STRESS DISTRIBUTION AROUND OVAL SHAPED HOLE IN FINITE LAMINATED PLATE SUBJECTED TO IN-PLANE LOADING

M M Chauhan[1*], D S Sharma [1],
[1] Mechanical Engineering Department, Institute of Technology, Nirma University, S G Highway, Ahmedabad, Gujarat, India 382481

ABSTRACT

In the present study, an attempt is made to find the stress distribution around oval shaped hole in finite composite plate subjected to in-plane loading. Muskhelishvili's complex variable approach along with the least square boundary collocation method is employed to obtain the solution. A conformal mapping function is used to map the oval shape in finite region onto the unit circle. Uniaxial and equi-biaxial in-plane loading are considered to obtain the stress field around the hole. Effect of size of plate, material properties, stacking sequence etc. are studied and presented. Comparison with the FEA solution is also presented for some cases.

Keywords: Boundary collocation method, Finite Plate, Laminated composite plate, Stress analysis

1. INTRODUCTION

Composite laminated plates are widely used in aero, civil and mechanical engineering applications. Various shaped cutouts are made in the laminated plates to cater the service or operational requirements. The oval shape cutouts are common in electrical panels to facilitate wiring connections. Around these cutouts high stress concentration is observed when the plate is loaded. This may lead to the failure of components. Hence, it is necessary to understand the stress distribution around the hole boundary.

Muskhelishvili [1] introduced the complex variable approach to solve some basic problems of theory of elasticity. Lekhnistkii[2] used complex variable approach to find stress distribution around hole in infinite anisotropic plate. Savin [3] used the complex variable method to find the stresses around hole in isotropic infinite plates. Ukadgaonker and Rao[4] proposed a general solution of stresses around holes in anisotropic infinite plate. Simha and Mohapatra [5] and Sharma [6] have proposed the solution of stress distribution around irregular shaped hole and polygonal holes respectively in isotropic infinite plate using Muskhelishvili's [1] complex variable method.

The problem of finding stress distribution around oval shaped hole was reported by Lekhnistkii [2] for an infinite laminated plate using complex variable approach. Sharma, Chauhan and Dave[7] also addressed the problem of stress distribution around oval hole in infinite anisotropic plate subjected to various in-plane loading.

Ogonowski [8] attempted the problem of finding stress distribution around circular hole in finite plate using boundary collocation method proposed by Newman [9]. Ukadgaonker and Murali[10], Xu et al.[11] and Madenci et al[12] have solved the problems of stress distribution around crack, circular and elliptical hole and multiple circular hole in finite plate respectively using least square boundary collocation method along with complex variable method. Boundary collocation method is less time consuming and closely approximates the closed form solutions.

In the present study, an attempt is made to obtain the stress distribution around oval shaped hole in finite region of composite plate subjected to in-plane loading using least square boundary collocation method. A computer programme is prepared to solve various cases. Some results are compared with the FEA solution.

2. ANALYTICAL FORMULATION

The normal and shear stresses in the anisotropic media can be represented in terms of Muskhelishvili's [1] complex stress functions as follows.

$$\sigma_x = 2Re[s_1{}^2\phi'(z_1) + s_2{}^2\psi'(z_2)]$$
$$\sigma_y = 2Re[\phi'(z_1) + \psi'(z_2)]$$
$$\tau_{xy} = -2Re[s_1\phi'(z_1) + s_2\psi'(z_2)] \quad (1)$$

Where , $\phi'(z_1)$ and $\psi'(z_2)$ are the first derivative of the Muskhelishvili's[1] complex stress function $\phi(z_1)$ and $\psi(z_2)$ respectively which is analytic in the region covered by finite size square plate and the centrally located hole. s_1 and s_2 are complex constants of anisotropy obtained from strain-displacement condition, Airy's stress function and generalize Hooke's law.

Stress functions for a centrally located hole in a finite size square plate can be expressed in terms of Laurent series [1] as stated in Eqn. (2).

* mihir.chauhan@nirmauni.ac.in

$$\phi(z_1) = \alpha_1 \ln z_1 + \sum_{m=1}^{\infty} A_m {z_1}^{-m} + \sum_{m=1}^{\infty} B_m {z_1}^{m}$$

$$\psi(z_2) = \alpha_2 \ln z_2 + \sum_{m=1}^{\infty} C_m {z_2}^{-m} + \sum_{m=1}^{\infty} D_m {z_2}^{m} \qquad (2)$$

where, α_1, α_2, A_m, B_m, C_m and D_m are the unknown complex coefficients of series. For the traction free hole, logarithmic terms are dropped hereon [1].

The exterior of the oval shape is mapped on to the exterior of the unit circle in ξ- plane, using following mapping function.

$$z = \omega(\xi) = \frac{R}{2}\left[(1+c)\xi + \frac{(1-c)}{\xi} + \frac{2\varepsilon}{\xi^3}\right] \qquad (3)$$

where, R is the size factor, $\xi = e^{i\theta}$, c is positive and less than unity and ε is negative and less than unity. Set of combinations of c and ε are derived to obtain the oval shape of different sizes.

To determine the values of unknown complex coefficients of series, boundary collocation method is used along with least square method.

3. BOUNDARY COLLOCATION METHOD

The boundary collocation method is one of the effective methods to arrive at the solution of finite plate problems. In boundary collocation method, numbers of collocation points on exterior and interior boundaries are selected and boundary conditions are applied on to them.

If the terms of infinite series in the stress functions are truncated to M number of finite terms, then it leads to 4M unknowns coefficients to be found. To find these 4M unknowns, at-least 4M equations are required. N_1 numbers of collocation points are selected on exterior boundary and N_2 numbers of collocation points are selected on interior boundary and force boundary conditions, as stated in Eqn.(4), are imposed on each of these collocation points. This generates $2(N_1+N_2)$ equations which are solved simultaneously to find unknown coefficients of series stress functions.

$$\pm Fx = 2Re\left(\begin{bmatrix} s_1\phi(z_1) \\ +s_2\psi(z_2) \end{bmatrix}\right) - 2Re\left(\begin{bmatrix} s_1\phi({z_1}^0) \\ +s_2\psi({z_2}^0) \end{bmatrix}\right)$$

$$\mp Fy = 2Re\left(\begin{bmatrix} \phi(z_1) \\ +\psi(z_2) \end{bmatrix}\right) - 2Re\left(\begin{bmatrix} \phi({z_1}^0) \\ +\psi({z_2}^0) \end{bmatrix}\right) \qquad (4)$$

Fx and Fy are the forces acting on the boundary. Upper and lower sign refers to the exterior and interior boundary respectively. ${z_{1,2}}^0$ is any reference point on the boundary.

Generally, N_1 and N_2 are so selected that $2(N_1+N_2) >> 4M$. This over determined system of linear equations is then solved using least square method. Once the unknown complex coefficients of series are known, the stresses can be easily derived on the contour of the hole using Eqn. (1). Coordinates transformations are used to obtain the stresses in polar coordinates on the boundary of hole.

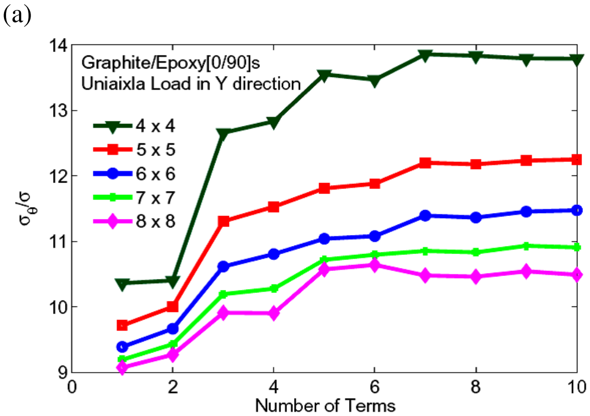
4. RESULTS AND DISCUSSION

A computer programme is prepared to solve the problem of finite laminated plate with oval shaped hole. Loading conditions, material properties and size of plate are input to the programme. For the present study 100 collocation points on hole boundary and 4*L (L = Length of plate) number of collocation points on exterior boundary are selected for boundary collocation. Here all points are equally spaced on the respective boundary.

The numerical solutions for certain problems are obtained for the Graphite/epoxy (E_1=181GPa, E_2=10.3GPa, G_{12}=7.17GPa and ν_{12}=0.28), Plywood (E_1=11.79GPa, E_2=5.89GPa, G_{12}=0.69GPa and ν_{12}=0.071) and Glass/epoxy (E_1=47.4GPa, E_2=16.2GPa, G_{12}=7.0GPa and ν_{12}=0.26). Some of the results are obtained for isotropic plate (E=207GPa, G=79.3GPa and ν=0.3) for comparison.

Numbers of terms in an infinite series of stress functions are the significant parameter to converge the solution. The convergence is affected by various parameters like material properties, stacking sequence, loading condition, size of plate, numbers of collocation points and shape of hole. Hence the convergence study is necessary for each case. Figure 1 shows the convergence curves for Graphite/Epoxy $[0/90]_s$ and Plywood plate of different sizes subjected to uniaxial load in Y-direction. It is observed that the convergence has been achieved up to 7 terms of series for both the laminates.

(a)



(b)

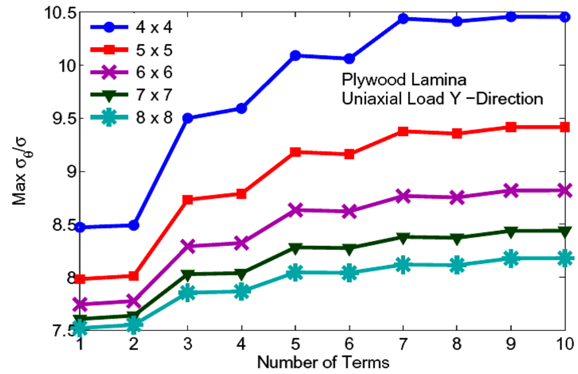


Figure 1: Effect of number of terms in series of stress function (a) Graphite/Epoxy (b) Plywood

4.1 Comparison with literature

To obtain the results of infinite plate from the present solution method, a large plate of size 100 x 100 (Length x Width) is considered with the centrally located oval shaped hole. Results obtained for the 100 x 100 size plate are compared with the results reported by Lekhnistkii [2] for the oval hole in an infinite plate subjected to uniaxial load in Y-direction. Table (1) shows the comparison of stress concentration values around oval shaped hole in various material obtained by present method with that of by Lekhnistkii [2]. The results are in a good agreement.

Table 1: Comparison of stress concentration values

	Isotropic Material		Ply wood	
	Present method	Lekhni-stkii[2]	Present method	Lekhni-stkii[2]
Max	4.722	4.44	7.213	6.39
Min	-0.915	-0.90	-1.235	-1.20

Figure 2 shows the comparison of stress distribution around oval shaped hole in 100x100 size plate of Plywood under action of uniaxial load in Y direction obtained by present method with the results published by Sharma, Chauhan and Dave [12] for infinite plate. The maximum stress concentration observed is 7.213 and 6.762 for 100x100 size plate through present method and for infinite plate from literature [12] respectively.

4.2 Finite isotropic plate subjected to uni-axial loading

Figure 3 shows the stress distribution around oval shaped hole in 10x10 size finite isotropic plate subjected to uniaxial load in Y direction. The maximum stress concentration observed is 5.0581 by present method while it is 5.096 through ANSYS.

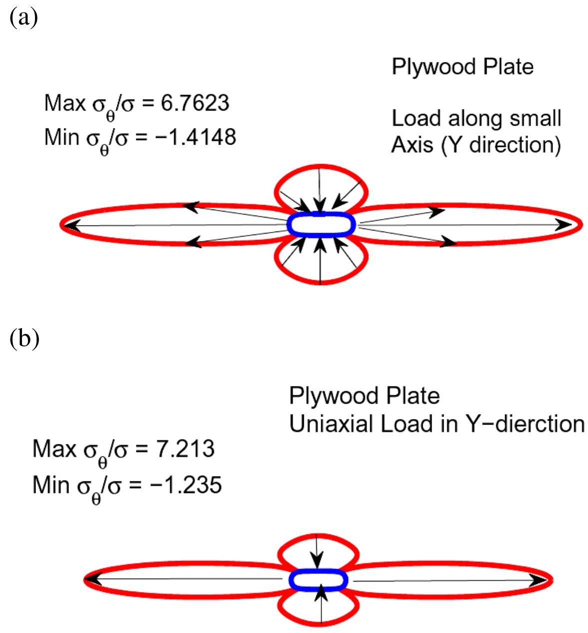


Figure 2: Comparison with literature for oval shaped hole in Plywood plate subjected to uniaxial load in Y-direction (a) Literature, (b) Present method

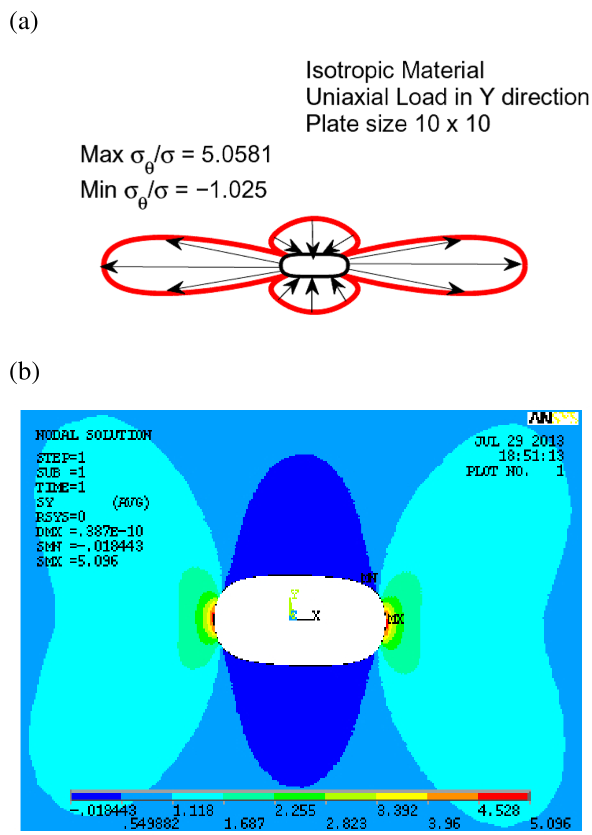


Figure 3: Stress distribution around Oval shaped hole in 10x10 size isotropic plate subjected to uniaxial load in Y-direction. (a) Present (b) ANSYS

4.3 Finite anisotropic plate subjected to uniaxial load

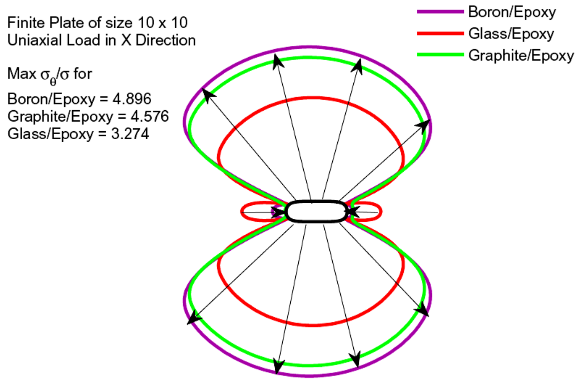


Figure 4: Stress distribution in finite anisotropic plate subjected to uniaxial load in x-direction

Figure 4 shows the stress distribution around oval shaped hole in 10 x 10 size finite plates of Graphite/Epoxy, Boron/Epoxy and Glass/Epoxy subjected to uniaxial load in X direction. The maximum stress concentration is 4.896, 4.576 and 3.274 in Boron/Epoxy, Graphite/Epoxy and Glass/Epoxy respectively.

4.4 Finite plate subjected to biaxial loading

Figure 5 shows the stress distribution around oval hole in 10 x 10 finite plate of different material. The maximum stress concentration is 8.154, 4.473 and 4.611 in Plywood, Glass/Epoxy and Isotropic material respectively.

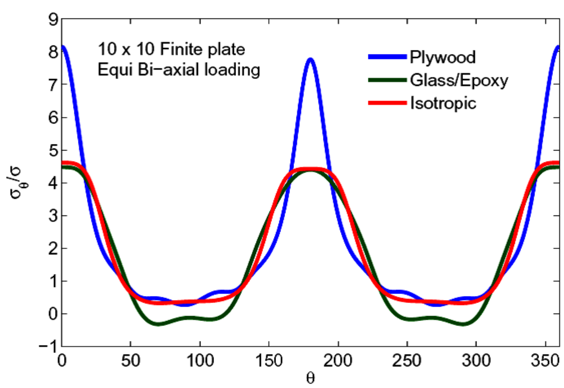


Figure 5: Stress distribution around oval hole in 10 x 10 size finite plate subjected to equi-biaxial loading

4.5 Effect of plate size

Figure 6 shows the effect of plate size on the stress distribution around oval shaped hole in a Graphite/Epoxy $[0/90]_s$ plate subjected to uniaxial load in Y-direction. Due to symmetry, only quarter part is shown in figure. It is obvious that stress concentration becomes more severe as the plate size decreases. As the plate size increases the stress concentration is approaching the value of infinite plate problem.

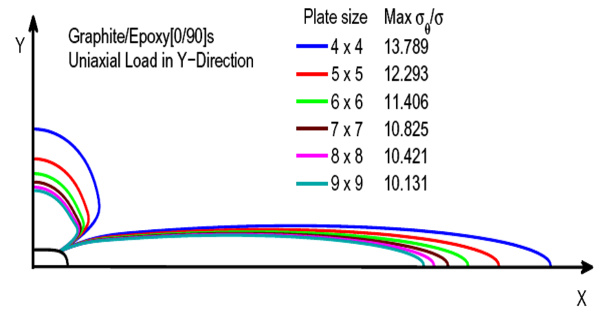


Figure 6: Effect of plate size on the stress distribution around oval shape hole in finite Graphite/Epoxy $[0/90]_s$

5. CONCLUSION

The presented solution can be used to find the stress distribution around centrally located oval shaped hole in isotropic/anisotropic finite plate subjected to in-plane loading. The present solution can also be useful to study the effect of various parameters on the stress concentration around hole. Different sizes of oval shape can also be generated by changing the values of constants c and ε in the mapping function.

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