Study on Dynamic Response Evaluation through various Numerical Methods

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ABSTRACT

Response of structure subjected to various types of arbitrary dynamic forces like Pulse, Ramp, Unit Impulse, Earthquake is obtained by employing suitable Numerical Integration Method, since closed form solution of such problems are not evident. Mostly used numerical integration method includes Newmark's Method and Runge-Kutta Method. Apart, many other numerical methods like Central Difference Method, Wilson Theta Method and Recursive Formula are also available. In the present study, dynamic response of linear Single Degree of Freedom (SDOF) system subject to dynamic force is obtained using various numerical methods. Dynamic forces considered are Harmonic Excitation (both at mass and base), Unit Impulsive Force and Earthquake Excitation. Convergence studies are carried out to arrive at appropriate time stepping interval for all numerical methods studied. Comparison among numerical solution obtained by various numerical methods is carried out. Apart, comparison is also carried out with closed form solution wherever possible. It is found that in case of Harmonic Excitation (at mass) and Unit Impulsive Force, Newmark's Method (Linear Acceleration Method) converges to the closed form solution at large time stepping interval and hence best suited for them. Similarly, 4^{th} Order Runge-Kutta Method proves the best suitable method for Harmonic Excitation (at base). In case of Earthquake Excitation time stepping size is very small, hence all methods are best suitable for Earthquake Excitation.

Keywords*— Dynamic forces; SDOF system; Numerical integration methods; Convergence Study*

I. INTRODUCTION

There are many realistic and practical problems whose "Closed Form Solution" cannot be obtain analytically. For such cases numerical methods are used to obtain solution. Numerical methods are such methods that solve problems in form of numbers at different values of independent variable, rather than as an expression of independent variable. Further it is noted that numerical solutions are not exact but they are quite accurate, efficient and stable.

Dynamic response of the structure is expressed in terms of displacement, velocity and acceleration. Numerical methods are one of the most powerful techniques to evaluate the dynamic response of the structure. There are two approaches to evaluate the dynamic response: a) Numerical interpolation of excitation and b) Numerical integration techniques. The equation of the motion of the structure is expressed in form of 2nd order differential equation. When the system is linear both numerical and analytical methods are applicable but when the system is non-linear only the numerical integration methods are applicable to solve the differential equation of the motion. There are many numerical methods available for evaluation of the dynamic response such as: Duhamel integration, Recurrence formula method [4], Newmark's method [5], 4^{th} order Runge-Kutta method [2], Central difference method [3], Wilson Theta method etc.

In the present study Linear SDOF system subjected to various dynamic forces like Harmonic periodic excitation (at mass and base), Unit impulsive force and Earthquake excitation. Dynamic response of SDOF system is determined using various numerical integration methods [1]. Apart closed form solutions are also obtained whenever possible. Convergence study is carried out to determine time stepping size for numerical integration methods. The method which converges to closed form solution for large time step size, is best suitable method for particular type of loading.

II. NUMERICAL ANALYSIS

Problem Statement

A Linear SDOF system as shown in Figure 1. has following properties: [3]
Mass (m) = 44360 kg
Stiffness (k) = 1751268 N/m
Damping ratio (ς) = 0.05
Natural time period (T) = 1 sec
Initial conditions: $x(0) = 0$ and $\dot{x}(0) = 0$
Type of loading: (a) Harmonic periodic excitation (at mass)
(b) Harmonic periodic excitation (at base)
(c) Unit impulsive force
(d) Earthquake excitation

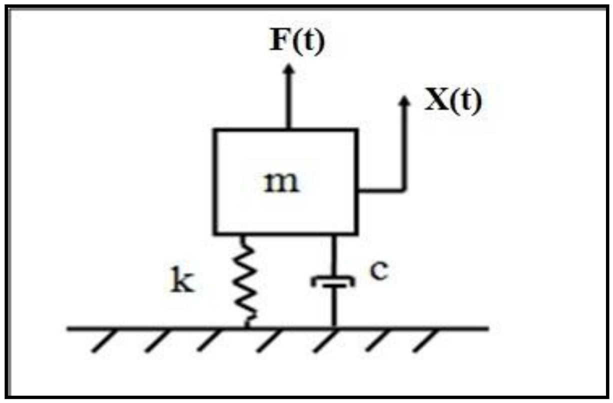


Figure 1. Linear SDOF system.

Dynamic response of the linear SDOF system shown in Figure 1 is evaluated for above four type of loading conditions by using closed form solution and five numerical methods. Stability of numerical integration methods for different types of dynamic forces is established in form of time stepping convergence and hence computational effort.

<u>A. Harmonic periodic excitation (at mass)</u>

System shown in Figure 1 is subjected to harmonic periodic excitation at its mass. Harmonic periodic excitation is shown in Fig. 2. Duration of loading is 20 sec. Magnitude and frequency of force are 44482 N and 6.28 rad/sec, respectively. Time step size is taken as 0.01 sec.

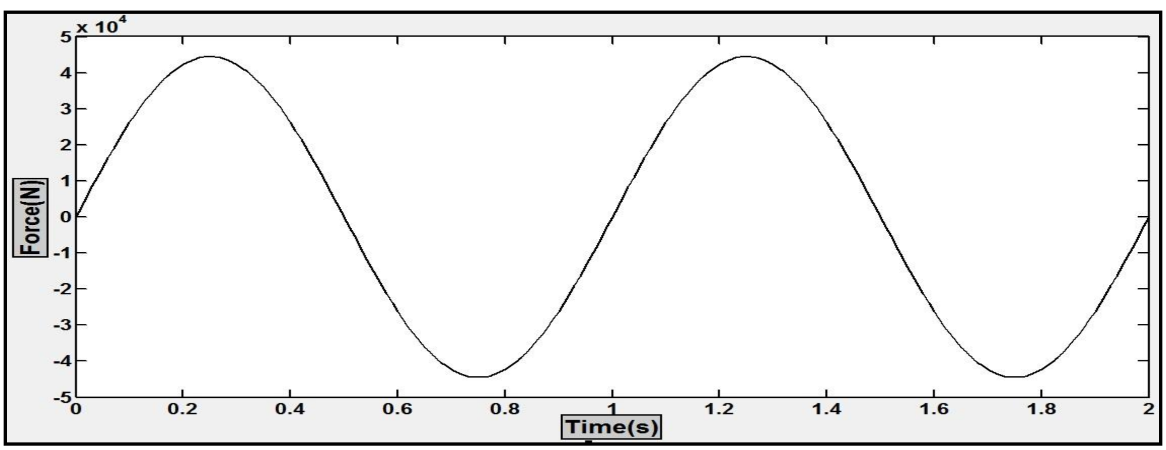


Figure 2. Harmonic periodic excitation applied at mass level of the system

Dynamic response of the system is evaluated using numerical methods and closed form solution is also obtained. Dynamic response of the system in terms of displacement and acceleration is obtained and shown in Figure 3.

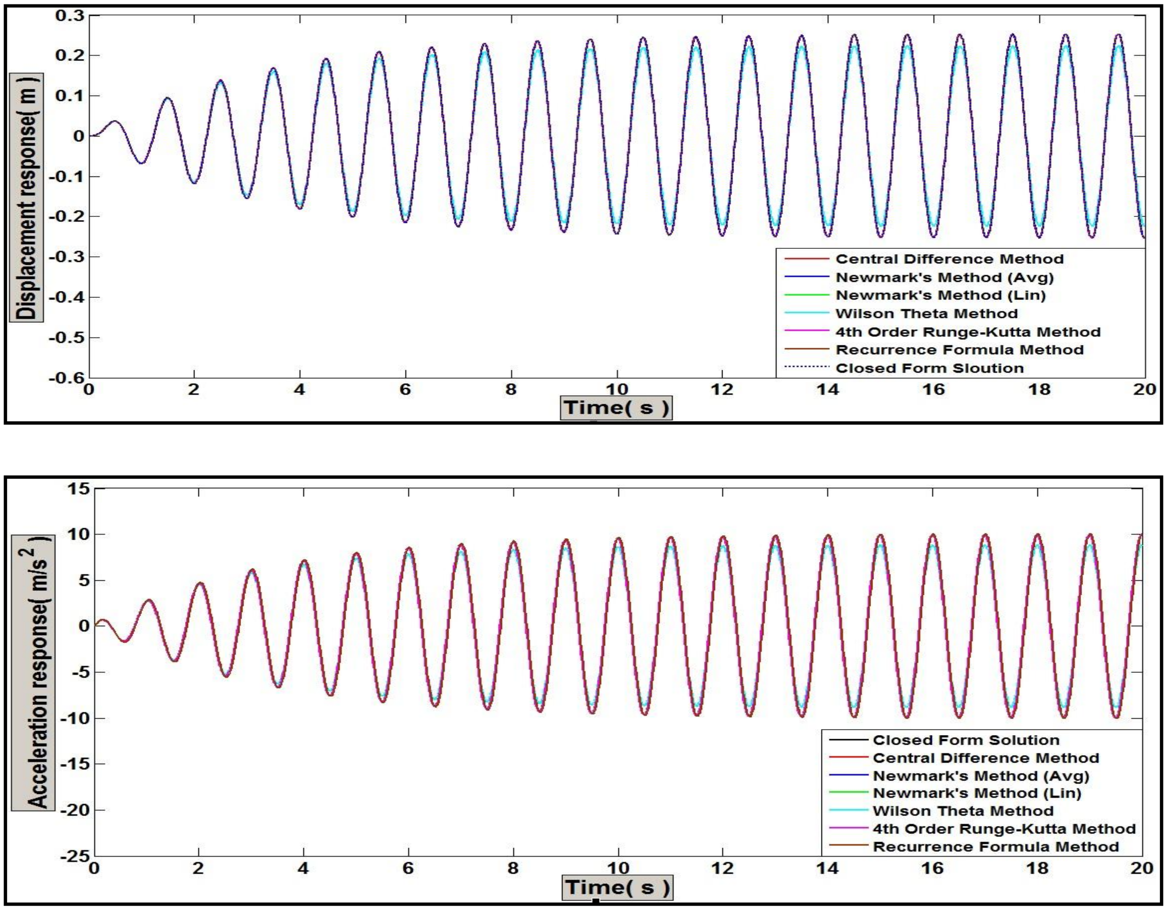


Figure 3. Displacement and Acceleration response of system subjected to harmonic periodic excitation (at mass)

It is evident that dynamic responses are evaluated by numerical integration methods are follows same pattern as closed form solution. Steady state is occured after 14.25 sec and steady state displacement is 0.2921 m. The input force is harmonic in nature so the response of the system is also harmonic in nature which is clearly seen in Figure 3.

The peak displacement is 0.11862 m as obtained from closed form solution. Newmark's method (Lin) gives exact solution for time stepping size of 0.05 sec. While Wilson Theta method, Central difference method, Newmark's method (Avg), 4th order Runge-Kutta method and Recurrence formula method give exact solution for time stepping size of 0.0001 sec, 0.001 sec, 0.003 sec, 0.003 sec and 0.009 sec respectively. So Newmark's method (Lin) is best suitable for harmonic periodic excitation (at mass) type of loading.

B. Harmonic periodic excitation (at base)

System shown in Figure 4 is subjected to harmonic periodic excitation at its base. Harmonic periodic excitation is shown in Figure 4. Duration of loading is 20 sec. Loading specification is same as mention earlier in section II-A.

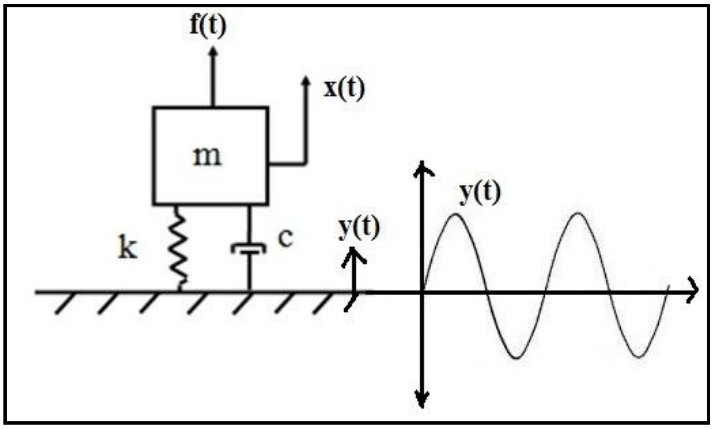
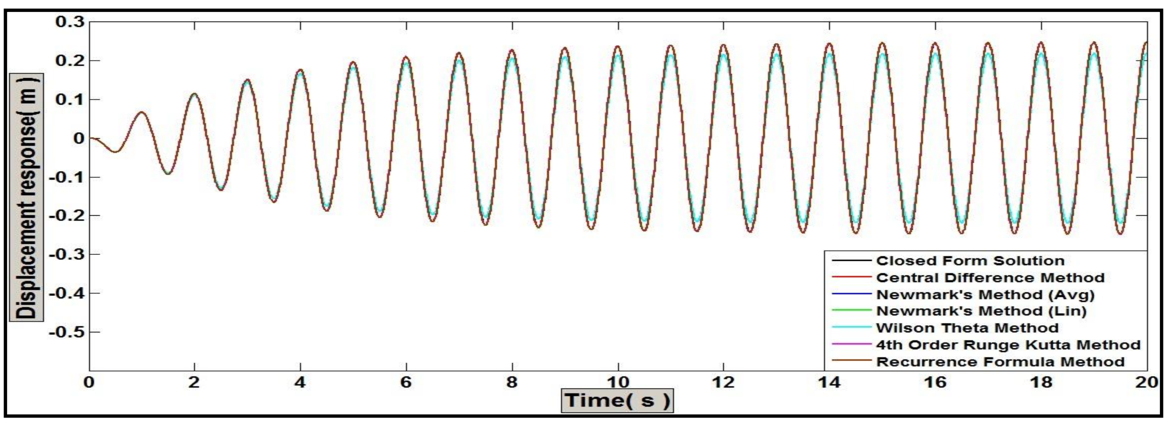


Figure 4. Harmonic periodic excitation applied at the base level of the system

Displacement and acceleration response of the system are obtained using various numerical methods. Figure 5. shows displacement and acceleration response of Linear SDOF system subjected to harmonic periodic excitation at base.



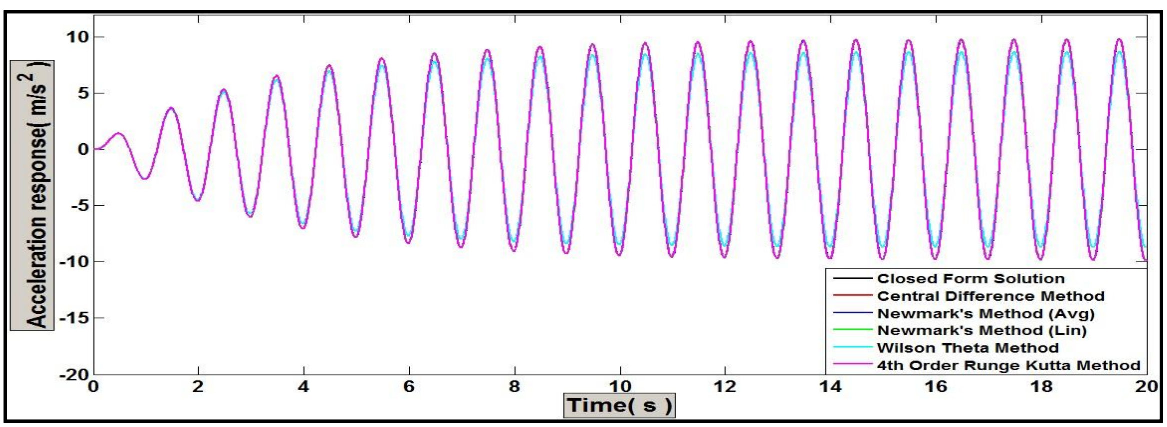


Figure 5. Displacement and acceleration response of the system subjected to harmonic periodic excitation(at base).

It is found that dynamic responses are evaluated by numerical integration methods are follows same pattern as closed form solution. Steady state is occured after 15 sec and steady state displacement is 0.2442 m. The input force is harmonic in nature so the response of the system is also harmonic in nature which is clearly seen in Figure 5.

The peak displacement is 0.11605 m as obtained from closed form solution. 4th order Runge-Kutta method gives exact solution for time stepping size of 0.008 sec. While, Central difference method, Newmark's method (Lin) and Recurrence formula method give exact solution for time stepping size of 0.005 sec. Wilson Theta method and Newmark's method (Avg) give exact solution for time stepping size of 0.0001 sec and 0.003 sec, respectively. So 4th order Runge-Kutta method is best suitable for harmonic periodic excitation (at base) type of loading.

C. Unit impulsive force

Response of SDOF system shown in Figure 1 is derived for unit impulsive force. Force is applied to the system in form of small impulse which can be represented as initial velocity of magnitude of 1/m. Therefore initial conditions of system are: $x(0)=0$ and $\dot{x}(0) = 1/m$. Time stepping size used is 0.1 sec.

Dynamic response obtained in terms of displacement and acceleration for SDOF system through different numerical methods and closed form solution is shown in Figure 6.

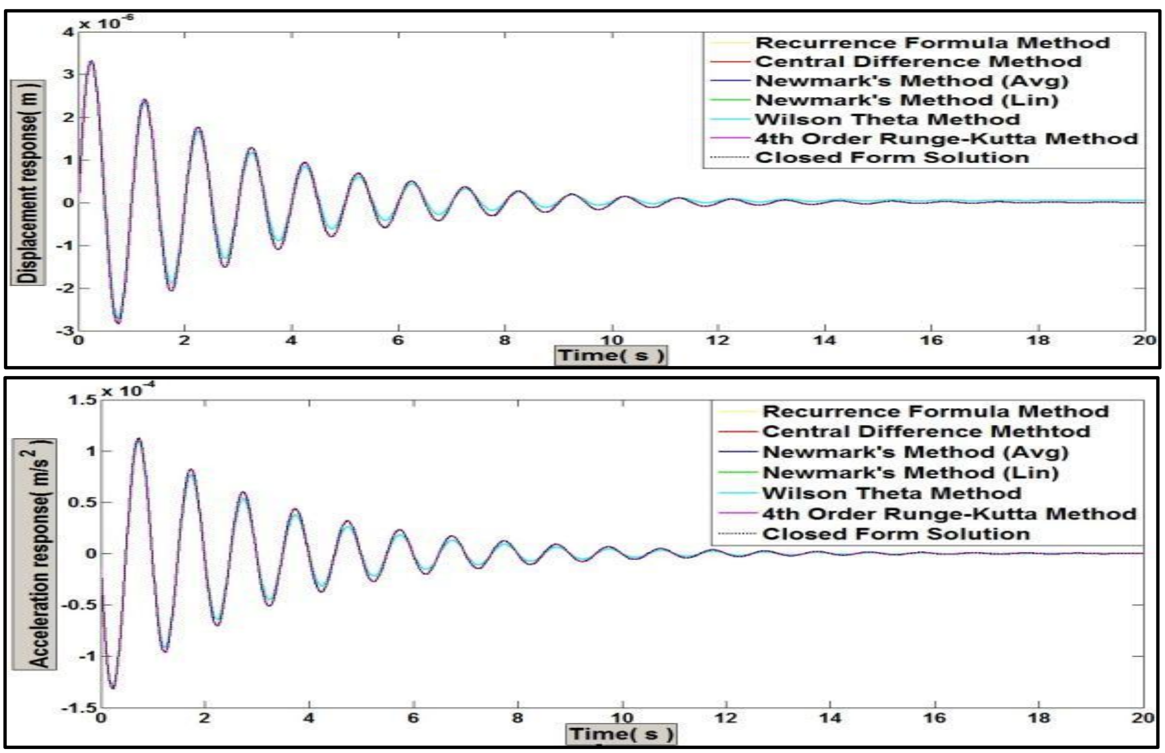


Figure 6. Displacement and acceleration response of the system subjected to unit impulsive force.

It is evident that for unit impulse force the response of the system reduces logarithmically. Response is similar to free vibration response with 1/m value of initial velocity. All numerical integration methods give dynamic response almost similar to closed form solution. The closed form solution shows that peak displacement of the system is 0.00032 m. Newmark's method (Lin) gives exact peak displacement for time stepping size of 0.007 sec which is larger than other numerical integration methods. So Newmark's method (Lin) is best suitable for unit impulsive type of loading.

D. Earthquake excitation

Finally, Linear SDOF system shown in Figure 1. is subjected to Northridge earthquake (1994) excitation which is characterized as pulse type. Ground acceleration details are as follows:

Station Name: County Hospital, Parking Lot, Northridge.
Component: 90º
Peak Ground Acceleration (PGA): 0.6041 g
Duration of earthquake (t): 40 sec
Time step size (Δt): 0.02 sec

Northridge ground acceleration excitation is shown in Figure 7.

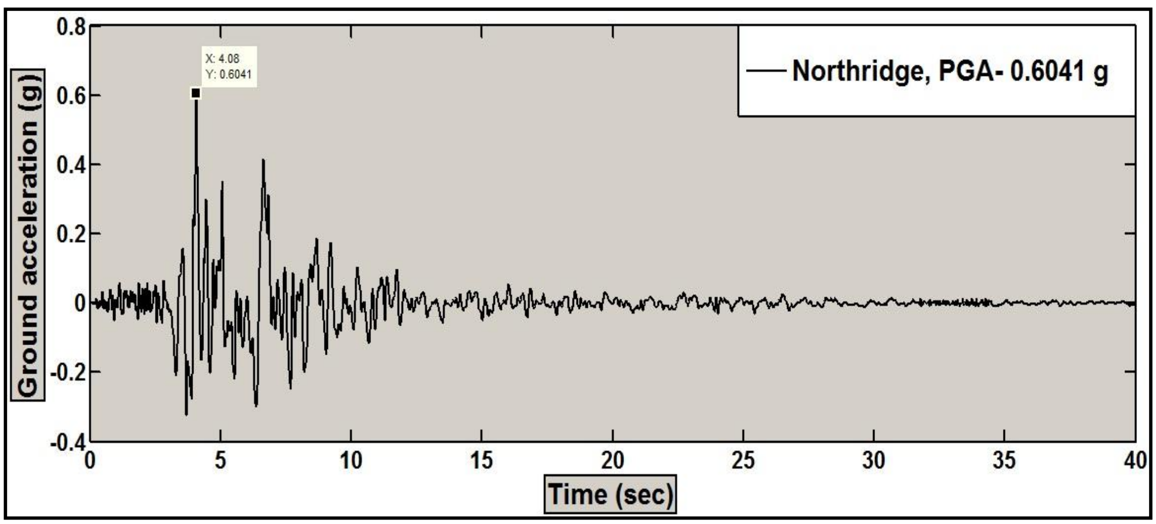


Figure 7. Northridge earthquake ground motion, 1994

Dynamic response of system obtained in terms of displacement and acceleration, is as shown in Figure 8.

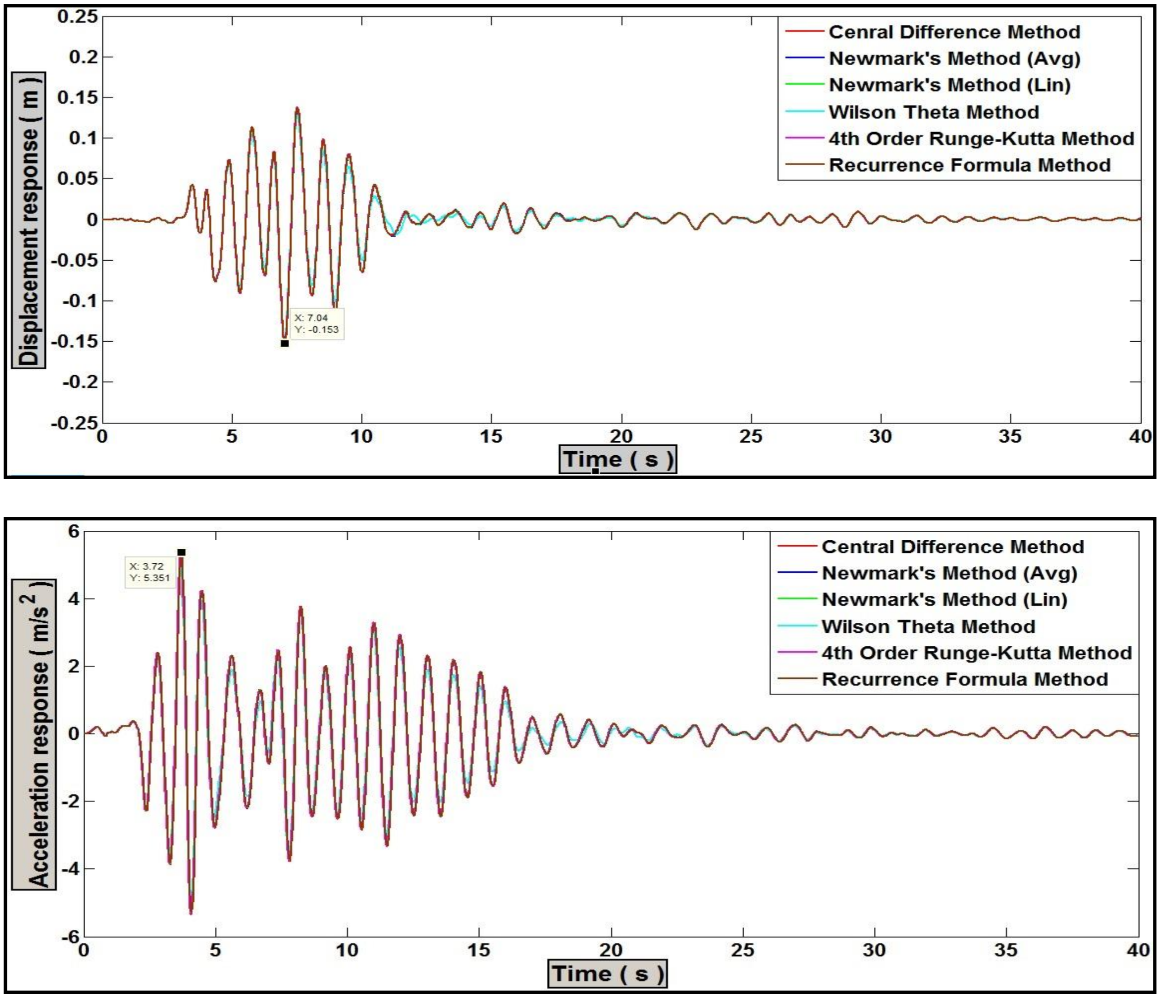


Figure 8. Displacement and Acceleration response of system subjected to Northridge earthquake excitation

It is found that all numerical methods give dynamic response almost similar. In earthquake excitation time stepping size is very small and constant. In Northridge earthquake it is 0.01 sec. So for this small time stepping size it is found that all numerical methods give accurate results except Wilson Theta method. The reason behind performance of all numerical methods being identical is due to fact that Earthquake excitation loading itself has data at high sampling rate, i.e. low time interval.

Thus, response of SDOF system subjected to Earthquake excitation any numerical method considered for the study may be used except Wilson Theta methods.

III. CONVERGENCE STUDY

Convergence study is necessary to check the stability and accuracy of the numerical methods. In this study maximum displacement response of SDOF system is considered for comparison of different methods. The method which converges to the exact solution at large time stepping size is best method in terms of stability and accuracy. Convergence study is performed for all type of loading except earthquake excitation, as for later the time stepping size is fix and depends on acceleration measurements. As time interval for earthquake excitation measurement is significantly small, it gives stable and accurate solution. However it has been found that small time stepping size do not yield accuracy for Wilson Theta Method.

Representative convergence study plot obtained for harmonic periodic excitation (at mass) is shown in Figure 9.

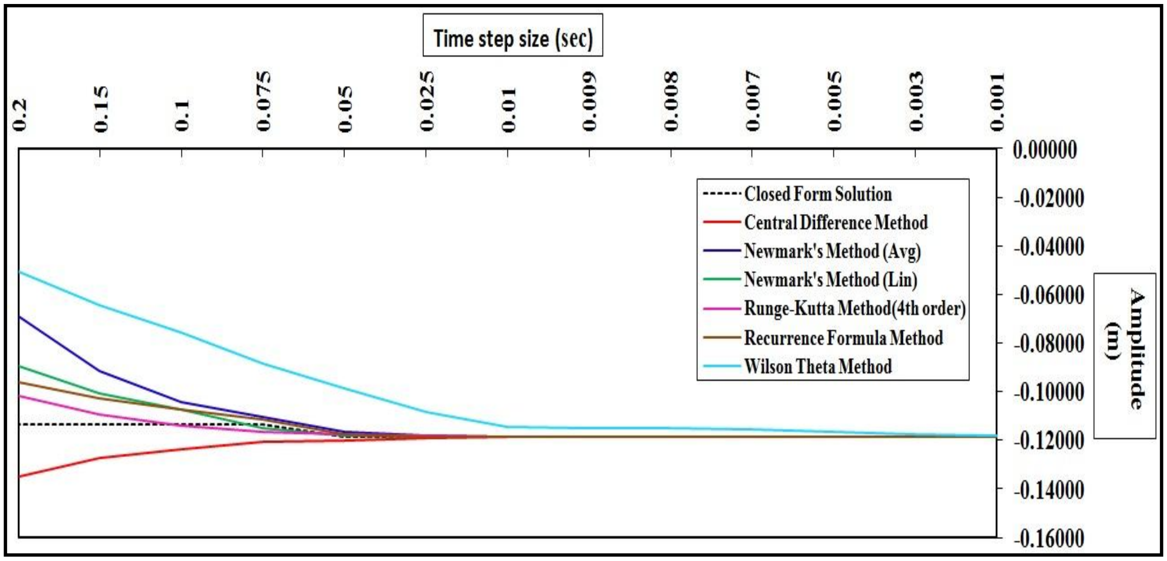


Figure 9. Convergence plot for harmonic periodic excitation (at mass).

Figure 9 shows that Newmark's Method (Lin) converges to closed form solution for large time stepping size which is 0.01 sec. Thus, it is best suitable method for harmonic periodic excitation (at mass) type of loading condition.

Table 1. shows most suitable methods for different types of loading applied to SDOF system based on convergence study.

Table 1. Best suitable methods for various types of loading

Sr.No.	Type of Loading	Best Suitable Method
1.	Harmonic Periodic Excitation (at mass)	Newmark's Method (Linear)
2.	Harmonic Periodic Excitation (at base)	4^{th} Order Runge-Kutta Method
3.	Unit Impulsive Force	Newmark's Method (Linear)

Note that, for SDOF system subjected to earthquake excitation any numerical method may be used except Wilson Theta method.

IV. CONCLUSIONS

Dynamic response of Linear SDOF system subjected to various types of forces like harmonic periodic excitation (at mass and base), unit impulsive force and earthquake excitation are considered. Dynamic response of the system in form of displacement and acceleration is obtained for each type of loading using Numerical integration methods. Various Numerical Integration methods like Central Difference Method, Newmark's Method, 4th Order Runge-Kutta Method, Recurrence Formula Method and Wilson Theta Method are used for the present study. Apart, closed form solutions are also obtained, whenever it is necessary. Comparison among various numerical methods and closed form solution is carried out. Suitability of numerical methods is established based on convergence study of time stepping size used for numerical methods.

Study reveals that Newmark's method (Lin) yields dynamic response of system as obtained through closed form solution when subjected to Harmonic periodic excitation (at mass) and impulsive force. However, 4^{th} order Runge-Kutta method proves to be better for SDOF system subjected to Harmonic excitation at base. For SDOF system subjected to Earthquake excitation any numerical method may be used. It is observed that Wilson Theta method performs worst among all the numerical methods employed in the present study.

V. REFERENCES

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