

Optimize the Capacity of Freeze Drying for Banana Chips

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Optimize the Capacity of Freeze Drying for Banana Chips

Major Project Report

Submitted in partial fulfillment of the requirements

For the Degree of

Master of Technology in Mechanical Engineering

(Thermal Engineering)

By

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Declaration

This is to certify that

1. The thesis comprises my original work towards the degree of Master of Technology in Thermal Engineering at Nirma University and has not been submitted elsewhere for a degree or diploma.
2. Due acknowledgement has been made in the text to all other material used.

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RAVIKUMAR B. THAKER

Abstract

In 21st century, there is a large demand of development of pharmaceutical and food products because it is directly related with human hygiene. So the quality of final product should not be compromised with process of manufacturing. The Freeze-drying is considered to be an attractive dehydration method of preserving the quality of high value food products. Since it is concerned with the human hygiene. But, it is an expensive operation, which calls for efficient tools capable of maximizing its the capacity. In the present work, freeze drying process is studied for banana chips. The mathematical models of the drying process are discretized and solved using Finite Volume Methodology. The drying time for different thicknesses of banana chips is determined in an attempt to find the optimum thickness for maximum rate for production.

In Chapter 1 basic concept of the freeze drying process is reported. Chapter 2 describes the literature review regarding the optimization of the freeze drying process and the experimental data of the yoghurt, skim milk. In Chapter 3 mathematical formulation of the freeze drying process is discussed. Discretization of the fundamental governing equations of the freeze drying process and solution algorithm are reported in Chapter 4. The results of the simulations are reported in Chapter 5. In Chapter 6 conclusions derived from the result are discussed. The scope for the future work is given in Chapter 7.

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Nomenclature

C_g	Specific heat of gas, kJ/kgK
C_w	Specific heat of water vapour, kJ/kgK
C_{pw}	Concentration of water vapor, kg/m^3
C_{pw}^o	Initial Concentration of water vapor, kg/m^3
C_{sw}	Concentration of sorbed water, kg/m^3
C_{sw}^*	Equilibrium concentration of bound water, kg/m^3
C_p	Specific heat of dried layer, kJ/kgK
C_p	Specific heat of frozen layer, kJ/kgK
E	Activation Energy, kJ/kg
F	Emmissivity of surface
L	Product thickness, m
M_w	Molecular weight of water vapor, $kg/kgmole$
N_{in}	Mass flux of inert gas, kg/m^2s
N_t	Total mass flux of water and inert gas, kg/m^2s
N_w	mass flux of water vapor, kg/m^2s
R	gas constant, kJ/kgK
T	Temperature of Product, K
T_{up}	Temperature of upper plate, K
T_X	Temperature of sublimation Front, K
T_I	Temperature in the dried layer, K
T_{II}	Temperature in frozen layer, kJ/kg
X	Position of Interface, m
k	Rate Constant, s^{-1}
k_g	Desorption Rate, s^{-1}
k_I	Thermal conductivity of dried layer, W/mK
k_{II}	Thermal conductivity of frozen layer, W/mK
p_w	Partial pressure of water vapor, N/m^2
p_w^o	Initial Partial pressure of water vapor, N/m^2
p_{wX}	Partial pressure of water vapor at interface kJ/hr

q_I	Heat Flux at $x=0, W/m^2$
q_{II}	Heat Flux at the bottom of the tray, W/m^2
ΔH_V	Enthalpy of vaporisation, kJ/Kg
ΔH_S	Enthalpy of sublimation, kJ/Kg
α_I	Thermal diffusivity of dried product, m^2/s
α_{II}	Thermal diffusivity of frozen product, m^2/s
ρ_I	Density of dried product, kg/m^3
ρ_{II}	Density of frozen product, kg/m^3
ε_p	porosity of the product
σ	Stefan boltzmann constant, W/m^2K^4

Chapter 1

Introduction

In 21st century, there is a large demand of development of pharmaceutical and food products because it is directly related with human hygiene. So the quality of final product should not be compromised with process of manufacturing. In food and beverages industry, Freeze drying is one of the processes used for manufacturing of banana chips, maintaining quality of apple and strawberry for a long time, removal of water from vaccines and storing it in a vial etc. Many other applications like stabilization of living materials such as microbial cultures, preservation of whole animal specimens for museum display, restoration of books and other items damaged by water and the concentration and recovery of reaction products. Freeze drying has its own advantages over other drying processes because it produces highest quality of the food product obtainable by any drying method[1].

1.1 Freeze Drying

Certain biological materials, pharmaceuticals, and foodstuffs, which may not be heated even to moderate temperatures in ordinary drying, may be freeze dried. The substance to be dried is usually frozen. In freeze drying, the water or another solvent is removed as a vapor by sublimation from the frozen material in a vacuum chamber. After the solvent sublimates to a vapor, it is removed from the drying chamber where the drying process occurs. As a rule, freeze drying produces the highest quality food product obtainable by any drying method. A prominent factor is the structural rigidity afforded by the frozen substance at the surface where sublimation occurs. This rigidity to a large extent prevents collapse of the solid matrix remaining after drying. The result is a porous, nonshrunken structure in the dried product that facilitates rapid and almost complete rehydration when water is added to the substance at a later time.

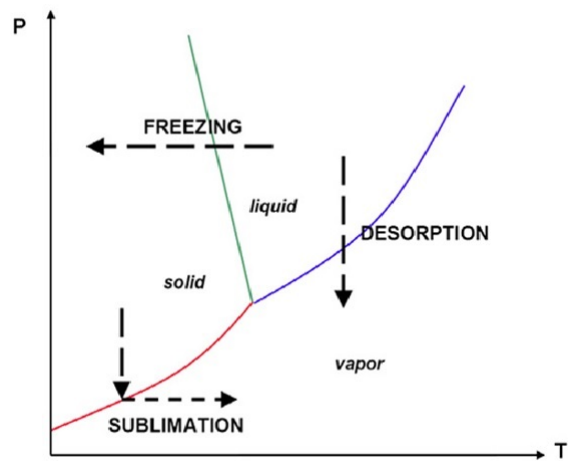


Figure 1.1: Freeze drying Principle on phase diagram of water[2]

Freeze drying of food and organic materials also has the advantage of little loss of flavor and smell. The low processing temperatures, the relative absence of liquid water, and the quick move of any nearby area of the material dried from a completely hydrated to an nearly completely dehydrated state minimize the degradative reactions that typically happen in ordinary drying processes, for example nonenzymatic browning, protein denaturation, and enzymatic reactions. In any food material, some nonfrozen water, which is called bound or sorbed water, will almost unavoidably be present during freeze drying, however there is very often a rather sharp transition temperature for the still wet region during drying, below which the product quality enhances uniquely. This improvement shows that sufficient water is frozen to give the beneficial product attributes of freeze drying. However, freeze drying is an costly form of dehydration for foods because of the slow drying rate and the use of vacuum. The expense of processing is offset to some extent by the absence of any need for refrigerated handling and storage. Freeze drying or lyophilization is a process in which water is frozen, followed by its removal from the product, initially by sublimation (primary drying) and after that by desorption (secondary drying). Freeze drying is a process of drying in which water is sublimed from the product after it is frozen which is shown in Fig.(.). It is a drying process appropriate to manufacture of specific pharmaceuticals, food and biologicals that are unstable in water solutions for long storage periods, but that are stable in the dry state. The term “lyophilization” describes a process that helps to create the product dry. The primary principle included in freeze drying is a phenomenon called sublimation, where water passes directly from solid state (ice) to the vapor state without passing through the liquid state. Sublimation of water can take place at pressures and temperature below triple point i.e. 4.58 mm of Hg and 0.01 degree Celsius. The freeze drying requires very high vacuum in order to produce a satisfactory drying rate.

The material to be dried is first frozen and then subjected under a high vacuum and then heated by conduction or radiation or by both so that frozen liquid sublimates leaving only solid, dried components of the original liquid and it also not required the refrigeration system and can be stored at room temperature. In general in this process surface temperature of the product increased up to 30-40 degree celsius and chamber pressure in the range of 0.01-0.6 mm of Hg[1].

1.1.1 Freezing

Freezing is the first step of a freeze drying process, and the performance of the overall freeze drying process depends significantly on this stage. At the end of the freezing step which is shown in Fig.(1.1), About 65%-90% of the initial moisture is in the frozen state and the rest remains in the structure of the product in liquid state as per the properties of product. To freeze a substance it must be cooled to a temperature at which the water and the solids are fully crystallized or at which areas of crystallized ice and solids are enclosed in zones in which amorphous concentrated solids and water remain in a solid-state[1].

1.1.2 Primary Drying

After the freezing stage, The frozen product put in the drying chamber is evacuated and the chamber pressure is reduced to a value that allows the ice sublimation to take place which is shown in Fig. (1.1). This says that the start of primary drying stage. Selection of the chamber pressure depends on the ultimate frozen temperature of the material. The pressure must be lower than the triple point of the water in order to sublime the ice. This includes careful control of two variable temperature and pressure involved in the freeze drying process. For the sublimation purpose required the heat energy which is provide from conduction and radiation both modes of heat transfer. From this increases the temperature of the product by 60-65 degree celcius. Primary drying takes more time so it is main objective for the controlling of the drying process. In some product like fruit not required the secondary drying because not required in the powder form like skim milk. So for all the product primary drying is necessary because all product passes through this phase of freeze drying[1].

Heat enter in to the product by the many ways like:

- By direct contact between the container base and the shelf, so here the shape of the container is important.
- By conduction across the container base and then through the frozen mass to the drying front (also called the sublimation interface)

- By gaseous convection between the product and residual gas molecules in the chamber.
- By radiation, this is low due to low temperature encountered in freeze-drying.

1.1.3 Secondary Drying

After primary freeze-drying is complete, and all ice has sublimed, bound moisture is still present in the product. The product appears dry, but the residual moisture content may be as high as 7-8% continued drying is necessary at warmer temperature to reduce the residual moisture content to optimum values. This process is called 'Isothermal Desorption' as the bound water is desorbed from the product which is shown in Fig.(1.1). Secondary drying is normally continued at a product temperature higher than ambient but compatible with the sensitivity of the product. In contrast to processing conditions for primary drying which use low shelf temperature and a moderate vacuum, desorption drying is facilitated by raising shelf temperature and reducing chamber pressure to a minimum. Care should be exercised in raising shelf temperature too highly; since, protein polymerization or biodegradation may result from using high processing temperature during secondary drying. Secondary drying is usually carried out for approximately 1/3 or 1/2 the time required for primary drying. The general practice in freeze-drying is to increase the shelf temperature during secondary drying and to decrease chamber pressure to the lowest attainable level. Also, the water remaining during secondary drying is more strongly bound, thus requiring more energy for its removal. Decreasing the chamber pressure to the maximum attainable vacuum has traditionally been thought to favor desorption of water[1].

1.2 Basic Component of freeze dryer

1.2.1 Vacuum Chamber

This is the vacuum tight box, here and there called the lyophilization chamber. The chamber contains shelves for handling item which is Fig.(1.2). The chamber can likewise fit with a stoppering system. It is commonly made of stainless steel and as a rule very cleaned within and protected and clad on the outside. The entryway locking course of action by a water driven or electric engine[1].

1.2.2 Shelves

A little research solidify dryer may have stand out shelf yet all others will have a few. The shelf outline is made more difficult due to the few capacities it needs to perform

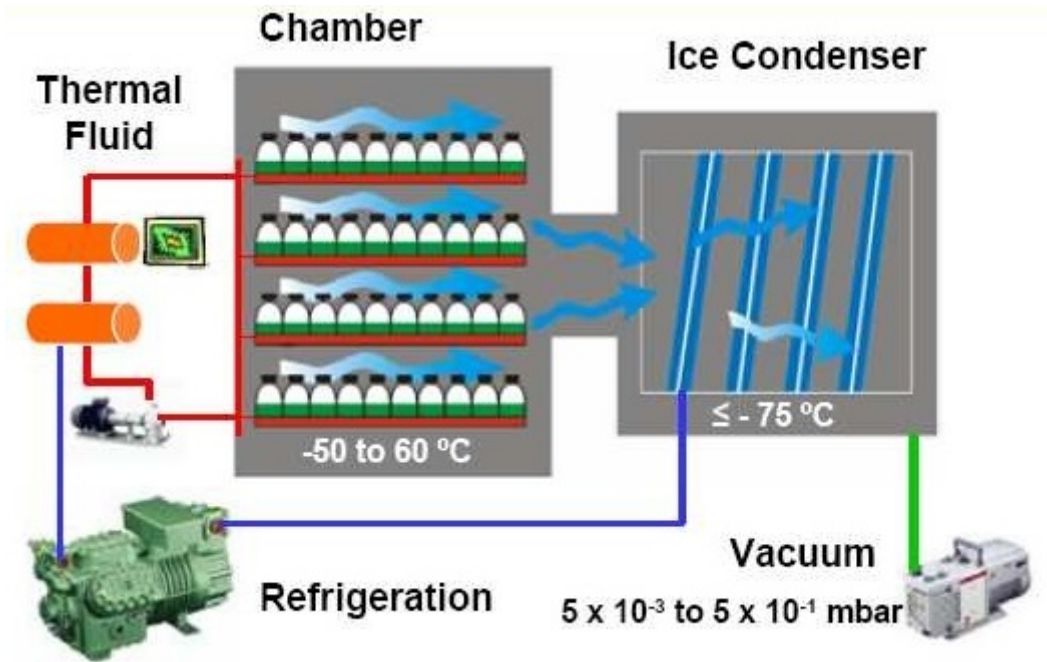


Figure 1.2: Component of Freeze dryer[1]

which is shown in Fig.(1.2). The shelf is work as a heat exchanger, expelling vitality from the product during solidifying, and supplying vitality to the item amid the essential and auxiliary drying fragments of the stop drying cycle. The shelf will be joined with the silicone oil system through either altered or adaptable hoses. Shelf can be fabricated in sizes up to $4m^2$ [1].

1.2.3 Process condenser

The process condenser is sometimes called as just the condenser or the cold trap. It is intended to trap the solvent, which is normally water, during the drying process. The process condenser will consist of coils or sometimes plates which are refrigerated to permit temperature which is shown in Fig.(1.2). These refrigerated coils or plates may be in a vessel separate to the chamber, or they could be situated within the same chamber as the shelves. Hence there is designation “external condenser” and “internal condenser”. Physically, the external condenser is traditionally placed behind the chamber, but it may be along the side, below or above. The position of the condenser does not affect trapping performance. For an internal condenser the refrigerated coils or plates are put under the shelves on smaller machines, and behind the shelves on larger machines, but again there is no performance constraint, only the geometry of the chamber[1].

1.2.4 Shelf fluid system

The freeze-drying process requires that the product is first frozen and then energy in the form of heat is supplied throughout the drying phases of the cycle. This energy exchange is traditionally done by circulating a fluid through the shelves at a desired temperature. The temperature is set in an external heat exchange system consisting of cooling heat exchangers and an electrical heater. The fluid circulated is normally silicone oil. This will be pumped around the circuit at a low pressure in a sealed circuit by means of a pump which is shown in Fig.(1.2)[1].

1.2.5 Refrigeration system

The product to be freeze dried is either frozen before into the dryer or frozen whilst on the shelves. A certain amount of energy is needed to this system. Compressors or sometimes-liquid nitrogen supplies the cooling energy. Most often multiply compressors are required and the compressor may perform two duties, one to cool the shelves and the second to cool the process condenser[1].

1.2.6 Vaccum System

To remove solvent in a reasonable time, vacuum must be applied during the drying process. The vacuum level required will be typically in the range of 50 to 100t bar. To achieve such a low vacuum, a two stage rotary vacuum pump is used which is shown in Fig.(1.2). For large chambers, more number of pumps may be used[1].

1.2.7 Control system

Control may be generally fully automatic for production machines. The control components required are as mentioned above, shelf temperature, pressure and time. A control program will set up these values as required by the product or the process. The time may vary from a few hours to several days. Other data such as a product temperatures and process condenser temperatures can also be recorded and stored [1].

1.3 Method of freeze drying

1.3.1 Manifold Freeze drying

In the manifold method, flasks or vials are individually attached to the ports of a drying chamber. The product either frozen in a freezer, by direct submersion in a low temperature bath, or by shell freezing, depending on the nature of the product and the volume to



Figure 1.3: Manifold freeze dryer[3]

be freeze dried. The prefrozen product is quickly attached to the drying chamber or manifold to prevent warming. The vacuum must be created in the product container quickly, and the operator relies on evaporative cooling to maintain the low temperature of the product. This procedure can only be used for relatively small volumes and product with high eutectic and collapse temperatures. Manifold drying has several advantages over batch tray drying. Since the vessels are joined to the manifold individually, each vial or flask has a direct path to the collector which is shown in Fig.(1.3). This removes some of the competition for molecular space created in a batch system, and is most ideally realized in a cylindrical drying chamber where the distance from the collector to each product vessel is the same. Heat input can be affected by simply exposing the vessels to surrounding temperature or by means of a circulating bath. For some products, where precise temperature control is required, manifold drying may not be suitable[1].

1.3.2 Batch or Bulk Freeze drying

In a batch drying, large numbers of comparative sized product or vessels containing like product are placed together in a tray dryer which is as shown in Fig.(1.4). Bulk drying is generally carried out in a tray dryer like batch drying. However, the product is poured into a bulk pan and dried as a single unit. Although the product is spread through out the entire surface area of the shelf and may be the same thickness as product in vials, the lack of empty spaces within the product mass changes the rate of heat input. The heat input is limited basically to that provided by contact with the shelf.

The product is usually prefrozen on the shelf of the tray dryer. Precise control of the product temperature and the amount of heat supplied to the product during drying can



Figure 1.4: Batch or Bulk Freeze dryer[3]

be maintained. Generally all vials in the batch are treated during drying process, although some variation in the system can occur. Slight difference in heat input from the shelf can be expressed in different areas. Vials located in the front portion of the shelf may radiantly through the clear door. These slight variations can result in small difference in residual moisture. Batch drying allows closure of all vials in a lot at the same time, under the same atmospheric condition. The vials can be stoppered in a vacuum, or backfiling with inert gas. Stoppering of all vials at the same time ensures a uniform environment in each vial and uniform product stability during storage. Batch drying is used to prepare large numbers of ampoules or vials of one product and is commonly used in the pharmaceutical industry[2].

1.4 Application of Freeze drying

1.4.1 Pharmaceutical and biotechnology

Pharmaceutical companies often use freeze-drying to increase the life of products, such as vaccines and other injectables. By removing the water from the material and sealing the material in a vial, the material can be easily stored, shipped, and later reconstituted to its original form for injection[1].

1.4.2 Food Industry

Freeze-drying is used to preserve food and make it very lightweight and easily stored for long time without cold storage. The process has been popularized in the forms of

freeze-dried ice cream, apple, skim milk, yoghurt, banana etc.[1].

1.4.3 Technological Industry

In chemical synthesis, products are often freeze-dried to make them more stable, or easier to dissolve in water for subsequent use. In bioseparations, freeze-drying can be utilized also as a late-stage cleaning procedure, because it can effectively remove solvents. Furthermore, it is capable of concentrating substances with low molecular weights that are too small to be removed by a filtration layer[1].

1.5 Advantages of freeze drying

- Chemical decomposition is minimized.
- Removal of water without excessive heating.
- Enhanced product stability in a dry state.
- Ease of processing a liquid, simplifies aseptic handling.
- More compatible with sterile operations than dry powder fling.

1.6 Limitation of freeze drying

- The high investment, operating and maintenance costs.
- The understanding of the process and the equipment requires a team of skilled and permanently trained workers.

1.7 Closure

In this chapter the fundamentals of the freeze drying process as well as the method of freeze drying its application, advantages and limitations are discussed. Freeze drying is the process of removal of moisture from a product by first freezing it and then subjecting it to a pressure below the tripple point of water such that the moisture is removed due to sublimation process.

Chapter 2

Literature Review

Estefania Lopez-Quiroga et al. have done that the freeze drying is the one of the attractive method for drying because get the heighest quality of product but this process is expansive so its required the optimum control of freeze drying process. For this purpose theoretical modelling has been done for better understanding of process dynamics and their effect on the product and drying time. For this solve the governing equation By this analysis get the parameter which is to be control of freeze drying process. The purpose is to minimize the freeze drying cycle time with satisfying the product satbility. In this analysis product is skim milk and constrain is at the end of the process only 2% water is present in the skim milk product for this finding the optimum parameter of skim milk product like tickness of product, chamber pressure and shelf temperature.

Here shelf temperature and chamber pressure range is given below for calculation:

$$223 \text{ K} \leq T_L \leq 323 \text{ K}$$

$$10 \text{ Pa} \leq P_c \leq 60 \text{ Pa}$$

From this range doing case study on the skim milk product for different thickness like 3 mm and 6 mm. with use of finite element method to gether with the arbitrary langrangian-eulerian method in COMSOL Multiphysics. Also doing optimization with both one and two control variable[2].

From one control varaiable (shelf temperature) optimization get 17.71% reduction in process time and for the two control variable(shelf temperature and chamber pressure) optimization get 25.5 % reduction in process time[2].

W.J. Mascarenhasa et al. have done the computational model for finite element analysis of the freeze drying process in two dimensional axisymmetric space. This model calculate the time vise variation of partial pressure of water vapor, the temperature and the concentration of sorbed water. This type of solution doing with the help of energy and material balance apply to the product which is frozen and put in to the vaccum chamber. In this the problem of moving boundary can be solved with ALE approach used

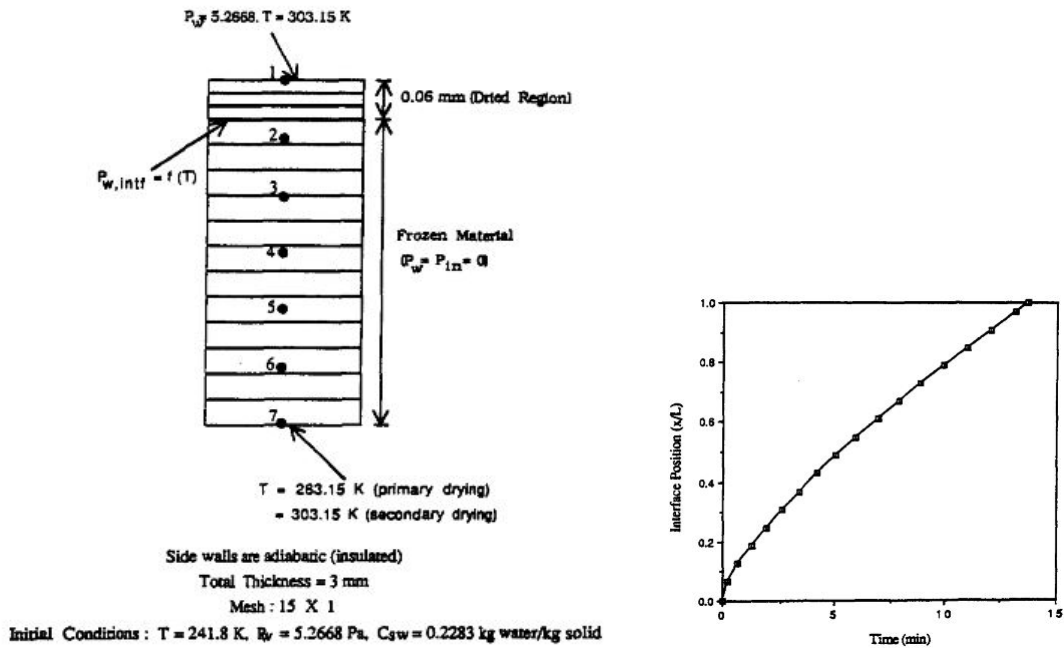


Figure 2.1: Finite element mesh with boundary condition and interface position with time of skim milk[5]

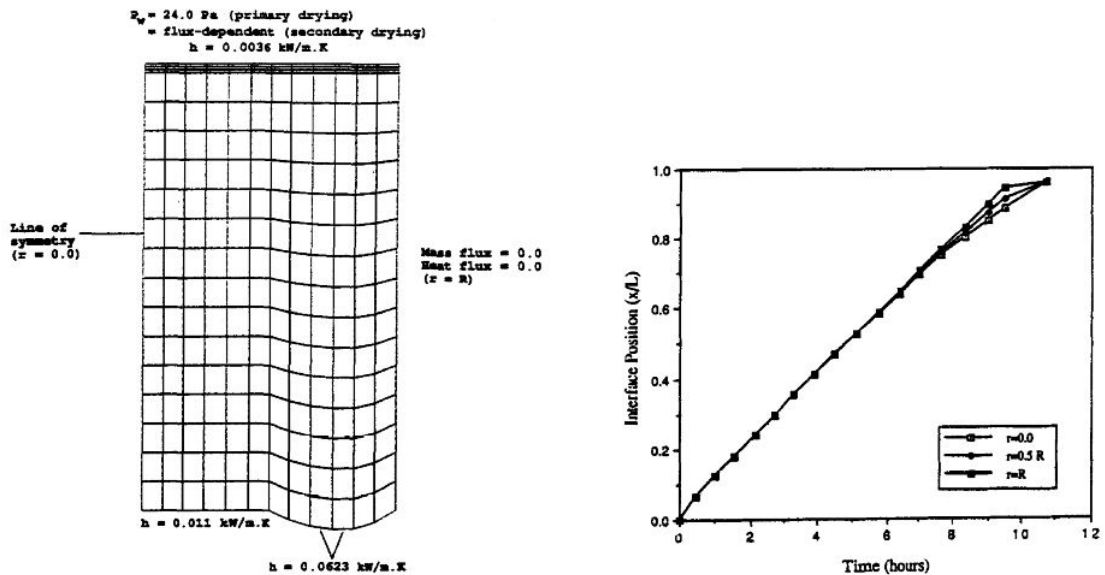


Figure 2.2: Finite element mesh with boundary condition and interface position with time of BST[5]

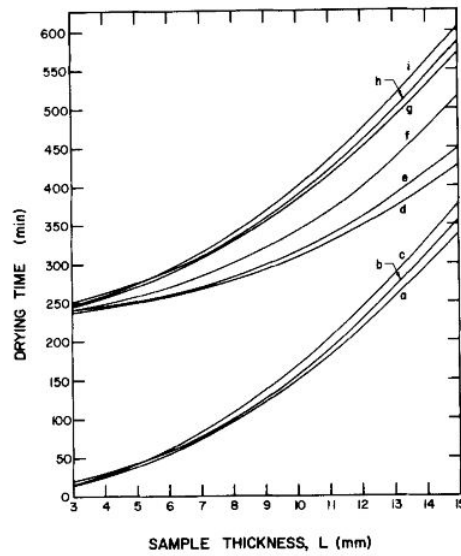


Figure 2.3: Effect of sample size on drying time when chamber pressure 0.1 mm Hg and condenser temperature 215 K[6]

to incorporate the effects of the moving mesh upon the governing equations. For the non linear parabolic equation used the two-step Runge-Kutta scheme is used. Than after solve the various problem like Freeze drying of skim milk product and protein, Bovine somatotrapin(BST). Result get of the thickness of product like 3mm and 6 mm which is shown in below Fig.(2.1) and Fig.(2.2) respectively[5].

M. J. MILLMAN et al.have done freeze drying analysis with the sorption-sublimation model in this sujested that the pressure keeps at the lowest value and also study the operational policy from the get result of the drying time by supplying the heat from the upper and lower heating plates by radiation and conduction respectively. At this time temperature of both plates different by satisfying the constrain of scorch and melting temperature of product. The analysis says that 80% of the heat used in the free water removal and remaining heat is used to remove the the bound water from the product. Here the bount water constrain take 5%[6]. R J LITCHFIELD et al. have done the work on the energy and material balance on the frozen produch put in the vaccum chamber with adsorption-sublimation model and get the drying time of the different thickness product with experimental validation in this doing analysis on the turkey meat which as shown Fig.(2.4) [7].

Kyuya Nakagawa et al. have done the matematical modelling to predict drying kinetics of the freeze drying process in multi-dimensiona. Here analysis is takes place on the apple peeled cube.From this subject the weight loss during the drying of product as function of time.Here in below figure given the mechanism of heat transfer takes palace during freeze drying. Also the result comparision with the experimental trial [8].

Table 2.1: Drying curve[6]

Operating Condition	Free Water removal phase	Final Avg. Water Wt. Fraction (0.05 Kg of water/Kg of Solid) min	Final MaxWater Wt. Any Point in Fraction(0.05 Kg of water/Kg of solid)min
Radiation to upper dried surface; conduction through a film layer at $x = L$	a	d	g
Radiation to upper dried surface; conduction through a film layer at $x = L$	b	e	h
No radiation to upper dried surface	c	f	i

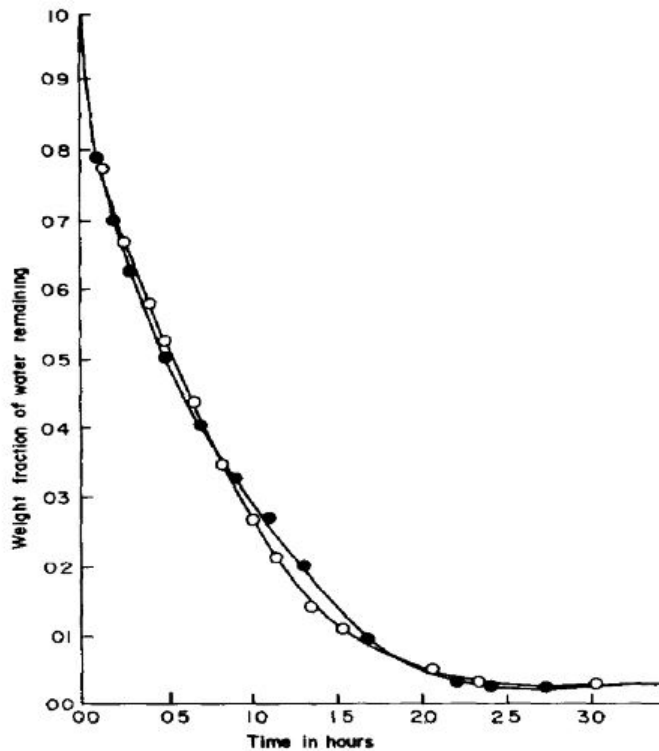


Figure 2.4: Freeze drying of turkey meat, surface temperature = 60°C , pressure = 119.9 N/m^2 , no inerts Sandall's experimental results. Run No 38, on absolute moisture basis, adsorption-sublimation model[7]

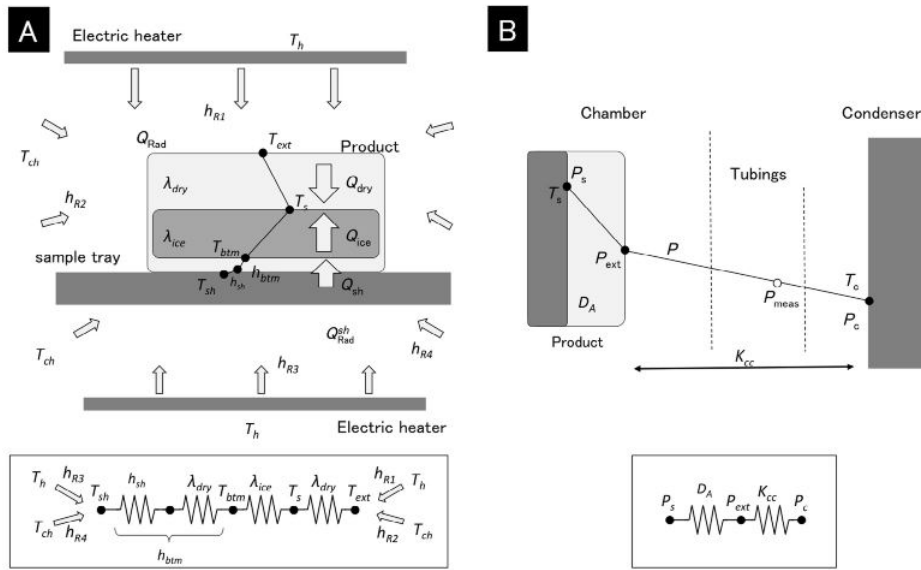


Figure 2.5: Schematic illustration of heat transfer (A) and mass transfer (B))[8]

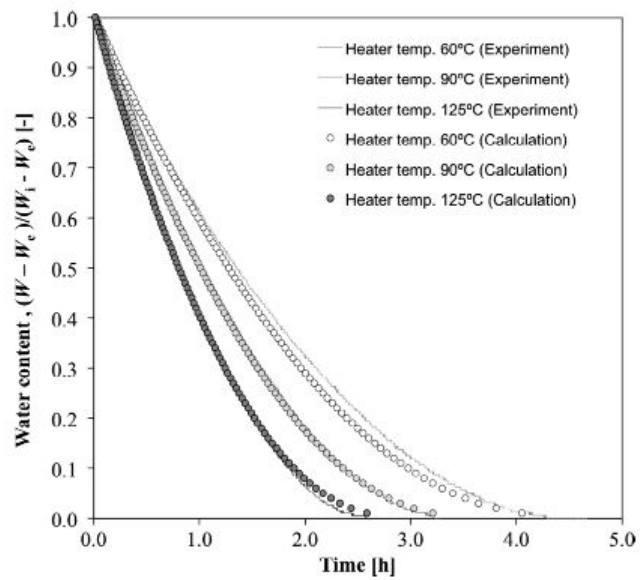


Figure 2.6: Comparison of Experimentally obtained and simulated drying curves[8]

Chokri Hammami et al. have done analysis on the finding the freeze drying process variable for strawberries. We also know that the preservation of biological and food product is done by many types of drying process. From the many types of drying process our aim to find the process variable of freeze drying process because it is having advantages like gives highest rehydration capacity, shape, colour, texture etc. are more qualitative than the other process. So that the doing experiment on the pilot freeze dryer having surface area of stainless steel plate of 0.15 m^2 and also same surface area of condenser plate. The trial is performed till the water vapor pressure reaches its lowest constant value 5 Pa and in this experiment on the strawberries the 99% of water is removed. From getting result of experiment on strawberries use surface response method allow us to graphically determine optimal working conditions. Here get the result at 30 Pa and shelf temperature 50°C the freeze drying time range from 60-65 h when the loading density 18 kg/m^2 . But reduced to 54 h with using variable heating plate temperature and also not affected the product quality[9].

N. K. Sharma et al. have done the analysis on the production rate of freeze dried yoghurt by taking effect of product thickness, chamber pressure and heating condition. Yoghurt is nothing but the food made from the curdled milk. For the experiment take the three sample thickness 3.8, 6.2 and 9.4 mm tested in the chamber pressure of 0.01 mm Hg and 0.5 mm Hg. So get the result at 6.2 mm and 0.01 mm Hg chamber pressure get the maximum production rate of yoghurt[10]. Trelea I.C et al. have done the dynamic modelling of freeze drying process for getting the stability variable of product like product surface temperature and also the partial pressure of water vapor. After getting the result done the optimization of the freeze drying process. With use of interactive tool for finding the different operating condition. Hence get the physical property of the product is main operating condition for optimum result[11].

Alves O. et al. have done In the research on the comparison of energy required for conventional freeze drying and non-conventional freeze drying. In non-conventional drying here used the heat pump which require less energy compared to conventional freeze drying process. It is possible to convert the paste into powder with the use of non conventional drying. So it is beneficial for drying of paste type product into the powder form[12]. Michael J. Pikal have done that the collapse temperature is not a unique property but it increases with increasing rate of sublimation and surface area of product which is proved by both experimentally and theoretically. The result from this analysis is get when the time $t=0 \text{ sec}$ the secondary drying occurs by the evaporation as shown in Fig.(2.7)[13].

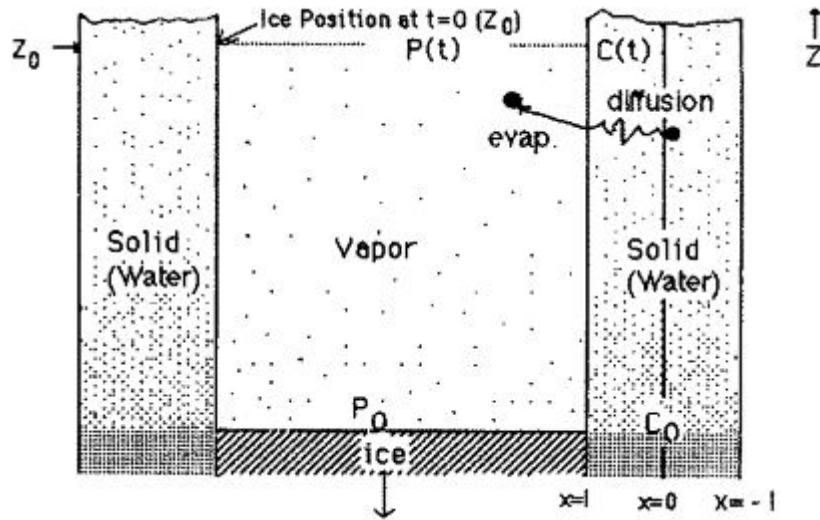


Figure 2.7: A schematic of secondary drying during primary drying.[13]

R J Litchfield And A I Liapis had said Mathematical model which give the idea about unsteady state analysis because the freeze drying is the unsteady state process for this purpose applied the quasi-steady state approach and find the optimal policy than used for unsteady state problem. Here control variables are energy input and the chamber pressure and constraints are the product melting point of frozen interface and scorch point[14].

A I Liapis and R. Bruttini had give the contribution for predicting the dynamic behaviour of the primary and secondary drying of pharmaceutical crystalline and amorphous solutes. For this pupose give the mathematical model of the desorptiuon rate constant(k_g) from the experimental result. Which is $k_g = k * exp\left(\frac{-E}{RT}\right)$ [15].

2.1 Motivation of study

Freeze drying is an important process used for preservation of food product and vaccines/medicines. One of the important product is banana chips. The batch size and drying time for banana chips depend on the thickness used. There is an optimumu thickness for which of the rate of production is maximum. This problem is chosen for the present study.

2.2 Objective of Work

The objectives are to carryout mathematical formulation of the freeze drying process for banana chips. To solve the governing differential equations, derived using the mathematical formulation, numerically using finite volume method considering different thicknesses

of banana chips. To determine an optimum thickness which yields the maximum rate of production for a batch.

2.3 Closure

The literature review regarding the optimization of the freeze drying process, theoretically and experimentally, of the different product like skim milk, yoghurt, etc. is presented. Also, the information about the mathematical modelling and solution methodology is derived.

Chapter 3

Mathematical Formulation

In this section, a qualitative description and a mathematical model of the freeze drying process are given.

3.1 System Formulation

The schematic of the problem is shown in Fig.(2.7). For simplicity of analysis, one-dimensional problem is considered and the higher dimension problem is left for future work. A banana chip of thickness L is considered. It is assumed to be kept in a drying chamber at a low pressure (below tripple point of water) / high vaccum. Hence the partial pressure of the air is neglected for the analysis.

Two distinct zones/layers, i.e.,dried layer(I) and frozen layer(II) as shown in Fig.(2.7) are formed during the drying process. Heat q_I is supplied to the surface of the dried layer by radiation; this heat is then transferred by conduction to the frozen layer. Heat q_{II} is supplied by heatng plate and is conducted through the bottom of the tray and through the frozen material reach the sublimation interface (at $x=X$) between the two layers. The term N_w in Fig.(2.7) represents the mass flux of water vapor in the dried layer.

3.1.1 Mathematical Formulation for Primary Drying stage

In the primary drying stage sublimation occurs as a process of heat directed to the sublimation interface through the dried (I) and frozen (II) layers. The subsequent water vapor is transported by convection and diffusion through the permeable dried layer, enters the vacuum chamber, and nally gathers upon the condenser plate. The following assumption are made in the creation of mathematical model[16].

(a) only one-dimensional heat and mass ows, normal to the interface and surfaces, are considered;

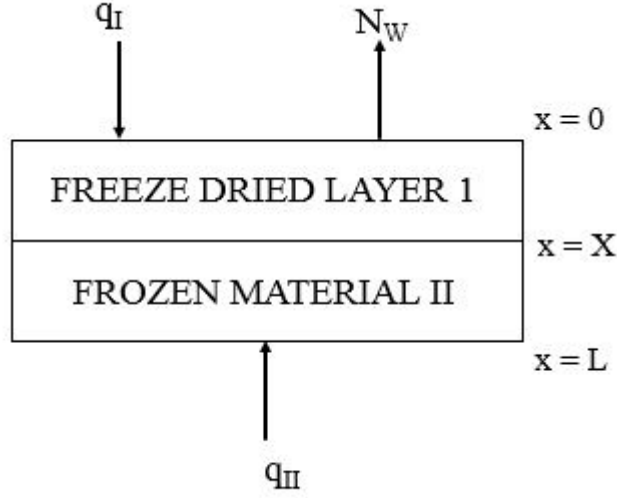


Figure 3.1: Schematic of a product during freeze-drying[6]

(b) sublimation occurs at an interface parallel to and at a distance X from the surface of the sample.

(c) the thickness of the interface is taken to be infinitesimal

(d) no inert gas in the chamber during freeze drying.

(e) The frozen region consider to be constant thermal conductivity, density, and specific heat.

Applying the energy balance in the dried layer, we have

$$\frac{\partial T_I}{\partial t} = \alpha_I \frac{\partial^2 T_I}{\partial x^2} - \frac{C_w}{\rho_I C_p} \left(\frac{\partial (N_w T_I)}{\partial x} \right) + \frac{\Delta H_V}{\rho_I C_{pI}} \left(\frac{\partial C_{sw}}{\partial t} \right), \quad 0 \leq x \leq X \quad (3.1)$$

Here, T_I , α_I , ρ_I and C_{pI} are the temperature, thermal diffusivity, density and specific heat respectively in the dried layer, and C_w , C_{sw} and ΔH_v are the specific heat of water vapor, concentration of sorbed bound water and heat of vaporization of sorbed bound water respectively.

Similarly, apply the energy balance in the frozen layer of product which is given below:

$$\frac{\partial T_{II}}{\partial t} = \alpha_{II} \frac{\partial^2 T_{II}}{\partial x^2}, \quad X \leq x \leq L \quad (3.2)$$

The initial and boundary conditions for Eq.(3.1) and Eq.(3.2) are,

at $t = 0$,

$$T_I = T_{II} = T_X = T^o, \quad 0 \leq x \leq L \quad (3.3)$$

at $x = 0$,

$$q_I = -k_I \frac{\partial T_I}{\partial x}, \quad t > 0 \quad (3.4)$$

and

$$q_I = \sigma F (T_{up}^4 - T_I^4), \quad t > 0 \quad (3.5)$$

For radiation heat transfer to the upper dried surface, at $x=X$

$$\begin{aligned} k_{II} \frac{\partial T_{II}}{\partial x} - k_I \frac{\partial T_I}{\partial x} + V (\rho_{II} C_p T_{II} - \rho_I C_p T_I) + N_w C_w T_X \\ = -\Delta H_S N_w, \quad 0 < t \leq t_{X=L} \end{aligned} \quad (3.6)$$

at $x=X$,

$$T_I = T_X = T_{II}, \quad t > 0 \quad (3.7)$$

at $x=L$,

$$q_{II} = k_{II} \frac{\partial T_{II}}{\partial x}, \quad t > 0 \quad (3.8)$$

Applying the material balance equation in the dried layer,

$$\varepsilon_p \frac{\partial C_{pw}}{\partial t} + \frac{\partial C_{sw}}{\partial t} + \frac{\partial N_w}{\partial t} = 0 \quad (3.9)$$

Here, C_{pw} represents the concentration of water vapor in the dried layer and ε_p is the porosity of dried layer. The modelling of the terms $\frac{\partial C_{sw}}{\partial t}$ and N_w is given below

$$\frac{\partial C_{sw}}{\partial t} = k_g (C_{sw}^* - C_{sw}) \quad (3.10)$$

Assuming $C_{sw}^* = 0$, because in high vacuum isothermal water desorption of bound water in the equilibrium condition can be neglected. Eq. (3.10) can be rewritten as,

$$\frac{\partial C_{sw}}{\partial t} = -k_g C_{sw} \quad (3.11)$$

The mass flux of water vapor N_w can be modelled as,

$$N_w = -k_w \frac{\partial C_{pw}}{\partial x} \quad (3.12)$$

The initial and boundary conditions of Eq.(3.9) are given below,
at $t = 0$,

$$C_{pw} = 0, \quad x > 0 \quad (3.13)$$

at $x = 0$,

$$C_{pw} = C_{pw}^o = M_w \left(\frac{p_w^o}{RT_I} \right), \quad t \geq 0 \quad (3.14)$$

at $x = X$,

$$C_{pw} = C_{pwX} = M_w \left(\frac{p_wX}{RT_X} \right), \quad 0 < t \leq t_{X=L} \quad (3.15)$$

The partial pressure of water vapor P_w is the function of temperature of interface T_X also getting from the design of the condenser.

The velocity of the sublimating moving interface, V is related to the rate of sublimation and can be determined through a material balance across the interface: is called fluid velocity or water vapor velocity at the interface.

$$V = \frac{dX}{dt} = - \left(\frac{N_w}{\rho_{II} - \rho_I} \right) \quad (3.16)$$

3.2 Closure

Mathematical modeling of the freeze drying process is presented. The fundamental governing differential equations describing the freeze drying process are reported along with the boundary conditions. Solution of these equations gives the temperature field in the dried and frozen layer, the concentration of sorbed/bound water as well as that of the water vapor in the dried layer. Using this solutions the drying time can also be computed.

Chapter 4

Discretization

The finite volume method is used to carryout the discretization with the use of central differencing and implicit approach. A control volume P along with the neighbouring volumes / nodes S and N is shown in Fig.(4.1) with s and n referring to the faces separating these volumes. The nodes are assumed to be uniformly distributed, hence the volume of Δx is assumed to be constant for each node. The standard finite volume procedure is used for the discretization, which is given below.

Discretization of Eq. (3.1):

$$\int_{\Delta x} \int_{\Delta t} \frac{\partial T_I}{\partial t} dt dx = \alpha_I \int_{\Delta x} \int_{\Delta t} \frac{\partial^2 T_I}{\partial x^2} dt dx$$

$$- \frac{C_w}{\rho_I C_{pI}} \int_{\Delta x} \int_{\Delta t} \left(\frac{\partial (N_w T_I)}{\partial x} \right) dt dx + \frac{\Delta H_V}{\rho_I C_{pI}} \int_{\Delta x} \int_{\Delta t} \left(\frac{\partial C_{sw}}{\partial t} \right) dt dx \quad (4.1)$$

$$(T_I - T_I^o) \Delta x = \alpha_I \left(\left(\frac{\partial T_I}{\partial x} \right)_s - \left(\frac{\partial T_I}{\partial x} \right)_n \right) \Delta t$$

$$- \left(\frac{C_w}{\rho_I C_{pI}} \right) \left((N_w T_I)_s - (N_w T_I)_n \right) \Delta t + \left(\left(\frac{\Delta H_v}{\rho_I C_{pI}} \right) (C_{sw} - C_{sw}^o) \right) \Delta x \quad (4.2)$$

$$(T_I - T_I^o) \Delta x = \alpha_I \left(\left(\frac{T_{IS} - T_{IP}}{\delta x_s} \right) - \left(\frac{T_{IP} - T_{IN}}{\delta x_n} \right) \right) \Delta t$$

$$- \left(\frac{C_w}{\rho_I C_{pI}} \right) \left(\left(\frac{(N_w T_I)_S + (N_w T_I)_P}{2} \right) - \left(\frac{(N_w T_I)_P + (N_w T_I)_N}{2} \right) \right) \Delta t$$

$$+ \left(\left(\frac{\Delta H_v}{\rho_I C_{pI}} \right) (C_{sw} - C_{sw}^o) \right) \Delta x \quad (4.3)$$

Eq. (4.3) can be expressed in the standard form as

$$a_p \phi_p = a_s \phi_s + a_n \phi_n + S \quad (4.4)$$

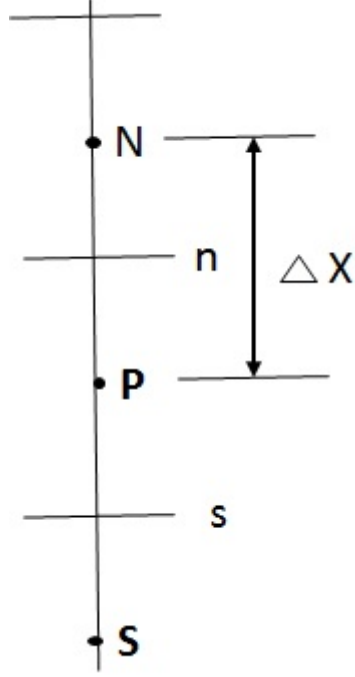


Figure 4.1: 1-Dimensional Discretization

where,

$$a_s = \alpha_I \frac{\Delta t}{\delta x_s} \quad (4.5)$$

$$a_n = \alpha_I \frac{\Delta t}{\delta x_n} \quad (4.6)$$

$$a_p = a_s + a_n + \Delta x \quad (4.7)$$

$$S = (C_{sw} - C_{sw}^o) x \left(\frac{\Delta H_V}{\rho_I C_{pI}} \right) + T_{Ip}^o \Delta x - \left(\frac{C_w \Delta t}{\rho_I C_{pI}} \right) \left(\frac{(N_w T_I)_S - (N_w T_I)_N}{2} \right) \quad (4.8)$$

Discretization of Eq. (3.2):

$$\int_{\Delta x} \int_{\Delta t} \frac{\partial T_{II}}{\partial t} dt dx = \alpha_{II} \int_{\Delta x} \int_{\Delta t} \frac{\partial^2 T_{II}}{\partial x^2} dt dx \quad (4.9)$$

$$(T_{IIp} - T_{IIp}^o) \Delta x = \alpha_{II} \left(\left(\frac{T_{IIs} - T_{IIp}}{\delta x_s} \right) - \left(\frac{T_{IIp} - T_{IIn}}{\delta x_n} \right) \right) \Delta t \quad (4.10)$$

Eq. (4.10) can be expressed in the standard form as given in Eq. (4.4) with

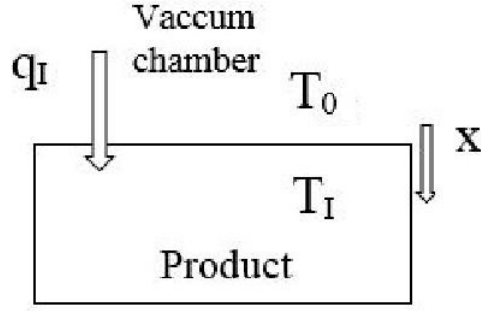


Figure 4.2: Conduction in the dried layer

$$a_s = \left(\frac{\alpha_{II}}{\Delta x} \right) \Delta t \quad (4.11)$$

$$a_n = \left(\frac{\alpha_{II}}{\Delta x} \right) \Delta t \quad (4.12)$$

$$a_p = a_s + a_n + \Delta x \quad (4.13)$$

$$S = T_{IIp}^o \Delta x \quad (4.14)$$

Discretization of Eq. (3.4): Withe reference to Fig. (4.2), we can write:

$$q_I = -k_I \left(\frac{T_I - T_0}{\Delta x/2} \right) = k_I \left(\frac{T_0 - T_I}{\Delta x/2} \right) \quad (4.15)$$

where T_0 is the boudnary node temperature at the top surface.

Discretization of Eq. (3.8): Withe reference to Fig.(4.3), we can write:

$$q_{II} = k_I \left(\frac{T_0 - T_{II}}{\Delta x/2} \right) \quad (4.16)$$

where T_0 is the boudnary node temperature at the bottom surface.

Discretization of interface condition

$$\begin{aligned} k_{II} \left(\frac{T_X - T_{IIs}}{\Delta x} \right) - k_I \left(\frac{T_{In} - T_X}{\Delta x} \right) + V (\rho_{II} C_{pII} T_{II} - \rho_I C_{pI} T_I) \\ + N_w C_{pus} T_X = -\Delta H_S N_w \end{aligned} \quad (4.17)$$

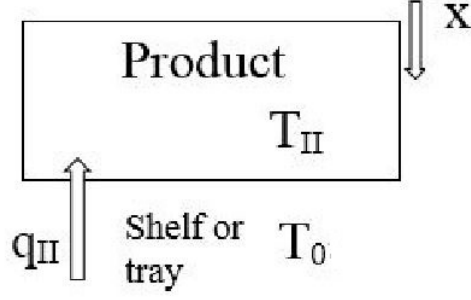


Figure 4.3: Conduction in the frozen layer

Discretization of Eq. (3.9) Substituting Eq.(3.12) in Eq. (3.9) and performing the integrations,

$$\varepsilon_p \int_{\Delta x} \int_{\Delta t} \frac{\partial C_{pw}}{\partial t} dt dx + \int_{\Delta x} \int_{\Delta t} \frac{\partial C_{sw}}{\partial t} dt dx - k_w \int_{\Delta x} \int_{\Delta t} \frac{\partial^2 C_{pw}}{\partial x^2} dt dx = 0 \quad (4.18)$$

$$\varepsilon_p (C_{pw} - C_{pw}^o) \Delta x + (C_{sw} - C_{sw}^o) \Delta x - k_w \left(\left(\frac{\partial C_{pw}}{\partial x} \right)_s - \left(\frac{\partial C_{pw}}{\partial x} \right)_n \right) \Delta t = 0 \quad (4.19)$$

$$\varepsilon_p (C_{pw} - C_{pw}^o) \Delta x + (C_{sw} - C_{sw}^o) \Delta x - k_w \left(\left(\frac{C_{pwS} - C_{pwp}}{\delta x_s} \right) - \left(\frac{C_{pwp} - C_{pwN}}{\delta x_n} \right) \right) \Delta t = 0 \quad (4.20)$$

Equation (4.20) can be expressed in the standard form given in Eq. (4.4) with

$$a_s = \left(\frac{k_w}{\Delta x} \right) \Delta t \quad (4.21)$$

$$a_n = \left(\frac{k_w}{\Delta x} \right) \Delta t \quad (4.22)$$

$$a_p = a_s + a_n + \varepsilon_p \Delta x \quad (4.23)$$

$$S = \varepsilon_p (C_{pwp}^o) \Delta x + (C_{sw} - C_{sw}^o) \Delta x \quad (4.24)$$

4.1 Solution Algorithm

The solution algorithm is presented with a flow chart in Fig.(4.4) this can be described as follows.

1. Give the input parameters such as time step, heat input, thickness of banana and its properties.
2. Give the initial condition such as concentration of water in the banana and temperature of frozen banana.
3. Mesh the solution domain.
4. Specify the boundary conditions.
5. Solve the discretization equations to obtain solutions of Cpw , T_I , T_{II} , N_w , C_{sw} , V . To solve the equations use a suitable convergence criterion, for example 0.0001.
6. After a certain number of time steps the value of C_{sw} reduces to the one required and the final solution is obtained.
7. Obtain solutions for different thicknesses and compute the drying time for each thicknesses.
8. Based on the thickness calculate the batch size (kg) and divide it by the corresponding drying time to obtain the rate of production.

4.2 Closure

A finite volume discretization of the governing equations described in Chapter 3 is carried out. A solution algorithm is presented and discussed.

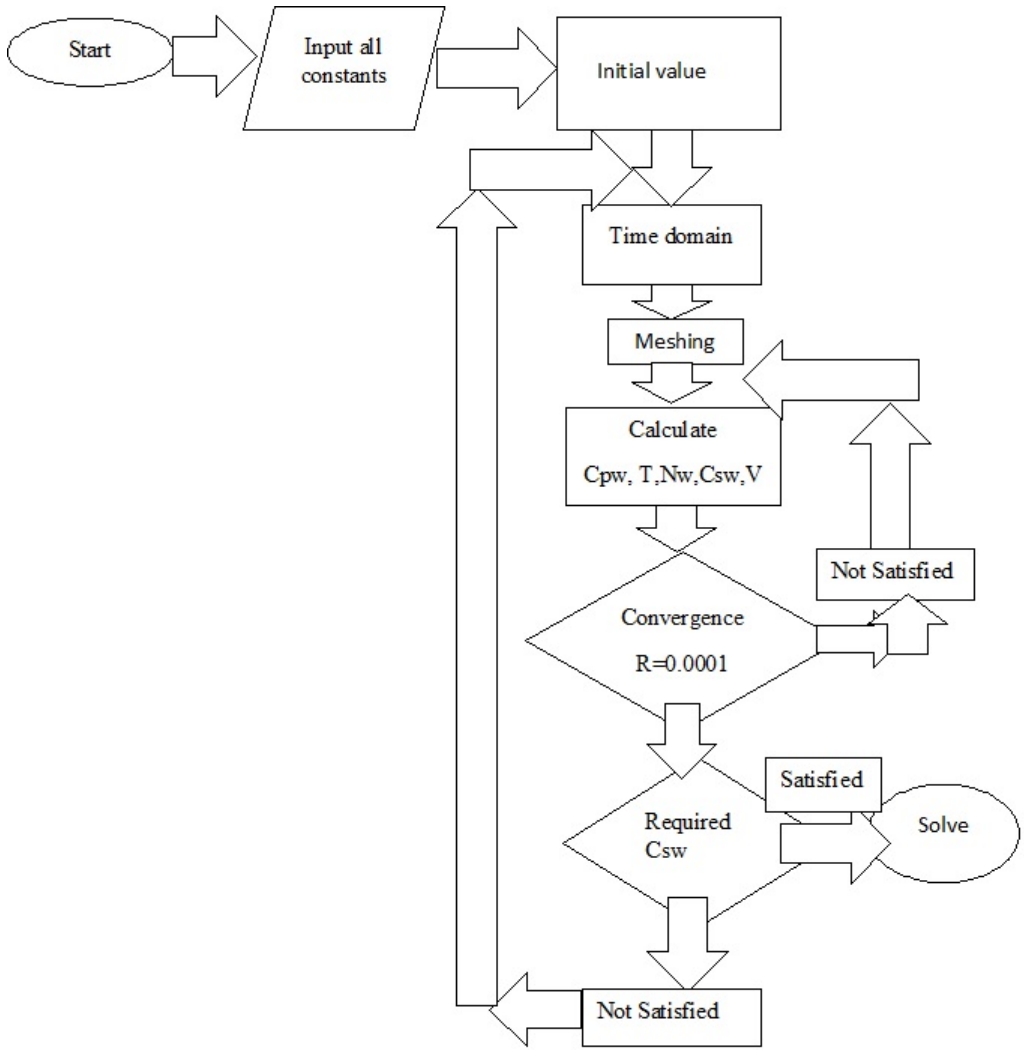


Figure 4.4: Flowchart

Chapter 5

Results

Using the solution methodology discussed in Chapter 4, the discretized governing equations are solved for different thicknesses, i.e., 1 mm, 1.5 mm, 2 mm, 2.5 mm, 3 mm, 3.5 mm, 4 mm of banana chips and the results are reported here. The values of the properties and the constants used for the solution are reported in Appendix A.

The plot of drying time as a function of thickness is shown in Fig.5.1. It is observed that the drying time increases with the thickness. Using the drying time information, the production rate is calculated for these thicknesses and is plotted in Fig.5.2. The production rate initially decreases with increase in thickness. It is minimum for a thickness of 1.5 mm. With further increase in thickness, the production rate increases. However, the maximum thickness that can be used is limited by the suitability for eating. Therefore, a thickness of 2.5 mm to 3.0 mm is recommended to be used for freeze drying of banana chips for maximum rate of production.

The plot of temperature versus axial distance is given in Fig.5.3 for different thicknesses of banana chips. It can be observed that higher temperatures will be reached for smaller thickness for the same heat flux. In the case of 1 mm thickness the temperature is found to be 303 K.

The plot of concentration of water vapor C_{pw} versus axial distance is given in Fig.5.4 for different thicknesses of banana chips. It can be observed that C_{pw} increases with the increase in thickness due to the lower temperatures in the dried region for larger thickness.

Now we shown the result of different thickness with the temperature as shown in Fig.5.3. Temperature of the banana chips is same as the ambient temperature for 1mm thickness in the dried layer but in the frozen layer is same as frozen banana temperature. But the temperature changes occur in the more thick banana such as 1.5 mm, 2 mm, 2.5 mm, 3 mm and 3.5 mm which is shown in Fig.5.3.

The plot of concentration of sorbed water (C_{sw}) as a function of axial distance at different times is shown in Fig.5.5 for a representative case of 1 mm thickness. It can be observed that initially the drop in C_{sw} is small until the time of 1500 seconds. With

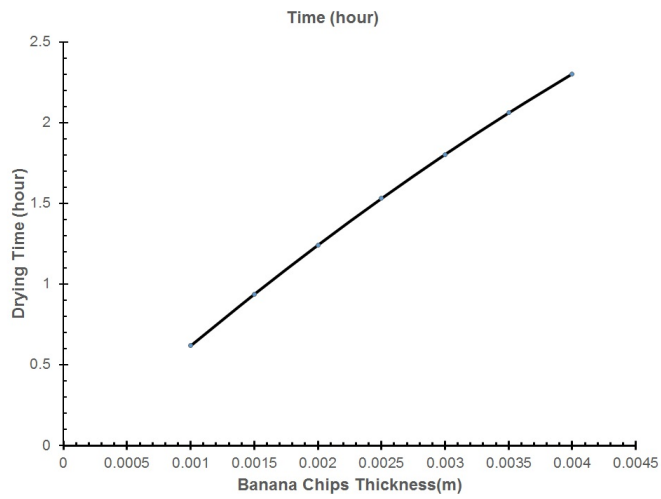


Figure 5.1: Drying Time as a function of thickness

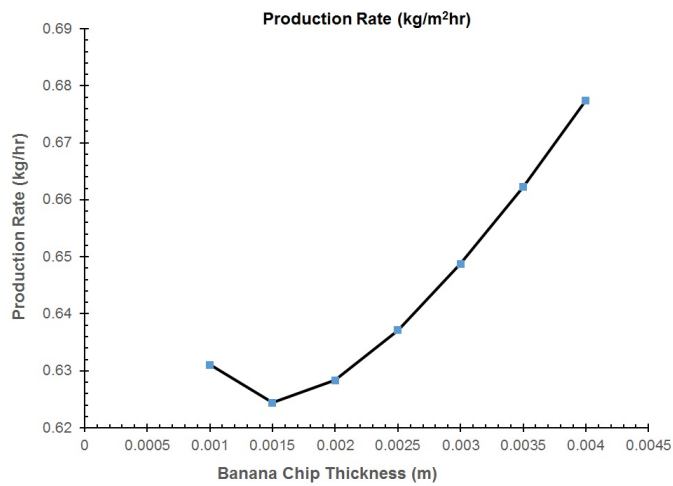


Figure 5.2: Production rate of banana Chips as a function of thickness

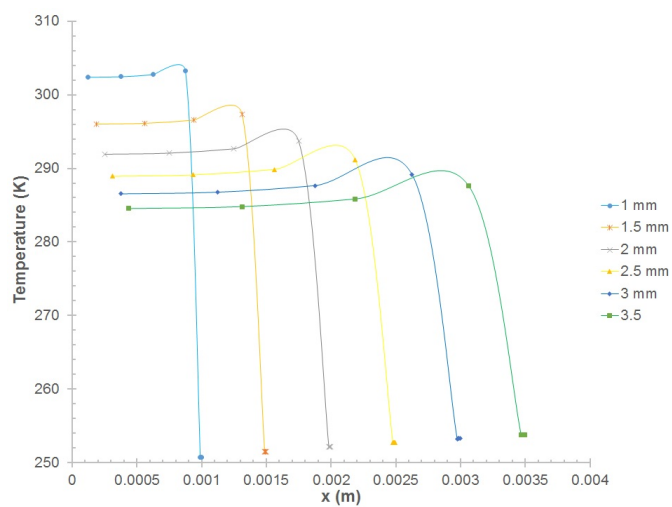


Figure 5.3: Temperature For Different Thickness of Banana

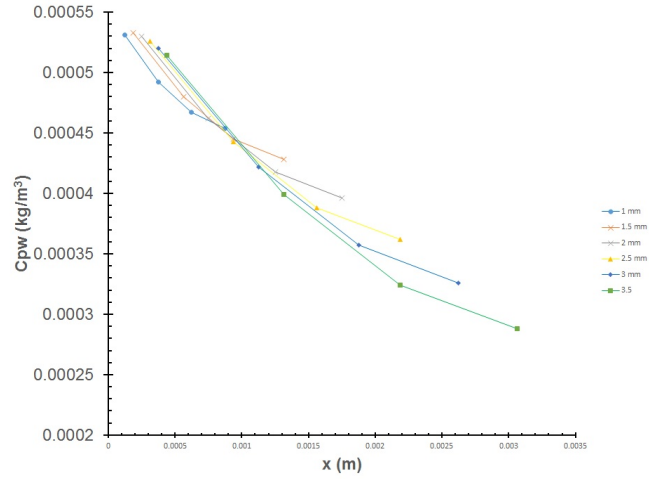


Figure 5.4: Concentration of Water Vapor in Dried Layer

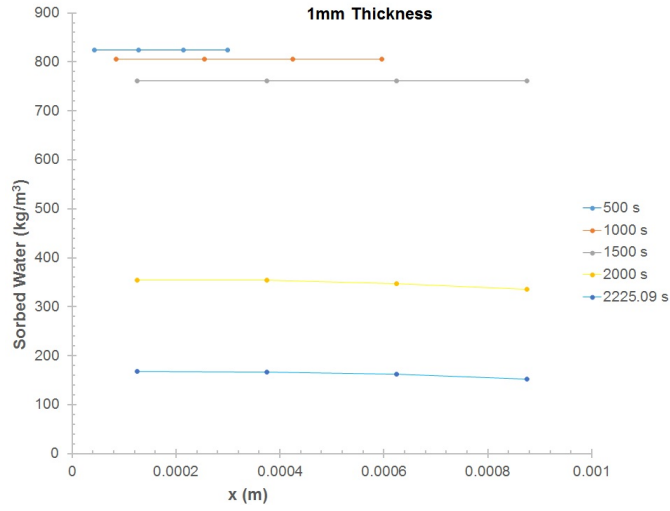


Figure 5.5: Concentration of Sorbed Water in Dried Layer At Different Time

further increase in time the rate of drop increases significantly. The plot of temperature as a function of axial distance at different times is shown in Fig.5.6 for a case of 1 mm thickness. As observed the temperature in the dried region increases from 253 K at time $t = 0$ s to 303 K at time 2225.09 sec. It can also be observed that the frozen region vanishes with time. The plot of concentration of water vapor (C_{pw}) as a function of axial distance at different times is shown in Fig.5.7 for a representative case of 1 mm thickness. It can be observed that C_{pw} decreases with increase in time due to increasing temperature.

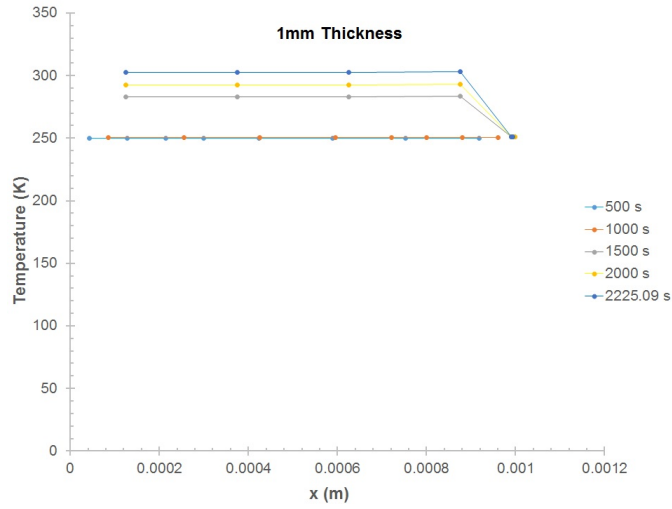


Figure 5.6: Temperature Profile in Dried Layer At Different Time

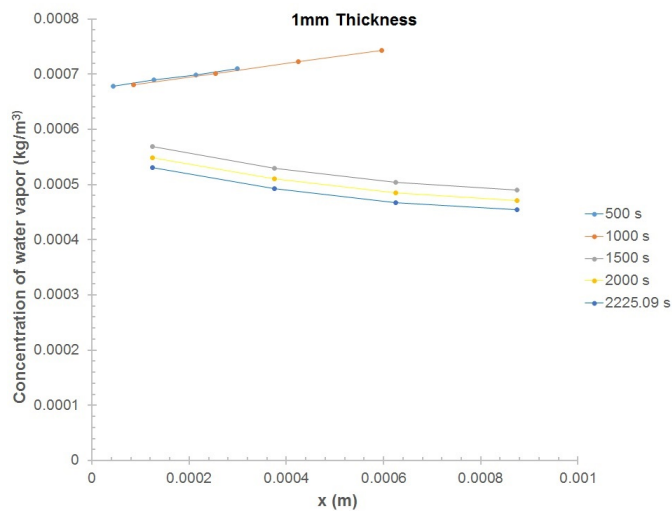


Figure 5.7: Concentration of Water vapor At Different Time

Chapter 6

Conclusions

From the present study following conclusions are made.

- As the thickness of banana chip increases, the drying time increases.
- For a thickness of 1.5 mm the rate of production is minimum. Beyond this thickness the rate of production increases monotonically. However, a thickness of 2.5 mm to 3.0 mm is recommended for maximum rate of production from the point of view of suitability for eating.
- The temperature reached at the end of drying is maximum for minimum chip thickness.
- Concentration of water vapor increases with the increasing the thickness of banana chip due to decrease in temperature as a result of increase in thickness.

Chapter 7

Future Scope

Scope of future work are mentioned below.

- Experimental validation of the results derived in the present work using numerical methods can be carried out.
- Extension of the procedure to two and three dimensions can be carried out to capture the physics in a better way.
- The problem of optimization of the thickness for maximum production rate can also be solved for other food products.
- Effects of varying other parameters such as chamber pressure and shelf temperature can also be verified on the drying time / production rate.

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Appendix A

Table 1: Constant Parameter For Banana[17]

ρ_I	390.54 kg/m^3
ρ_{II}	1122.68 kg/m^3
C_{pI}	1.050 kJ/kgK
C_{pII}	1.86 kJ/kgK
C_w	1.855 kJ/kgK
ε_p	0.7962
ΔH_v	2791.2 kJ/kg
k_I	0.3835 W/mK
k_{II}	1.8225 W/mK
k	$2.7021 \times 10^7 \text{ s}^{-1}$
E	3130.81 kJ/kg
R	461.89 kJ/kgK
k_w	$0.004 \text{ m}^2/\text{s}$
q	1100 W/m^2