FE Based Eccentricity Analysis of Rogowski Coil

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***Abstract*—The Rogowski Coil (RC) has air as core around which coil is wound. It works on the principle of Faraday's and Ampere's law. RC can be used to measure both high speed impulse and alternating currents. RC offers some very important properties like wide bandwidth, galvanic isolation, linearity, lightness and cheap. Presence of all these properties makes RC better than conventional current transducers. It also offers many other applications viz., energy management, protection systems, CT calibration, current sharing, resistance, welding process, measurement of partial discharges and earth resistance of transmission towers. It can measure currents ranging from mili-amperes (mA) to mega-amperes (MA) without any saturation. The construction and measurement conditions decide the application of RC.**

The eccentricity of coil is the main parameter considered for modelling of RC. The eccentricity effect becomes prominent when the conductor gets displaced away from the centre due to the deviation from circular shape. Change in conductor position affects the measurements done, so this analysis ensures the accuracy of RC. In this paper, eccentricity analysis of RC is done using ANSYS software tool. The observed deviation of mutual inductance during the analysis can be justified with the theoretical existing data.

Keywords: Current Transformers, Distributed Parameters, Rogowski coil, FE analysis, Eccentricity

I. INTRODUCTION

A Rogowski Coil (RC) is a current measuring device. It has air as core. In RC, a toroid is formed out of a conducting wire; the toroid is laid such that both the terminals of wire are at the same end in order to fulfil the conditions required for application of Ampere's law. This assembly surrounds the conductor in order to measure the alternating current easily. The rigidity of winding and the coil diameter decides high degree of immunity and the low sensitivity against external fields, thus, the voltage induced. This induced voltage is in proportion to the derivative of current in the conductor. At output, an integrator circuit is placed in order to obtain a smooth sinusoidal without distortion and oscillations.

Due to recent developments of microprocessor-based measurements and implementation of electronic devices in the circuits, lower current measurement has been made possible. Integrator circuits were not required in traditional split-core current transformers, due to which measurement of very slow changing currents became possible. These currents have frequency components as low as 1Hz.

RC can be used for current monitoring. Which is very useful in case of electromagnetic launchers, in precision welding systems or arc melting furnaces. Also, it is used for short circuit testing of generators as well as, in sensors for protection systems in industrial plants. High linearity of RCs makes it useful in measurement of harmonic current. Some of the advantages of RC are as follows:

- To keep output current constant, secondary turns need to be increased in conventional transformers. Although, the RC used is much smaller for equivalent rating.
- In RC there is no worry of secondary winding opening.
- RC construction is cheap.
- Even large currents does not affect the linearity, as it does not have any iron core to saturate.
- RC offers excellent transient response.
- It has a large bandwidth, that ranges from 1Hz to 1 GHz
- It is isolated from main circuit and hence ensure its own safety in case of any short circuit.

The paper consists of six sections. Section II and III, describes the basics of RC and integrator circuit. Model parameters and various models are discussed in section IV. After that, its simulation and results are given in section V. Conclusion is provided in Section VI.

II. BASICS OF ROGOWSKI COIL

A simple RC comprises of toroid of windings, as shown in Fig 1, encircling the current path. The relationship between the magnetic field induced and the current flowing through the RC is given by Ampere's law [1]:

$$I(t) = \frac{1}{\mu_0} \oint \vec{B}(t).\vec{dS} \qquad (1)$$

where, S signifies the distance along the coil and the variations in magnetic field B leads to voltage induction in the windings.

Given that, cross-section for the windings and number of turns per unit length of the conductor are constant, Faraday's law can be used to get an unique relationship between induced voltage u(t) across terminals of the RC and current flowing is I(t). It can be given as follows [1]:

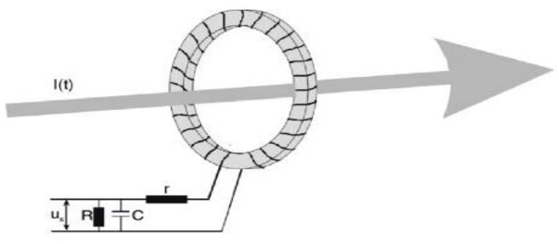

Fig. 1: A Simple RC with R-C Integrator [1]

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$$u(t) = \frac{d\emptyset}{dt} = \int \vec{B}(t).\vec{d}A = \frac{A}{s}\mu_0 I(\dot{t}) \qquad (2)$$

where, windings cross-section is given by A and s is the number of turns per unit length.

Eqn. 2 does not depend on current distribution. Placement of an integrator block is obligatory at the output end of the coil in order to derive current value from the voltage induced u (t). A simple integrator made up of R-C is as shown in Fig 1. Another simple R-C circuit which is designed as an integrator is shown in Fig 2. In the figure, L represents each winding's self-inductance and resistance and capacitance of the integrator circuit is given by R and C. If the current flowing in the circuit is i, induced voltage can be written as follows [1]:

$$u(t) = \frac{d\emptyset}{dt} = L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t i dt' \qquad (3)$$

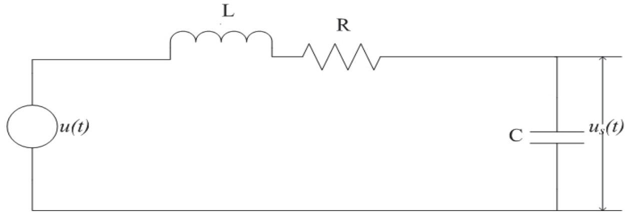

Fig. 2: Equivalent Circuit Diagram of a Single Turn Winding with R-C Integrator [1]

In the Fourier transform spectrum of current, *considering ω as the highest frequency, keeping value of Lω << R, Eqn. 3 can be rewritten as [1]:*

$$u(t) = \frac{d\emptyset}{dt} = Ri + \frac{1}{C}\int_0^t i dt' \qquad (4)$$

If time of measurement t, is significantly less than RC value, then us (t) can be given by:

$$u_s(t) = \frac{1}{C}\int_0^t i dt' = \frac{1}{RC}\int_0^t u(t')dt' = \frac{A\mu_0}{sRC} I(t) = \mu_0 \frac{NA}{S_m}\frac{1}{RC} I(t) \qquad (5)$$

Here, N represents no. of windings, A represents c/s of the coil, Sm represents mean length of the coil.

Here, Lω << R condition should be satisfied in the Eqn. 4 and 5. But, in case of presence of high frequency components in current wave spectrum of the pulse currents to be measured, above condition is not fulfilled. In such condition, if the coil resistance r, and external *resistor R, is less than Lω, and also R << 1/ωC, then Eqn.* 3 can be rewritten as follows [1]:

$$\frac{d\emptyset}{dt} \approx L\frac{di}{dt} \Rightarrow \frac{\emptyset}{L} = \frac{\frac{\mu_0 NAI}{s}}{\frac{\mu_0 N^2 AI}{s}} = \frac{I}{N}; \; u_R(t) = iR = \frac{R}{N} I \qquad (6)$$

It is quite clear from these equations that to find *lowest measureable frequency, r + R << Lω condition is* suitable, whereas, for highest measureable frequency LC resonant frequency of coil is responsible. Thus, for high frequency measurements, the integrator in RC is an intrinsic one. Other problem is caused due to capacitive coupling between the casing of coil and the coil windings, which results in an error at the output. Hence, it is omitted so as to increase the accuracy. This coupling effect is a prominent cause of error in case of high frequency measurement. So that, the coil acts as a x'mission line and the induced voltage of several points reach the coil terminal with delay time. Therefore, in the output of the coil, improper excitation of an individual winding can cause a remarkable oscillation. Although, the output has direct relation with the current distribution in the coil.

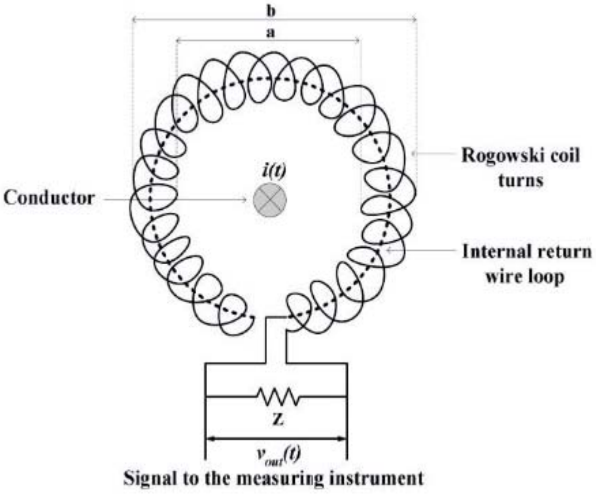

Fig. 3: Structure of RC [6]

Even though the RC has a simple structure, some special considerations are to be taken care of. Like problem is created due to huge magnetic flux between two terminals of the coil. To increase the exemption of the coil from stray fields and to decrease the flux, the RC have to consider the two interconnected electric paths, which are in opposite direction. When both of the paths are considered in the direction as the winding is laid, then in order to increase the output of the coil, they should be wound reversely. When one of the path is along with winding and the other is along a simple wire, the second wire can be reversed through the first wound path, which can be separated from one end for measuring, where the primary path cannot be opened. This configuration is shown in Fig. 3.

III. INTEGRATOR CIRCUIT FOR ROGOWSKI COIL

An integrator circuit is required in normal operation mode, at the output of RC. Noise reduction is one of the advantage of the integrator, because of low-pass filter characteristic [4]. Various techniques are available for implementation of the integrator. Integrator circuit is designed by using operational amplifier. As mentioned, for high frequencies (>100MHz) RC-base integrator is appropriate, while, for low frequency (< 100MHz) measurements, Op Amp-based integration method is useful [9].

The basic Op Amp integration circuit is shown in Fig 4. Ignoring the R1, the integrator gain would become (1+G) R2 C, where the open-loop gain of Op Amp is given by G. Considering such case, saturation condition is seen at output if there is any noise in the input. Addition of R1 to the basic circuit, decreases the integrator DC gain down to R1/R2. For integration of low frequencies, mid-frequencies and high frequencies, an Op Amp circuit, an RC circuit and coil in self-integration mode are used respectively. So, the proposed scheme supports a wider frequency bandwidth [2].

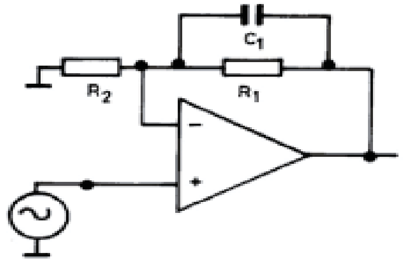

Fig. 4: Basic OP-AMP Integrator Circuit [1]

IV. ROGOWSKI COIL MODELLING

Modelling of RC can be done by three possible models. These can be listed as:

- Lumped Model.
- Distributed Model.
- Numerical Model.

Depending on the application of the RC, one of these model is selected for analysis purpose.

All these models for analysis of RC are discussed here. Once the comparison of these models is done, one of the model is proposed for the eccentricity analysis of RC.

A. Lumped Model

The lumped model consists of a resistor, an inductor, and a capacitor as shown in Fig 5. Lumped model of RC is acceptable for low frequency analysis only [1].

The parameters of this model are computable as follows [1]:

$$R_l = \rho_C \frac{l_w}{\pi d^2} \tag{7}$$

$$L_l = \frac{\mu_0 N^2 d_{rc}}{2\pi} \log\frac{b}{a} \tag{8}$$

$$C_l = \frac{4\pi^2 \varepsilon_0 (b+a)}{\log\left(\frac{b+a}{b-a}\right)} \tag{9}$$

In the above equations, length of coil wire is given by *lω,, N is no. of turns, ρc is electrical* resistivity of the coil wire, radius of this wire is given by d, drc is the diameter of each loop in the coil,, and a, b are the internal and external radius of the coil respectively.

Z is computed as an external impedance in the coil, which includes the damping resistor, capacitance of coupling cable and measurement instrument impedance.

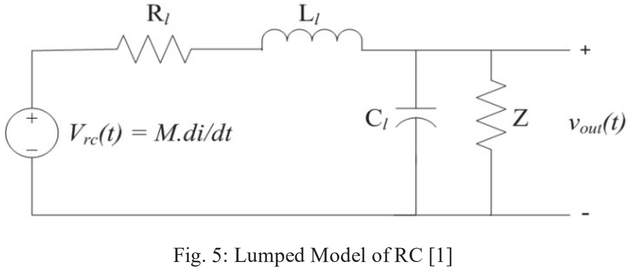

Fig. 5: Lumped Model of RC [1]

These equations are considered in ideal mode, but, while constructing coil, there are some deficiencies; e.g. turns are not uniformly distributed along the coil, the return path is not along the proper winding axis, the cross-section area is non uniform, and because of bending, it turns into an ellipse. Since, practically all of these imperfections exist, the considered formulas does not give accuracy as expected. Exact parameter determination is very important, for high-frequency applications, and hence, the bandwidth selection and choice of the damping resistor. In addition, issues like skin effect and proximity effect causes the volatile distribution of current in the coil. To consider these, some parasitic inductors and capacitors have to be considered in the model. All these parameters cannot be defined using the computational methods accurately. Hence, once the coil construction is done, it is advised to measure these parameters.

B. Distributed Model

The lumped model is not applicable for high-frequency measurements, so distributed model is used to replace it. The distributed model used for the RC modelling is based on transmission line model theory. It is as shown in Fig. 7.

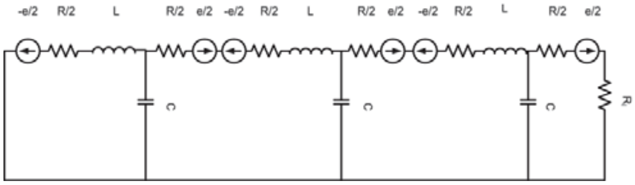

Fig. 7: Distributed Model of RC [9]

This model is used for following purposes [7]:

- To reduce the negative impact of reflection by decreasing the stray capacitances.
- Coil parameters optimization, particularly the shunt capacitance in order to reach self-integration mode.
- To find the termination resistance effect.

The electrical distance is considered to be made up of n number of small divisions, in distributed model of RC. Every division has its own series inductance L, given by L = L1/n, a stray capacitance, given by a shunt capacitor C = C1/n, a resistance, given by resistor R = R1/n, and a voltage source e, where e = ut/n representing the incident and reflected travelling wave.

For measurement of currents with severe variations, most important factors include, the coil sensitivity, the travelling time of the wave, front time of the wave and pulse width of the current.

The resistance of termination resistor should be kept as low as possible in order to achieve self-integrating mode. Theoretically, for self-integrating mode the best possible state is short circuit condition. But, this is practically not possible. Inter-turn capacitances and shunt capacitances should be kept minimum to obtain the best transient response. The shunt capacitor is the stray capacitance present between the primary conductor and the coil windings, whereas, the inter-turn capacitance is the capacitance between the coil loops. These capacitances can be decreased by various techniques. One of the methods involves putting a shield around the RC, in order to reduce the shunt capacitors [3].

For a coil wound around a non-magnetic former and with a cross-sectional area A, Ampere's law can be given by:

$$i(t) = \oint \vec{H}.\overrightarrow{dl} \tag{10}$$

According to Faraday's law, in a differential length dl, the induced electromotive force (EMF) edl is given by [9]:

$$e_{dl} = -\frac{d\psi_m}{dt} = -\frac{d}{dt}\left(\vec{B}\vec{ds}\right) = -\mu_0 A \frac{dH}{dt}\cos\theta \tag{11}$$

Magnetic flux density is considered constant. In the equation, the magnetic flux density, magnetic field and permeability of free space is given by B, H and μ respectively. Coil is positioned perpendicular to the *current carrying conductor (i.e., θ = 0). So, if N' turns are* considered for dl along l length, then,

$$e_t = \int_0^l e_{dl} N' dl = -\mu_0 A N' \frac{d}{dt}\int_0^t H\cos\theta\, dl = -\mu_0 A N' \frac{di(t)}{dt} = -M\frac{di(t)}{dt} \tag{12}$$

Here, the mutual inductance, M can be obtained by dividing et, the EMF induced in the coil and i(t), the difference occurred in the primary current, that is to be measured. Induced voltage for a coil wound over a circular nonmagnetic former can be given by equation 11. *Where, the area of former is given by, A = πr2, with r as* minor radius, N gives the number of turns (where N' = *N/2πR0), and R0 is the mean* major radius.[6]

$$e_t = -\frac{\mu_0 N r^2}{2R_0}\frac{di(t)}{dt} \tag{13}$$

Usually the design of RC sensor is given according to the integral parameter model. But, when very high frequency is considered, distributed parameter model is to be used. Because the coil has distributed inductance and capacitance, the electromagnetic wave in the coil has slower speed than in air. Then, the limit frequency considered in distributed parameter model is much lower than the limit frequency calculated for the mechanical size of the coil. A distributed parameter model is shown in Fig. 8.

C. Numerical Model

The RC is a linear mutual inductor, which is linked with the magnetic field generated by the current i(t) flowing in the primary conductor. The electro-motive force e(t), induced in the coil is given by [8]:

$$e(t) = M\frac{d(i)}{dt} \tag{14}$$

Where, M is the mutual inductance coefficient present between the coil and the primary conductor.

To investigate the most important constructive and geometrical parameters, a 3D modelling approach is employed. In this attention is focused on the use of Numerical model, which uses the Biot-Savart's law to deduce the magnetic vector potential distribution.

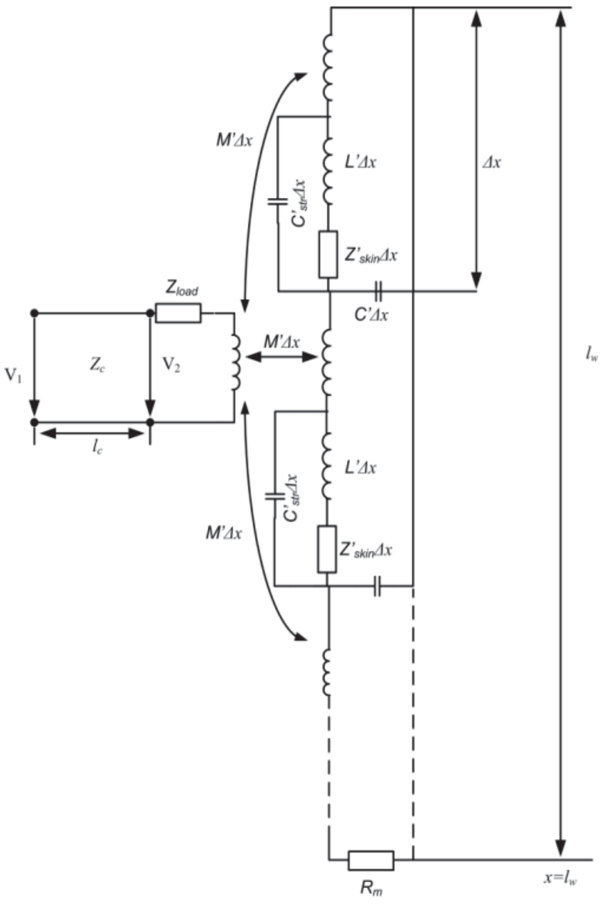

Fig. 8: Distributed Parameters Model of RC [8]

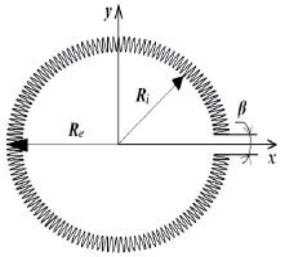

Fig. 9: Geometrical Features of the RC [8]

The magnetic flux linked with the coil is calculated by the line integral of the magnetic potential vector along the coil turns. The two-dimensional (2D) model is used to validate the 3D results, according to the Finite Element Method (FEM)[7].

The modelling analysis allows the analysis of RC's behavior as a function of the [9]:

- Position of the power conductor with respect to the coil center.
- Eccentricity of coil.
- with respect to the base plane of the coil
- *opening angle β values*
- non-orthogonal orientation of the primary conductor with respect to axis

Among various available models for modeling of RC, distributed parameters model is discussed over here for the use of modeling of RC.

V. SIMULATION RESULTS

FEA is carried out using ANSYS Magnetostatic module considering the above distributed parameter model as shown in above Fig. 8. The coil and conductor are shown in Fig. 11, at rated current that is, 1000 A. In this, the electromagnetic analysis is carried out and magnetic flux density is recorded. The maximum flux density is located on the source conductor in which rated current is given. Magnetic flux density decreases gradually as we move away from the source conductor. It reaches the maximum value at source conductor. Magnetic flux density also decreases as we move gradually towards end of the coil area. The magnetic flux density distribution in source conductor and RC can be seen in Fig. 12. The magnetic flux density variation in the coil and conductor at different positions of the source conductor is shown in Fig. 12, 14 & 16. Here current of 1000 A magnitude is applied at one end of conductor and voltage of 0 V is applied at other end in order to consider that designed circuit is closed circuit so that current gets a path to flow.

D. *Inputs for Analysis*

Graphs in Fig. 10 shows the input which is given for analysis purpose. Where current of 1000 A is applied at one end of the conductor and voltage of magnitude 0 V is applied at other end of conductor.

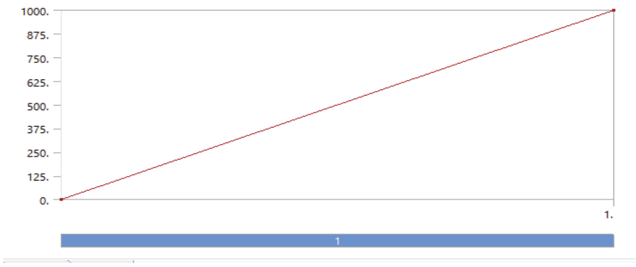

Fig. 10: Current Application for RC 1000A Applied for 1sec

E. *Magnetostatic Analysis*

Source conductor is placed in the middle of the coil area first, as shown in Fig. 11. Fig. 12 shows the magnetic flux density distribution in coil and conductor. Electromagnetic analysis is carried out and magnetic flux density is recorded at all the contacts and all other conducting parts. As shown, the magnetic flux density varies from minimum 0 T to maximum 0.05103 T. Magnetic flux density value is displayed with different colors from blue color indicating minimum flux density to red color indicating maximum flux density.

Now, source conductor position is away from middle point as shown in Fig. 13 and 15. As shown, changing the position of conductor, magnetic flux density varies marginally in the spherical regions shown by the enlarged views in Fig. 14, which is from minimum 0 T to maximum 0.068546 T. The magnetic flux density varies from minimum 0 T to maximum 0.018865 T as shown in the Fig. 16.

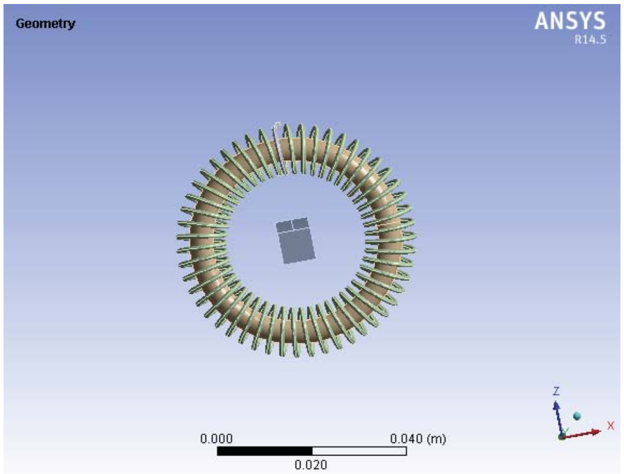

Fig. 11: Geometry of RC

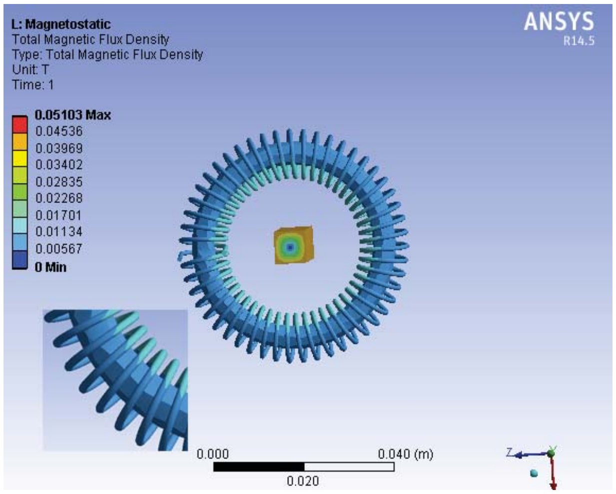

Fig. 12: Magnetic Flux Density Distribution Plot for Middle Placing of Conductor

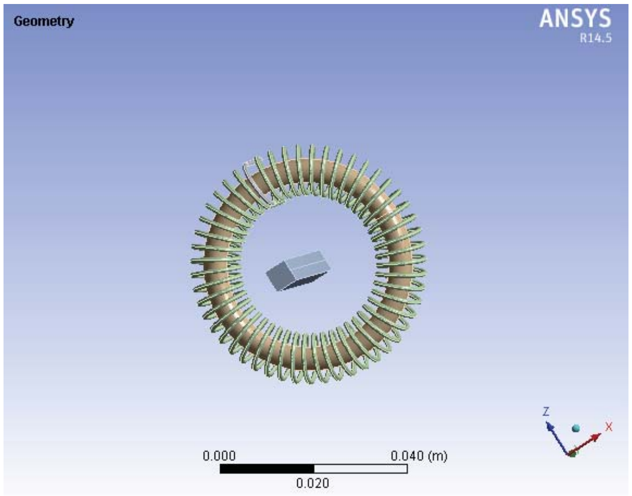

Fig. 13: Geometry for Second Position of the Conductor

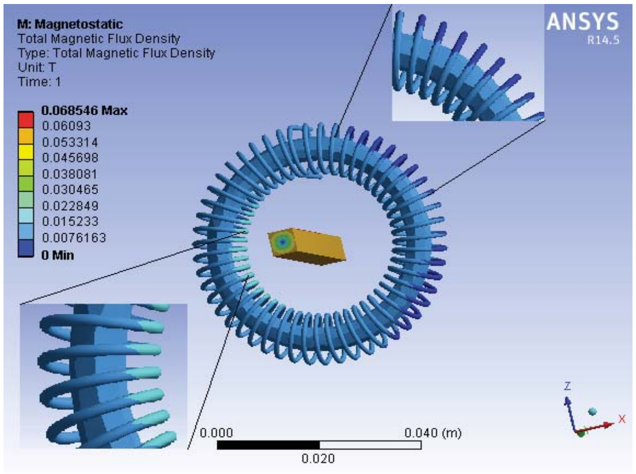

Fig. 14: Magnetic Flux Density Distribution Plot for Middle Placing of Conductor

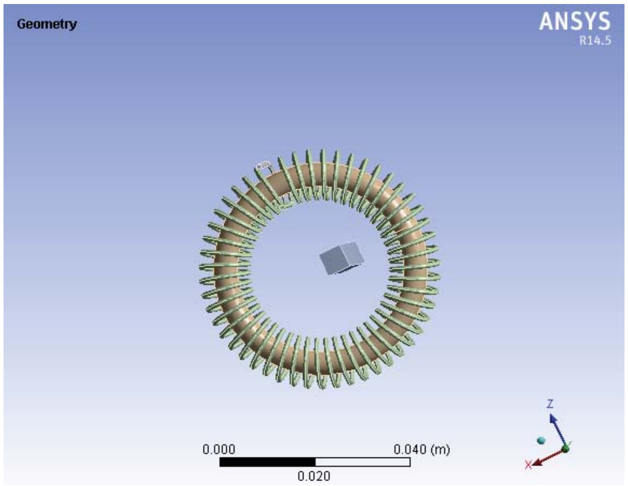

Fig. 15: Geometry for Third Position of the Conductor

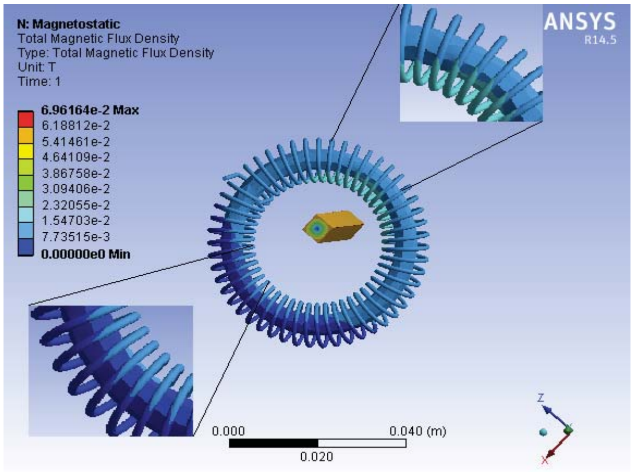

Fig. 16: Magnetic Flux Density Distribution Plot for Third Position of Conductor

According to various position of the conductor, equally distributed flux density distribution between coil and conductor is obtained in middle position of the conductor from where we can get accurate measurement.

VI. CONCLUSION

The paper presented is the detailed study of RC analysis. RC is an air-core coil, which can measure high impulse currents. The advantage of RC coil over the conventional Current Transformers (CTs) is, there won't be any saturation as the air-core is being used for the measurement of high frequency impulse currents. This paper focuses the distributed model through the various conductor positions using the ANSYS FEM based analysis. These variation in the position deviates the flux distribution resulting the deviations in the measurements for which, detailed modelling of RC coil is needed. From modelling analysis of this measurement, proper justification for RC is obtained. By considering any base for this measurement, different position of the conductor gives proper measurement of high frequency impulse current. According to simulation results, middle positon of the conductor can give proper output of Rogowski Coil.

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