_T v_{qs0} - X_T v_{ds0})\}] \quad (10)$$

$$A_{23} = -A_{14} \quad (11)$$

$$A_{24} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-X_m(R_r + K_{p1})] \quad (12)$$

$$A_{25} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_m I_{dso} + X_{rr} I_{dro})X_m] \quad (13)$$

$$A_{26} = 0 \quad (14)$$

$$A_{27} = A_{16} \quad (15)$$

$$A_{28} = 0 \quad (16)$$

$$A_{31} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-R_T X_m + \{(\frac{k_{p2}X_{ss}}{X_m v_{s0}\omega_s})(R_T V_{ds0} + X_T V_{qs0})\}] \quad (17)$$

$$A_{32} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(X_m X_{ss} - S_o X_m X_{ss})\omega_s + X_T X_{rr} + \{(\frac{k_{p2} X_{ss}}{X_m v_{s0} \omega_s}) (R_T V_{qs0} - X_T V_{ds0})\}] \quad (18)$$

$$A_{33} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-X_{ss}(R_r + K_{p2})] \quad (19)$$

$$A_{34} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_m^2 + S_o X_{ss} X_{rr})\omega_s] \quad (20)$$

$$A_{35} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(X_m I_{qso} - X_{rr} I_{qro})X_{ss}] \quad (21)$$

$$A_{36} = \frac{\omega_b X_{ss}}{X_{rr}X_{ss} - X_m^2} \quad (22)$$

$$A_{37} = 0 \quad (23)$$

$$A_{38} = \frac{\omega_b X_{ss} K_{p2}}{X_{rr}X_{ss} - X_m^2} \quad (24)$$

$$A_{41} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_m X_{ss} - S_o X_m X_{ss})\omega_s - X_T X_{rr} + \{(\frac{k_{p1} k_{opt} X_{ss} \omega_s \omega_{ro}^2}{v_{s0}^3 X_m}) (R_T V_{ds0} - X_T V_{qs0})\}] \quad (25)$$

$$A_{42} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-X_m R_T + \{(\frac{k_{p1} k_{opt} X_{ss} \omega_s \omega_{ro}^2}{v_{s0}^3 X_m}) (R_T V_{qs0} - X_T V_{ds0})\}] \quad (26)$$

$$A_{43} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(X_m^2 - S_o X_{ss} X_{rr})\omega_s] \quad (27)$$

$$A_{44} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [-X_{ss}(R_r + K_{p1})] \quad (28)$$

$$A_{45} = \frac{\omega_b}{X_{rr}X_{ss} - X_m^2} [(-X_m I_{dso} + X_{rr} I_{dro})X_{ss} + 2(\frac{K_{p1} X_{ss}}{X_m})] \quad (29)$$

$$A_{46} = 0 \quad (30)$$

$$A_{47} = A_{36} \quad (31)$$

$$A_{48} = 0 \quad (32)$$

$$A_{51} = \frac{X_m I_{qro}}{2H} \quad (33)$$

$$A_{52} = \frac{-X_m I_{dro}}{2H} \quad (34)$$

$$A_{53} = \frac{-X_m I_{qso}}{2H} \quad (35)$$

$$A_{54} = \frac{X_m I_{dso}}{2H} \quad (36)$$

$$A_{55} = 0 \quad (37)$$

$$A_{56} = 0 \quad (38)$$

$$A_{57} = 0 \quad (39)$$

$$A_{58} = 0 \quad (40)$$

$$A_{61} = \frac{K_{i2}}{V_{so}\omega_s X_m} \{(R_T V_{ds0} + X_T V_{qs0})\} \quad (41)$$

$$A_{62} = \frac{K_{i2}}{V_{so}\omega_s X_m} \{(R_T V_{qs0} - X_T V_{ds0})\} \quad (42)$$

$$A_{63} = -K_{i2} \quad (43)$$

$$A_{64} = 0 \quad (44)$$

$$A_{65} = 0 \quad (45)$$

$$A_{66} = 0 \quad (46)$$

$$A_{67} = 0 \quad (47)$$

$$A_{68} = K_{i2} \quad (48)$$

$$A_{71} = \left\{ \left( \frac{k_{i1} k_{opt} X_{ss} \omega_s \omega_{ro}^2}{v_{s0}^3 X_m} \right) (R_T V_{ds0} - X_T V_{qs0}) \right\} \quad (49)$$

$$A_{72} = \left\{ \left( \frac{k_{i1} k_{opt} X_{ss} \omega_s \omega_{ro}^2}{v_{s0}^3 X_m} \right) (R_T V_{qs0} - X_T V_{ds0}) \right\} \quad (50)$$

$$A_{73} = 0 \quad (51)$$

$$A_{74} = K_{i1} \quad (52)$$

$$A_{75} = \left\{ \frac{2k_{i1} k_{opt} X_{ss} \omega_s \omega_{ro}}{v_{s0} X_m} \right\} \quad (53)$$

$$A_{76} = 0 \quad (54)$$

$$A_{77} = 0 \quad (55)$$

$$A_{78} = 0 \quad (56)$$

$$A_{81} = \left\{ \left( \frac{-k_{p3}}{v_{s0}} \right) (R_T V_{ds0} + X_T V_{qs0}) \right\} \quad (57)$$

$$A_{82} = \left\{ \left( \frac{k_{p3}}{v_{s0}} \right) (R_T V_{qs0} - X_T V_{ds0}) \right\} \quad (58)$$

$$A_{83} = 0 \quad (59)$$

$$A_{84} = 0 \quad (60)$$

$$A_{85} = 0 \quad (61)$$

$$A_{86} = 0 \quad (62)$$

$$A_{87} = 0 \quad (63)$$

$$A_{88} = 0 \quad (64)$$