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STRESSES AROUND ELLIPTICAL/OVAL SHAPED HOLE IN FINITE LAMINATED PLATE

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***Abstract*--In this paper, a general solution for stresses around elliptical and oval shaped hole in finite laminated plate is presented. The plate is subjected to in-plane loading and stress functions are obtained using Muskhelishvili's complex variable approach. The complex stress functions are represented in terms of Laurent series and constants of the series are derived using boundary collocation method. Effect of plate size, hole geometry, material properties, hole location and loading condition on stress concentration is presented. The results obtained through present method are compared with finite element solution and with the literature.**

I. INTRODUCTION

The laminated composite plates are widely used in many engineering applications nowadays. Different shapes of cut-outs are purposefully made in the plate for the necessity of service. Under the loading condition these cut-outs exhibit high stress concentration. Many researchers like Savin [1], Lekhnitskii [2], Ukadgaonker & Rao [3], Sharma [4]-[6], Patel and Sharma [7] etc. have attempted the problem of finding stress concentration around different shapes of holes in isotropic/ anisotropic plate subjected to various loading conditions using Muskhelishvili's [8] complex variable approach. All these solutions are applicable to infinite plates only.

For the finite plate, Ogonowski [9], Madenci et al [10], Xu et al [11], Pan et al [12], Chauhan and Sharma [13] and few more have presented the solutions for stresses around circular, elliptical or rectangular hole.

The attempt is made here to present the generalized solution for stress concentration around elliptical/ oval shaped hole in finite anisotropic plate subjected to in-plane loading. The complex variable approach in conjunction with boundary collocation method is used to derive the stress functions. The influence of plate size, material properties, fiber orientation, hole geometry, hole location and loading angle on stress concentration is also presented. The results obtained through present method are compared with finite element solution and the literature.

II. ANALYTICAL FORMULATION

For an anisotropic thin plate, the stress components are represented in terms of complex stress functions as,

$$\sigma_x = 2\,\mathrm{Re}\left[\sum_{j=1}^{2}\mu_j^2\phi_j'(z_j)\right]$$
$$\sigma_y = 2\,\mathrm{Re}\left[\sum_{j=1}^{2}\phi_j'(z_j)\right] \tag{1}$$
$$\tau_{xy} = 2\,\mathrm{Re}\left[\sum_{j=1}^{2}\mu_j\phi_j'(z_j)\right] \qquad (j=1,2),$$

where σ_x and σ_y are normal stresses in x and y direction, τ_{xy} is the shear stress in xy plane, $\phi_j(z_j)$ are complex stress functions [8], $\phi_j'(z_j)$ is the first derivatives of complex stress function, $z_j = x + \mu_j y$ define the point on z_j plane, μ_j are constants of anisotropy. Constants of anisotropy (μ_j) are the roots of characteristics equation obtained by applying generalized Hooke's law, compatibility conditions and Airy's stress function to anisotropic plate [2].

A. *Mapping Function*

Elliptical or Oval shaped hole in z-plane is mapped conformally on to the unit circle in ζ-plane by a mapping function,

$$z = w(\zeta) = \left(C\zeta + \sum_k \frac{m_k}{\zeta^k}\right) \tag{2}$$

where $\zeta = e^{i\theta}$, C and m_k are the constants that define the different geometry of the hole as shown in Table 1.

TABLE I. CONSTANTS OF MAPPING FUNCTION

Hole	Constants
Ellipse	$C=1$; $k=1$; $m_1 = \frac{a-b}{a+b}e^{2i\alpha}$
Oval	$C=1-c$; $k=1,3$; $m_1=(1-c)e^{2i\alpha}$; $m_3 = 2\varepsilon e^{4i\alpha}$

where a and b are semi major and minor axis of ellipse, $c=-9\varepsilon$ and ε is the constant. Here α is the hole orientation angle.

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Introducing the constants of anisotropy, (2) takes the form,

$$z_j = a_j\left(C\zeta_j + \sum_k \frac{m_k}{\zeta_j^k}\right) + b_j\left(\frac{C}{\zeta_j} + \sum_k m_k\zeta_j^k\right) \tag{3}$$

where $a_j = 1 + i\mu_j$ and $b_j = 1 - i\mu_j$. The mapping function (3) map the elliptical/ oval shape hole on to the unit circular hole around which the analytic complex function is easy to obtain.

B. Stress function

The complex stress functions for the finite plate with hole can be represented in terms of Laurent series [9] as,

$$\phi_j(\zeta_j) = \sum_{n=1}^{N}\left(A_{jn}\zeta_j^{\,n} + B_{jn}\zeta_j^{\,-n}\right) \tag{4}$$

where A_{jn} and B_{jn} are the constants of series, ζ_j are the mapped coordinates of corresponding z_j point obtained by solving (3). The constants of the series stress functions are obtained from the boundary conditions.

C. Boundary condition

A finite anisotropic plate with hole is subjected to in-plane loading on the boundary of the plate and the hole is considered traction free. The forces on the boundary of the plate can be represented in terms of complex stress function [10] as,

$$\begin{aligned} \pm F_x &= 2\,\mathrm{Re}\left[\mu_j\left(\phi_j(\zeta_j) - \phi_j(\zeta_j^0)\right)\right] \\ \mp F_y &= 2\,\mathrm{Re}\left[\left(\phi_j(\zeta_j) - \phi_j(\zeta_j^0)\right)\right] \end{aligned} \tag{5}$$

where ζ_j are the mapped coordinate obtained by solving (3), $F_x = \sigma_{nx}(y - y_0)$ and $F_y = \sigma_{ny}(x - x_0)$ here x_0 and y_0 are the coordinates of reference point while x and y are the coordinates of collocation point at which the forces are evaluated. Upper and lower signs in (5) correspond to the outer and inner boundary respectively. σ_{nx} and σ_{ny} are the applied loading in X and Y direction respectively and can be obtained as,

$$\begin{aligned} \sigma_{nx} &= \frac{\sigma}{2}\left[(\lambda+1) + (\lambda-1)\cos 2\beta\right] \\ \sigma_{ny} &= \frac{\sigma}{2}\left[(\lambda+1) - (\lambda-1)\sin 2\beta\right] \end{aligned} \tag{6}$$

where σ is applied load per unit length, λ is loading factor can be set 0 or 1 for uniaxial and equi-biaxial loading respectively and β is loading angle.

III. Solution Methodology

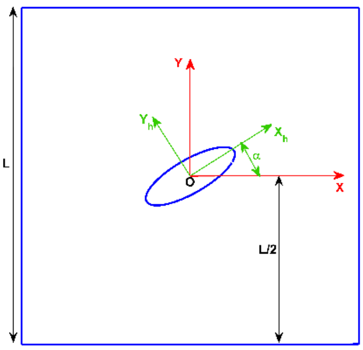
A boundary collocation method is employed in the present solution to derive the constants of the series stress function. The solution begins with the generation of P_1 and P_2 number of collocation points on the boundary of finite plate and hole respectively in z-plane. Using mapping function (3), the corresponding mapped coordinates ζ_j are obtained for each collocation points. Substituting ζ_j and (4) into (5), total 2P ($P=P_1+P_2$) number of boundary equations are obtained in terms of unknown constants (A_{jn}, B_{jn}) of series stress functions. If the infinite series of the stress functions (4) are truncated to a finite number N, total 4N number of unknowns are required to be obtained from 2P boundary equations. In general the large number of collocations points are selected to define the boundary accurately and that results in the overdetermined system of equations (i.e. 2P>>4N). The overdetermined system of equations is solved using least square method. Once the unknowns of the series are known, the stress components can be derived using (1). The stress transformation is used to derive the stress components in polar coordinates.

IV. Results and Discussion

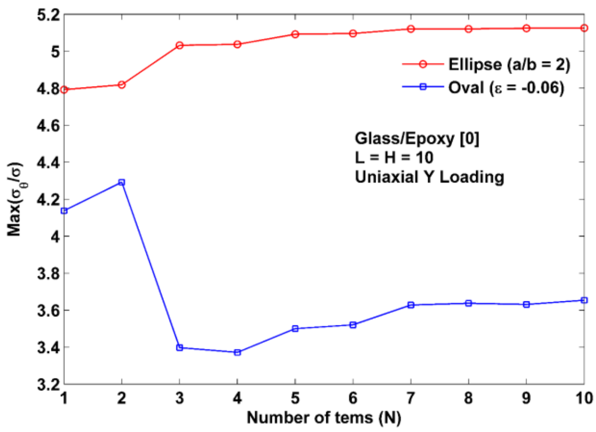
A computer programme is prepared based on the analytical solution presented in previous section. The loading condition, material properties, hole geometry and plate size are input to the programme. Fig. 1(a) shows the geometrical parameters of plate and hole. An iterative process of increasing the number of terms in stress functions is employed to achieve the convergence (Refer Fig. 1(b)). The materials considered here are as shown in Table 2.

TABLE II. MATERIAL PROPERTIES

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}
Graphite/Epoxy	182	10.3	7.17	0.28
Glass/Epoxy	47.4	16.2	7	0.26
Plywood	11.79	5.89	0.69	0.071
Boron/Epoxy	282.77	23.79	10.35	0.27
Isotropic steel	E = 205		G = 80	ν =0.26



(a)



(b)

Figure 1. (a) Geometry of the plate with hole (b) Convergence

Table 3 shows the comparison of stress concentration factors obtained through present method for large dimension of plate (L = 100) with that of results of infinite plate available in the literature for uniaxial Y loading. The results are in good agreement.

TABLE III. COMPARISON WITH LITERATURE (MAX(σ_θ / σ))

Hole geometry	Material	Present method	Literature
Ellipse (a/b = 2)	Glass/Epoxy	4.641 (L = 100)	4.70 [6]
Oval (ε = -0.04)	Plywood	6.812 (L = 100)	6.39 [2]
Oval (ε =-0.04)	Isotropic	4.490 (L = 100)	4.44 [2]

Fig. 2 shows the comparison of the stress distribution around elliptical and oval shaped hole in finite plate subjected to unit uniaxial Y loading obtained through present method with that of finite element solution through ANSYS. The maximum stress around elliptical hole with a/b = 2 in Glass/Epoxy plate (L = 8) is 5.408 by present method and 5.566 by ANSYS. For the oval shape (ε = -0.03) hole in Plywood plate (L = 10), maximum stress is 8.424 and 8.451 by present method and ANSYS respectively. The results obtained by present method are closely agree with that of finite element solution.

Fig. 3 shows the effect of plate size on the stress concentration around centrally located elliptical or oval hole in different materials. It is observed that as the plate size increases, the stress concentration decreases and it approaches the values of infinite plate. It can be evident from Fig. 3 that the present method is capable to produce the satisfactory results of infinite plate also by considering large dimension of the plate.

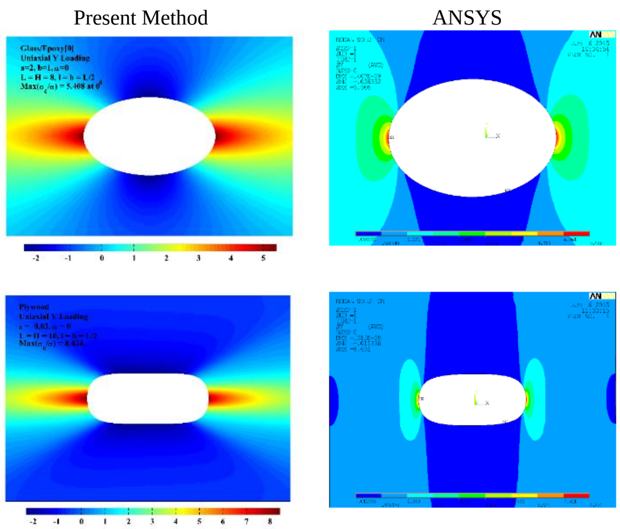
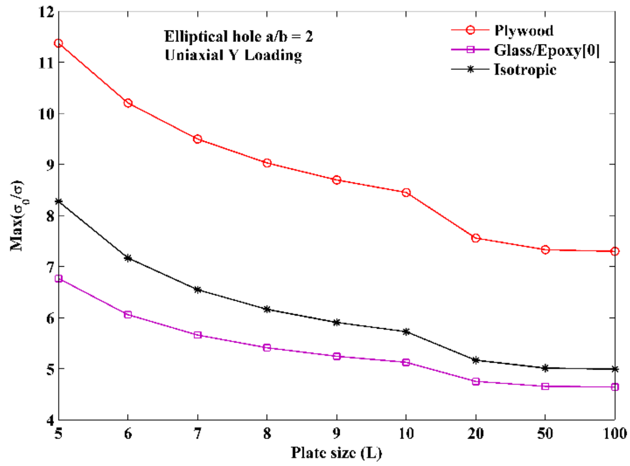


Figure 2. Comparison with finite element solution



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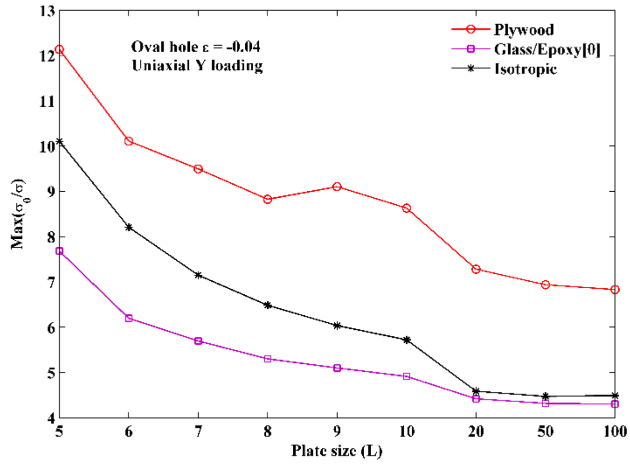
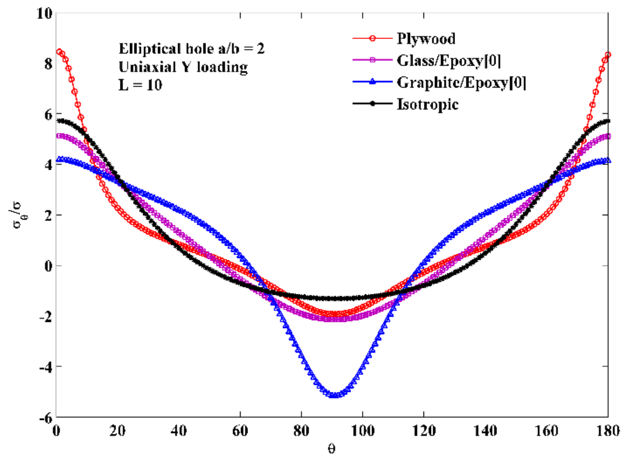


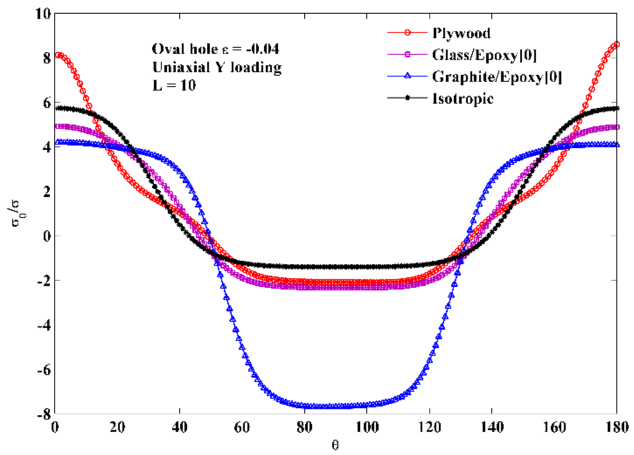
(b)

Figure 3. Effect of plate size (a) Elliptical hole (a/b = 2), (b) Oval hole (ε =-0.04)

Fig. 4 shows the stress distribution around elliptical hole (a/b = 2) and oval shaped hole (ε = -0.04) in finite plate (L = 10) of different materials subjected to uniaxial Y loading. The highest stress concentration is observed for Plywood plate while the Graphite/Epoxy plate shows high compressive stress due to their different material properties and anisotropy.



(a)



(b)

Figure 4. Effect of material properties (a) Elliptical hole (a/b = 2), (b) Oval hole (ε =-0.04)

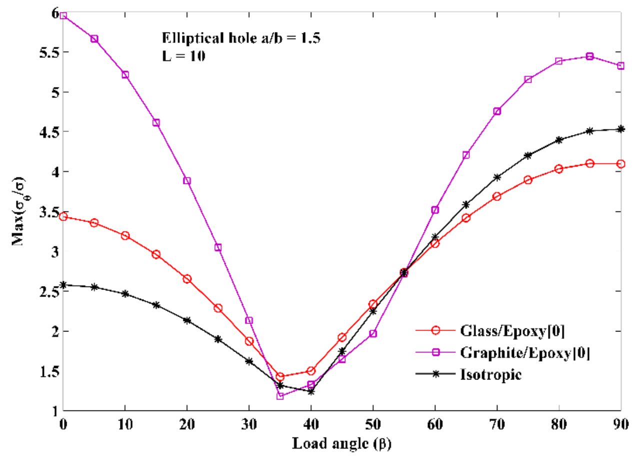
The effect of different loading conditions can be observed from Table 4. The maximum stress concentration is observed for uniaxial Y loading while the minimum is observed for uniaxial X loading. Table 4 also shows the effect of stacking sequence on stress concentration around hole. The ply with [90] shows the high stress concentration. Addition of 0^0 ply and 45^0 ply decreases the stress concentration for uniaxial Y loading.

TABLE IV. STRESS CONCENTRATION AROUND HOLE IN GLASS/EPOXY FINITE PLATE (L = 10)

	Ellipse (a/b = 1.5)			**Oval (ε = -0.03)**		
	Uni-X	***Uni-Y***	***Equi-bi***	***Uni-X***	***Uni-Y***	***Equi-bi***
[0]	3.342	4.096	3.351	2.162	6.323	5.630
[90]	2.380	6.749	4.489	2.189	11.511	9.326
$[0/90]_s$	2.931	5.439	4.128	1.857	8.707	7.506
$[0/45/90]_s$	2.933	4.990	5.434	3.265	7.248	8.319

The effect of load angle on stress concentration is shown in Fig. 5 for elliptical and oval shaped hole in Glass/Epoxy[0], Graphite/Epoxy[0] and Isotropic finite plate (L = 10). The minimum stress concentration is observed for load angle ranges from 30^0 to 40^0 for different materials.

The geometry of the hole has significant effect on stress concentration around hole as shown in Fig. 6. For the elliptical hole the ratio of semi major axis to semi minor axis (a/b) is varied and the maximum stress concentration around hole is plotted against ratio ((a-b)/(a+b)) as shown in Fig. 6(a). For ((a-b)/(a+b)) = 0, the hole is circular and as the ratio ((a-b)/(a+b)) approaches 1, the hole will become crack and shows very high stress concentration. For the oval hole, the parameter ε has small negative values but as its value increases and approaches zero, the hole will become crack and exhibit very high stress concentration as shown in Fig. 6(b).



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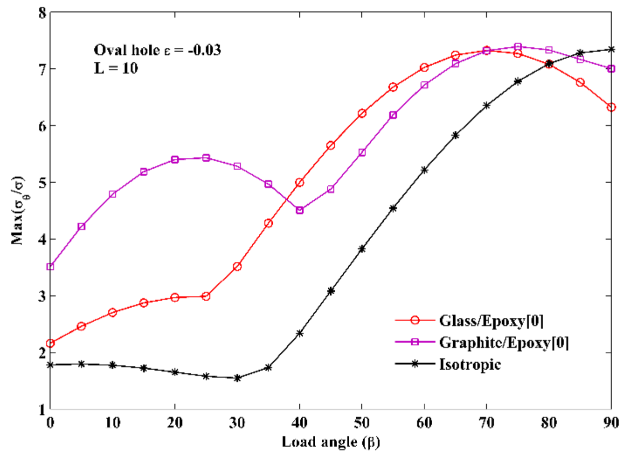

(b)

Figure 5. Effect of load angle (a) Elliptical hole, (b) Oval hole

By varying the angle α , the hole can be orientated at different angle. The effect of hole orientation on stress concentration is shown in Fig. 7 for elliptical and oval hole in Isotropic plate (L = 10) subjected to uniaxial X loading. The maximum stress concentration is observed for 90^0 orientation.

V. CONCLUSION

A generalized solution to derive the stress distribution around elliptical/ oval shaped hole in finite anisotropic plate is carried out using complex variable approach in conjunction with boundary collocation method. The results are closely agree with literature and finite element solution. The present method is time efficient and capable to accommodate different geometries of plate boundary. The solution is capable to produce the results of infinite plate also by considering large plate dimension. The results of isotropic materials can also be obtained by selecting suitable material properties. The present method is suitable for parametric study also. The stress distribution around the hole is significantly influenced by plate size, material properties, stacking sequence, loading condition, hole geometry and hole orientation.

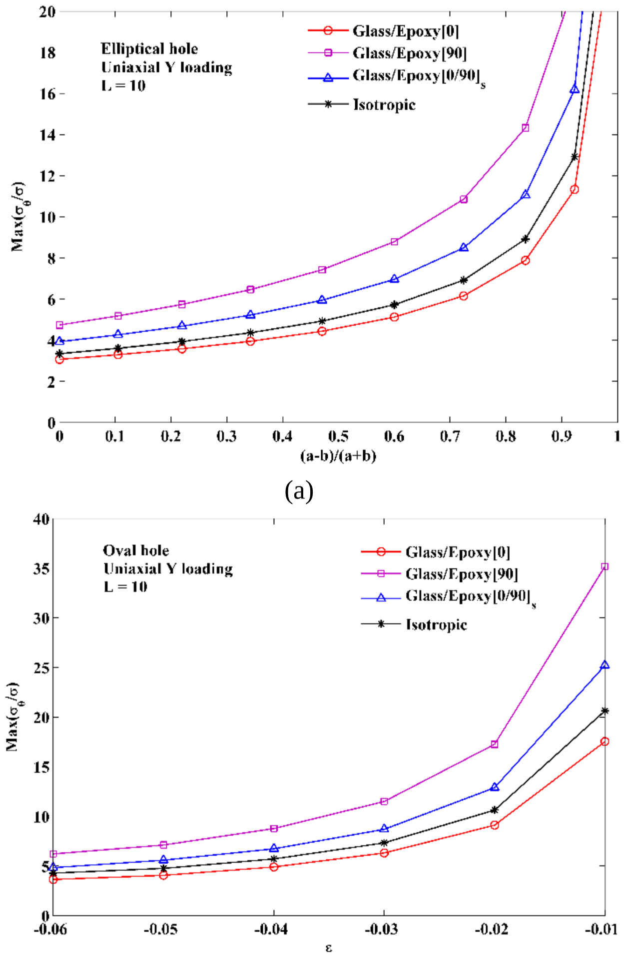

Figure 6. Effect of hole geometry (a) Elliptical hole (b) Oval hole

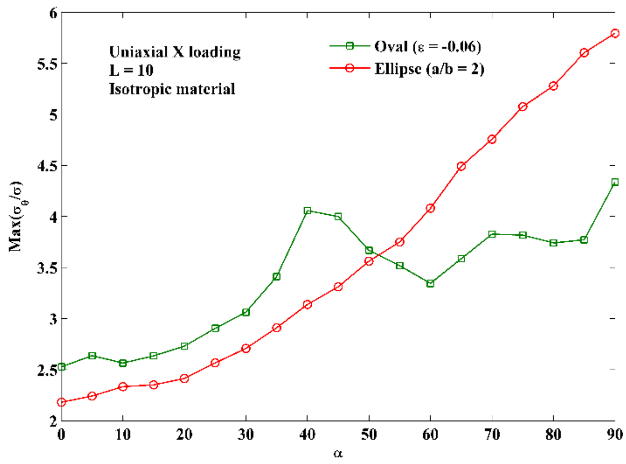

Figure 7. Effect of hole orientation

References

[1] Savin GN, Stress Concentration around Holes, Pergamon Press, New York, 1961.

[2] Lekhnitskii SG, Anisotropic Plates, Gordon and Breach Science Publisher, New York, 1968.

[3] Ukadgaonker VG, and Rao DKN, “A general solution for stress distribution around holes in symmetric laminates under in-plane loading”, Composite Structures, vol. 49, pp. 339-354, 2000.

[4] Sharma DS, “Stress distribution around polygonal holes”, Int. J. Mech. Sci., vol. 65, pp. 115-124, 2012.

[5] Sharma DS, “Moment distribution around polygonal holes in infinite plate”, Int. J. Mech. Sci., vol. 78, pp. 177-182, 2014

[6] Sharma DS, “Stress distribution around circular/ elliptical/ triangular holes in infinite composite plate”, Engineering Letters, vol. 20, pp. 1-9, 2012.

[7] Patel NP, and Sharma DS, “Bending of composite plate weakened by square hole”, Int. J. Mech. Sci., vol. 94-95, pp. 131-139, 2015.

[8] Muskhelishvili NI, Some Basic Problem of Mathematical Theory of Elasticity, P. Noordhoof Ltd, The Netherlands, 1962.

[9] Ogonowski JM, “Analytical study of finite geometry plate with stress concentration”, 21st AIAA/ASME/ASCE/AHS SDM Conference, 1980.

[10] Madenci E, Ileri L, and Kudva JN, “Analysis of finite composite laminates with holes”, Int. J. Solids Structures, vol. 30(6), pp. 825-834, 1993.

[11] Xu X, Sun L, and Fan X, “Stress concentration of finite composite laminates weakened by multiple elliptical holes”, Int. J. Solids Structures, vol. 32(20), pp. 3001-3014, 1995.

[12] Pan Z, Cheng Y, and Liu J, “Stress analysis of a finite plate with a rectangular hole subjected to uniaxial tension using modified stress functions”, Int. J. Mech. Sci., vol. 75, pp. 265-277, 2013.

[13] Chauhan MM, and Sharma DS, “Stresses in finite anisotropic plate weakened by rectangular hole”, Int. J. Mech. Sci., vol. 101-102, pp. 272-279, 2015.