# Failure Strength of Orthotropic Lamina with Various Shaped Cutouts

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**Abstract**

**In this paper, the failure strength of an orthotropic lamina with different shaped cutouts is presented for in-plane loading. The failure strength of laminate is determined on first ply failure basis by various failure criteria like Hashin – von-Mises, Tsai – Hill, Hashin – Rotem, Tsai – Wu etc. Effect of various parameters like cutout shape, material properties, fiber angle, angle of loading etc. on the failure strength is studied and presented.**

## 1. Introduction

Composite materials find wide applications in civil, mechanical and aerospace industries nowadays. For certain service requirement, various shaped cutouts are made in plate like structural components made of composite material. These cutouts weaken the laminate and hence it is important to know the strength of this laminate with this cutout under the action of various loading conditions. Failure strength of the laminate can be derived using various failure criterions like Tsai – Hill theory, Hashin – Rotem theory, Tsai – Wu, Hashin – von-Mises etc described by Jones [1].

Muskhelishvili [2] has introduced the complex variable formulation for the solution of basic problems of theory of elasticity. Lekhnistkii [3] implemented the complex variable formulation proposed by Muskhelishvili for anisotropic plates. Based on this formulation Ukadgaonker and Rao [4], Rao et al [5], Sharma [6] etc. obtained stress distribution around rectangular, square, circular/ elliptical and irregular shaped hole. Ukadgaonker and Rao [4], Rao et al [5] obtained failure strength of laminate using various theories of failure.

In the present study, Muskhelishvili's complex variable approach and first ply failure theory is used to obtain the failure strength of laminate subjected to in-plane load with various shaped cut outs. The effect of various parameters like shape of cutout, fiber angle, angle of loading, material property etc. on strength of the laminate is studied and presented.

## 2. Complex variable Formulation

The anisotropic plate of numbers of orthotropic laminae stacked symmetrically is considered to be subjected to in-plane loads. Using classical lamination plate theory, the mean value of strain along the thickness of plate is related to stress by generalized Hooke's law.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} \\ a_{12} & a_{22} & a_{26} \\ a_{16} & a_{26} & a_{66} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{1}$$

where, $\sigma_x$, $\sigma_y$ and $\tau_{xy}$ are mean value of stresses and $a_{ij}$ = compliance coefficients.

Stress components can be written in terms of Airy's stress function ($U$) as follows.

$$\sigma_x = \frac{\partial^2 U}{\partial y^2}$$
$$\sigma_y = \frac{\partial^2 U}{\partial x^2}$$
$$\tau_{xy} = -\frac{\partial^2 U}{\partial x \partial y} \tag{2}$$

Considering Airy's stress function, generalized Hooke's law along with strain displacement compatibility conditions, the following characteristic equation is obtained.

$$a_{22}s^4 - 2a_{26}s^3 + (2a_{12} + a_{66})s^2 - 2a_{16}s + a_{66} = 0 \tag{3}$$

The roots of the characteristic equation eq. (3) are as follows.

$$\begin{aligned} s_1 &= \alpha_1 + i\beta_1 \\ s_2 &= \alpha_2 + i\beta_2 \\ s_3 &= \alpha_1 - i\beta_1 \\ s_4 &= \alpha_2 - i\beta_2 \end{aligned} \tag{4}$$

The Airy's stress function *U(x, y)* can be written as follows.

$$U(x, y) = 2Re[F_1(z_1) + F_2(z_2)] \tag{5}$$

Here, $z_1 = x + s_1 y$ and $z_2 = x + s_2 y.$

Introducing the analytic stress function $\phi(z_1)$ and $\psi(z_2)$ as follows.

$$\begin{aligned} \frac{\partial F}{\partial z_1} &= \phi(z_1) \\ \frac{\partial F}{\partial z_2} &= \psi(z_2) \end{aligned} \tag{6}$$

Substituting equation (6) in to (5) and in turns (5) in to (2), the stress components can be written in terms of $\phi(z_1)$ and $\psi(z_2)$ as follows.

$$\begin{aligned} \sigma_x &= 2Re[{s_1}^2\phi^{'}(z_1) + {s_2}^2\psi'(z_2)] \\ \sigma_y &= 2Re[\phi^{'}(z_1) + \psi'(z_2)] \\ \tau_{xy} &= -2Re[s_1\phi^{'}(z_1) + s_2\psi'(z_2)] \end{aligned} \tag{7}$$

Stresses given in Cartesian coordinate system by equation (7) can be transformed to polar coordinate by following transformation.

$$\begin{Bmatrix} \sigma_\theta \\ \sigma_r \\ \tau_{r\theta} \end{Bmatrix} = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{8}$$

where *m* and *n* are direction cosines.

To consider various in-plane loading condition, arbitrary biaxial loading condition is adopted as proposed bay Gao [7].

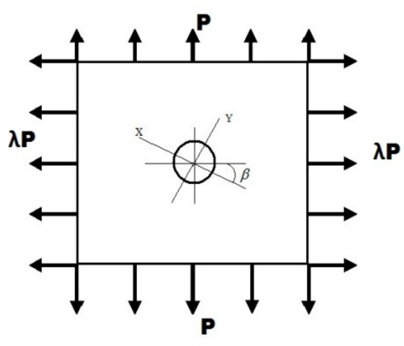


**Figure 1: Arbitrary biaxial loading condition**

Referring to figure1, the in-plane biaxial loading condition is considered as follows:

$$\sigma_{x'}{}^{\infty} = \lambda p; \sigma_{y'}{}^{\infty} = p \text{ and } \tau_{x'y'}{}^{\infty} = 0 \text{ at } |z| \to \infty \,, \tag{9}$$

where $\sigma_{x'}{}^{\infty}$ and $\sigma_{y'}{}^{\infty}$ are stresses applied at infinity.

## 3. Mapping Function

The region outside the hole of various shapes in z-plane can be conformally mapped on to the unit circle in $\xi$- plane by mapping function. Following generalized mapping function is used.

$$z = w(\xi) = R\left(\xi + \sum_{n=1}^{K} \frac{C_n}{\xi^n}\right) \tag{10}$$

where $C_n$ are constant of mapping function. Values of $C_n$ for various shaped holes are tabulated in Table 1. $\xi = e^{i\theta}$ representing unit circle in $\xi$- plane. R is size of hole.

| Hole Shape | n | $c_n$ |
|---|---|---|
| Circle | 0 | - |
| Ellipse | 1 | $\frac{a-b}{a+b}$, a, b are semi major and minor axis of ellipse |
| Square | [3 7 11 15 19] | [-1/6 1/56 -1/176 1/384 -7/4864] |
| Rectangle, Side ratio - 2 | [1 3 5 7 9 11] | [0.304604 -0.1512027 -0.027634 0.008685 0.00902146 0.0006092] |

**Table 1: Constants for mapping function for various shapes [5].**

Due to affine transformation, the mapping function for anisotropic material is as follows.

$$z_j = w_j(\xi) = \frac{R}{2}\left(a_j\left(\frac{1}{\xi} + \sum_{n=1}^{K} c_n \xi^n\right) + b_j\left(\xi + \sum_{n=1}^{K} c_n \xi^{-n}\right)\right)$$

where $a_j = (1 + is_j);\ b_j = (1 - is_j);\quad j = 1,2$ (11)

## 4. Stress function

The stress function for the infinite plate with hole is obtained as follows. The stress functions $\phi_1(z_1)$ and $\psi_1(z_2)$ for the infinite domain without hole are obtained under remotely applied loading condition.

$$\phi_1(z_1) = B^* z_1$$
$$\psi_1(z_2) = (B'^* + iC'^*)z_2 \tag{12}$$

where

$$B^* = \frac{\sigma_x{}^\infty + (\alpha_2{}^2 + \beta_2{}^2)\sigma_y{}^\infty + 2\alpha_2\tau_{xy}{}^\infty}{2\left((\alpha_2 - \alpha_1)^2 + (\beta_2{}^2 - \beta_1{}^2)\right)}$$

$$B'^* = \frac{(\alpha_1{}^2 - \beta_1{}^2 - 2\alpha_1\alpha_2)\sigma_y{}^\infty - \sigma_x{}^\infty - 2\alpha_2\tau_{xy}{}^\infty}{2\left((\alpha_2 - \alpha_1)^2 + (\beta_2{}^2 - \beta_1{}^2)\right)}$$

$$C'^* = \frac{([\alpha_1 - \alpha_2]\sigma_x{}^\infty + [\alpha_2(\alpha_1{}^2 - \beta_1{}^2) - \alpha_1(\alpha_2{}^2 - \beta_2{}^2)]\sigma_y{}^\infty + [(\alpha_1{}^2 - \beta_1{}^2) - (\alpha_2{}^2 - \beta_2{}^2)]\tau_{xy}{}^\infty)}{2\beta_2\left((\alpha_2 - \alpha_1)^2 + (\beta_2{}^2 - \beta_1{}^2)\right)} \tag{13}$$

Using equation (12), boundary conditions on a fictitious hole are determined as follows.

$$f_1 = 2Re[\phi_1(z_1) + \psi_1(z_2)]$$

$$f_2 = 2Re[s_1\phi_1(z_1) + s_2\psi_1(z_2)] \tag{14}$$

Substituting equation (11) into (12) and finally equation (12) into (14), boundary conditions can be obtained. The infinite plate with hole without loading is now considered and the stress function $\phi_0(z_1)$ and $\psi_0(z_2)$ are obtained by applying negative of the boundary conditions (14) on the hole boundary and using Schwarz integral formula.

$$\phi_0(\xi) = \frac{i}{4\pi(s_1-s_2)}\int_\gamma \left(s_2 f_1{}^0 - f_2{}^0\right)\frac{t+\xi}{t-\xi}\frac{dt}{t} + \lambda_1 \tag{15}$$

$$\psi_0(\xi) = \frac{-i}{4\pi(s_1-s_2)}\int_\gamma \left(s_1 f_1{}^0 - f_2{}^0\right)\frac{t+\xi}{t-\xi}\frac{dt}{t} + \lambda_2 \tag{16}$$

where $\gamma$ is unit circle boundary in $\xi$- plane, t is value of $\xi$ on hole boundary, $\lambda_1$and $\lambda_2$are imaginary constants, $f_1{}^0 = -f_1$ and $f_2{}^0 = -f_2$.

By evaluating the integral in equation (15) and (16), the stress functions are obtained as follows.

$$\phi_0(\xi) = \left\{\frac{a_3}{\xi} + b_3\sum_{n=1}^{N}\frac{c_n}{\xi^n}\right\}$$

$$\psi_0(\xi) = -\left\{\frac{a_4}{\xi} + b_4\sum_{n=1}^{N}\frac{c_n}{\xi^n}\right\} \tag{17}$$

where,

$$a_3 = \left\{\frac{1}{s_1-s_2}\right\}[s_2(K_1+\overline{K_2}) - (K_3+\overline{K_4})]$$

$$b_3 = \left\{\frac{1}{s_1-s_2}\right\}[s_2(K_2+\overline{K_1}) - (K_4+\overline{K_3})]$$

$$a_4 = \left\{\frac{1}{s_1-s_2}\right\}[s_1(K_1+\overline{K_2}) - (K_3+\overline{K_4})]$$

$$b_4 = \left\{\frac{1}{s_1-s_2}\right\}[s_1(K_2+\overline{K_1}) - (K_4+\overline{K_3})]$$

$$K_1 = \left(\frac{R}{2}\right)[B^*a_1 + (B'^* + iC'^*)a_2]$$

$$K_2 = \left(\frac{R}{2}\right)[B^*b_1 + (B'^* + iC'^*)b_2]$$

$$K_3 = \left(\frac{R}{2}\right)[s_1B^*a_1 + s_2(B'^* + iC'^*)a_2]$$

$$K_4 = \left(\frac{R}{2}\right)[s_1B^*b_1 + s_2(B'^* + iC'^*)b_2]$$

Finally the stress function for the infinite plate with hole with remote loading is obtained by superimposing the two stress functions.

$$\phi(z_1) = \phi_1(z_1) + \phi_0(z_1)$$

$$\psi(z_2) = \psi_1(z_2) + \psi_0(z_2) \tag{18}$$

## 5. Failure Theories

The maximum load that can be applied to the material before the failure is termed as failure strength of the material. Different failure theories are used to determine the failure strength of each lamina in a laminate and the minimum values among all these is considered as the failure strength of the laminate according to First Ply Failure theory.

Various failure theories are explained as follows. Here $S_l$, $S_l{}'$ are considered as tensile and compressive strength of lamina in longitudinal direction, $S_t$ and $S_t{}'$ are tensile and compressive strength of lamina in transverse direction and $S_s$ is shear strength.

**5.1 Hashin – von – Mises theory (H-vM):**

According to H-vM theory the failure strength $s_f$ of the orthotropic lamina is as follows.

$$\left(\frac{\sigma_\theta}{\sigma}\right)^2\left\{\frac{\sin^4(\theta-\varphi)}{S_l{}^2}+\frac{\cos^4(\theta-\varphi)}{S_t{}^2}+\frac{\sin^2(\theta-\varphi)\cos^2(\theta-\varphi)}{S_s{}^2}-K^*\frac{\sin^2(\theta-\varphi)\cos^2(\theta-\varphi)}{S_lS_t}\right\}=\frac{1}{s_f{}^2} \quad (19)$$

where $K^*=\frac{E_1(1+\nu_{21})+E_2(1+\nu_{12})}{2\sqrt{E_1E_2(1+\nu_{21})1+\nu_{12})}}$.

**5.2 Tsai – Hill theory (T – H):**

As per Tsai – Hill theories of failure, strength can be obtained as follows.

$$\left(\frac{\sigma_1}{\sigma}\right)^2\frac{1}{S_l{}^2}+\left(\frac{\sigma_2}{\sigma}\right)^2\frac{1}{S_t{}^2}+\left(\frac{\tau_6}{\sigma}\right)^2\frac{1}{S_s{}^2}-\left(\frac{\sigma_1\sigma_2}{\sigma^2}\right)^2\frac{1}{S_l{}^2}=\frac{1}{s_f{}^2} \quad (20)$$

where $\sigma_1$, $\sigma_2$, $\tau_6$ are the transformed stress components along fibers orientation from $\sigma_x$, $\sigma_y$, $\tau_{xy}$ as follows.

$$\sigma_1=\sigma_x cos^2\varphi+\sigma_y sin^2\varphi+2\tau_{xy}sin\varphi cos\varphi$$

$$\sigma_2=\sigma_x sin^2\varphi+\sigma_y cos^2\varphi-2\tau_{xy}sin\varphi cos\varphi$$

$$\tau_6=(\sigma_y-\sigma_x)sin\varphi cos\varphi+(cos^2\varphi-sin^2\varphi)\tau_{xy} \quad (21)$$

**5.3 Hashin – Rotem theory (H – R):**

According to Hashin – Rotem theory, the failure strength can be determined as:

$$\left(\frac{\sigma_2}{\sigma}\right)^2\frac{1}{S_t{}^2}+\left(\frac{\tau_6}{\sigma}\right)^2\frac{1}{S_s{}^2}=\frac{1}{s_f{}^2} \quad (22)$$

**5.4 Tsai – Wu theory (T – W):**

According to Tsai – Wu theory, the failure strength of lamina can be derived from following equation.

$$\sigma f_1\left(\frac{\sigma_1}{\sigma}\right)+\sigma f_2\left(\frac{\sigma_2}{\sigma}\right)+\sigma^2 f_{11}\left(\frac{\sigma_1}{\sigma}\right)^2+\sigma^2 f_{22}\left(\frac{\sigma_2}{\sigma}\right)^2+\sigma^2 f_{66}\left(\frac{\tau_6}{\sigma}\right)^2+2\sigma^2 f_{12}\left(\frac{\sigma_1\sigma_2}{\sigma^2}\right)=1 \quad (23)$$

where $\left({}^{\sigma_1}/_{\sigma}\right)$, $\left({}^{\sigma_2}/_{\sigma}\right)$ and $\left({}^{\tau_6}/_{\sigma}\right)$ are the normalized values of stresses in principal fiber directions. By designating the value of $\sigma$ that causes failure as $s_f$ and rewriting equation (23) as:

$$as_f{}^2+bs_f-1=0 \quad (24)$$

where

$$a=f_{11}\left(\frac{\sigma_1}{\sigma}\right)^2+f_{22}\left(\frac{\sigma_2}{\sigma}\right)^2+f_{66}\left(\frac{\tau_6}{\sigma}\right)^2+2f_{12}\left(\frac{\sigma_1\sigma_2}{\sigma^2}\right)$$

$$b=f_1\left(\frac{\sigma_1}{\sigma}\right)+f_2\left(\frac{\sigma_2}{\sigma}\right)$$

$$f_{11}=\frac{1}{S_lS_l{}'},\quad f_{22}=\frac{1}{S_tS_t{}'},\ \ f_{66}=\frac{1}{S_s{}^2}$$

$$f_1=\frac{1}{S_l}-\frac{1}{S_l{}'},\quad f_2=\frac{1}{S_t}-\frac{1}{S_t{}'},\ f_{12}\cong 0.5(f_{11}f_{22})^{1/2}$$

Solution of equation (24) gives the failure strength of lamina.

## 6. Results and Discussion

A computer programme is prepared to determine the stress distribution around the cut outs and to determine the failure strength by different failure theories for different material. The programme is also capable to study the effect of various parameters.

Table 2 shows material properties of various materials used in the present study.

| Property | Glass/ Epoxy | Boron/ Epoxy | Graphite/ Epoxy |
|---|---|---|---|
| $E_1$ (MPa) | 47.4 x $10^3$ | 282.77 x $10^3$ | 181x$10^3$ |
| $E_2$ (MPa) | 16.2 x $10^3$ | 23.79 x $10^3$ | 10.3 x $10^3$ |
| $\nu_{12}$ | 0.26 | 0.27 | 0.28 |
| $G_{12}$ (MPa) | 7 x $10^3$ | 10.35 x $10^3$ | 7.17 x $10^3$ |
| $S_l$ (MPa) | 1062 | 1260 | 1500 |
| $S_l'$ (MPa) | 610 | 2500 | 1500 |
| $S_s$ (MPa) | 72 | 67 | 68 |
| $S_t$ (MPa) | 31 | 61 | 40 |
| $S_t'$ (MPa) | 118 | 202 | 246 |

**Table 2: Material properties [8]**

### 6.1 Failure strength of lamina

The results presented here is for the plate of Graphite/Epoxy material of $[0/90]_s$ with circular hole and subjected to equi-biaxial loading. Failure strength of each lamina is determined around a hole using various failure criterions. Table 3 shows the values of failure strength for $0^0$ lamina around a circular hole in $[0/90]_s$ laminate.

| Angle (Deg.) | Failure Strength (MPa) | | |
|---|---|---|---|
| | Tsai_Hill | Hashin - Rotem | Tsai-Wu |
| 0 | **10.22** | **10.22** | **10.22** |
| 5 | 11.16 | 11.16 | 11.15 |
| 10 | 13.95 | 13.95 | 13.89 |
| 15 | 18.48 | 18.48 | 18.31 |
| 20 | 24.55 | 24.55 | 24.16 |
| 25 | 31.93 | 31.93 | 31.14 |
| 30 | 40.30 | 40.30 | 38.88 |
| 35 | 49.29 | 49.28 | 46.96 |
| 40 | 58.50 | 58.49 | 54.98 |
| 45 | 67.49 | 67.49 | 62.51 |
| 50 | 75.85 | 75.86 | 69.21 |
| 55 | 83.20 | 83.24 | 74.85 |
| 60 | 89.33 | 89.43 | 79.39 |
| 65 | 94.38 | 94.59 | 83.17 |
| 70 | 99.25 | 99.73 | 87.29 |
| 75 | 106.82 | 107.99 | 94.65 |
| 80 | 125.87 | 129.52 | 113.88 |
| 85 | 191.28 | 214.77 | 180.11 |
| 90 | 383.14 | 2.84 x $10^{17}$ | -383.14 |
| 95 | 191.28 | 214.77 | 180.11 |

| | | | |
|---|---|---|---|
| 100 | 125.87 | 129.52 | 113.88 |
| 105 | 106.82 | 107.99 | 94.65 |
| 110 | 99.25 | 99.73 | 87.29 |
| 115 | 94.38 | 94.59 | 83.17 |
| 120 | 89.33 | 89.43 | 79.39 |
| 125 | 83.20 | 83.24 | 74.85 |
| 130 | 75.85 | 75.86 | 69.21 |
| 135 | 67.49 | 67.49 | 62.51 |
| 140 | 58.50 | 58.49 | 54.98 |
| 145 | 49.29 | 49.28 | 46.96 |
| 150 | 40.30 | 40.30 | 38.88 |
| 155 | 31.93 | 31.93 | 31.14 |
| 160 | 24.55 | 24.55 | 24.16 |
| 165 | 18.48 | 18.48 | 18.31 |
| 170 | 13.95 | 13.95 | 13.89 |
| 175 | 11.16 | 11.16 | 11.15 |
| 180 | **10.22** | **10.22** | **10.22** |

**Table 3: Strength of $0^0$ lamina around the circular hole in $[0/90]_s$ laminate**

As shown in table 3, the minimum value of strength is 10.22 MPa at $0^0$ and $180^0$. It can also be noted that the Tsai – Hill and Hashin –Rotem theories are giving exactly same results while Tsai – Wu varies a little.

### 6.2 Effect of loading condition

Failure strength of a laminate subjected to different loading condition is determined. Considering Graphite/ Epoxy $[0/90]_s$ infinite plate with circular hole subjected to various loading condition like Uniaxial load in x direction, Uniaxial load in y direction, equi-biaxial load and shear load, failure strength is calculated and represented in table 4.

| **Loading condition** | **Failure Strength around circular hole (MPa)** | | |
|---|---|---|---|
| | **T – H** | **H – R** | **T – W** |
| Uniaxial in Y – Direction | 8.13 | 8.13 | 8.13 |
| Uniaxial in X – Direction | 8.13 | 8.13 | 8.13 |
| Equi-Biaxial | 10.21 | 10.21 | 10.21 |
| Shear | 12.49 | 12.49 | 12.38 |

**Table 4: Strength of Graphite/Epoxy $[0/90]_s$ laminate with circular hole for various loading condition**

From the table 4, it is seen that the strength is minimum for uni-axial load i.e. 8.13 while it is maximum i.e. 12.49 for shear load for cross ply.

For Graphite/Epoxy $[45/-45]_s$ laminate with circular hole, the maximum strength is13.86 MPa under uniaxial loading and minimum strength is 6.76 MPa under shear load.

#### 6.3 Effect of loading angle

To study the effect of loading angle on the failure strength an infinite laminated plate of Graphite/ Epoxy $[0/90]_s$ with elliptical hole is considered to be subjected to uniaxial load at an angle $\alpha$ with positive y – axis. Figure 2 shows the failure strength of laminate for different loading angle from $0^0$ to $90^0$. Maximum strength is observed when the loading is in the direction of major axes of ellipse i.e. in x direction, while the minimum strength is observed when loading is in y-direction i.e. in line of minor axes of ellipse.

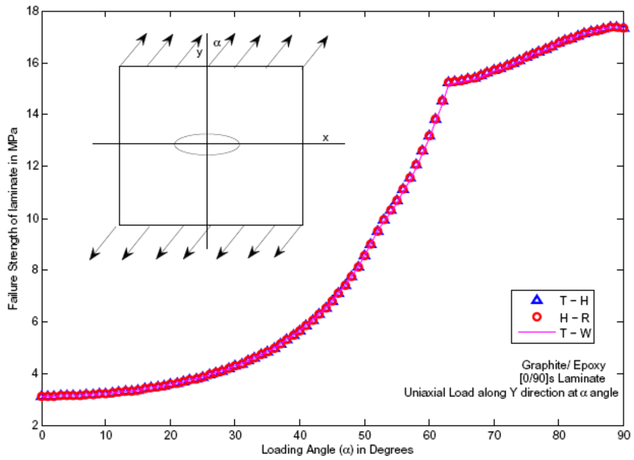


**Figure 2: Effect of loading angle on failure strength**

### 6.4 Effect of material properties

Failure strength of different material subjected to uniaxial loading condition is listed in table 5 for cross ply and angle ply. It is observed that Graphite/Epoxy $[0/90]_s$ with circular hole gives minimum strength while subjected to uniaxial load and Boron/ Epoxy $[45/\text{-}45]_s$ with circular hole gives maximum strength for uniaxial load.

| Failure Criterion | Failure Strength (MPa) | | | | | |
|---|---|---|---|---|---|---|
| | Graphite/ Epoxy | | Boron/ Epoxy | | Glass/ Epoxy | |
| | $[0/90]_s$ | $[45/\text{-}45]_s$ | $[0/90]_s$ | $[45/\text{-}45]_s$ | $[0/90]_s$ | $[45/\text{-}45]_s$ |
| T – H | 8.13 | 13.86 | 11.96 | 20.38 | 8.83 | 13.96 |
| H – R | 8.13 | 13.86 | 11.96 | 20.38 | 8.83 | 13.96 |
| T – W | 8.13 | 13.81 | 11.96 | 20.23 | 8.83 | 13.86 |

**Table 5: Effect of material properties on failure strength for uni-axial load**

### 6.5 Effect of fiber orientation angle

Table 6 shows the comparison of the failure strength evaluated by various failure criteria for the Graphite/ Epoxy lamina with square hole with rounded corners, subjected to equi-biaxial loading. The maximum strength is observed for $0^0$ and $90^0$ fiber angles while minimum strength is observed for $45^0$ fiber angle.

| Fiber angle | Failure Strength (MPa) | | |
|---|---|---|---|
| | Tsai – Hill | Hashin – Rotem | Tsai - Wu |
| $0^o$ | 6.087 | 6.0867 | 5.9316 |
| $30^o$ | 4.0692 | 4.0691 | 4.0321 |
| $45^o$ | 3.7958 | 3.7958 | 3.7958 |
| $60^o$ | 4.0692 | 4.0691 | 4.0321 |
| $90^o$ | 6.087 | 6.0867 | 5.9317 |

**Table 6: Effect of fiber angle on failure strength around square hole (Biaxial Load)**

### 6.6 Effect of hole shape

Cutout shape has significant effect on strength of laminate. Effect of various shapes of cutouts on the failure strength of laminate is described in table 7. Table 7 shows the failure strength of Graphite/Epoxy $[0/90]_s$ and $[45/\text{-}45]_s$ laminate with different shape of cut outs subjected to uniaxial loading. The minimum failure strength is observed for square hole due to high stress concentration at sharp corners while the maximum strength is observed for circular hole in $[45/-45]_s$ laminate.

| Failure Criterion | Failure Strength (MPa) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Circle | | Ellipse | | Rectangle (Side ratio:2) | | Square | |
| | $[0/90]_s$ | $[45/-45]_s$ | $[0/90]_s$ | $[45/-45]_s$ | $[0/90]_s$ | $[45/-45]_s$ | $[0/90]_s$ | $[45/-45]_s$ |
| T – H | 8.13 | 13.86 | 3.13 | 5.73 | 4.083 | 3.90 | 5.10 | 2.86 |
| H – R | 8.13 | 13.86 | 3.13 | 5.73 | 4.083 | 3.90 | 5.10 | 2.86 |
| T – W | 8.13 | 13.81 | 3.13 | 5.73 | 4.084 | 3.88 | 5.10 | 2.86 |

**Table 7: Effect of cutout shape on failure strength**

## 7. Conclusion

The present generalized solution is capable to determine the failure strength of a composite laminated plate subjected to various in-plane loading condition. Material properties, loading condition, geometry of cutout, fiber angle etc. parameters have significant effect on failure strength on the failure strength of the laminate.

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